

2014 Autumn Semester, course for graduate student

Lecture notes: Physics of Laser-Plasma Interaction

# VI. Parametric instabilities in underdense plasma (次临界密度等离子体中的参量不稳定性)

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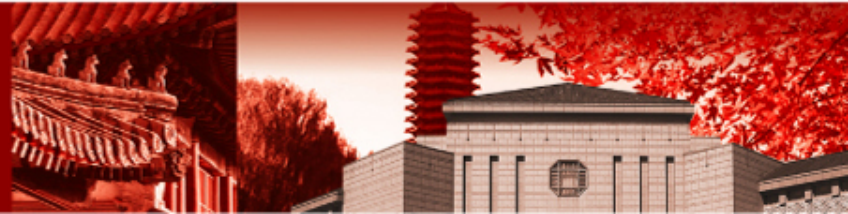
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# Coupling between laser and the plasma oscillators

表-1 激光等离子体中的主要相互作用过程

不稳定过程	能量守恒	发生区域	后果
共振吸收	$\omega_0 = \omega_{emw}$	$\sim n_{cr}$	超热电子
受激 Brillouin 散射	$\omega_0 = \omega_s + \omega_{isw}$	$< n_{cr}$	减少激光能量沉积
受激 Raman 散射	$\omega_0 = \omega_s + \omega_{epw}$	$\leq n_{cr} / 4$	超热电子, 减少激光能量沉积
双等离子体衰变	$\omega_0 = \omega_{epw} + \omega_{epw}$	$\sim n_{cr} / 4$	超热电子
离子声衰变	$\omega_0 = \omega_{isw} + \omega_{epw}$	$n_{cr}$	超热电子

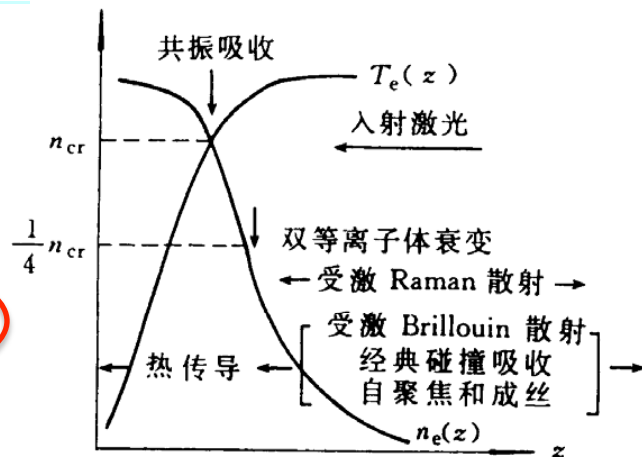


图 2.3 激光等离子体主要耦合过程发生的温度密度区

为了实现高增益靶丸内爆，必须最大限度地限制超热电子的产生。

$$\begin{aligned} \omega_l &= \omega_1 + \omega_2 \\ k_l &= k_1 + k_2 \end{aligned} \quad \text{Manley-Rowe 关系式}$$

激光强度:  $10^{14}$ - $10^{16}$  W/cm<sup>2</sup>

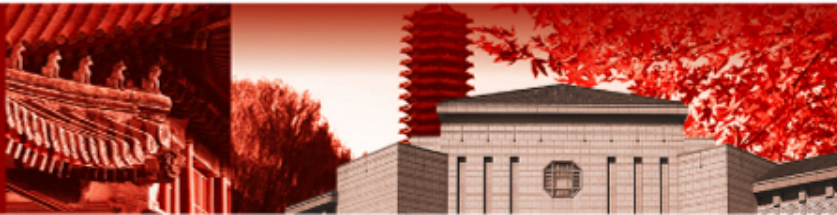
$$\omega_{ek}^2 = \omega_{pe}^2 + 3k_{ek}^2 v_e^2$$

$$\omega_0^2 = \omega_{pe}^2 + k_0^2 c^2$$

$$\omega_{is}^2 = k_{is}^2 C_s^2$$



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# Outline

## VI. 1 受激Raman散射

(Stimulated Raman Scattering)

## VI. 2 双等离子体衰变

(Two Plasma Decay)

## VI. 3 受激Brillouin散射

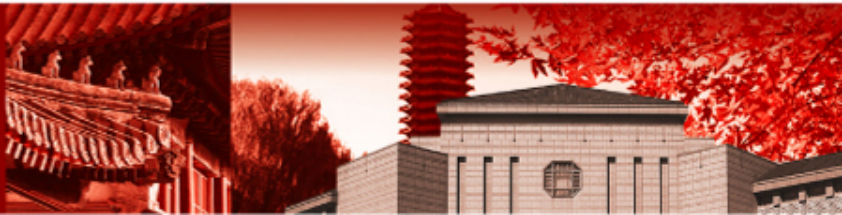
(Stimulated Brillouin Scattering)

## VI. 4 激光的自聚焦与成丝不稳定性

(Self-focusing and filamentation instability)



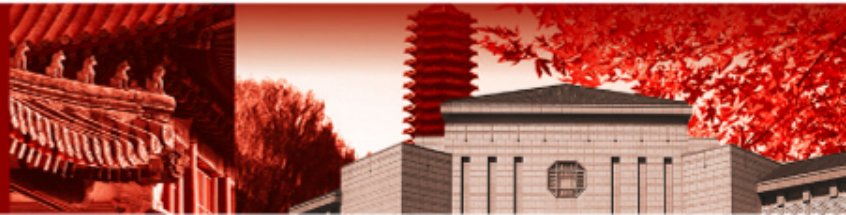
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# VI.1 受激Raman散射 (Stimulated Raman Scattering)



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# (1) 什么是受激Raman散射?

