

2014 Autumn Semester, course for graduate student

Lecture notes: Physics of Laser-Plasma Interaction

# V. Parametric excitation of electron and ion waves

## (电子和离子波的参量激发)

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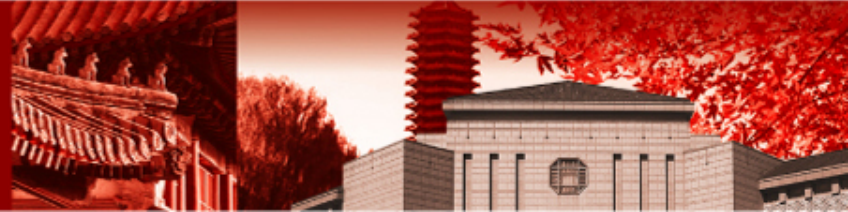
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# Outline

V. 1 参量振荡、参量激发、参量共振、参量放大

(parametric oscillator, excitation, resonance, and amplifier)

V. 2 共振吸收

V. 3 由离子密度涨落引起的耦合

V. 4 不稳定性图象

V. 5 不稳定性分析

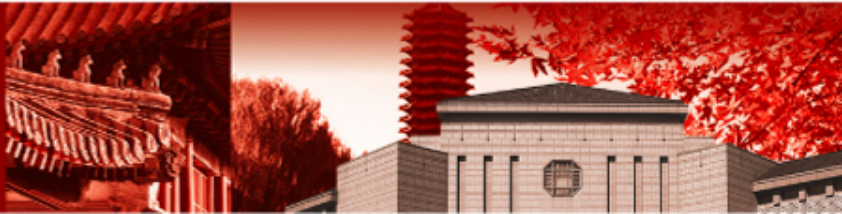
V. 6 色散关系

V. 7 空间不均匀性引起的不稳定的阈值

V. 8 非相干波对不稳定性的影响



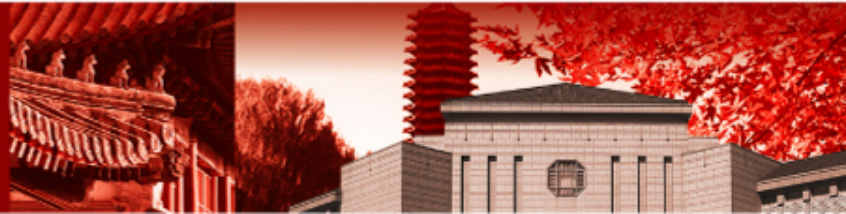
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# V.1 参量振荡、参量激发、参量共振、参量放大 (parametric oscillator, excitation, resonance, and amplifier)

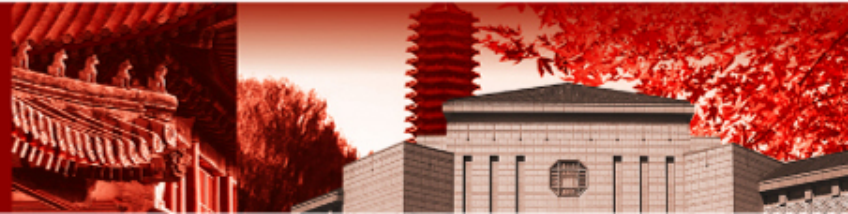


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# What are parametric instability?

- **Parametric Oscillator:** A parametric oscillator is a harmonic oscillator whose parameters oscillate in time. For example, a well known parametric oscillator is a child pumping a swing (秋千) by periodically standing and squatting to increase the size of the swing's oscillations.
- **Parametric instabilities:** In plasma, there are some eigen modes such as the electron plasma oscillation, ion acoustic mode. They are easily excited by the modulated drivers. A pump laser can drive such plasma oscillations automatically from noise. This is called a parametric instability.
- Obviously there is some **threshold amplitude** for the laser since the plasma oscillation also damps.





# 参量振荡 (parametric oscillation)

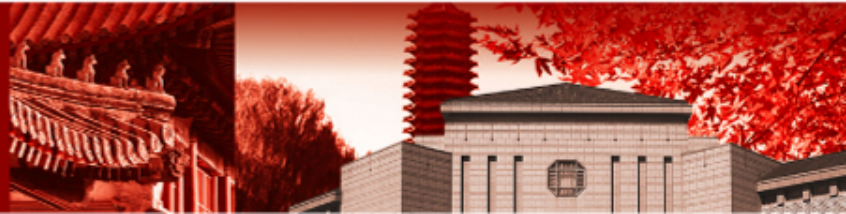
$$\frac{d^2x}{dt^2} + \beta(t) \frac{dx}{dt} + \omega^2(t)x = 0$$

This equation is linear in  $x(t)$ . By assumption, **the parameters  $\omega^2$  and  $\beta$  depend only on time** and do *not* depend on the state of the oscillator. In general,  $\beta(t)$  and/or  $\omega^2(t)$  are assumed to vary periodically, with the same period  $T$ .

Remarkably, **if the parameters vary at roughly twice the natural frequency of the oscillator**, the oscillator phase-locks to the parametric variation and absorbs energy at a rate proportional to the energy it already has. Without a compensating energy-loss mechanism provided by  $\beta$ , **the oscillation amplitude grows exponentially**. (This phenomenon is called **parametric excitation, parametric resonance or parametric pumping**.)



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# Transformation of the equation

By making a change of variables  $q(t) \stackrel{\text{def}}{=} e^{D(t)} x(t)$

where  $D(t)$  is a time integral of the damping:  $D(t) \stackrel{\text{def}}{=} \frac{1}{2} \int^t d\tau \beta(\tau)$ .

This change of variables eliminates the damping term  $\frac{d^2 q}{dt^2} + \Omega^2(t) q = 0$

where the transformed frequency is defined  $\Omega^2(t) = \omega^2(t) - \frac{1}{2} \left( \frac{d\beta}{dt} \right) - \frac{1}{4} \beta^2$ .

In general, the variations in damping and frequency are relatively small perturbations

$$\beta(t) = \omega_0 [b + g(t)] \quad \omega^2(t) = \omega_0^2 [1 + h(t)]$$

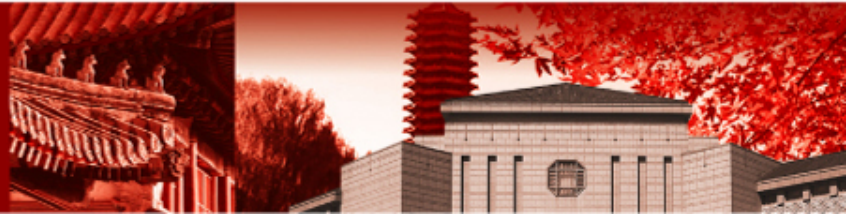
where  $\omega_0$  and  $b\omega_0$  are constants, namely, the time-averaged oscillator frequency and damping, respectively. The transformed frequency can be written in a similar way:  $\Omega^2(t) = \omega_n^2 [1 + f(t)]$ ,

where  $\omega_n$  is the [natural frequency](#) of the damped harmonic oscillator  $\omega_n^2 \stackrel{\text{def}}{=} \omega_0^2 \left( 1 - \frac{b^2}{4} \right)$

and  $\omega_n^2 f(t) \stackrel{\text{def}}{=} \omega_0^2 \left\{ h(t) - \frac{1}{2\omega_0} \left( \frac{dg}{dt} \right) - \frac{b}{2} g(t) - \frac{1}{4} g^2(t) \right\}$ .

Thus, our transformed equation can be written

$$\frac{d^2 q}{dt^2} + \omega_n^2 [1 + f(t)] q = 0.$$



# A solution of the transformed equation

Let us assume that  $f(t)$  is sinusoidal, specifically  $f(t) = f_0 \sin 2\omega_p t$

where the pumping frequency  $2\omega_p \approx 2\omega_n$  but need not equal  $2\omega_n$  exactly. The solution  $q(t)$  of our transformed equation may be written

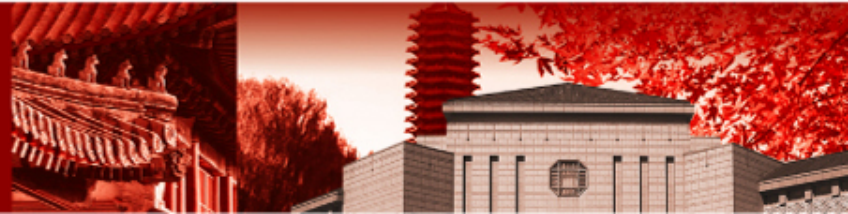
$$q(t) = A(t) \cos \omega_p t + B(t) \sin \omega_p t$$

where we have factored out the rapidly varying components (  $\cos \omega_p t$  and  $\sin \omega_p t$  ) to isolate the slowly varying amplitudes  $A(t)$  and  $B(t)$  .

Substituting this solution into the transformed equation and retaining only the terms first-order in  $f_0 \ll 1$  yields two coupled equations

$$2\omega_p \frac{dA}{dt} = \left( \frac{f_0}{2} \right) \omega_n^2 A - (\omega_p^2 - \omega_n^2) B$$

$$2\omega_p \frac{dB}{dt} = - \left( \frac{f_0}{2} \right) \omega_n^2 B + (\omega_p^2 - \omega_n^2) A$$



# A solution of the transformed equation

We may decouple and solve these equations by making another change of variables

$$A(t) \stackrel{\text{def}}{=} r(t) \cos \theta(t)$$

$$B(t) \stackrel{\text{def}}{=} r(t) \sin \theta(t)$$

which yields the equations

$$\frac{dr}{dt} = (\alpha_{\max} \cos 2\theta) r$$

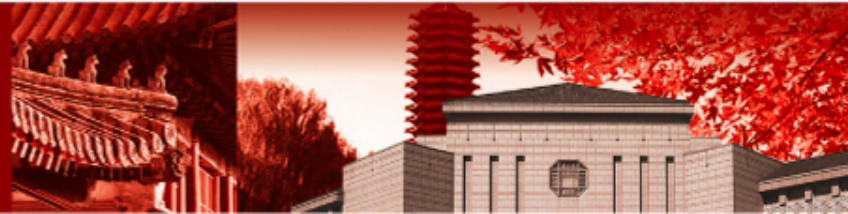
$$\frac{d\theta}{dt} = -\alpha_{\max} [\sin 2\theta - \sin 2\theta_{\text{eq}}]$$

where we have defined for brevity  $\alpha_{\max} \stackrel{\text{def}}{=} \frac{f_0 \omega_n^2}{4\omega_p}$  and  $\sin 2\theta_{\text{eq}} \stackrel{\text{def}}{=} \left( \frac{2}{f_0} \right) \epsilon$

and the detuning  $\epsilon \stackrel{\text{def}}{=} \frac{\omega_p^2 - \omega_n^2}{\omega_n^2}$



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# A solution of the transformed equation

The  $\theta$  equation does not depend on  $r$ , and linearization near its equilibrium position  $\theta_{eq}$  shows that  $\theta$  decays exponentially to its equilibrium  $\theta(t) = \theta_{eq} + (\theta_0 - \theta_{eq}) e^{-2\alpha t}$

where the decay constant  $\alpha \stackrel{\text{def}}{=} \alpha_{\max} \cos 2\theta_{eq}$

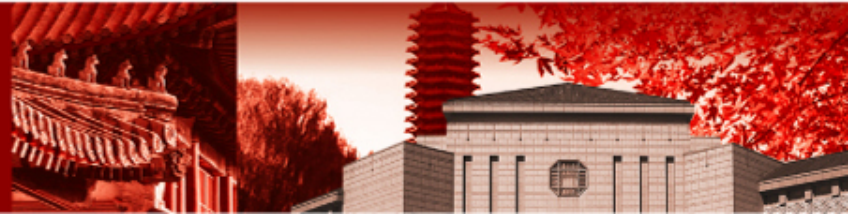
In other words, the parametric oscillator phase-locks to the pumping signal  $f(t)$ . Taking  $\theta(t) = \theta_{eq}$  (i.e., assuming that the phase has locked), the  $r$  equation

becomes  $\frac{dr}{dt} = \alpha r$

whose solution is  $r(t) = r_0 e^{\alpha t}$ ; the amplitude of the  $q(t)$  oscillation diverges exponentially. However, the corresponding amplitude  $R(t)$  of the *untransformed* variable  $x \stackrel{\text{def}}{=} q e^{-D(t)}$  need not diverge.

$$R(t) = r(t) e^{-D(t)} = r_0 e^{\alpha t - D(t)}$$

The amplitude  $R(t)$  diverges, decays or stays constant, depending on whether  $\alpha t$  is greater than, less than, or equal to  $D(t)$ , respectively.



# A solution of the transformed equation

The maximum growth rate of the amplitude occurs when  $\omega_p = \omega_n$ . At that frequency, the equilibrium phase  $\theta_{eq}$  is zero, implying that  $\cos 2\theta_{eq} = 1$  and  $\alpha = \alpha_{max}$ . As  $\omega_p$  is varied from  $\omega_n$ ,  $\theta_{eq}$  moves away from zero and  $\alpha < \alpha_{max}$ , i.e., the amplitude grows more slowly. For sufficiently large deviations of  $\omega_p$ , the decay constant  $\alpha$  can become purely imaginary since

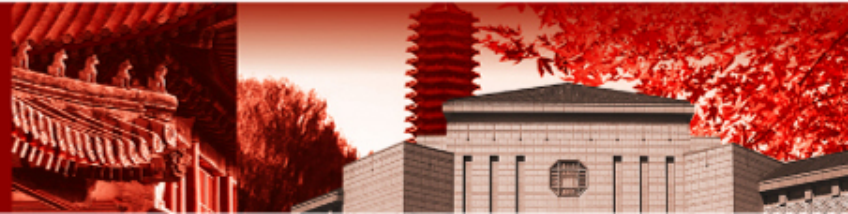
$$\alpha = \alpha_{max} \sqrt{1 - \left(\frac{2}{f_0}\right)^2 \epsilon^2}$$

If the detuning  $\epsilon$  exceeds  $f_0/2$ ,  $\alpha$  becomes purely imaginary and  $q(t)$  varies sinusoidally. Using the definition of the detuning, the pumping frequency  $2\omega_p$  must lie between

$$2\omega_n \sqrt{1 - \frac{f_0}{2}} \quad \text{and} \quad 2\omega_n \sqrt{1 + \frac{f_0}{2}}$$

in order to achieve exponential growth in  $q$ .

It shows that the spread in pumping frequencies that result in exponentially growing  $q$  is approximately  $\omega_n f_0$ .





# Intuitive derivation of parametric excitation

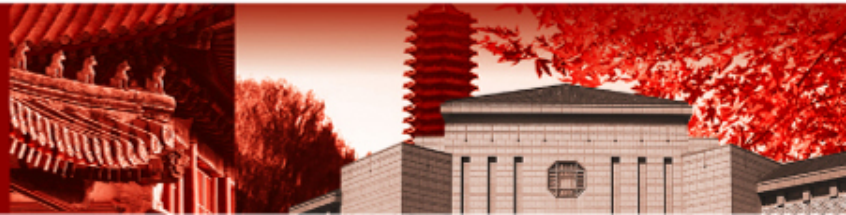
$$\frac{d^2 q}{dt^2} + \omega_n^2 q = -\omega_n^2 f(t) q$$

which represents a simple harmonic oscillator being driven by a  $q$  signal  $-\omega_n^2 f(t) q$  that is proportional to its response .

Assume that  $q(t) = A \cos \omega_p t$  already has an oscillation at frequency  $\omega_p$  and that the pumping  $f(t) = f_0 \sin 2\omega_p t$  has double the frequency and a small amplitude  $f_0 \ll 1$  . Their product  $f(t)q(t)$  produces two driving signals, one at frequency  $\omega_p$  and the other at frequency  $3\omega_p$

$$f(t)q(t) = \frac{f_0}{2} A (\sin \omega_p t + \sin 3\omega_p t)$$

Being off-resonance, the  $3\omega_p$  signal is attenuated and can be neglected initially. By contrast, the  $\omega_p$  signal is on resonance, serves to amplify  $q$  and is proportional to the amplitude  $A$  . Hence, the amplitude of  $q$  grows exponentially unless it is initially zero.





# Parametric resonance

Parametric resonance takes place when the external excitation frequency equals to twice the natural frequency of the system.

This effect is different from regular resonance because it exhibits the instability phenomenon.

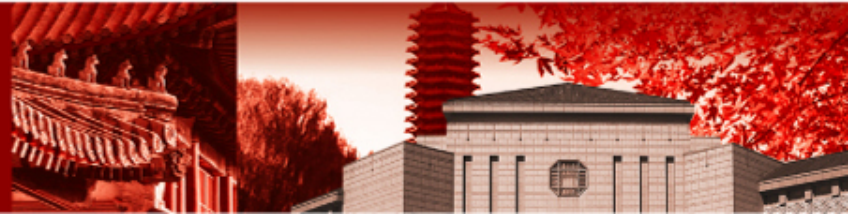
If a pendulum's length changes with time.