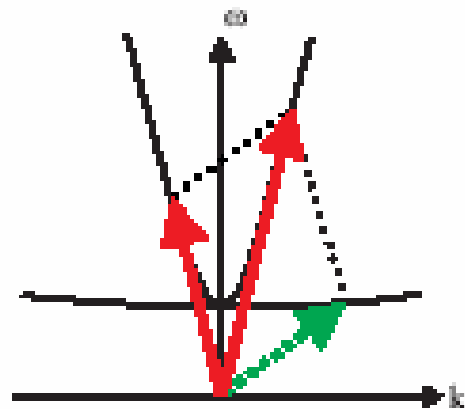
## 三波耦合:

大振幅光波

散射光波

等离子体波

$$\omega_{ek}^2 = \omega_{pe}^2 + 3k^2 v_e^2$$



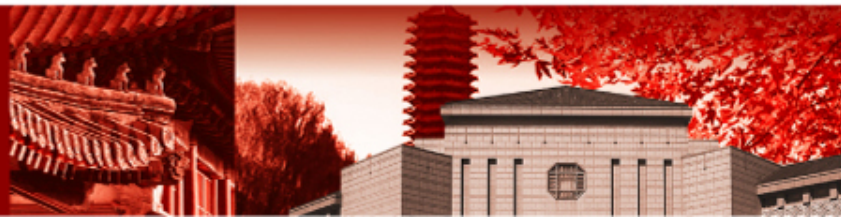
$$\begin{aligned} \omega_0 &= \omega_s + \omega_{ek} \\ \vec{k}_0 &= \vec{k}_s + \vec{k} \end{aligned} \quad \text{Manley - Rowe 关系式}$$

在等离子体中光波的最小频率是 $\omega_{pe}$ ，所以 $\omega_0 \geq 2\omega_{pe}$ ，即 $n \leq n_{cr}/4$ 。

在这个过程中，入射光的一部分能量被散射，一部分沉积在电子等离子体波中，份额是 $\omega_{ek} / \omega_0$ 。这部分能量随电子等离子体波阻尼(或波破)而加热等离子体



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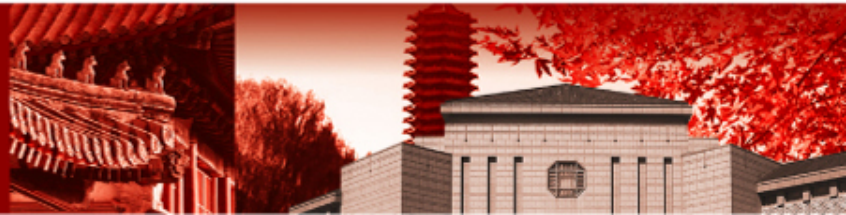
## (2) 受激Raman散射的后果

电子等离子体波具有很高的相速度（光波速度量级），当等离子体波受到阻尼或波破时，随波前进的电子将变成具有很高能量的超热电子。

这些电子的射程很长，可以预热激光靶芯的热核燃料，影响压缩和对称性，降低耦合效率，不能实现高增益激光聚变，所以**Raman**不稳定性受到特别重视和关注。



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### (3) 受激Raman散射的不稳定性机理

光波电场  $E_L$

密度涨落  $\delta n$

$$\vec{v}_L = \frac{e\vec{E}_L}{m_e\omega_0}, \text{产生横流 } \delta\vec{J} = -e\vec{v}_L\delta n.$$

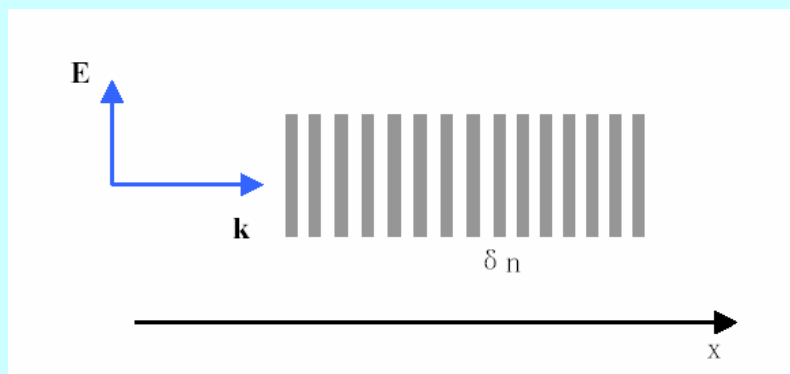
(电子等离子体波)

如果波数和频率适当匹配，这个横流将产生一个振幅为 $\delta E$ 的散射光波，散射光波和入射光波相互作用产生一个波压的变化

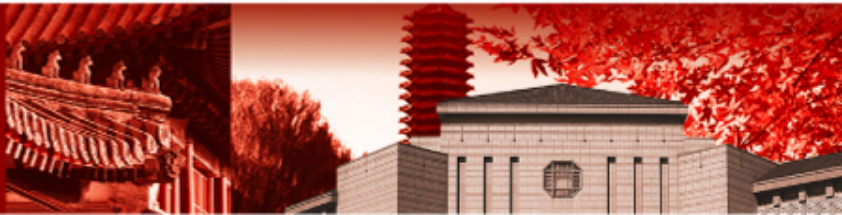
$$\nabla\left(\frac{E^2}{8\pi}\right) = \frac{\nabla(E_L \cdot \delta E)}{4\pi}.$$

把等离子体从高压区推到低压区→密度涨落 正反馈 不稳定性发生

$\delta n \rightarrow \delta J \rightarrow \delta E \rightarrow F_p \rightarrow \delta n$  入射波、散射波、静电波三波相互作用



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# (4) 受激Raman散射的耦合方程



$$\begin{aligned}\nabla \cdot \vec{E} &= 4\pi\rho \\ \nabla \times \vec{B} &= \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{J} \\ \nabla \times \vec{E} &= -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} &= 0\end{aligned}$$



$$\begin{aligned}\vec{B} &= \nabla \times \vec{A} \\ \vec{E} &= -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} - \nabla \cdot \Phi \\ \nabla^2 \Phi + \frac{1}{c} \frac{\partial}{\partial t} \nabla \cdot \vec{A} &= -4\pi\rho \\ \nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} - \nabla \left( \nabla \cdot \vec{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t} \right) &= -\frac{4\pi}{c} \vec{J}\end{aligned}$$

Lorentz gauge

$$\nabla \cdot \mathbf{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t} = 0$$

$$\begin{aligned}\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} &= -4\pi\rho \\ \nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} &= -\frac{4\pi}{c} \mathbf{J}\end{aligned}$$

Lorentz's gauge invariance

Coulomb gauge (transverse gauge)

$$\nabla \cdot \mathbf{A} = 0$$

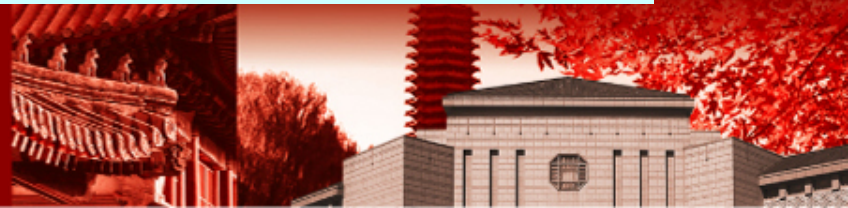
$$\begin{aligned}\nabla^2 \Phi &= -4\pi\rho \\ \nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} &= \frac{1}{c} \frac{\partial}{\partial t} \nabla \cdot \Phi - \frac{4\pi}{c} \vec{J}\end{aligned}$$

$\Phi$  satisfies Poisson's equation

$\mathbf{A}$  comes from the transverse  $\mathbf{J}_t$



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## (4) 受激Raman散射的耦合方程

$$\vec{A}, \Phi \quad \vec{B} = \nabla \times \vec{A}$$
$$\vec{E} = -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} - \nabla \Phi$$

代入Ampere定律,

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t},$$

选择库仑规范

$$\nabla \cdot \vec{A} = 0$$

有

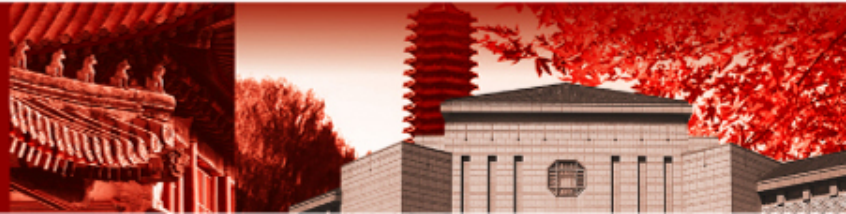
$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \vec{A} = \frac{4\pi}{c} \vec{J} - \frac{1}{c} \frac{\partial}{\partial t} \nabla \Phi,$$

横向部分  $\vec{J}_t$             和光波相联系

纵向部分  $\vec{J}_l$             和等离子体静电波相联系



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## (4) 受激Raman散射的耦合方程

纵向部分  $J_t$  可通过Poisson方程和电荷守恒方程与 $\Phi$ 联系起来

$$\frac{\partial}{\partial t} \rightarrow$$

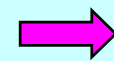
$$\nabla^2 \Phi = -4\pi\rho,$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0.$$

$$\nabla \cdot \left( \frac{\partial}{\partial t} \nabla \Phi - 4\pi \vec{J} \right) = 0.$$

$$\text{因为 } \nabla \cdot \vec{J}_t = 0, \frac{\partial}{\partial t} \nabla \Phi = 4\pi \vec{J}_t$$