For small amplitudes and by linearising, the stability of the periodic solution is given by :  $\ddot{u} + (a + B \cos t)u = 0$

where  $u$  is some perturbation from the periodic solution. Here the  $B \cos(t)$  term acts as an 'energy' source and is said to parametrically excite the system. The Mathieu equation describes many other physical systems to a sinusoidal parametric excitation



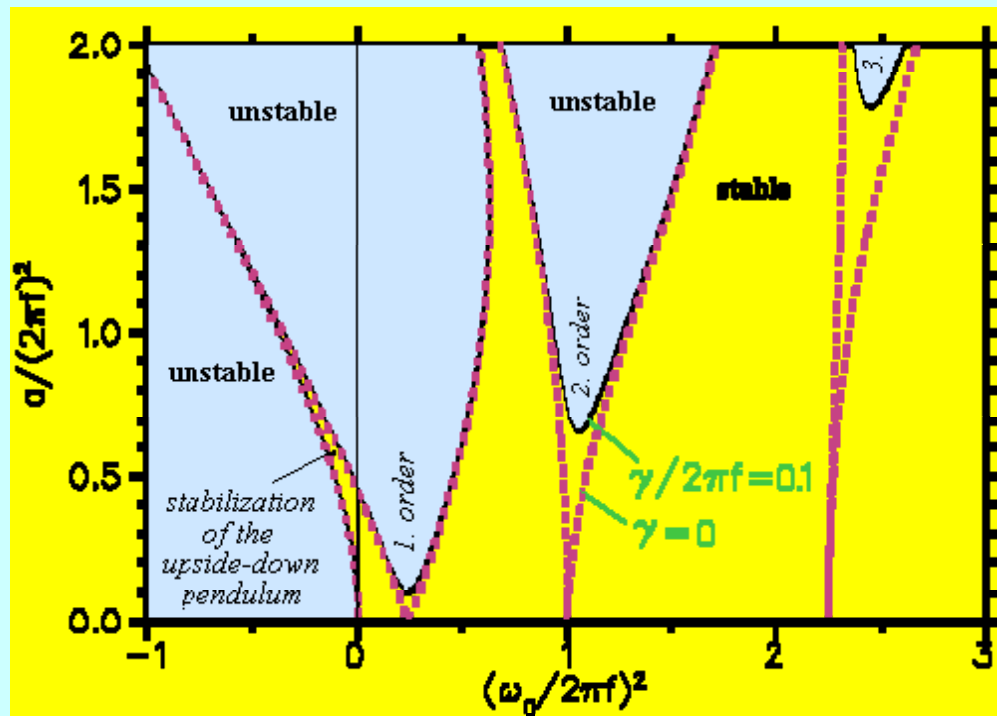
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# Parametric resonance

The vertically driven pendulum. But vertical driving can change stability into instability and vice versa.

$$\frac{d^2\varphi}{dt^2} + \gamma \frac{d\varphi}{dt} + (\omega_0^2 + a \cos 2\pi f t) \varphi = 0$$



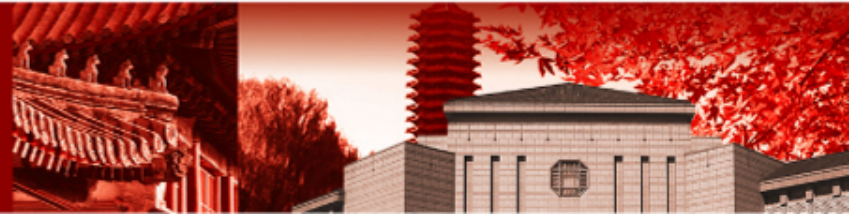
An infinitesimal driving amplitude  $a$  destabilizes the down-hanging pendulum if the **parametric resonance condition**

$$f_0 \equiv \frac{\omega_0}{2\pi} = \frac{f}{2} n$$

is fulfilled, where  $n$  is an integer defining the **order** of parametric resonance



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# Parametric amplifier

The parametric oscillator equation can be extended by adding an external driving force :

$$\frac{d^2x}{dt^2} + \beta(t) \frac{dx}{dt} + \omega^2(t)x = E(t).$$

Let the damping be constant  $\beta(t) = b\omega_0$  and assume that the external driving force is at the mean resonance frequency, i.e.,  $E(t) = E_0 \sin \omega_0 t$ . The equation becomes

$$\frac{d^2x}{dt^2} + b\omega_0 \frac{dx}{dt} + \omega_0^2 [1 + h_0 \sin 2\omega_0 t] x = E_0 \sin \omega_0 t$$

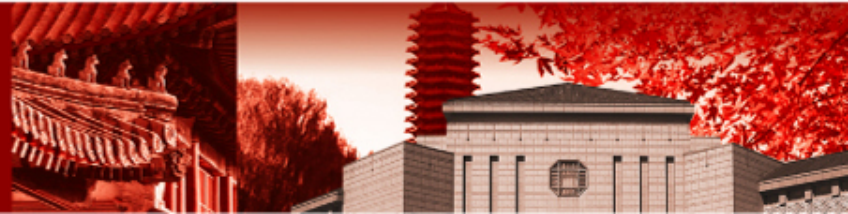
whose solution is roughly  $x(t) = \frac{2E_0}{\omega_0^2 (2b - h_0)} \cos \omega_0 t$ .

As  $h_0$  approaches the threshold  $2b$ , the amplitude diverges.

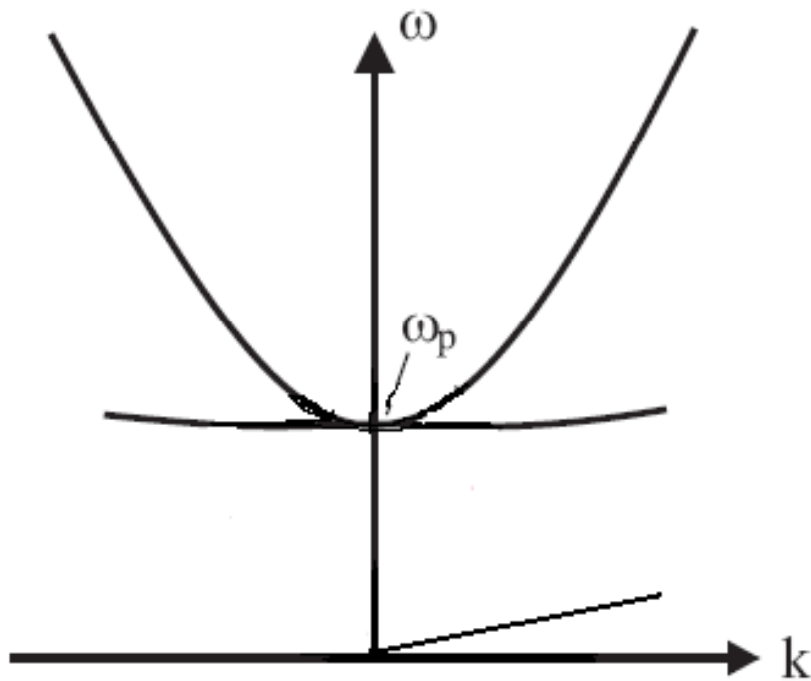
When  $h \geq 2b$ , the system enters parametric resonance and the amplitude begins to grow exponentially, even in the absence of a driving force.



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# Plasma oscillators: electron plasma oscillator, ion acoustic oscillator



$$\begin{aligned}\omega^2 &= \omega_{pe}^2 + \gamma_e k^2 v_e^2 = \omega_{pe}^2 + 3k^2 v_e^2 \\ &= \omega_{pe}^2 (1 + 3k^2 \lambda_D^2)\end{aligned}$$

$$\omega = \pm k C_s,$$

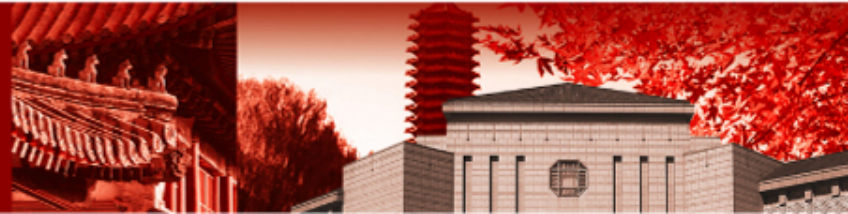
$$C_s = \sqrt{\frac{Z T_e + 3 T_i}{m_i}}.$$

离子声速

$$\omega^2 = \omega_{pe}^2 + c^2 k^2$$



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# Coupling between laser and the plasma oscillators

(参量共振形式的等离子体不稳定性)

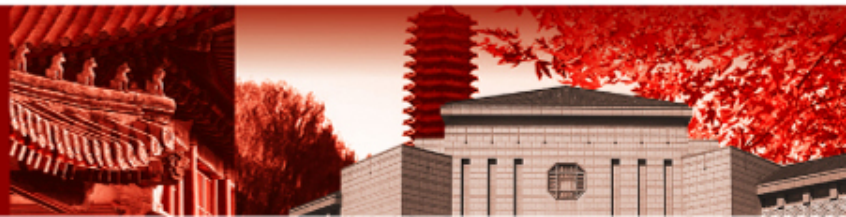
激光和等离子体相互作用可以产生耦合,产生另外的波模

$$\left. \begin{array}{l} \omega_l \\ \omega_1 \end{array} \right\} \rightarrow \left. \begin{array}{l} \omega_2 = \omega_l - \omega_1 \\ \omega_3 = \omega_l + \omega_1 \end{array} \right\} \rightarrow \left. \begin{array}{l} \omega_2 \\ \omega_1 \end{array} \right\} \rightarrow \left. \begin{array}{l} \omega_1 = \omega_l - \omega_2 \\ \omega_4 = \omega_l + \omega_2 \end{array} \right\} \omega_1 \text{模被加强}$$

倘若驱动模得到的能量大于它由于阻尼损失掉的能量,它就能增长,发生参量共振形式的等离子体不稳定性。



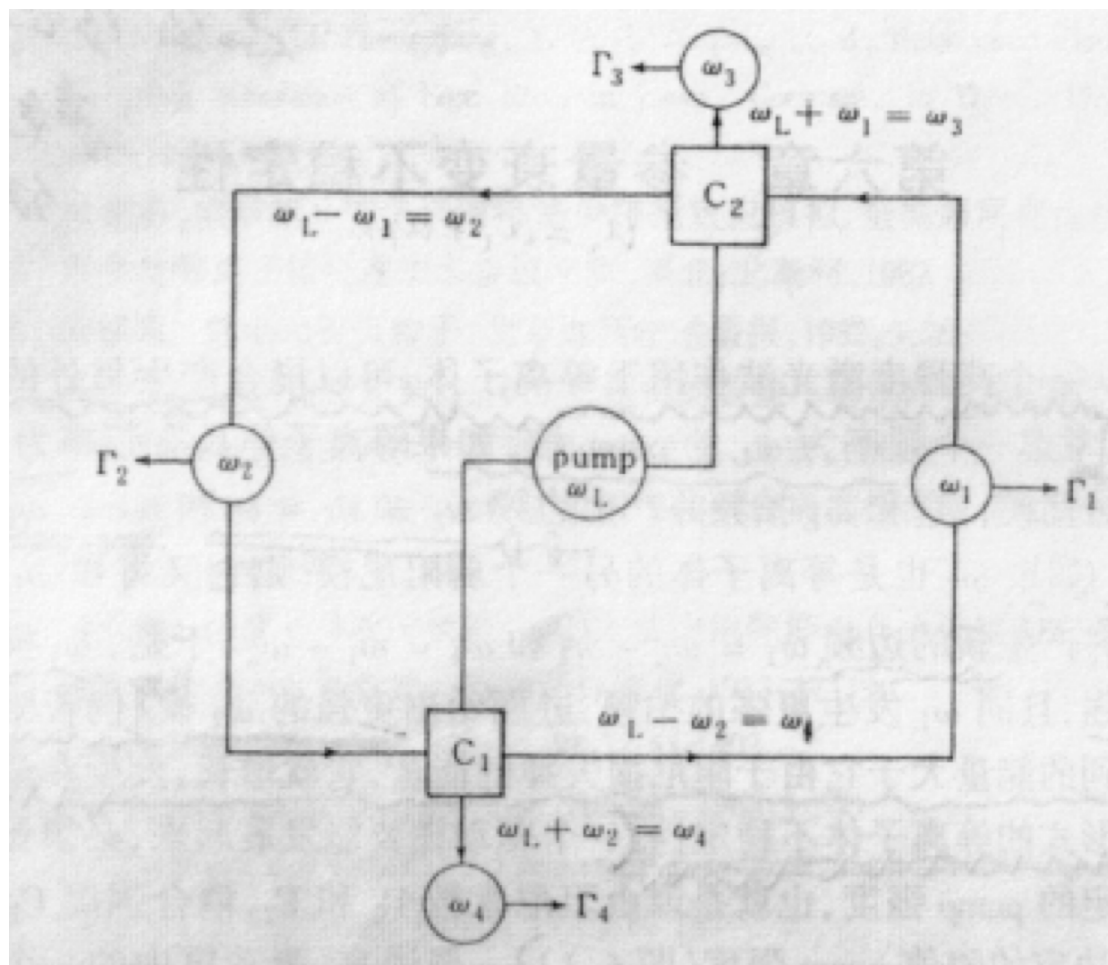
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# Coupling between laser and the plasma oscillators

(参量共振形式的等离子体不稳定性)



- 一个驱动模要能发展起来, 必须要有一定的pump强度:

由阻尼常数  $\Gamma_1$  和  $\Gamma_2$ , 耦合强度  $C_1$  和  $C_2$  决定的阈值pump强度。

- 驱动模接近共振时, 阈值最低:

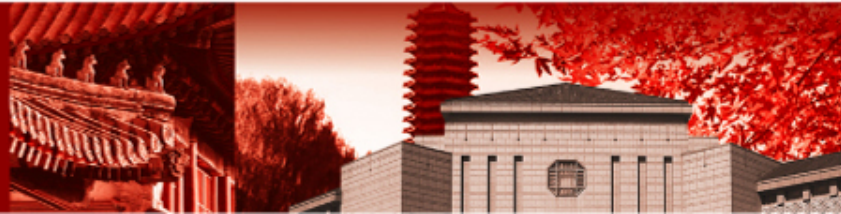
$$\omega_l = \omega_1 + \omega_2$$

$$\vec{k}_l = \vec{k}_1 + \vec{k}_2$$

Manley-Rowe relations



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# Coupling between laser and the plasma oscillators

激光等离子体, 无磁化等离子体

强激光: pump波L

驱动模: 电子等离子体 (纵) 波e ← 电子密度涨落

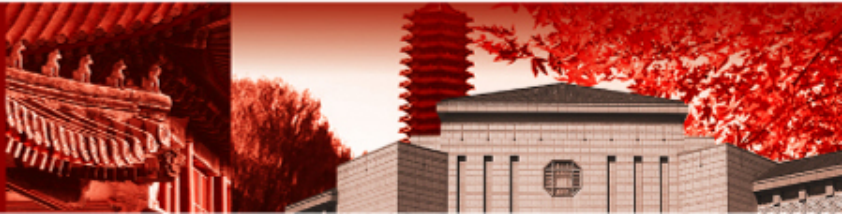
离子声 (纵) 波I ← 离子密度涨落

另一个电磁 (横) 波L' (共振吸收)

- (1)  $L \rightarrow e+i$  参量衰变不稳定性(PDI) (有效吸收激光能量)
- (2)  $L \rightarrow L'+e$  受激Raman散射(SRS) (减少激光能量吸收, 反射)
- (3)  $L \rightarrow L'+i$  受激Brillouin散射 (SBS) (减少激光能量吸收, 反射)
- (4)  $L \rightarrow e+e$  双等离子体衰变(TPD) (有效吸收激光能量)



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# Coupling between laser and the plasma oscillators

表-1 激光等离子体中的主要相互作用过程

不稳定过程	能量守恒	发生区域	后果
共振吸收	$\omega_0 = \omega_{epw}$	$\sim n_{cr}$	超热电子
受激 Brillouin 散射	$\omega_0 = \omega_s + \omega_{lsw}$	$< n_{cr}$	减少激光能量沉积
受激 Raman 散射	$\omega_0 = \omega_s + \omega_{epw}$	$\leq n_{cr} / 4$	超热电子, 减少激光能量沉积
双等离子体衰变	$\omega_0 = \omega_{epw} + \omega_{epw}$	$\sim n_{cr} / 4$	超热电子
离子声衰变	$\omega_0 = \omega_{lsw} + \omega_{epw}$	$\sim n_{cr}$	超热电子

为了实现高增益靶丸内爆，必须最大限度地限制超热电子的产生。

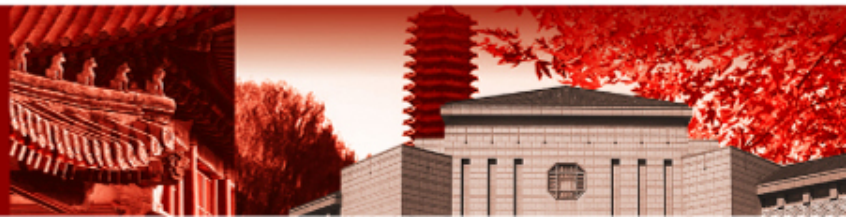
$$\omega_l = \omega_1 + \omega_2$$

$$k_l = k_1 + k_2$$

Manley-Rowe 关系式



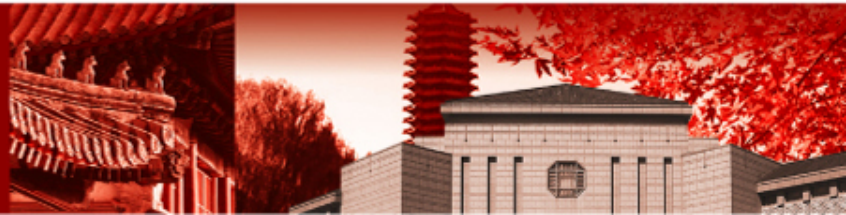
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## V.2 共振吸收



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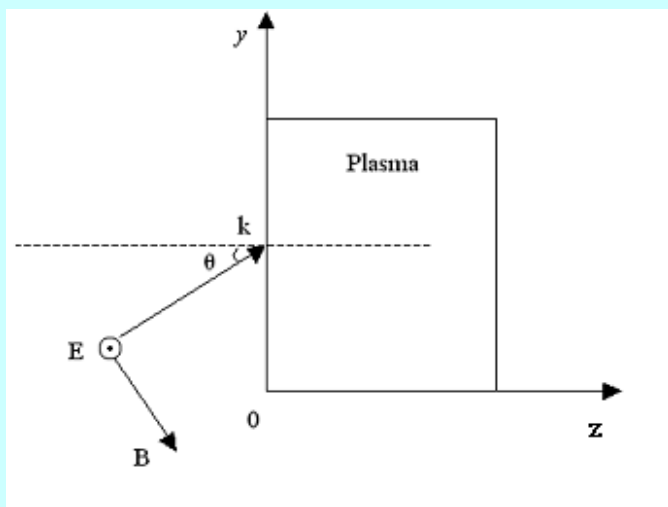


# 共振吸收

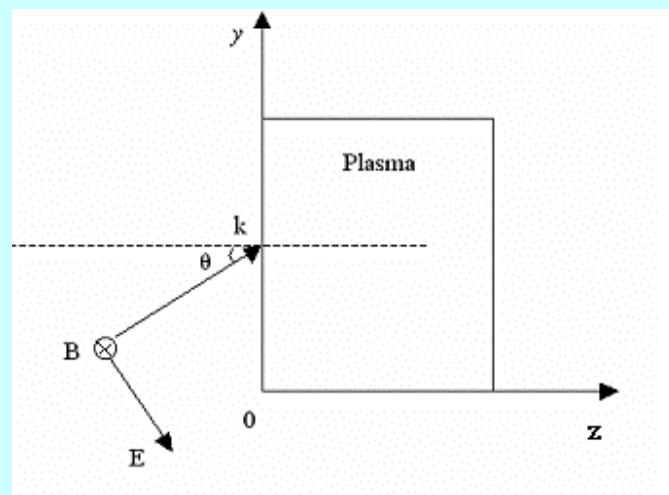
**入射平面：** 考虑一束线偏振激光斜入射到等离子体表面，激光波矢与等离子体密度梯度方向组成入射平面。

**S偏振：** 如果激光的偏振方向垂直于激光波矢与等离子体密度梯度方向组成的入射平面，这种光称为S偏振。

**P偏振：** 如果激光的偏振方向处于激光波矢与等离子体密度梯度方向组成的入射平面，这种光称为P偏振。



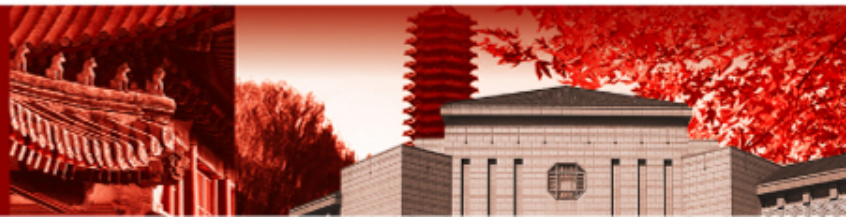
S偏振



P偏振



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# 共振吸收

斜入射P偏振光入射到非均匀等离子体中，电场沿等离子体密度方向有分量，电子在这一电场作用下发生振荡，引起电荷分离，导致电子密度涨落。如果涨落的频率与电子等离子体频率接近，涨落与激光共振作用，在 $\epsilon=1-\omega_{pe}^2/\omega^2=0$ 点上共振激发电子等离子体波，这一现象称为共振吸收。显然，共振吸收发生在临界面上。

共振吸收是一种重要的反常吸收机制，特别对长波长激光。对钕玻璃基频激光（波长为1.05微米），有实验观察表明，共振吸收可达40%。由于共振吸收直接把激光能量转化为电子等离子体波能量，然后通过Landau阻尼变为几十keV的超热电子，对激光聚变十分不利。

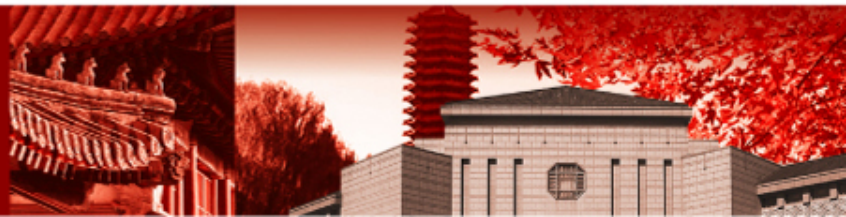
共振吸收，从机制上说是一种**线性吸收**，因为只要给定临界面附近的等离子体密度，高频激光电场引起的电荷分离和激发起来的静电振荡的振幅都是和激光电场的一次方成正比。

**非线性效应：**超热电子的产生

有质动力



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# 共振吸收

Maxwell equations

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$-\nabla \times \nabla \times \mathbf{E} = \nabla^2 \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) \stackrel{1}{=} \frac{1}{c} \frac{\partial^2 \epsilon \mathbf{E}}{\partial t^2}$$

transverse wave

longitudinal wave

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla^2 \mathbf{E} - \frac{1}{c} \frac{\partial^2 \epsilon \mathbf{E}}{\partial t^2} = 0$$

$$\nabla \times \mathbf{E} = 0$$

$$\epsilon = 0$$

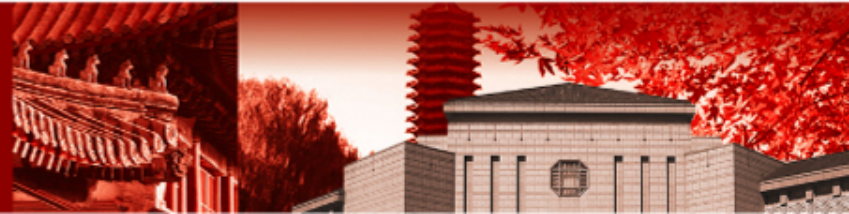
$$\nabla \cdot \mathbf{D} = \epsilon \nabla \cdot \mathbf{E} + \nabla \epsilon \cdot \mathbf{E}$$

homogen. plasma  $\nabla \epsilon \cdot \mathbf{E} = 0$   
waves are purely **EM** or **ES**

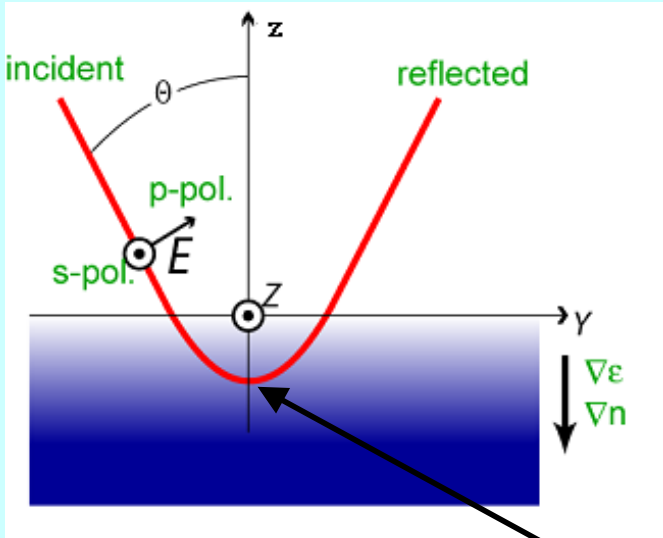
inhomogen. plasma  $\nabla \epsilon \cdot \mathbf{E} \neq 0$   
waves are of **mixed** nature



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# 共振吸收

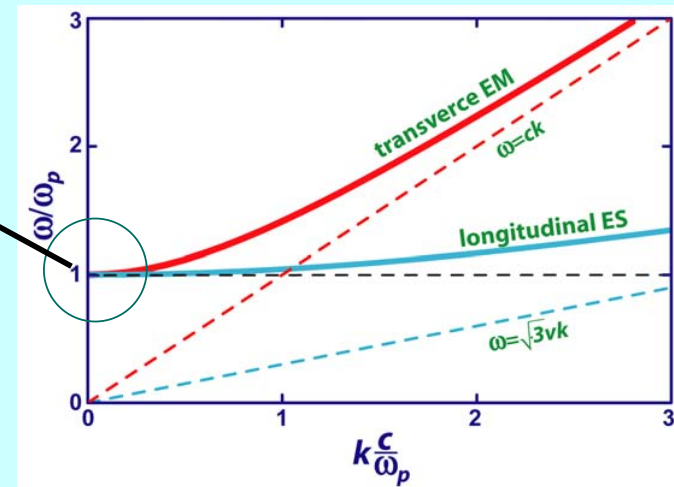


$$\omega^2 / \omega_{pe}^2 = 1 + c^2 k^2 / \omega_{pe}^2 \quad \text{电磁波的色散关系}$$

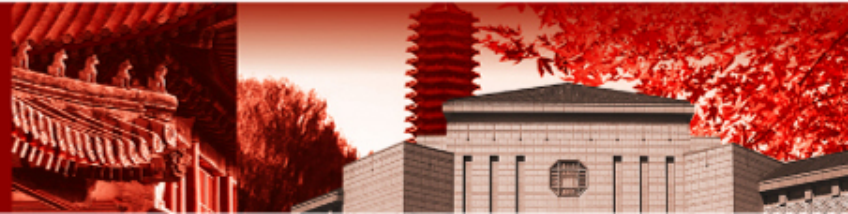
$$\omega^2 / \omega_{pe}^2 = 1 + 3k^2 v_e^2 / \omega_{pe}^2 \quad \text{静电波色散关系}$$

p-polarisation  $\nabla \epsilon \cdot \mathbf{E} \neq 0$  - EM waves are coupled to ES waves!

s-polarisation  $\nabla \epsilon \cdot \mathbf{E} = 0$  - no coupling to ES plasma waves



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# 共振吸收

激光斜入射到电子密度为 $n_e(\mathbf{z})$ 的等离子体平板上，真空等离子体表面在 $\mathbf{z}=0$ 处，入射角： $\theta$ ，定义为激光传播矢量与密度梯度方向之间的夹角。

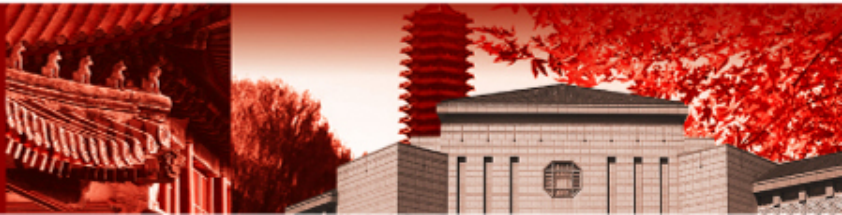
入射面： $\mathbf{y-z}$ 平面  $k_x = 0, \frac{\partial}{\partial x} = 0,$

在真空与等离子体界面处，  $k_y = \frac{\omega}{c} \sin \theta, k_z = \frac{\omega}{c} \cos \theta,$

证明入射光波的电矢量在入射面内外对光的传播有重要的影响。

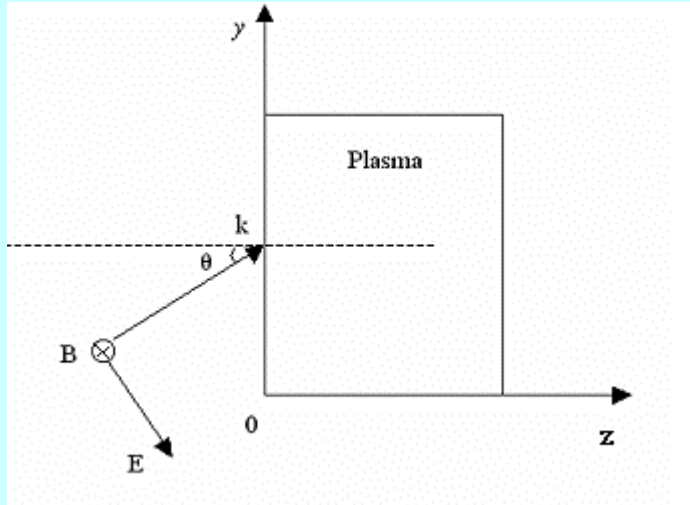


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# 斜入射的S极化光



S偏振

Ex

电场的波动方程为

$$\frac{\partial^2}{\partial y^2} E_x + \frac{\partial^2}{\partial z^2} E_x + \frac{\omega^2}{c^2} \varepsilon(z) E_x = 0,$$

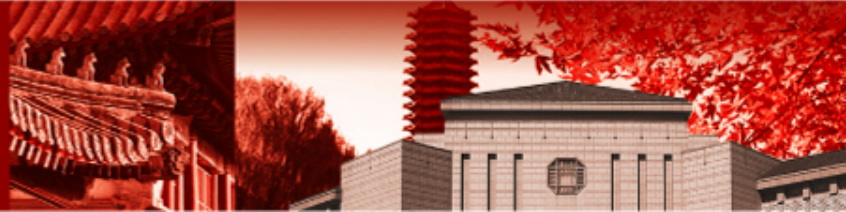
$\varepsilon(z)$ 只是 $z$ 的函数， $k_y$ 必定是守恒的，

$$k_y = \omega \sin \theta / c,$$

$$E_x = E(z) \exp\left(\frac{i\omega y \sin \theta}{c}\right),$$



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# 斜入射的S极化光

这样

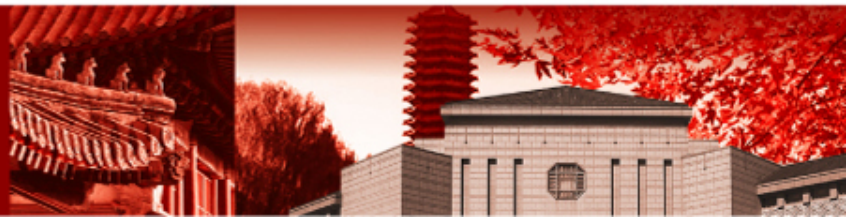
$$\frac{d^2 E(z)}{dz^2} + \frac{\omega^2}{c^2} [\varepsilon(z) - \sin^2 \theta] E(z) = 0,$$

显然, 当  $\varepsilon(z) = \sin^2 \theta$  光波发生反射, 因为  $\varepsilon(z) = 1 - \omega_{pe}^2(z) / \omega^2$ , 所以在等离子体频率  $\omega_{pe} = \omega \cos \theta$  时发生反射。

斜入射激光在比临界密度低的地方反射, 在  $n_e = n_c \cos^2 \theta$  反射。

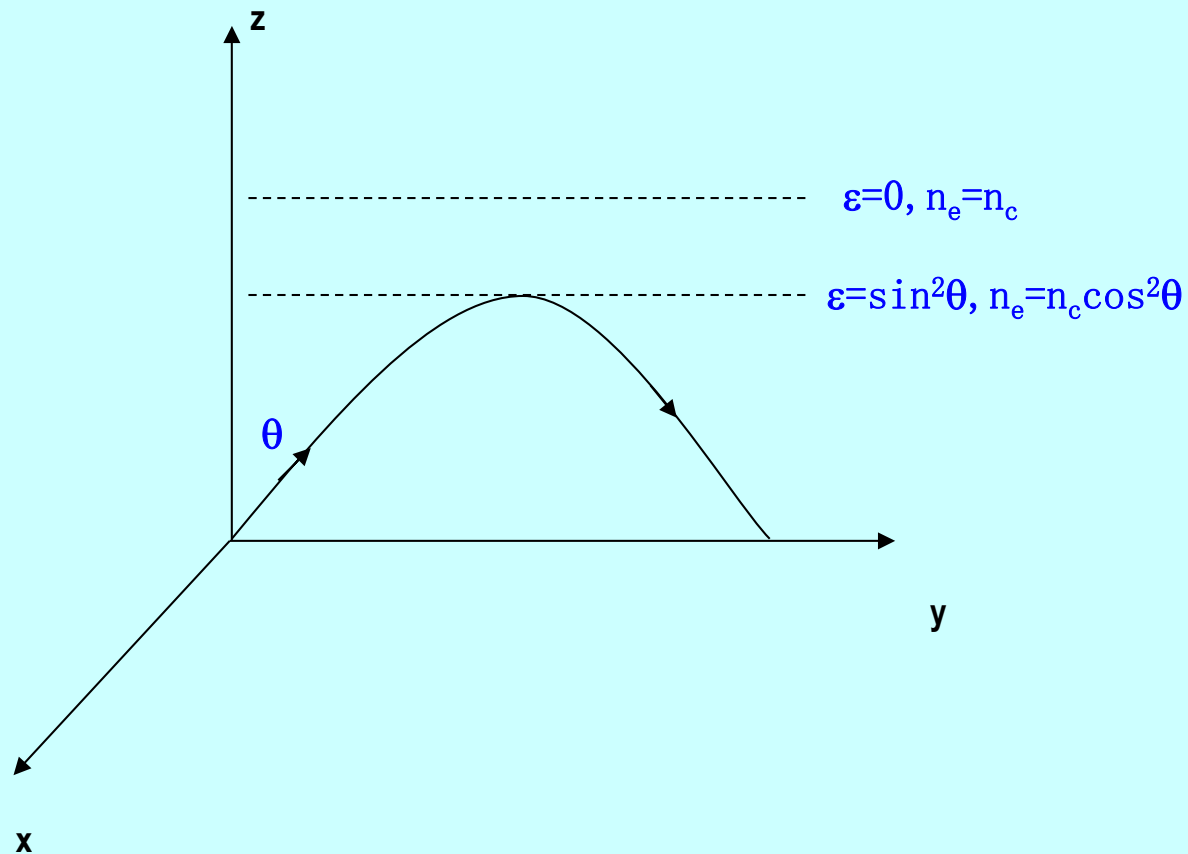


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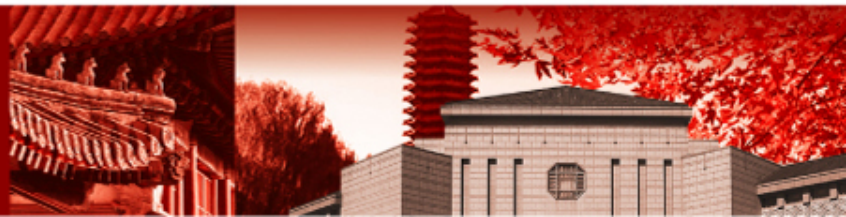