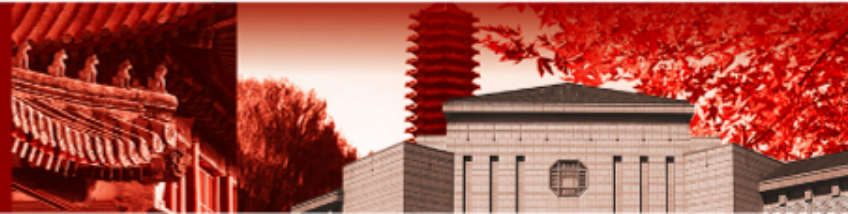
$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \vec{A} = \frac{4\pi}{c} \vec{J} - \frac{1}{c} \frac{\partial}{\partial t} \nabla \Phi,$$



$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \vec{A} = 4\pi \vec{J}_t.$$



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## (4) 受激Raman散射的耦合方程

考虑均匀等离子体 (和共振吸收?)

$$\vec{A} \cdot \nabla n_e = 0. \quad \rightarrow \quad \vec{J}_t = -n_e e \vec{u}_t, \quad \vec{u}_t: \text{电子在光波电场中的振荡速度}$$

$$|\vec{u}_t| \ll c, \quad \frac{\partial \vec{u}_t}{\partial t} = -\frac{e}{m} \vec{E}_t = \frac{e}{m_e c} \frac{\partial \vec{A}}{\partial t} \rightarrow \vec{u}_t = \frac{e}{m_e c} \vec{A}.$$

可以得到光波在等离子体中的传播方程

$$\left( \frac{\partial^2}{\partial t^2} - c^2 \nabla^2 \right) \vec{A} = -\frac{4\pi e^2}{m_e} n_e \vec{A},$$

$$\vec{A} = \vec{A}_L + \vec{A}', \quad n_e = n_0 + n_e'.$$

$$\left( \frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \omega_{pe}^2 \right) \vec{A}' = -\frac{4\pi e^2}{m_e} n_e' \vec{A}_L,$$

大振幅光波  $\vec{A}_L$  +

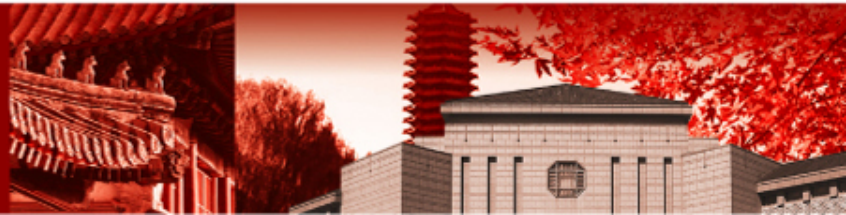
横向电流

小振幅密度涨落耦合

$\rightarrow$  散射光波



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## (4) 受激Raman散射的耦合方程

推导与电子等离子体波相联系的密度涨落方程

离子：不动的电中性背景

电子：热流体

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{u}_e) = 0.$$

$$\frac{\partial \vec{u}_e}{\partial t} + \vec{u}_e \cdot \nabla \vec{u}_e = -\frac{e}{m_e} \left( \vec{E} + \frac{\vec{u}_e \times \vec{B}}{c} \right) - \frac{\nabla P_e}{n_e m_e}.$$

纵向      横向



令

$$\vec{u}_e = \vec{u}_L + \frac{e\vec{A}}{m_e c},$$

$$\frac{\partial \vec{u}_L}{\partial t} = \frac{e}{m_e} \nabla \Phi - \frac{1}{2} \nabla \left( \vec{u}_L + \frac{e\vec{A}}{m_e c} \right)^2 - \frac{\nabla P_e}{n_e m_e}.$$

绝热状态方程(高频波)

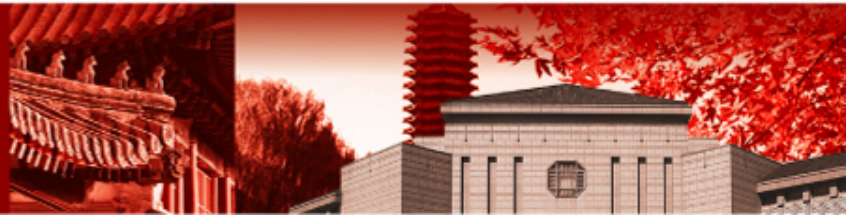
$$\frac{P_e}{n_e^3} = \text{常数},$$



有质动力 ~纵向和横向电场强度的梯度



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## (4) 受激Raman散射的耦合方程

线性化  $n_e = n_0 + n_e', \vec{u}_L = \vec{u}', \vec{A} = \vec{A}_L + \vec{A}', \Phi = \Phi'$

则  $\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{u}_e) = 0.$   $\rightarrow$   $\frac{\partial n_e'}{\partial t} + n_0 \nabla \cdot \vec{u}' = 0.$

$\frac{\partial \vec{u}_L}{\partial t} = \frac{e}{m_e} \nabla \Phi - \frac{1}{2} \nabla (\vec{u}_L + \frac{e\vec{A}}{m_e c})^2 - \frac{\nabla P_e}{n_e m_e}.$   $\rightarrow$   $\frac{\partial \vec{u}'}{\partial t} = \frac{e}{m_e} \nabla \Phi' - \frac{e^2}{m_e^2 c^2} \nabla (\vec{A}_L \cdot \vec{A}') - \frac{3v_e^2}{n_0} \nabla n_e'$