# 斜入射的S极化光

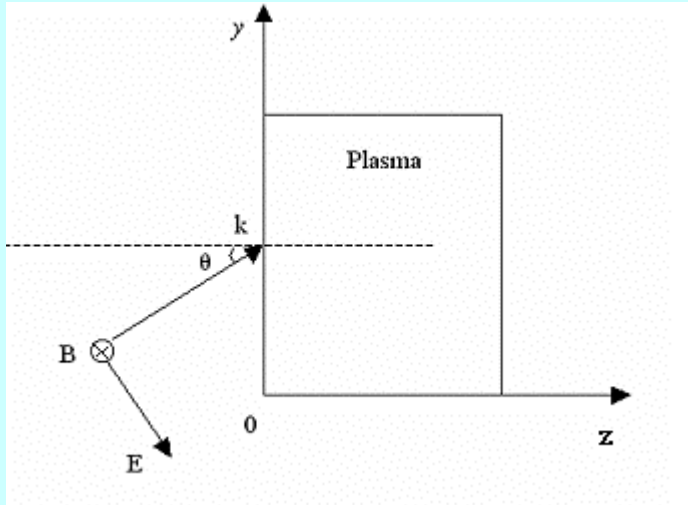
对于斜入射，激光在 $n_e=n_c\cos^2\theta$ 反射，不能传至临界面。



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# 共振吸收---斜入射的P极化光



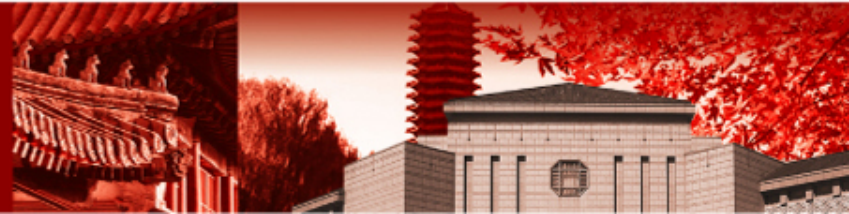
P偏振

如果入射光波的电矢量在入射面内，这种情况称为**P极化**。在这种情况下，电矢量的一个分量能使电子沿着密度梯度方向进行振荡，即

$$\vec{E} \cdot \nabla n_e \neq 0,$$

这种振荡产生电荷密度涨落，它可以被等离子体共振加强。

考虑一个平面电磁波以入射角 $\theta$ 入射在密度为 $n_e(z)$ 的不均匀等离子体平板上，现在电矢量在入射面内，即电矢量具有 $E_y, E_z$ 两个分量。



# 共振吸收---斜入射的P极化光

Poisson方程给出

$$\nabla \cdot (\varepsilon \vec{E}) = 0,$$

$$\nabla \cdot (\varepsilon \vec{E}) = \nabla \varepsilon \cdot \vec{E} + \varepsilon \nabla \cdot \vec{E} = 0$$

于是

$$\nabla \cdot \vec{E} = -\frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial z} E_z,$$

$$\varepsilon(z) = 1 - \omega_{pe}^2(z) / \omega^2,$$

注意，当 $\varepsilon=0$ 时是共振响应，即 $\omega_{pe}=\omega$ 。

**物理解释：**不同密度区域的电子振荡之间产生电荷密度涨落

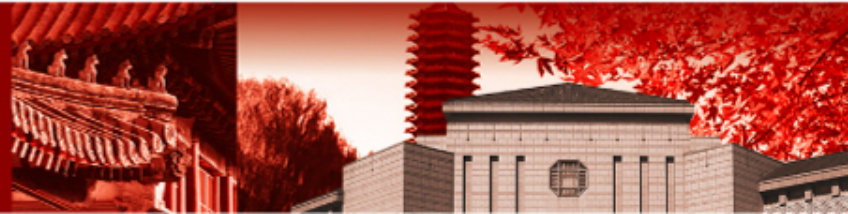
$$\delta n = n_e(\vec{x} + \vec{x}_{os}) - n_e(\vec{x}) \approx \vec{x}_{os} \cdot \nabla n_e$$

$\vec{x}_{os}$ 是电子在光波电场中振荡的幅度

$$\vec{x}_{os} = \frac{e\vec{E}}{m\omega^2}$$



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# 共振吸收---斜入射的P极化光

当 $\omega_{pe}=\omega$ 时，这个电荷密度涨落的频率恰好是等离子体共振响应的频率，因此在 $\epsilon=0$ 即临界密度处，一个电子等离子体波被共振激发。

即使一个斜入射光波在小于临界密度处( $n_e=n_c \cos^2\theta$ )被反射，它的场仍然能以隧道效应穿进临界密度区并在那里激发共振。

下面来计算Ez，将P极化激光通过磁场表示。

注意

$\vec{B} = \hat{x}B_x$ ，利用 $k_y = \frac{\omega}{c} \sin \theta$ 守恒，我们表示出

$$\vec{B} = \hat{x}B(z) \exp\left[-i\omega t + \frac{i\omega y}{c} \sin \theta\right]$$

将上式代入Ampere定律，得到电场和磁场的关系

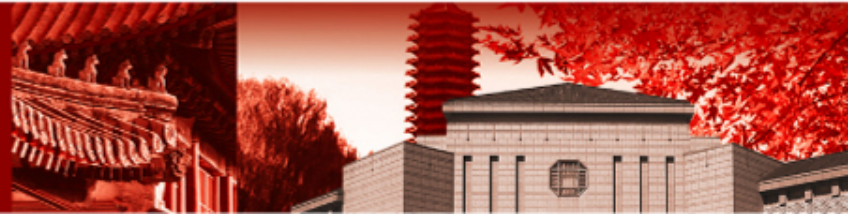
$$\nabla \times \vec{B} = -\frac{i\omega}{c} \epsilon \vec{E},$$

它的z分量是

$$E_z = \frac{\sin \theta B(z)}{\epsilon(z)},$$



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# 共振吸收---斜入射的P极化光

因为 $E_z$ 在临界密度处取峰值，所以我们将共振驱动场近似为 $E_d/\epsilon(z)$ ，这里 $E_d$ 是在共振点计算出的。  
( $E_d$ —光波电场的分量，在临界密度处沿着密度梯度方向振荡电子，驱动共振)

## $E_d$ 的计算:

假设等离子体线性分布， $n_e = n_{cr} z/L$ ，可用反转点 ( $z = L \cos^2 \theta$ ) 的磁场值  $B(z = L \cos^2 \theta)$  乘以一个从反转点到临界面的衰减因子  $e^{-\beta}$  来表示临界面的磁场  $B(z = L)$ 。

反转点磁场：(利用S极化波的Airy函数解进行计算)

$$B(z = L \cos^2 \theta) \approx 0.9 E_{FS} \left(\frac{c}{\omega L}\right)^{1/6}, \text{ 衰减因子 } \beta \text{ 取为}$$

$$\beta = \int_{L \cos^2 \theta}^L \frac{1}{c} \sqrt{\omega_{pe}^2 - \omega^2 \cos^2 \theta} dz = \frac{2\omega L}{3c} \sin^3 \theta$$

$$\text{垂直入射时, } B(\xi = 0) = 0.92 \left(\frac{c}{\omega L}\right)^{1/6} E_{FS}$$

$$\text{色散关系: } \omega^2 = \omega_{pe}^2 + k^2 c^2,$$

$$n_e = n_{cr} \frac{z}{L},$$

$$\text{反转点: } z = L \cos^2 \theta$$

$$\text{临界点: } z = L$$

$$k = \frac{1}{c} \sqrt{\omega_{pe}^2 - \omega^2 \cos^2 \theta}$$



$$\text{不定积分公式: } \int \sqrt{ax + b} dx = \frac{2}{3a} \sqrt{(ax + b)^3}$$



# 共振吸收---斜入射的P极化光

对于线性密度等离子体，积分可得到

$$\beta = \frac{2\omega L}{3c} \sin^3 \theta,$$

$$B(z=L) \approx e^{-\beta} B(z=L \cos^2 \theta) = 0.9 \left(\frac{c}{\omega L}\right)^{1/6} \exp\left[-\frac{2\omega L \sin^3 \theta}{3c}\right].$$

定义：

$$\tau = \left(\frac{\omega L}{c}\right)^{1/3} \sin \theta,$$

$$E_d = \frac{E_{FS}}{\sqrt{\frac{2\pi\omega L}{c}}} \varphi(\tau),$$

这里  $\varphi(\tau) \sim 2.3\tau \exp\left(-\frac{2\tau^3}{3}\right)$ ,

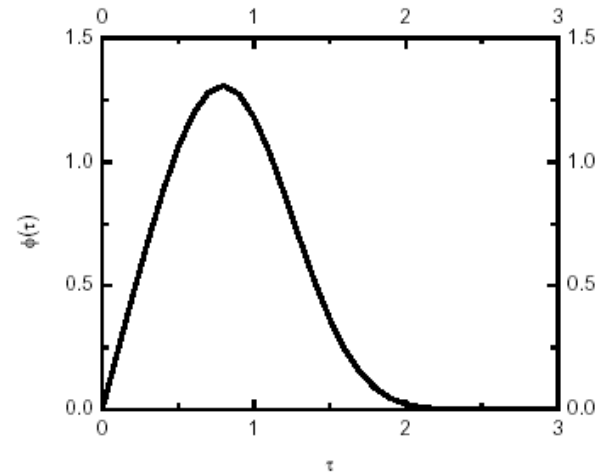
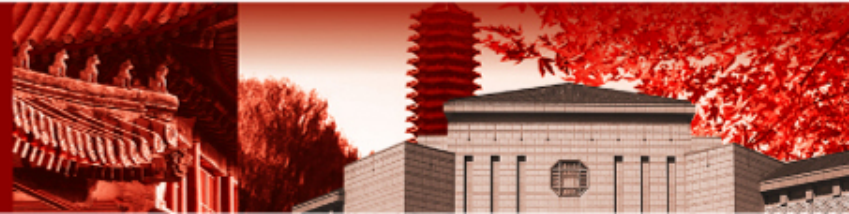


图-2.1 函数 $\phi(\tau)$ 随 $\tau$ 的变化



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# 共振吸收---斜入射的P极化光

## 讨论:

$$\tau = \left(\frac{\omega L}{c}\right)^{1/3} \sin \theta$$

### 1. $\theta$ 的影响

- $\theta=0$ 时,  $\tau=0$ ,  $\varphi(\tau)=0$ , 不发生共振, 此时在密度梯度方向没有激光电场分量;
- $\theta$ 很大时,  $\varphi(\tau)$ 很小, 这时是因为激光传播的反转点和临界面之间的距离太大, 指数因子变得太小的缘故;

### 2. $L$ 的影响

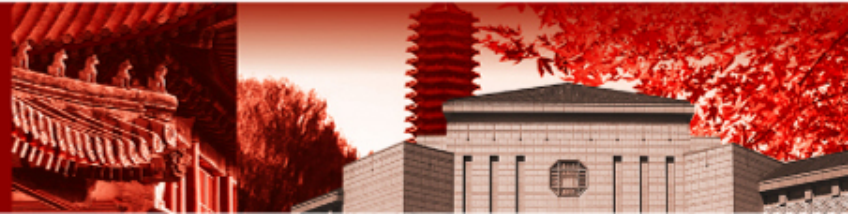
- 当 $\theta$ 固定时,  $L$ 较大, 密度变化平缓就意味着反转点远离临界面, 隧道效应大大减弱;
- 当 $L$ 太小时, 共振区域又太短, 也影响共振吸收;

### 3. 最佳值:

$$\left(\frac{\omega L}{c}\right)^{1/3} \sin \theta \approx 0.8$$



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# 共振吸收---斜入射的P极化光

和数值解比较：合理地符合。

- $\tau \rightarrow 0$ 是， $B(z=L)$ 变成精确解，所以粗估对于 $\tau \ll 1$ ,也变得很精确。
- 因为入射场从截断密度处行临界密度穿透时逐渐变弱是这里的主要物理效应，所以即使对于 $\tau \gg 1$ ， $\phi(\tau)$ 的表达式也是定性正确的。

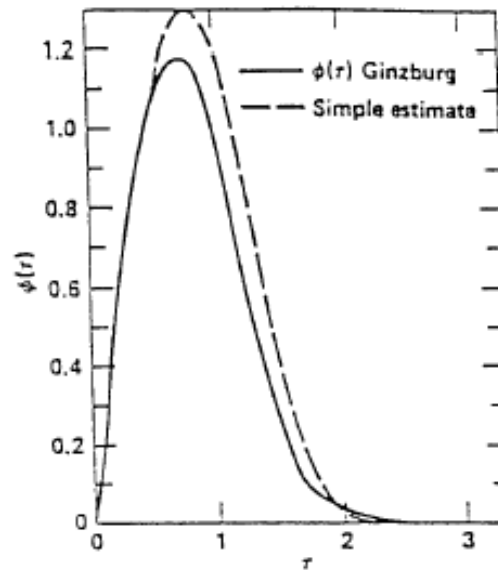


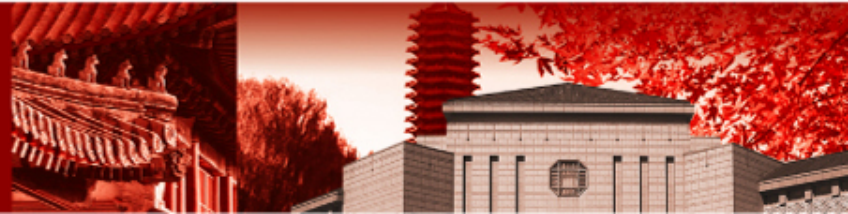
Fig. 1. A plot of the function  $\phi(\tau)$ , which characterizes the efficiency of resonance absorption. The solid line is from Ginzburg (1964). (Kruer, 1988.)

实线  $\phi(\tau)$  Ginzburg函数

虚线 粗估



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# 共振吸收---斜入射的P极化光

## 计算能量吸收:

共振驱动场近似为  $\mathbf{E}z = \mathbf{E}_d / \varepsilon(z)$

小阻尼:  $\nu$  ( $\nu / \omega \ll 1$ ),

可以表示电子-离子碰撞的耗散, 线性或非线性波-粒子相互作用, 甚至共振区之外的波的传播的耗散。

$$\frac{\partial \vec{u}_e}{\partial t} = -\frac{e}{m} \vec{E} - \nu \vec{u}_e$$

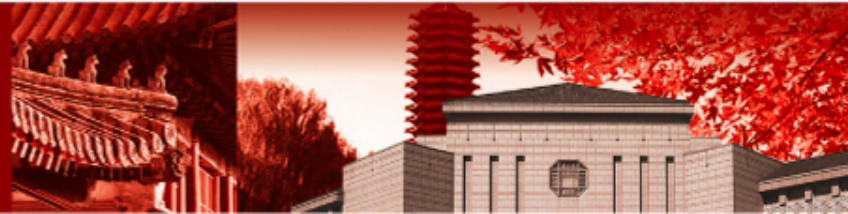
介电函数

$$\varepsilon(z) = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu)} \approx 1 - \frac{\omega_{pe}^2}{\omega^2} + i \frac{\omega_{pe}^2}{\omega^2} \frac{\nu}{\omega}$$

在 $z=L$ 处,  $\varepsilon \propto \nu$ , 所以 $\mathbf{E}z \propto \nu^{-1}$ , 共振区宽度 $\propto \nu$ 。



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# 共振吸收---斜入射的P极化光

吸收的能流

$$I_{abs} = \int_0^{\infty} \nu \frac{E_z^2}{8\pi} dz = \frac{\nu}{8\pi} \int_0^{\infty} \frac{E_d(z)^2}{|\varepsilon|^2} dz.$$

线性密度分布  $n_e = n_{cr} z/L$ ,

$$|\varepsilon|^2 = \left(1 - \frac{z}{L}\right)^2 + \left(\frac{\nu}{\omega}\right)^2 \frac{z^2}{L^2},$$

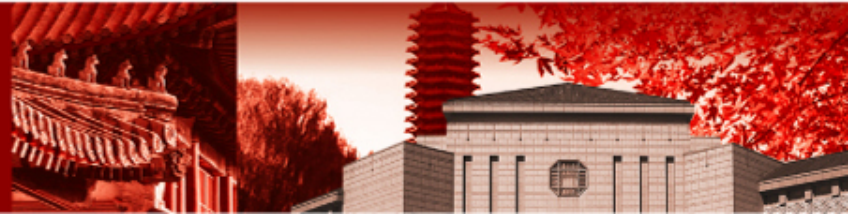
$$I_{abs} = \frac{\nu}{8\pi} \int_0^{\infty} \frac{E_d^2(z)}{\left(1 - \frac{z}{L}\right)^2 + \left(\frac{\nu}{\omega}\right)^2 \frac{z^2}{L^2}} dz$$

在共振函数狭窄范围内,  $E_d$ 近似为常数

$$I_{abs} \approx \frac{\nu E_d^2(z=L)}{8\pi} \int_0^{\infty} \frac{1}{\left(1 - \frac{z}{L}\right)^2 + \left(\frac{\nu}{\omega}\right)^2} dz,$$



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# 共振吸收---斜入射的P极化光

考虑到  $\nu/\omega \ll 1$ , 
$$\int_0^{\infty} \frac{1}{\left(1 - \frac{z}{L}\right)^2 + \left(\frac{\nu}{\omega}\right)^2} dz = \frac{L\pi\omega}{\nu},$$

所以 
$$I_{abs} \approx \frac{\nu E_d^2(z=L)}{8\pi} \frac{L\pi\omega}{\nu} = \frac{\omega L E_d^2}{8}.$$

根据能量守恒

$$I_{abs} = f_a \frac{c E_{FS}^2}{8\pi}.$$

**fa:** 吸收份额 在临界密度处激发一个静电波所引起的入射光波被吸收的份额  
用

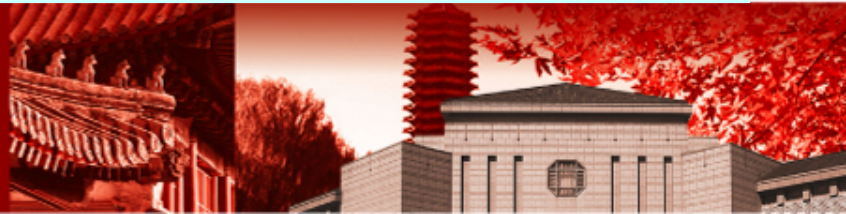
$$E_d = \frac{E_{FS}}{\sqrt{\frac{2\pi\omega L}{c}}} \varphi(\tau)$$

代入可得

$$f_a \approx \frac{\varphi^2(\tau)}{2}$$



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# 共振吸收---斜入射的P极化光

对于线性密度分布等离子体, 吸收份额最大值位于

$$\theta \approx \sin^{-1} \left[ 0.8 \left( \frac{c}{\omega L} \right)^{1/3} \right]$$

for  $\lambda = 1.06 \mu\text{m}$ ,  $L = 1 \mu\text{m}$ ,  $\rightarrow \theta \approx 26^\circ$

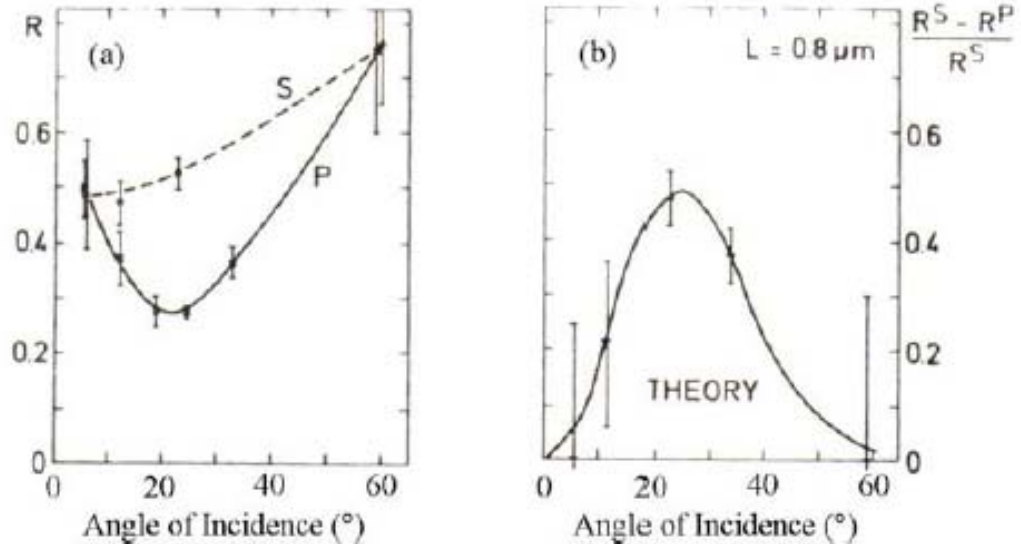


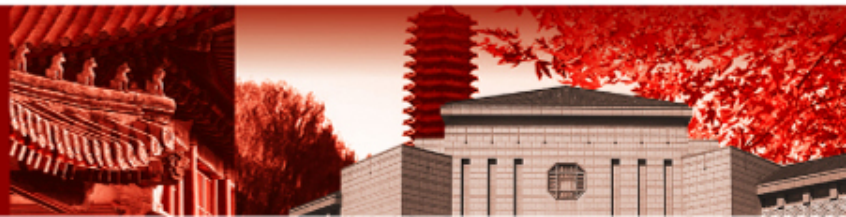
FIG. 11.6. Experimental results on resonance absorption obtained with 30 ps laser pulses at a wavelength of  $0.53 \mu\text{m}$  incident under an angle  $\alpha$  on a plasma layer with gradient scalelength of  $L = 0.8 \mu\text{m}$ . (a) Comparison of measured reflection rates for s-polarized and p-polarized light as a function of angle. (b) Comparison of the relative difference in reflection with calculated results (taken from Maaswinkel *et al.* 1979).



## V.3 由离子密度涨落引起的耦合



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# 共振吸收与反常碰撞吸收

**共振吸收：** 真空膨胀 → 密度梯度

$$\left(\frac{\partial^2}{\partial t^2} - 3v_e^2 \frac{\partial^2}{\partial x^2} + \omega_{pe}^2\right)n' = 0.$$

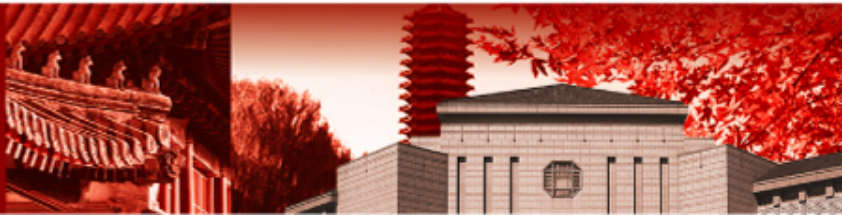
**P**极化激光斜入射等离子体时，电子在等离子体密度梯度方向振荡驱动电荷密度涨落，当振荡频率接近电子等离子体频率时，电子等离子体波被共振激发。

**离子涨落（？）** 
$$\frac{\partial^2 n_i'}{\partial t^2} - \frac{ZT_e + 3T_i}{m_i} \frac{\partial^2 n_i'}{\partial x^2} = 0$$

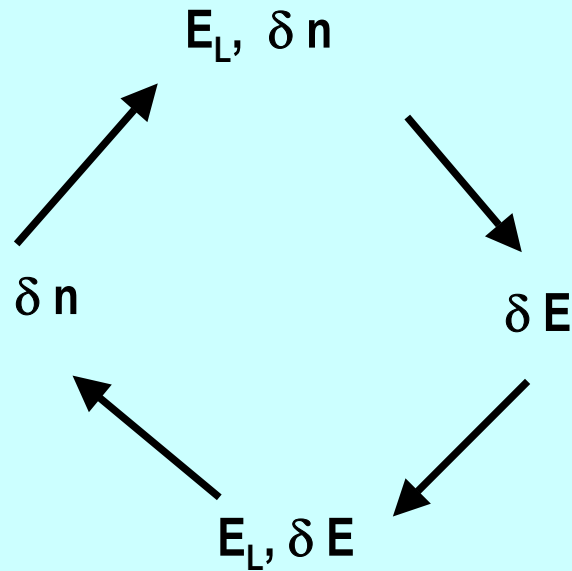
• 离子声波相联系的离子涨落 $\delta n_{ik}$  → 由于等离子体慢运动的准电中性，产生电子密度的空间变化 $\delta n$  → 电子在激光场中振荡，**pump**场的作用是移动有低频密度涨落的电子密度快速地通过给定的空间点 → 对于电子密度涨落引起附加的高频贡献，产生高频电子密度涨落 $\delta n_{ek}$  → 产生高频电场 $\delta E$ ，与高频**pump**场差频形成空间不均匀的场压强即有质动力 → 有质动力推动电子，并带动离子做新的低频运动，进一步加强原来的低频离子涨落 $\delta n_{ik}$ 。



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# 由离子声波引起的参量衰变不稳定性的直观解释



参量衰变不稳定性耦合机制

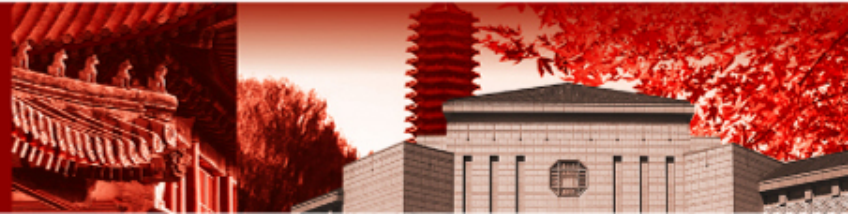
$$\vec{u}_{osc}^h = \frac{e\vec{E}_L}{m_e \omega} \rightarrow \delta \vec{J} = -e\vec{u}^h \delta n$$

$$\rightarrow \delta \vec{E}$$

$$\delta \vec{F}_p = \frac{1}{8\pi} \nabla |\vec{E}_L + \delta \vec{E}|^2 - \frac{1}{8\pi} \nabla |\vec{E}_L|^2$$
$$\approx \frac{1}{4\pi} \nabla (\vec{E}_L \cdot \delta \vec{E})$$



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# 一维模型研究光波通过离子密度涨落与电子等离子体波的耦合

振荡电场  $\mathbf{E} = \mathbf{E}_0 \exp(-i\omega_L t)$  离子波频率远小于光波频率

离子密度涨落  $n = n_0 + \Delta n \cos kx$   $n_0$ : 平均密度  $\Delta n$ : 密度调制的振幅

离子不动, 电子流体:  $n_e, u_e, P_e$  (静调制)

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x} (n_e u_e) = 0.$$

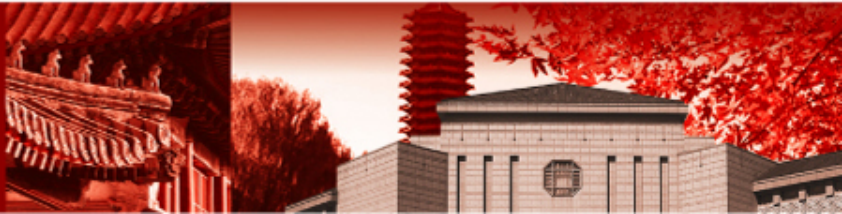
$$\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial t} (n_e u_e) + \frac{\partial}{\partial x} (n_e u_e^2) = -\frac{n_e e E}{m_e} - \frac{1}{m_e} \frac{\partial P_e}{\partial x} - \nu_{ei} n_e u_e.$$

运算后, 可得高频电子密度涨落方程

$$\frac{\partial^2 n_e}{\partial t^2} - \frac{\partial^2}{\partial x^2} (n_e u_e^2) - \frac{e}{m_e} \frac{\partial}{\partial x} (n_e E) - \frac{1}{m_e} \frac{\partial^2 P_e}{\partial x^2} - \nu_{ei} \frac{\partial}{\partial x} (n_e u_e) = 0$$



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# 一维模型研究光波通过离子密度涨落与电子等离子体波的耦合

线性化, 令

$$\begin{aligned} n_e &= n_0 + \Delta n \cos kx + n', \\ u_e &= u_0 + u', \\ E &= E_0 + E' \end{aligned}$$

$$\begin{aligned} n_0 &\gg \Delta n \gg n', \\ u_0 &\gg u', \\ E_0 &\gg E' \end{aligned}$$

$$\frac{\partial^2 n'}{\partial t^2} - \frac{e}{m_e} n_0 \frac{\partial E'}{\partial x} - 3v_e^2 \frac{\partial^2 n'}{\partial x^2} + v_{ei} \frac{\partial n'}{\partial t} = -\frac{eE_0}{m_e} \Delta n k \sin kx$$

这里假设 $\omega/k \gg v_e$ 时, 绝热状态方程近似

Poisson方程

$$\frac{\partial E'}{\partial x} = -4\pi e n',$$

消去 $n'$ ,有

$$\frac{\partial}{\partial x} \left[ \frac{\partial^2 E'}{\partial t^2} + \omega_{pe}^2 E' - 3v_e^2 \frac{\partial^2 E'}{\partial x^2} + v_{ei} \frac{\partial E'}{\partial t} \right] = -\frac{4\pi e^2}{m_e} E_0 \Delta n k \sin kx$$

$$\omega_{pe}^2 = \frac{4\pi n_0 e^2}{m_e}$$



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用连续性方程改写成: 
$$\frac{\partial^2 n_e}{\partial t^2} - \frac{\partial^2}{\partial x^2} (n_e u_e^2) - \frac{e}{m_e} \frac{\partial}{\partial x} (n_e E) - \frac{1}{m_e} \frac{\partial^2 P_e}{\partial x^2} + v_{ei} \frac{\partial n_e}{\partial t} = 0$$



# 一维模型研究光波通过离子密度涨落与电子等离子体波的耦合

对上式积分给出

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - 3v_e^2 \frac{\partial^2}{\partial x^2} + v_{ei} \frac{\partial}{\partial t}\right) E' = -\omega_{pe}^2 \frac{\Delta n}{n_0} E_0 \cos kx$$

离子密度涨落

Pump 场

该方程描述pump场（光波）同离子密度涨落相互作用激发一个电子等离子体波。

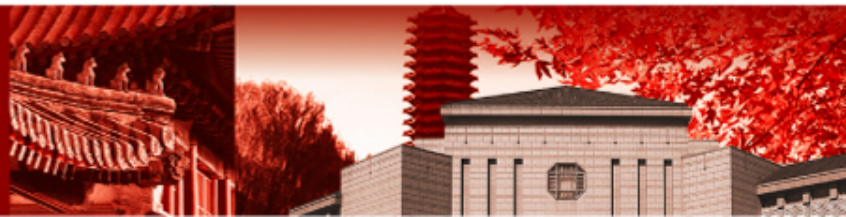
注意到pump场 $\sim \exp(-i\omega_L t)$ ，可以得到上式的解

$$E' = -\frac{\omega_{pe}^2}{\omega_L^2} \frac{\Delta n}{n_0} \frac{E_0 \cos kx}{\varepsilon(k, \omega_L)}. \quad \varepsilon(k, \omega_L) = 1 - \frac{\omega_{pe}^2 + 3k^2 v_e^2}{\omega_L^2} + i \frac{v_{ei}}{\omega_L}$$

电子等离子体波是密度涨落和泵浦波紧密相连，缺一不可。电场大小与密度涨落、泵浦波乘积成正比。



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# 一维模型研究光波通过离子密度涨落与电子等离子体波的耦合

因为pump波的能量耦合进驱动波中，所以pump波将受到阻尼，其能量阻尼率 $\nu^*$ 可以通过能量损失和由驱动波的能量吸收平衡得到，即

单位时间的能量吸收

$$\frac{\nu^* E_0^2}{8\pi} = \frac{k}{2\pi} \int_0^{\frac{2\pi}{k}} dx \frac{\nu_{ei} |E'|^2}{8\pi}, \quad \text{注意到} \quad \text{Im } \varepsilon = \frac{\nu_{ei}}{\omega_L}, n \sim n_{cr}$$



$$\frac{\nu^*}{\omega_L} = \frac{1}{2} \left( \frac{\Delta n}{n_{cr}} \right)^2 \frac{\text{Im } \varepsilon}{|\varepsilon(k, \omega_L)|^2}.$$

$$E' = -\frac{\omega_{pe}^2}{\omega_L^2} \frac{\Delta n}{n_0} \frac{E_0 \cos kx}{\varepsilon(k, \omega_L)}, \quad \varepsilon(k, \omega_L) = 1 - \frac{\omega_{pe}^2 + 3k^2 \nu_e^2}{\omega_L^2} + i \frac{\nu_{ei}}{\omega_L}$$



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# 一维模型研究光波通过离子密度涨落与电子等离子体波的耦合

$$\varepsilon(k, \omega_L) = 0$$

泵浦波和电子等离子体波共振耦合，电场十分大；

$$\varepsilon(k, \omega_L)$$

$\varepsilon$ 太小时， $\mathbf{E}'$ 很大，线性分析失效，必须考虑非线性效应。这种非线性效应是反复的模耦合。

$$\left| \frac{\Delta n}{n_{cr}} \right| > \left| \varepsilon(k, \omega_L) \right|$$

驱动波变得象pump波那样强，它的作用象pump波那样驱动一个 $2k$ 的波，它再与离子密度涨落拍频，驱动一个 $3k$ 的波，如此继续下去，可以得到一个驱动波的谱。

$$\left| \frac{\Delta n}{n_{cr}} \right| \sim \left| \varepsilon(Nk, \omega_L) \right|$$



极大的波数

即使是一个中等的密度的涨落，也可以有效地同长波长等离子体波耦合转换成短波长的等离子体波。



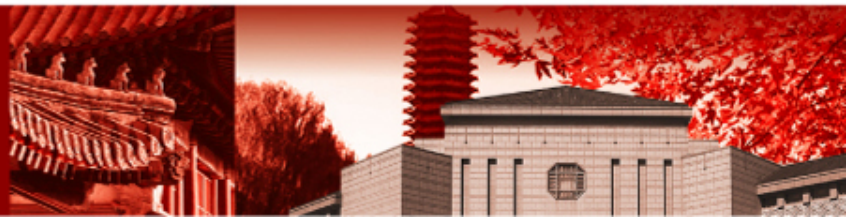
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$$\varepsilon(k, \omega_L) = 1 - \frac{\omega_{pe}^2 + 3k^2 v_e^2}{\omega_L^2} + i \frac{v_{ei}}{\omega_L}$$

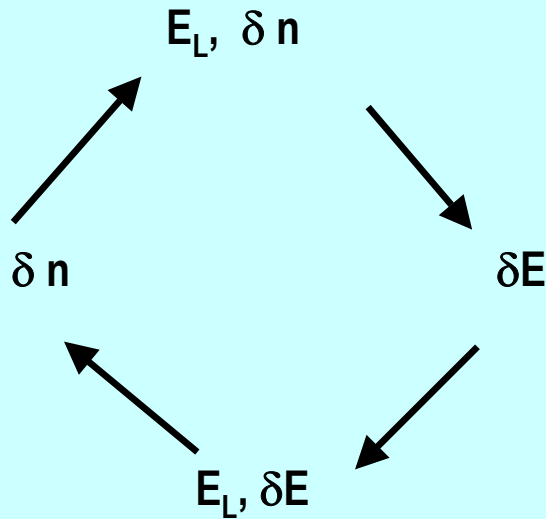
## V.4 不稳定性物理图像



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# 不稳定性物理图像



$$\vec{u}_{osc}^h = \frac{e\vec{E}_L}{m_e\omega} \rightarrow \delta\vec{J} = -e\vec{u}^h \delta n$$

$$\rightarrow \delta\vec{E}$$

$$\delta\vec{F}_p = \frac{1}{8\pi} \nabla |\vec{E}_L + \delta\vec{E}|^2 - \frac{1}{8\pi} \nabla |\vec{E}_L|^2$$

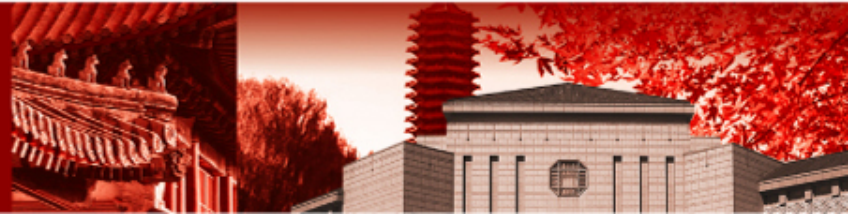
$$\approx \frac{1}{4\pi} \nabla(\vec{E}_L \cdot \delta\vec{E})$$