利用Poisson方程，消去 $\phi$

$v_e$  电子热速度

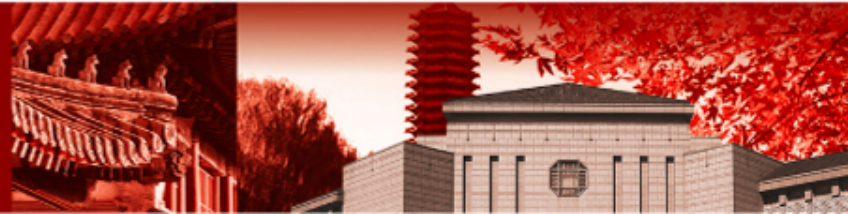
$$\nabla^2 \Phi' = 4\pi e n_e'$$

$$\left( \frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - 3v_e^2 \nabla^2 \right) n_e' = \frac{n_0 e^2}{m_e^2 c^2} \nabla^2 (\vec{A}_L \cdot \vec{A}')$$

该方程描述了由电磁波强度变化产生电子密度的涨落。



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## (5) 受激Raman散射的色散关系

三波耦合方程

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \omega_{pe}^2\right) \vec{A}' = -\frac{4\pi e^2}{m_e} n_e' \vec{A}_L,$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - 3v_e^2 \nabla^2\right) n_e' = \frac{n_0 e^2}{m_e^2 c^2} \nabla^2 (\vec{A}_L \cdot \vec{A}')$$

取

$$\vec{A}_L = \vec{A}_0 \cos(\vec{k}_0 \cdot \vec{x} - \omega_0 t) = \frac{\vec{A}_0}{2} [\exp(i\vec{k}_0 \cdot \vec{x} - i\omega_0 t) + \exp(-i\vec{k}_0 \cdot \vec{x} + i\omega_0 t)],$$

$$A' \sim \exp[i(kx - \omega t)], n' \sim \exp[i(kx - \omega t)]$$

并对上述两个方程作Fourier分析

$$(\omega^2 - k^2 c^2 - \omega_{pe}^2) \vec{A}'(\vec{k}, \omega) = \frac{4\pi e^2}{2m_e} \vec{A}_0 [n_e'(\vec{k} - \vec{k}_0, \omega - \omega_0) + n_e'(\vec{k} + \vec{k}_0, \omega + \omega_0)]$$

如果

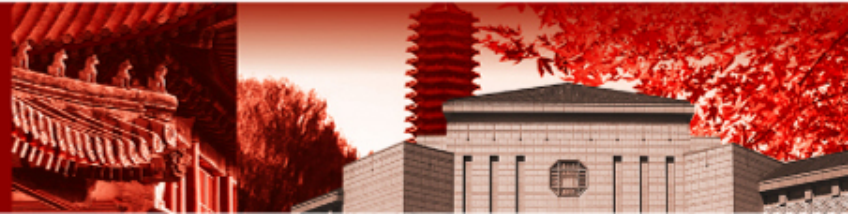
$$\omega_{ek}^2 = \omega_{pe}^2 + 3k^2 v_e^2$$

Bohm-Gross频率

$$(\omega^2 - \omega_{ek}^2) n_e'(\vec{k}, \omega) = \frac{k^2 e^2 n_0}{2m_e^2 c^2} \vec{A}_0 \cdot [\vec{A}'(\vec{k} - \vec{k}_0, \omega - \omega_0) + \vec{A}'(\vec{k} + \vec{k}_0, \omega + \omega_0)]$$



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## (5) 受激Raman散射的色散关系

消去 $\mathbf{A}'$ ，取 $\omega \sim \omega_{pe}$ ，并略去远离共振的哪些项 $n_e!(\mathbf{k}-2\mathbf{k}_0, \omega-2\omega_0)$ 和 $n_e!(\mathbf{k}+2\mathbf{k}_0, \omega+2\omega_0)$ ，就可以得到色散关系

$$\omega^2 - \omega_{ek}^2 = \frac{\omega_{pe}^2 k^2 v_{os}^2}{4} \left[ \frac{1}{D(\mathbf{k} - \mathbf{k}_0, \omega - \omega_0)} + \frac{1}{D(\mathbf{k} + \mathbf{k}_0, \omega + \omega_0)} \right].$$

$$D(\mathbf{k}, \omega) = \omega^2 - c^2 k^2 - \omega_{pe}^2.$$

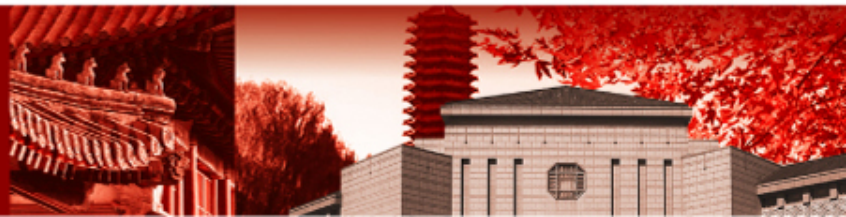
$$D(\mathbf{k} - \mathbf{k}_0, \omega - \omega_0) = (\omega - \omega_0)^2 - (\vec{k} - \vec{k}_0)^2 c^2 - \omega_{pe}^2$$

由色散关系可以得到受激Raman散射的增长率。

色散关系



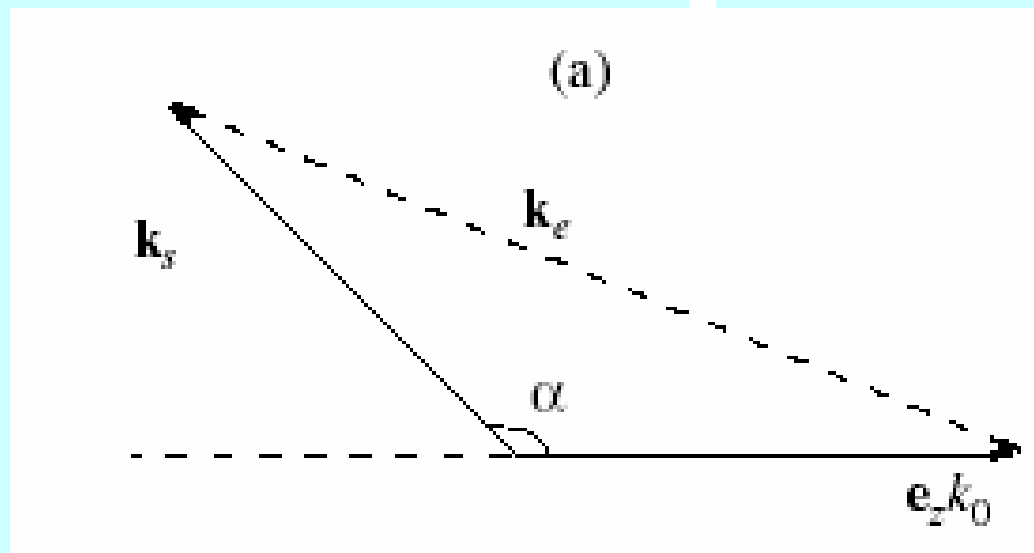
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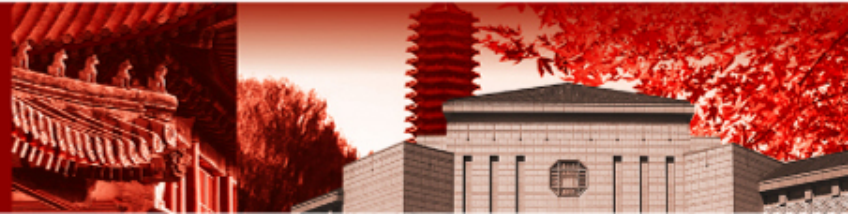
## (6) 后向受激Raman散射

讨论:

(1) 后向和侧向散射



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## (6) 后向受激Raman散射

忽略非共振的上移光波，可以得到  $D(k - k_0, \omega - \omega_0) = (\omega - \omega_0)^2 - (\vec{k} - \vec{k}_0)^2 c^2 - \omega_{pe}^2$

$$(\omega^2 - \omega_{ek}^2)[(\omega - \omega_0)^2 - (\vec{k} - \vec{k}_0)^2 c^2 - \omega_{pe}^2] = \frac{\omega_{pe}^2 k^2 v_{os}^2}{4}$$

取

$$\omega = \omega_{ek} + \delta\omega, \quad \text{且 } \delta\omega \ll \omega_{ek}$$

$$[(\omega_{ek} + \delta\omega)^2 - \omega_{ek}^2][(\omega_{ek} + \delta\omega - \omega_0)^2 - (\vec{k} - \vec{k}_0)^2 c^2 - \omega_{pe}^2] = \frac{\omega_{pe}^2 k^2 v_{os}^2}{4}$$



$$(\delta\omega)^2 = \frac{\omega_{pe}^2 k^2 v_{osc}^2}{16\omega_{ek}(\omega_{ek} - \omega_0)} = -\frac{\omega_{pe}^2 k^2 v_{osc}^2}{16\omega_{ek}(\omega_0 - \omega_{ek})} < 0$$



$$\omega_{ek}^2 = \omega_{pe}^2 + 3k^2 v_e^2$$

$$\omega^2 - \omega_{ek}^2 = \frac{\omega_{pe}^2 k^2 v_{os}^2}{4} \left[ \frac{1}{D(k - k_0, \omega - \omega_0)} + \frac{1}{D(k + k_0, \omega + \omega_0)} \right]$$

$$D(k, \omega) = \omega^2 - c^2 k^2 - \omega_{pe}^2$$

## (6) 后向受激Raman散射

当散射光共振时

$$(\omega - \omega_0)^2 - (\vec{k} - \vec{k}_0)^2 c^2 - \omega_{pe}^2 = 0$$

极大增长率出现，即当波数

$$\begin{aligned} k &= k_0 + \frac{\sqrt{(\omega_{ek} - \omega_0)^2 - \omega_{pe}^2}}{c} \approx k_0 + \frac{\sqrt{\omega_0^2 - 2\omega_{pe}\omega_0}}{c} \\ &= k_0 + \frac{\omega_0}{c} \sqrt{1 - \frac{2\omega_{pe}}{\omega_0}}, \end{aligned}$$