参量衰变不稳定性耦合机制

光波可以激发一种**电子波和离子波同时增长**的不稳定性，离子涨落与光波耦合产生一个等离子体波，等离子体波和光波拍频使电场强度产生空间变化，它通过有质动力加强离子密度涨落，因此形成一个正反馈图，不稳定性产生。



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# 不稳定性图像

均匀等离子体中的静离子涨落

$$n = n_0 + \Delta n \cos kx$$

光波电场近似为空间均匀场

$$E_d = E_0 \sin \omega_L t$$

激发的等离子体波的静电场

$$E' = -\frac{\omega_{pe}^2}{\omega_L^2} \frac{\Delta n}{n_0} \frac{E_0 \cos kx}{\varepsilon(k, \omega_L)}. \quad \varepsilon(k, \omega_L) = 1 - \frac{\omega_{pe}^2 + 3k^2 v_e^2}{\omega_L^2} + i \frac{v_{ei}}{\omega_L}$$

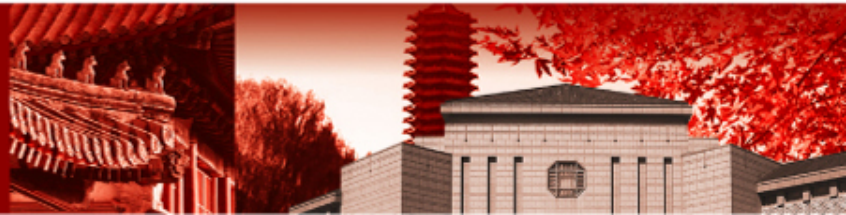
如果明显地包含时间依赖和忽略碰撞

$$\varepsilon(k, \omega_L) = \frac{\omega_L^2 - \omega_{ek}^2}{\omega_L^2}, \quad \omega_{ek}^2 = \omega_{pe}^2 + 3k^2 v_e^2$$

$$E' = \frac{\omega_{pe}^2}{\omega_L^2 - \omega_{ek}^2} \frac{\Delta n}{n_0} E_0 \cos kx \sin \omega_L t.$$



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# 不稳定性图像

电场强度和空间有关，电场强度的时间平均具有一种梯度。对于热的离子密度涨落的小振幅  $\Delta n$  的最低阶

$$\nabla \langle (E' + E_d)^2 \rangle = -\frac{\omega_{pe}^2}{\omega_L^2 - \omega_{ek}^2} \frac{\Delta n}{n_0} E_0^2 k \sin kx.$$

有质动力

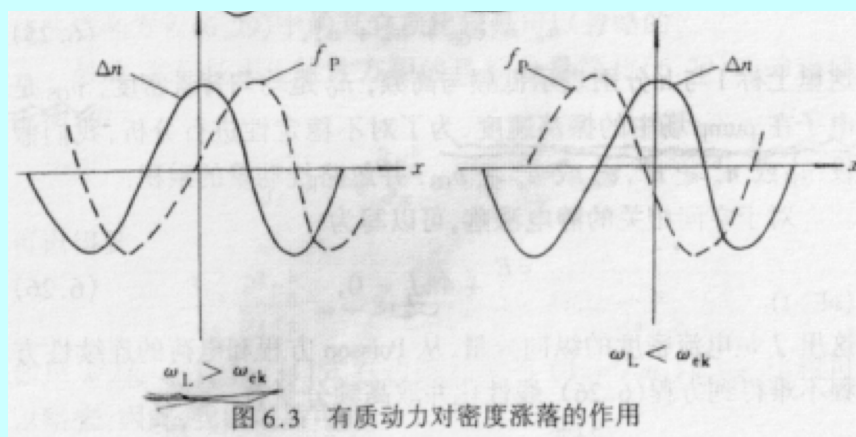
$$F_p = \frac{e^2 E_0^2}{2m_e \omega_L^2} \frac{\omega_{pe}^2}{\omega_L^2 - \omega_{ek}^2} \frac{\Delta n}{n_0} k \sin kx.$$

$$\omega_L > \omega_{ek}$$

$$\omega_L < \omega_{ek}$$

有质动力作用  
减小密度涨落

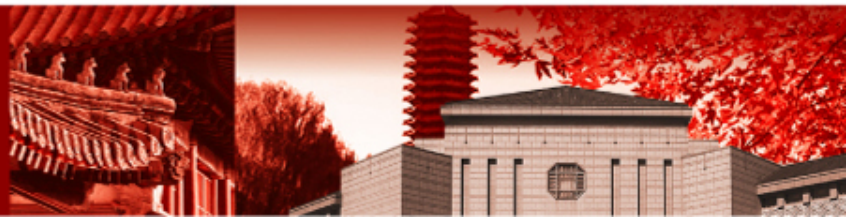
有质动力作用**加强**密度涨落，即推出更多的等离子体进入高密度区。



$$\vec{F}_p = -\frac{1}{4} \frac{e^2}{m_e \omega^2} \nabla \vec{E}^2(\vec{x})$$



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# 不稳定性图像

$$\omega = \omega_r + i\gamma = i\gamma$$

**振荡双流不稳定性：** 纯增长离子密度涨落将从噪声水平自发地发展，当它的振幅增大时，相联系的电子等离子体波也增长，这种不稳定性叫做振荡双流不稳定性。

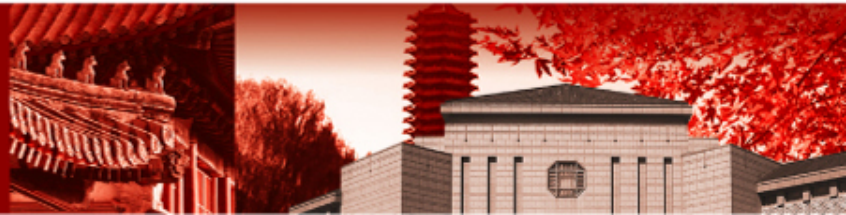
**离子声衰变不稳定性：** 如果考虑与离子声波相联系的离子密度涨落，这种涨落不再是静的，而是具有频率为 $kCs$ 。在这种情况下，光波最容易通过共振衰变为一个等离子体波加上一个离子声波，当三波频率完全匹配，即 $\omega = \omega_{ek} + kCs$ 时，这种不稳定性是最强的。因此，常常把这种不稳定性称作离子声衰变不稳定性。

$$\text{离子密度的小振幅扰动: } \frac{\partial^2 n_i'}{\partial t^2} - \frac{Z T_e + 3 T_i}{m_i} \frac{\partial^2 n_i'}{\partial x^2} = 0$$

$$\text{电子密度的小振幅扰动: } \left( \frac{\partial^2}{\partial t^2} - 3 v_e^2 \frac{\partial^2}{\partial x^2} + \omega_{pe}^2 \right) n' = 0.$$



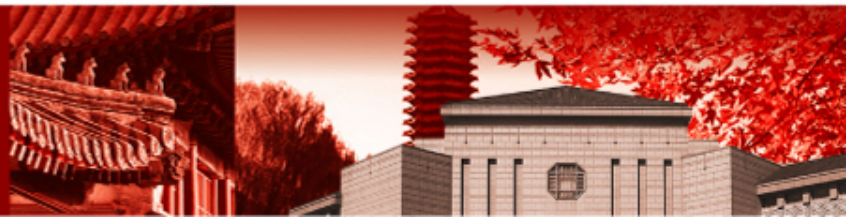
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## V.5 不稳定性分析



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# 不稳定性分析

## 一维扰动分析

pump场

$$E_d = E_0 \sin \omega_0 t \quad \omega_0 \sim \omega_{pe}$$

## 电子流体方程

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) = 0.$$

$$\frac{\partial}{\partial t}(n_e u_e) + \frac{\partial}{\partial x}(n_e u_e^2) = -\frac{n_e e E}{m_e} - \frac{1}{m_e} \frac{\partial P_e}{\partial x} - \nu_e n_e u_e,$$

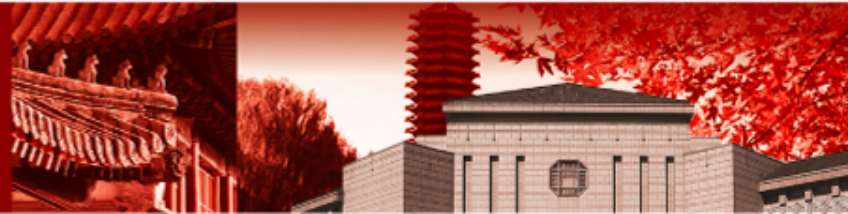
$\nu_e$ : 既模拟碰撞又模拟电子等离子体波的Landau阻尼

$$\frac{P_j}{n_j^\gamma} = \text{常数}, \quad \gamma = 3(\text{对于高频扰动}), \quad \gamma = 1(\text{对于低频扰动})$$

如果用 $\omega$ 和 $k$ 代表所考虑的物理过程的特征频率和特征波数，那么当 $\omega/k \ll vt$ 时，可作**等温状态方程**近似， $vt$ 是粒子运动的热速度。当 $\omega/k \gg vt$ 时，我们可直接忽略热流，作**绝热状态方程**近似。



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# 不稳定性分析

将电子涨落分为高频和低频分量

$$n_e = n_0 + n_e^l + n_e^h, \quad u_e = v_{os} + u_e^l + u_e^h.$$

设  $n_e^l$  或  $n_e^h \ll n_0$ ,  $u_e^l$  或  $u_e^h \ll v_{os}$ , 忽略扰动量的乘积

从Poisson方程和连续性方程

$$\frac{\partial E}{\partial t} + 4\pi J = 0.$$

**J**: 电流密度的纵向分量

线性化并取高频分量, 有 
$$\frac{\partial E^h}{\partial t} = 4\pi e(n_0 u_e^h + n_e^l v_{os}).$$

对运动方程线性化并取高频分量

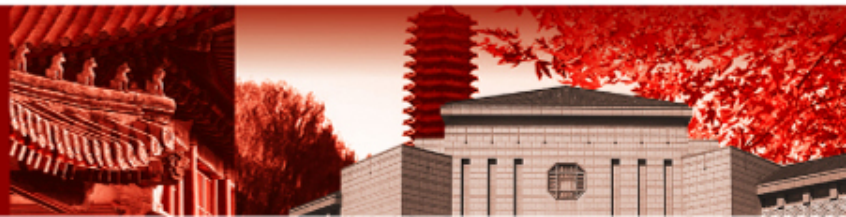
$$P_e^h = 3m_e v_e^2 n_e^h,$$

$$\frac{\partial u_e^h}{\partial t} + v_{os} \frac{\partial u_e^l}{\partial x} = -\frac{e}{m_e} E^h - \frac{3v_e^2}{n_0} \frac{\partial n_e^h}{\partial x} - v_e u_e^h.$$

$\frac{\partial}{\partial t} \rightarrow$



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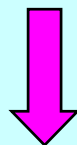
# 不稳定性分析

$$\frac{\partial^2 u_e^h}{\partial t^2} + v_e \frac{\partial u_e^h}{\partial t} + \frac{\partial}{\partial t} \left( v_{os} \frac{\partial u_e^l}{\partial x} \right) + \frac{3v_e^2}{n_0} \frac{\partial^2 n_e^h}{\partial x^2} = -\omega_{pe}^2 u_e^h - \frac{4\pi e^2}{m_e} n_e^l v_{os}.$$

↑  
小项

假设低频  $\ll \omega_0 \sim \omega_{pe}$ ,  $\frac{\partial}{\partial t} \left( v_{os} \frac{\partial u_e^l}{\partial x} \right) \sim \frac{\partial v_{os}}{\partial t} \frac{\partial u_e^l}{\partial x}$ .

再利用连续性方程的低频分量给出

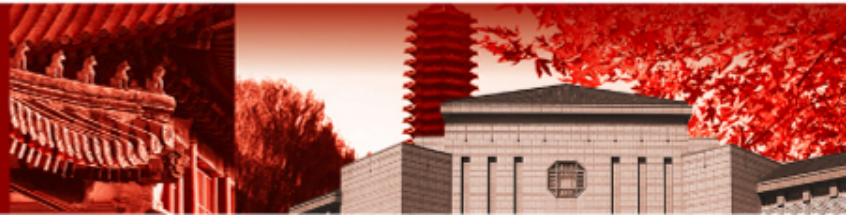


$$\frac{\partial v_{os}}{\partial t} \frac{\partial u_e^l}{\partial x} = -\frac{\partial v_{os}}{\partial t} \frac{1}{n_0} \left( \frac{\partial n_e^l}{\partial t} + v_{os} \frac{\partial u_e^h}{\partial x} \right).$$

$\sim \omega^l \quad \sim kv_{os} \ll \omega_{pe} \sim \omega_0$



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# 不稳定性分析

由连续性方程的高频分量  
可以得到

$$\frac{\partial n_e^h}{\partial t} + n_0 \frac{\partial u_e^h}{\partial x} + v_{os} \frac{\partial n_e^l}{\partial x} = 0,$$

↓ 小项

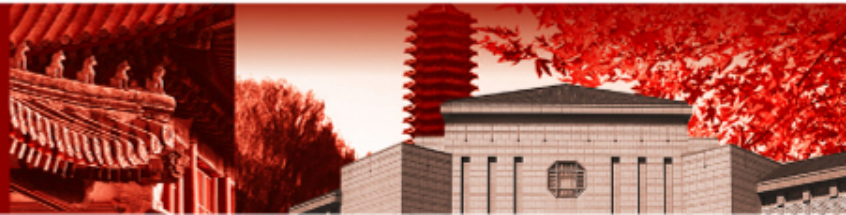
$$\frac{3v_e^2}{n_0} \frac{\partial^2 n_e^h}{\partial t \partial x} = -3v_e^2 \frac{\partial^2 u_e^h}{\partial x^2} - \frac{3v_e^2}{n_0} v_{os} \frac{\partial^2 n_e^l}{\partial x^2}.$$
$$\sim k^2 v_e^2 \quad \sim k^2 v_e^2 v_{os} \ll \frac{4\pi e^2}{m_e} n_e^l v_{os}$$

因此，得到电子运动的高频分量方程

$$\left( \frac{\partial^2}{\partial t^2} + \omega_{pe}^2 + v_e \frac{\partial}{\partial t} - 3v_e^2 \frac{\partial^2}{\partial x^2} \right) u_e^h = -\frac{4\pi e^2}{m_e} n_e^l v_{os}.$$



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# 不稳定性分析

为了得到一个低频涨落方程，我们必须考虑电子和离子二者的响应。

$Zm_e \ll M$ , 等温状态方程,

运动方程的低频分量变为

$$\frac{eE^l}{m_e} = -\frac{\partial}{\partial x} (v_{os} u_e^h) - \frac{v_e^2}{n_0} \frac{\partial n_e^l}{\partial x}.$$

低频电场传给有质动力，并且电子压力传给电子。

离子流体方程

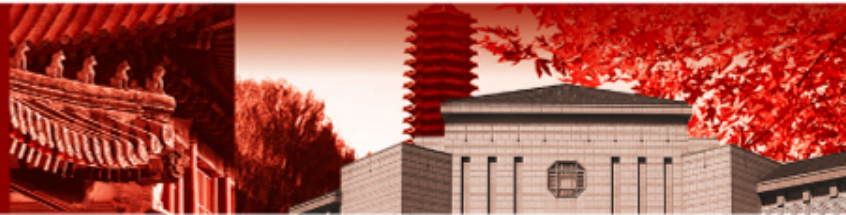
$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} (n_i u_i) = 0.$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = \frac{ZeE}{M} - \frac{1}{Mn_{0i}} \frac{\partial P_i}{\partial x} - \nu_i u_i,$$

$\nu_i$ : 既模拟碰撞又模拟离子涨落的Landau阻尼



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# 不稳定性分析

忽略离子对高频场的响应，假设  $n_i = n_{i0} + n_i^l$ ,  $u_i = u_i^l$ .

进行线性化，可得到线性化的连续性方程

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial n_i^l}{\partial t} + n_{0i} \frac{\partial u_i^l}{\partial x} = 0.$$

和线性化的运动方程

$$\frac{\partial}{\partial x} \rightarrow \frac{\partial u_i^l}{\partial t} = \frac{ZeE^l}{M} - \frac{3v_i^2}{n_{0i}} \frac{\partial n_i^l}{\partial x} - v_i u_i^l,$$

$$\frac{\partial^2 n_i^l}{\partial t^2} + v_i \frac{\partial n_i^l}{\partial t} + \frac{Zen_{0i}}{M} \frac{\partial E^l}{\partial x} - 3v_i^2 \frac{\partial^2 n_i^l}{\partial x^2} = 0.$$

前面已得到电子运动的低频分量方程

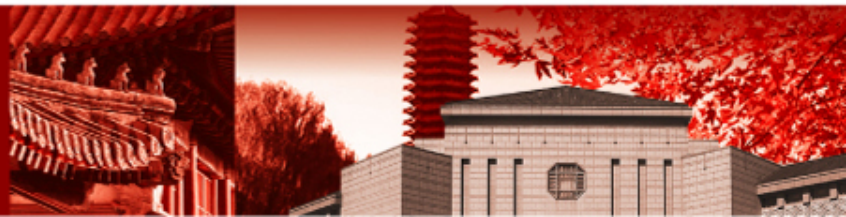
且

$$\frac{eE^l}{m_e} = -\frac{\partial}{\partial x} (v_{os} u_e^h) - \frac{v_e^2}{n_0} \frac{\partial n_e^l}{\partial x},$$

$$Zn_i^l = n_e^l. (\text{等离子体慢运动中的准电中性})$$



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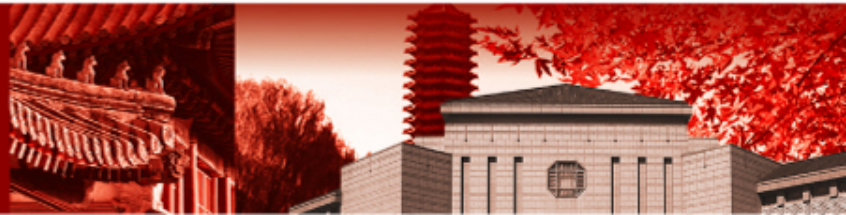
# 不稳定性分析

## 低频密度涨落方程

$$\frac{\partial^2 n_e^l}{\partial t^2} + v_i \frac{\partial n_e^l}{\partial t} - C_s^2 \frac{\partial^2 n_e^l}{\partial x^2} = \omega_{pi}^2 \frac{m_e}{4\pi e^2} v_{os} \frac{\partial^2 u_e^h}{\partial x^2},$$
$$C_s^2 = \frac{ZT_e + 3T_i}{M}, \omega_{pi}^2 = \frac{4\pi n_0 e^2 Z}{M}.$$



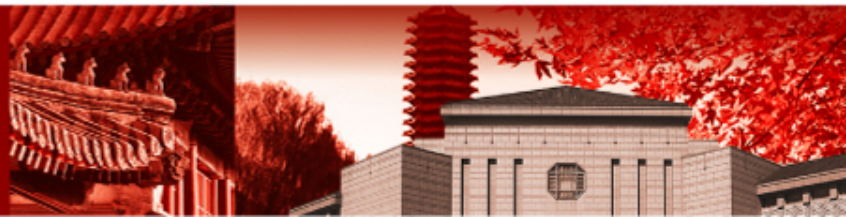
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## V.6 色散关系



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# 色散关系

$n_e^l$ 和 $u_e^h$ 的耦合方程 电子

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 + v_e \frac{\partial}{\partial t} - 3v_e^2 \frac{\partial^2}{\partial x^2}\right)u_e^h = -\frac{4\pi e^2}{m_e} n_e^l v_{os}.$$

离子

$$\frac{\partial^2 n_e^l}{\partial t^2} + v_i \frac{\partial n_e^l}{\partial t} - C_s^2 \frac{\partial^2 n_e^l}{\partial x^2} = \omega_{pi}^2 \frac{m_e}{4\pi e^2} v_{os} \frac{\partial^2 u_e^h}{\partial x^2}$$

设 $n_e^l$ 和 $u_e^h$ 随 $\exp(ikx-i\omega t)$ 变化,

$$v_{os}(t) = v_{os} \left[ \frac{\exp(i\omega_0 t) + \exp(-i\omega_0 t)}{2} \right], \quad \text{振幅 } v_{os} = \frac{eE_0}{m_e \omega_0}$$

进行时空Fourier变换给出

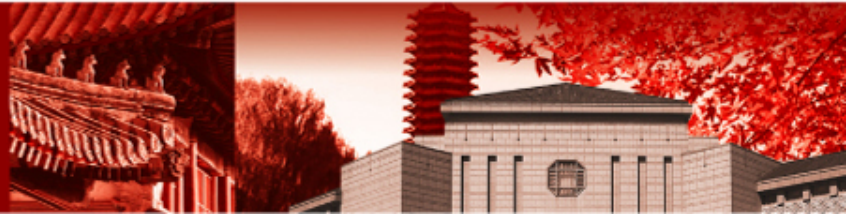
$$(\omega^2 + i\omega v_e - \omega_{ek}^2)u_{ek}^h(\omega) = \frac{4\pi e^2}{m_e} \frac{v_{os}}{2} [n_{ek}^l(\omega - \omega_0) + n_{ek}^l(\omega + \omega_0)],$$

$$(\omega^2 + i\omega v_i - k^2 C_s^2)n_{ek}^l(\omega) = \frac{\omega_{pi}^2 m_e}{4\pi e^2} \frac{k^2 v_{os}}{2} [u_{ek}^h(\omega - \omega_0) + u_{ek}^h(\omega + \omega_0)].$$

这里  $\omega_{ek}^2 = \omega_{pe}^2 + 3k^2 v_e^2$ .



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# 色散关系

假设  $\omega \ll \omega_0$ , 有

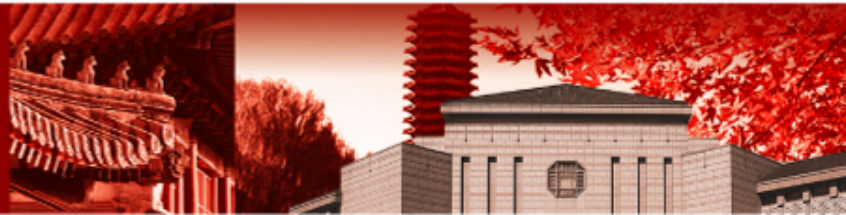
$$\begin{aligned}\bar{\varepsilon}(k, \omega \pm \omega_0) &= (\omega \pm 2\omega_0)(\omega \pm \delta) + i\nu_e(\omega \pm \omega_0) \\ &\cong \pm 2\omega_0(\omega \pm \delta) \pm i\nu_e\omega_0.\end{aligned}$$

$$(\omega^2 + i\omega\nu_i - k^2 C_s^2) \left[ \left( \omega + \frac{i\nu_e}{2} \right)^2 - \delta^2 \right] + \frac{\omega_{pi}^2 k^2 \nu_{os}^2}{4} \frac{\delta}{\omega_0} = 0.$$

振荡双流不稳定性和离子声衰变不稳定性可由该色散关系确定。



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# 色散关系

## 讨论：振荡双流不稳定性

1. 纯增长  $\omega = i\gamma$ , 振荡双流不稳定性

$$\delta = \omega_0 - \omega_{ek}$$

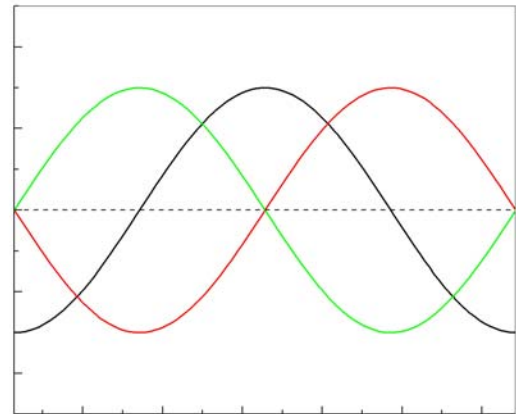
离子声波的Landau阻尼可忽略,  $\nu_i = 0$ ,

$$(\gamma^2 + k^2 C_s^2) \left[ \left( \gamma + \frac{v_e}{2} \right)^2 + \delta^2 \right] + \frac{\omega_{pi}^2 k^2 v_{os}^2}{4} \frac{\delta}{\omega_0} = 0.$$

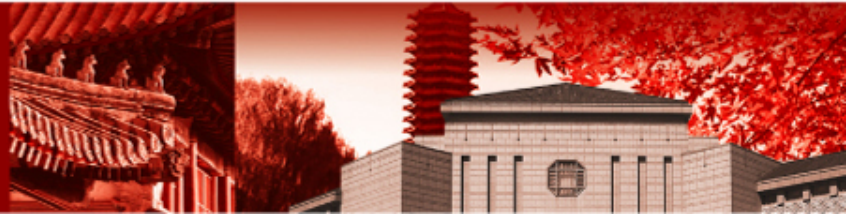
if  $\gamma > 0$ , 则要求  $\delta < 0$ ,  $\delta = \omega_0 - \omega_{ek} \Rightarrow \omega_0 < \omega_{ek}$

这正是前面讨论不稳定性的物理图象所期望的。

—  $n = n_0 + \Delta n \cos kx$   
—  $F_p$  for  $\omega_L > \omega_{ek}$  有质动力减小密度扰动  
—  $F_p$  for  $\omega_L < \omega_{ek}$  有质动力增强密度扰动



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$$(\gamma^2 + k^2 C_s^2) \left[ \left( \gamma + \frac{v_e}{2} \right)^2 + \delta^2 \right] + \frac{\omega_{pi}^2 k^2 v_{os}^2}{4} \frac{\delta}{\omega_0} = 0.$$

2.弱增长

$$\gamma \ll k C_s$$

有阈值



$$\frac{\partial}{\partial \delta}$$

$$\left( \gamma + \frac{v_e}{2} \right)^2 + \delta^2 + \frac{1}{4} \left( \frac{v_{os}}{v_e} \right)^2 \omega_0 \delta = 0.$$

$$k^2 C_s^2 \left[ \left( \gamma + \frac{v_e}{2} \right)^2 + \delta^2 \right] + \frac{\omega_{pi}^2 k^2 v_{os}^2}{4} \frac{\delta}{\omega_0} = 0.$$

我们可求出失配数 $\delta$ ，对上式取导数

$$\frac{\partial \gamma}{\partial \delta} = 0 \Rightarrow \delta = -\frac{1}{8} \left( \frac{v_{os}}{v_e} \right)^2 \omega_0.$$

将 $\delta$ 代入，就得到最大增长率

Langmuir波的  
Landau阻尼

$$\gamma = \frac{1}{8} \left( \frac{v_{os}}{v_e} \right)^2 \omega_0 - \frac{v_e}{2}$$

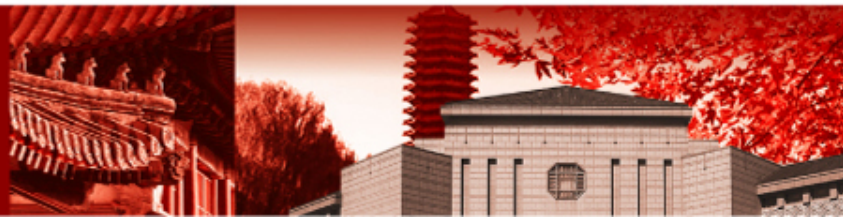
$$C_s^2 = \frac{Z T_e + 3 T_i}{M}, \omega_{pi}^2 = \frac{4 \pi n_0 e^2 Z}{M}.$$

$\gamma$ 可能大于0，也可能小于0，即可能是增长也可能是阻尼。

$\gamma > 0$  增长     $\gamma < 0$  阻尼



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# 色散关系

由于等离子体波的阻尼， **pump**场的振幅必须超过阈值才有净增长的出现，这个阈值由条件 $\gamma=0$  给出

$$\left(\frac{v_{os}}{v_e}\right)_{th}^2 = \frac{4v_e}{\omega_0}$$

$\gamma > 0$  要求

$$\left(\frac{v_{os}}{v_e}\right)_{th}^2 > \frac{4v_e}{\omega_0}$$

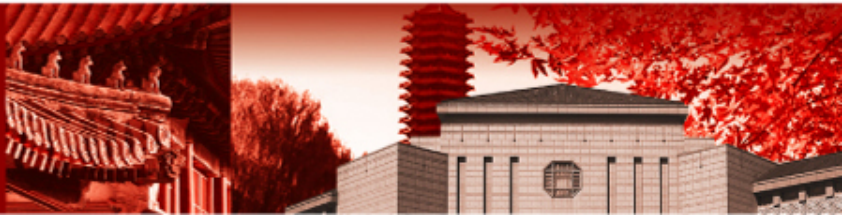
$$a_0 = \frac{eE_0}{m\omega_0 c} = \frac{v_{os}}{c}$$

$$v_{os} = \frac{eE_0}{m_e \omega_0}$$

$$a_0 = 8.5 \times 10^{-10} [I(W/cm^2) \lambda_L(\mu m)^2]^{1/2}$$



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# 色散关系

## 3. 强增长

$$\gamma \gg (kC_s, v_e)$$

$$(\gamma^2 + k^2 C_s^2) \left[ \left( \gamma + \frac{v_e}{2} \right)^2 + \delta^2 \right] + \frac{\omega_{pi}^2 k^2 v_{os}^2}{4} \frac{\delta}{\omega_0} = 0.$$



$$\frac{\partial}{\partial \delta}$$

$$\gamma^2 [(\gamma^2 + \delta^2)] + \frac{\omega_{pi}^2 k^2 v_{os}^2}{4} \frac{\delta}{\omega_0} = 0$$

$$\delta = \omega_0 - \omega_{ek}$$

$$\frac{\partial \gamma}{\partial \delta} = 0 \Rightarrow \delta = -\frac{\omega_{pi}^2 k^2 v_{os}^2}{8\omega_0 \gamma^2},$$

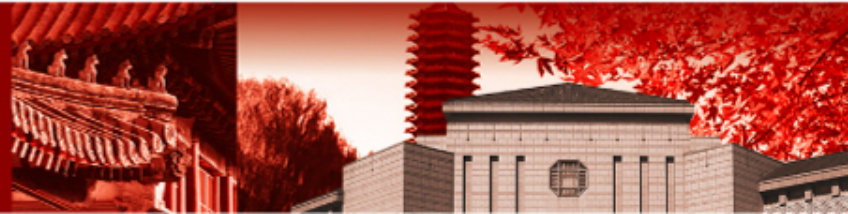
$$\gamma^3 = \frac{\omega_{pi}^2 k^2 v_{os}^2}{8\omega_0},$$

$$\gamma = \left( \frac{\omega_{pi}^2 k^2 v_{os}^2}{8\omega_0} \right)^{1/3} \approx \left( \frac{\omega_{pi}^2 k^2 v_{os}^2}{8\omega_0} \right)^{1/3} \approx \left( \frac{Zm_e}{M} \frac{k^2 v_{os}^2}{8} \omega_0 \right)^{1/3}.$$

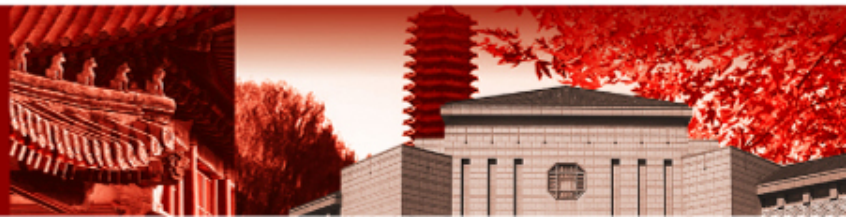
mismatch  $\delta = -\gamma.$



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**离子声衰变不稳定性：** 如果考虑与离子声波相联系的离子密度涨落，这种涨落不再是静的，而是具有频率为 $kCs$ 。在这种情况下，光波最容易通过共振衰变为一个等离子体波加上一个离子声波，当三波频率完全匹配，即 $\omega_0 = \omega_{ek} + kCs$ 时，这种不稳定性是最强的。因此，常常把这种不稳定性称作离子声衰变不稳定性。





# 色散关系

从色散关系出发

$$(\omega^2 + i\omega v_i - k^2 C_s^2) \left[ \left( \omega + \frac{i v_e}{2} \right)^2 - \delta^2 \right] + \frac{\omega_{pi}^2 k^2 v_{os}^2}{4} \frac{\delta}{\omega_0} = 0.$$

来研究离子声衰变不稳定性。

1、研究弱增长状态，  $\gamma \ll kC_s$

显然当离子声波和电子等离子体波接近共振时 ( $\omega_0 = \omega_{ek} + kC_s$ )，极大增长率出现。选取

$$\omega = kC_s + i\gamma$$

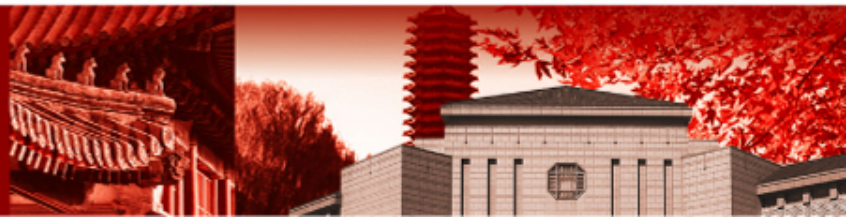
$$\delta = \omega_0 - \omega_{ek} = kC_s$$

$$\omega_0 + kC_s = 2kC_s + i\gamma, \omega_0 - kC_s = i\gamma,$$

$$\omega_0 + \delta = 2kC_s + i\gamma, \omega_0 - \delta = i\gamma.$$



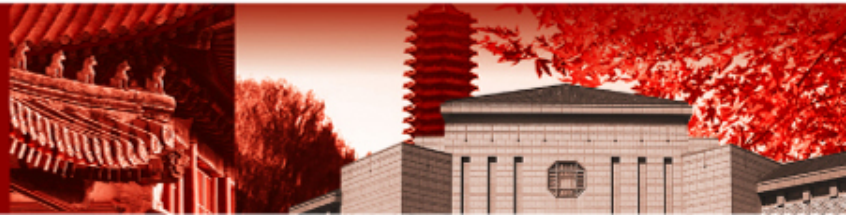
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$$(\omega^2 + i\omega v_i - k^2 C_s^2) \left[ \left( \omega + \frac{i v_e}{2} \right)^2 - \delta^2 \right] + \frac{\omega_{pi}^2 k^2 v_{os}^2}{4} \frac{\delta}{\omega_0} = 0.$$

$$\begin{aligned} \text{第一项: } & [(\omega + kC_s)(\omega - kC_s) + i\omega v_i] \left[ \left( \omega + \frac{i v_e}{2} + \delta \right) \left( \omega + \frac{i v_e}{2} - \delta \right) \right] \\ & = [(2kC_s + i\gamma)i\gamma + (kC_s + i\gamma)iv_i] \left[ 2kC_s + i\left(\gamma + \frac{v_e}{2}\right) \right] \left[ i\left(\gamma + \frac{v_e}{2}\right) \right] \\ & \approx [i2kC_s\gamma + ikC_s v_i] \left[ i2kC_s \left(\gamma + \frac{v_e}{2}\right) \right] \\ & = -4k^2 C_s^2 \left(\gamma + \frac{v_i}{2}\right) \left(\gamma + \frac{v_e}{2}\right) \\ & = -4k^2 C_s^2 \left[ \gamma^2 + \gamma \left(\frac{v_i}{2} + \frac{v_e}{2}\right) + \frac{v_i v_e}{4} \right] \end{aligned}$$