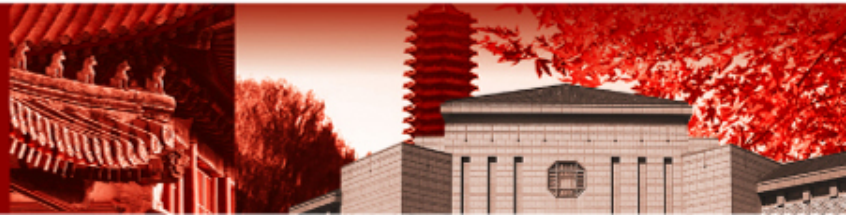
那么

$$\delta\omega = i\gamma$$

$$\gamma = \frac{kv_{osc}}{4} \left[ \frac{\omega_{pe}^2}{\omega_{ek}(\omega_0 - \omega_{ek})} \right]^{1/2}$$



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## (6) 后向受激Raman散射

对于极大增长率的后向散射

这个波数从  $n_e \leq n_{cr}/4$  的  $2k_0$  到  $n_e \sim n_{cr}/4$  的  $k=k_0$ .

$$k = k_0 + \frac{\omega_0}{c} \left(1 - \frac{2\omega_{pe}}{\omega_0}\right)^{1/2}; \quad \frac{2\omega_{pe}}{\omega_0} \sim 1, k \sim k_0;$$

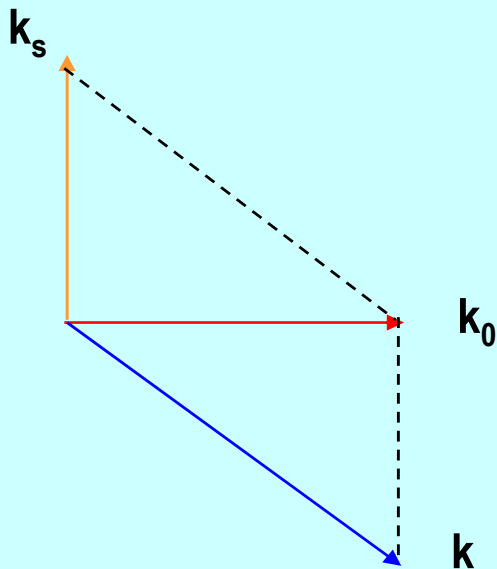
$$2\omega_{pe} / \omega_0 \ll 1, k \sim 2k_0;$$

对于  $90^\circ$  的侧向散射,  $(\vec{A}' \cdot \vec{A}_L = 0)$   
 于一条直线上。

因为入射光波和散射光波的电矢量不再处

当  $(\vec{A}' \cdot \vec{A}_L = 0)$  时, 这个增长率将为0。

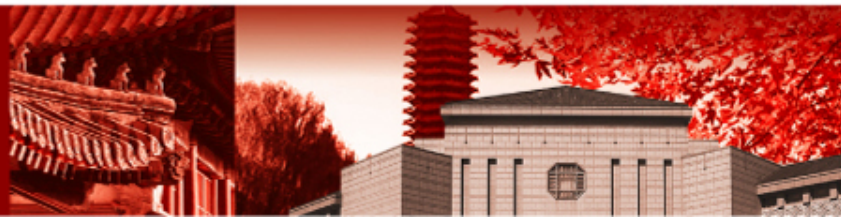
更一般的侧向散射情况,  $(\vec{A}' \cdot \vec{A}_L \neq 0)$  .



$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - 3v_e^2 \nabla^2\right) n_e' = \frac{n_0 e^2}{m_e^2 c^2} \nabla^2 (\vec{A}_L \cdot \vec{A}')$$



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## (7) 前向受激Raman散射

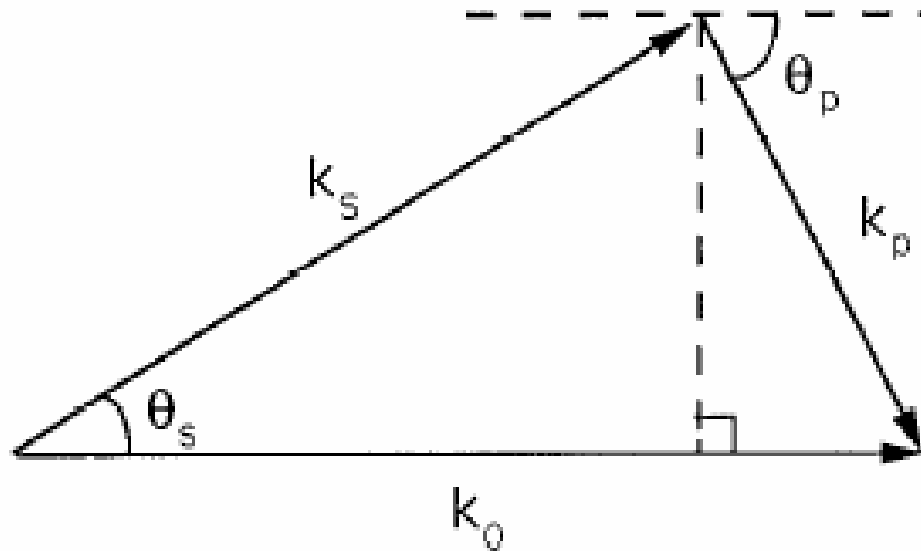
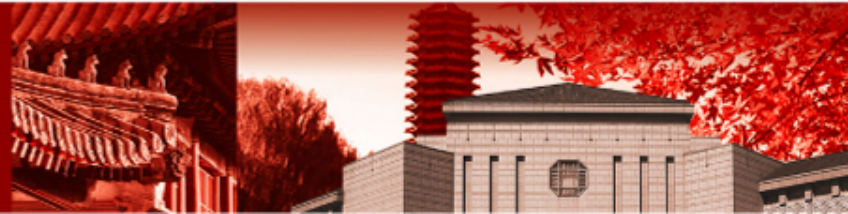