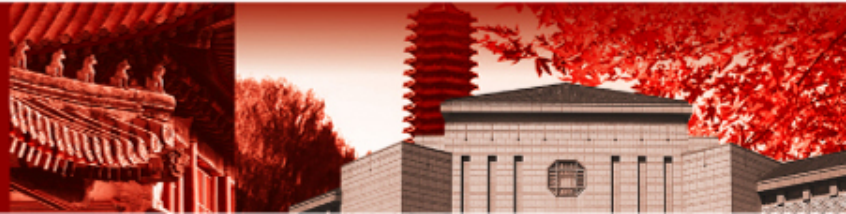
# 色散关系

代入上面方程中，得到关于增长率的一个二次方

$$4\gamma^2 + 2\gamma(v_i + v_e) + v_i v_e - \frac{1}{4} \left( \frac{v_{os}}{v_e} \right)^2 k C_s \omega_0 = 0$$



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# 色散关系

这种增长还是要求pump场超过阈值，可通过 $\gamma=0$ 得到

$$\left(\frac{v_{os}}{v_e}\right)_{th}^2 > \frac{4v_e}{\omega_0}$$

当增长率远大于碰撞频率，但远小于离子声频率时

$$\gamma \ll kC_s$$

$$\gamma \gg \nu_i, \nu_e$$

$$4\gamma^2 + 2\gamma(\nu_i + \nu_e) + \nu_i\nu_e - \frac{1}{4}\left(\frac{v_{os}}{v_e}\right)^2 kC_s\omega_0 = 0$$

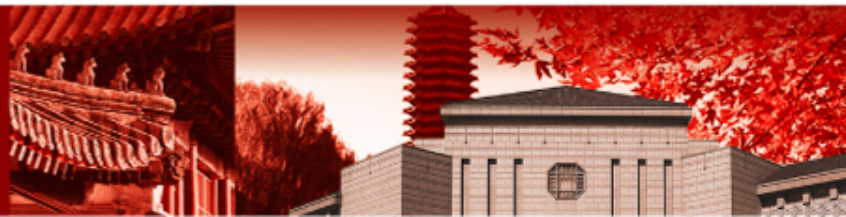


$$\gamma = \frac{1}{4} \frac{v_{os}}{v_e} (kC_s\omega_0)^{1/2}$$

弱增长状



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# 色散关系

## 2、对大振幅pump场, $\omega \gg kC_s$

离子波的频率可由pump场的强度来确定。在这种情况下, 离子波被称作准, 因为它不是未驱动等离子体的模。

假设  $\omega \gg kC_s$ , 略去阻尼项

$$(\omega^2 + i\omega v_i - k^2 C_s^2) \left[ \left( \omega + \frac{i v_e}{2} \right)^2 - \delta^2 \right] + \frac{\omega_{pi}^2 k^2 v_{os}^2}{4} \frac{\delta}{\omega_0} = 0.$$

令  $\omega^2 (\omega^2 - \delta^2) + \frac{\omega_{pi}^2 k^2 v_{os}^2}{4} \frac{\delta}{\omega_0} = 0.$

$$\frac{\partial \omega}{\partial \delta} = 0 \Rightarrow \delta = \frac{\omega_{pi}^2 k^2 v_{os}^2}{8\omega^2 \omega_0}.$$

$$\omega = |\omega| \exp(i\Phi), \delta = \alpha |\omega|$$

该方程的虚部给出

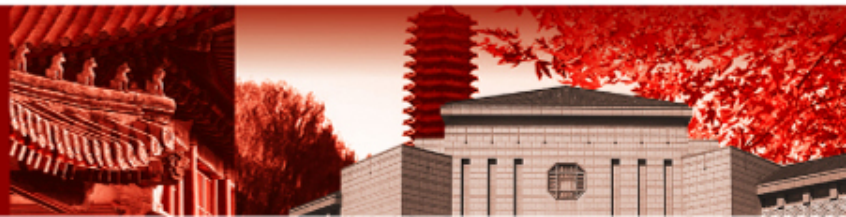
$$\sin \Phi = \frac{1}{2} \sqrt{2 - \alpha^2}$$

实部给出

$$|\omega| = \left( \frac{\omega_{pi}^2 k^2 v_{os}^2 \alpha}{4\omega_0} \right)^{1/3}$$



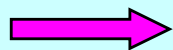
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# 色散关系

增长率  $\gamma = |\omega| \sin \Phi,$

当  $\alpha = \frac{1}{\sqrt{2}}$



$$\sin \Phi = \frac{1}{2} \sqrt{\frac{3}{2}}, \cos \Phi = \frac{1}{2} \sqrt{\frac{5}{2}}$$

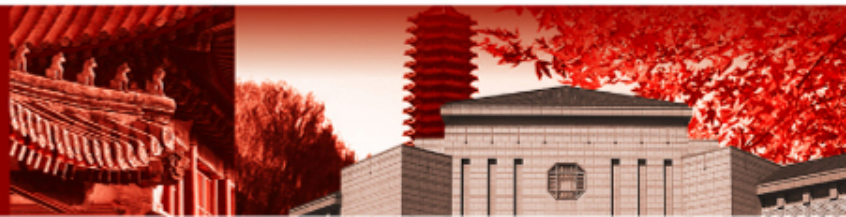
时，增长率取极大值。

$$\omega_r = |\omega| \cos \Phi \approx \frac{\sqrt{5}}{2} \left( \frac{\omega_{pi}^2 k^2 v_{os}^2}{16\omega_0} \right)^{1/3}$$

$$\gamma = |\omega| \sin \Phi \approx \frac{\sqrt{3}}{2} \left( \frac{\omega_{pi}^2 k^2 v_{os}^2}{16\omega_0} \right)^{1/3}$$



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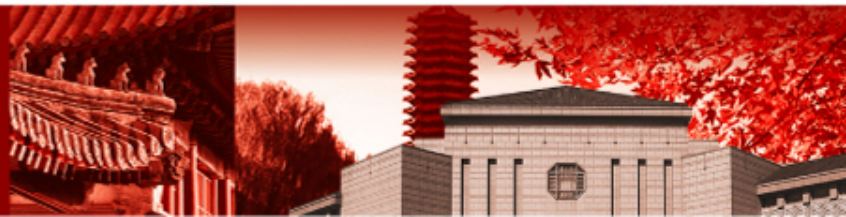




## V.7 空间不均匀性引起的不稳定的阈值



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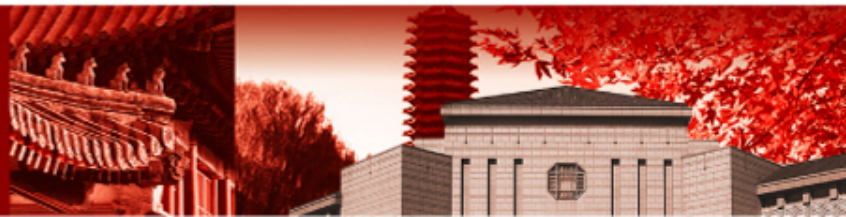
## 空间不均匀性引起的不稳定的阈值

不稳定性阈值往往不是由碰撞确定而是由空间不均匀性确定。在具有密度梯度的等离子体中，振荡双流不稳定性~~和离子声衰变不稳定性~~只有超过一个有限大小区域才能被激发。另外，不稳定性还可以传播出被激发的区域以外，成为另一种损失机制。

下面在我们的不稳定性讨论中将包含有密度梯度效应。



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# 空间不均匀性引起的不稳定的阈值

在具有密度梯度的等离子体中，振荡双流不稳定性和离子声衰变不稳定性只有超过一个有限大小区域才能被激发。

$$\omega_0 \sim \omega_{pe}$$

考虑垂直入射在**不均匀**等离子体中的光波的电矢量激发起等离子体波的不稳定性。

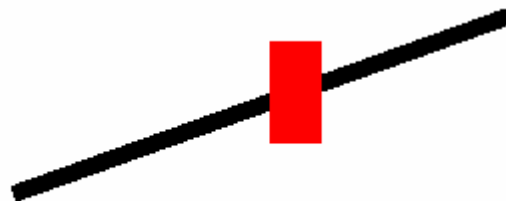
考虑振荡双流不稳定性。

$$\omega_0 < \omega_{ek}, \omega_{ek}^2 = \omega_{pe}^2 + 3k^2 v_e^2$$

假设：线性密度（在临界面附近），等离子体波具有波矢 $k_{\parallel}$ 沿着光波的电矢量方向，这种情况激发最强。但是，在密度较低的地方，波矢必须发展一个分量 $k_z$ 降低密度梯度。



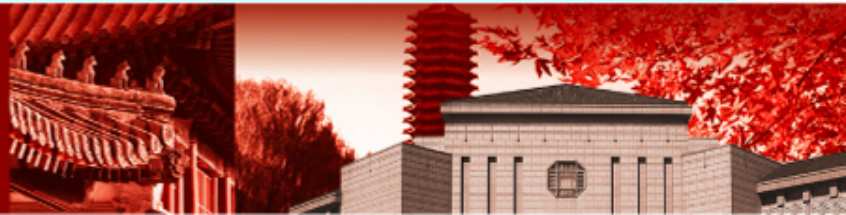
均匀等离子体的共振区



非均匀等离子体的共振区



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# 空间不均匀性引起的不稳定的阈值

热修正引起的频率增加  $\longleftrightarrow$  由于低密度引起的减少

$$3k_z^2 v_e^2 = \omega_{pe}^2 \frac{Z}{L}$$

$z=0$  相应于  $k=k_{\parallel}$  之处，随着  $k_z$  的增加，光波和等离子体之间的耦合系数减少，因为等离子体波开始越来越多地在垂直于 pump 场的电矢量方向传播。

可以用  $k=k_{\parallel}$  条件来估计相互作用区的大小

$$L_{INT} \approx \frac{3k_{\parallel}^2 v_e^2 L}{\omega_{pe}^2}$$

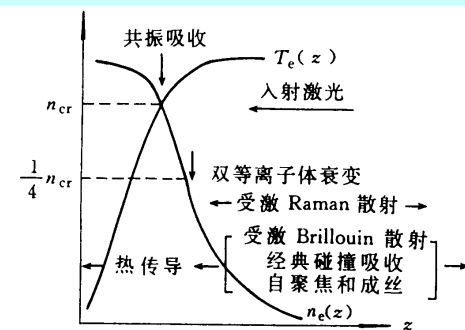
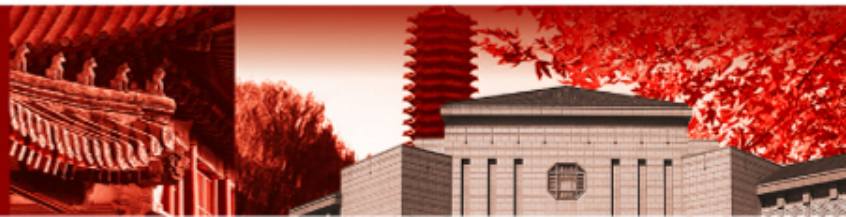


图 2.3 激光等离子体主要耦合过程发生的温度密度区



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# 空间不均匀性引起的不稳定的阈值

将波的能量传出这个区域的时间是

$$\tau = \int_0^{L_{INT}} \frac{dz}{v_{gz}}, v_{gz} = \frac{3k_z v_e^2}{\omega_{pe}},$$
$$\Rightarrow \tau = \frac{2k_{\parallel} L}{\omega_{pe}}.$$

$$\omega^2 = \omega_{pe}^2 + 3k^2 v_e^2$$

$$2\omega d\omega = 3v_e^2 2kdk,$$

$$v_g = \frac{d\omega}{dk} = 3 \frac{k}{\omega} v_e^2$$

有效阻尼率

$$\nu = 1/\tau,$$

$$\frac{\nu}{\omega_{pe}} = \frac{1}{2k_{\parallel} L}.$$

代入弱阻尼条件下的阈值方程

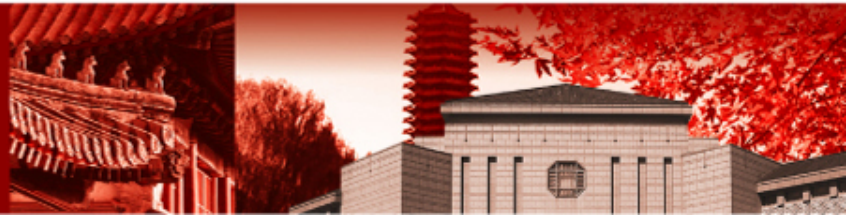
$$\left(\frac{v_{os}}{v_e}\right)_{th}^2 = \frac{4\nu_e}{\omega_0}$$

得到阈值为

$$\left(\frac{v_{os}}{v_e}\right)_{th}^2 \approx \frac{2}{k_{\parallel} L}$$



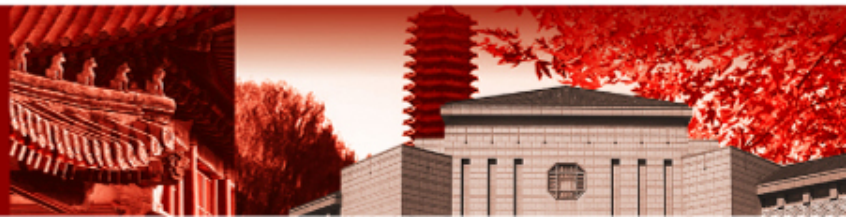
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## V.8 非相干波对不稳定性的影响



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# 非相干波对不稳定性的影响

- 空间相干性和时间相干性都着眼与光波场中各点（次波源）是否相干的问题。

- 时间相干性问题来源于光源发光过程在时间上的断续性。

谱线宽度

- 空间相干性问题来源于开展光源不同部分发光的独立性；

D 直径

相干时间:  $t_c \approx 1/\Delta\nu$ ,  $\Delta\nu$ : 谱线宽度

空间相干长度（横向）:  $b=D/n$ ,  $n$ : 倍数

空间相干性用区域的孔径角 $\Delta\theta_0$ 来描述，它与光源宽度 $b$ 的关系由空间相干性的

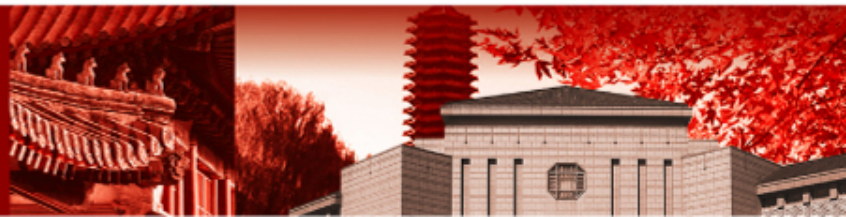
反比公式决定:  $b\Delta\theta_0 \sim \lambda$ ,

衍射极限对应的半宽度为 $\Delta\theta_0 = \lambda/D$ ,

激光的发散角:  $n\Delta\theta_0 = n\lambda/D$



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# 非相干波对不稳定性的影响

Pump波的时间非相干和空间非相干都将减少不稳定性的增长。

考虑一个频率接近  $\omega_{pe}$  并在它的振幅上具有无规调制的空间均匀pump场驱动的离子声衰变不稳定性

$$E = E_0 \alpha(t) \sin \omega_0 t \quad \omega_0 \sim \omega_{pe}$$

$\alpha(t)$ : 一个随机变量, 平均值为0, 且在单位1之间变化

当增长率远小于离子声频率时, 不稳定的振幅可以表示为

$$\bar{f} = \beta \exp\left[\gamma_0 \int_0^t \alpha(t') dt'\right], \quad \gamma_0: \text{无振幅调制时的增长率}, \beta \text{由初始条件确定。}$$

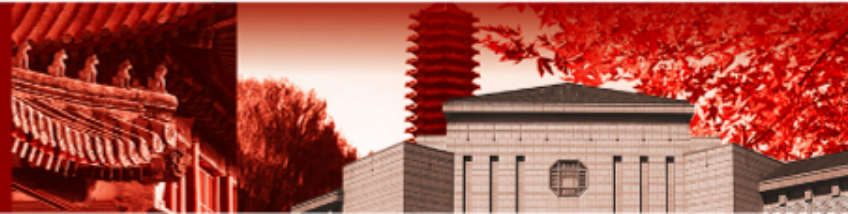
如果假设 $\alpha(t)$ 是Gaussian型, 则

$$\langle \bar{f} \rangle = \beta \exp\left[\frac{\gamma_0^2}{2} \int_0^t dt' \int_0^t \langle \alpha(t') \alpha(t'') \rangle dt''\right],$$

$\langle \rangle$  表示平均。



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# 非相干波对不稳定性的影响

有效频率带宽 $\Delta\omega$ 通过自关连函数定义。

$$\frac{1}{\Delta\omega} = \int_0^{\infty} d\tau \langle \alpha(t)\alpha(t+\tau) \rangle,$$

如果 $\Delta\omega \gg \gamma_0$ ，关于 $\langle f \rangle$ 的表示成为

$$\langle \bar{f} \rangle = \beta \exp\left(\frac{\gamma_0^2 t}{\Delta\omega}\right) = \beta \exp\left(\frac{\gamma_0}{\Delta\omega} \gamma_0 t\right)$$

$\langle \rangle$  表示平均。

由此可见，增长率按  $\gamma_0/\Delta\omega$  减少。

• **Pump** 波的强度是在带宽 $\Delta\omega$ 范围分布的，不稳定性的共振宽度是这个增长率。

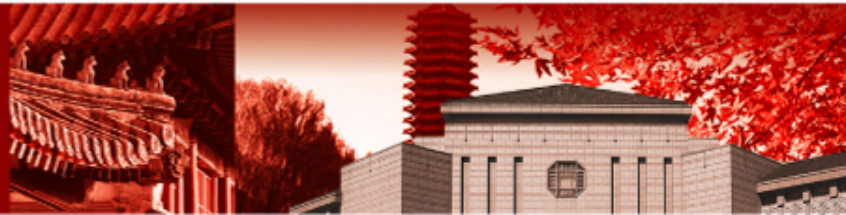
当 $\Delta\omega \gg \gamma_0$ 时，仅有**pump**波的某些份额才能与不稳定波进行耦合。

• **Pump** 波的**相位**无规则调制同样导致增长率的降低。

• **Pump** 场波矢的变宽或者等离子体的无规湍流，也可限制相干性，贡献于有效的带宽，限制不稳定性的增长。



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谢谢!



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