FIG. 1. Geometrical representation of momentum conservation for the near-forward scattering process. Here,  $k_0$  is the incident wave vector,  $k_s$  is the scattered light wave vector, and  $k_p$  is the plasma wave vector.



## (7) 前向受激Raman散射

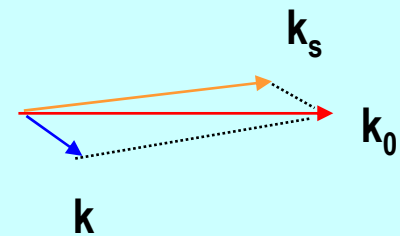
对于很低密度的前向散射情况,  $k \ll \omega_0/c$ , 这时上移和下移的光波是可以接近共振的, 这样

$$k = \frac{\omega_{pe}}{c}, \omega = \omega_{pe} + \delta\omega, \delta\omega \ll \omega_{pe},$$

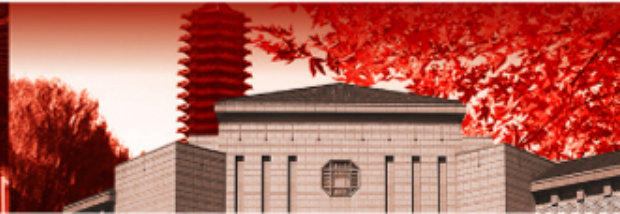
$$\begin{aligned} D(k \pm k_0, \omega \pm \omega_0) &= (\omega_{pe} + \delta\omega \pm \omega_0)^2 - (k \pm k_0)^2 c^2 - \omega_{pe}^2 \\ &\approx (\omega_{pe} \pm \omega_0)^2 + 2\delta\omega(\omega_{pe} \pm \omega_0) - k_0^2 c^2 - \omega_{pe}^2 \approx 2\delta\omega(\omega_{pe} \pm \omega_0) \end{aligned}$$

代入色散关系

$$\begin{aligned} (\omega_{pe} + \delta\omega)^2 - \omega_{ek}^2 &= \frac{\omega_{pe}^2 k^2 v_{os}^2}{4} \left[ \frac{1}{2\delta\omega(\omega_{pe} - \omega_0)} + \frac{1}{2\delta\omega(\omega_{pe} + \omega_0)} \right] \\ &= \frac{\omega_{pe}^2 k^2 v_{os}^2}{4} \frac{\omega_{pe}}{\delta\omega(\omega_{pe}^2 - \omega_0^2)} \approx -\frac{\omega_{pe}^3 k^2 v_{os}^2}{4} \frac{1}{\omega_0^2 \delta\omega} \end{aligned}$$



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## (7) 前向受激Raman散射

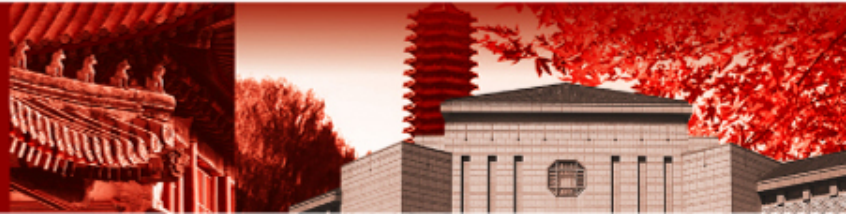
$$(\delta\omega)^2 = -\frac{\omega_{pe}^4}{8\omega_0} \frac{v_{os}^2}{c^2} < 0$$

令  $\delta\omega = i\gamma$  ， 得到极大增长率

$$\gamma = \frac{\omega_{pe}^2}{2\sqrt{2}\omega_0} \frac{v_{os}}{c}.$$



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## (8) 受激Raman散射的阈值

考虑阻尼效应后的三波耦合方程

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \omega_{pe}^2\right) \vec{A}' = -\frac{4\pi e^2}{m_e} n_e' \vec{A}_L + \nu_s \frac{\partial \vec{A}'}{\partial t},$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - 3\nu_e^2 \nabla^2\right) n_e' = \frac{n_0 e^2}{m_e c^2} \nabla^2 (\vec{A}_L \cdot \vec{A}) + \nu_e \frac{\partial n_e'}{\partial t}.$$

$\nu_s$ : 散射光波的能量阻尼率;

$\nu_e$ : 电子等离子体波的能量阻尼率.

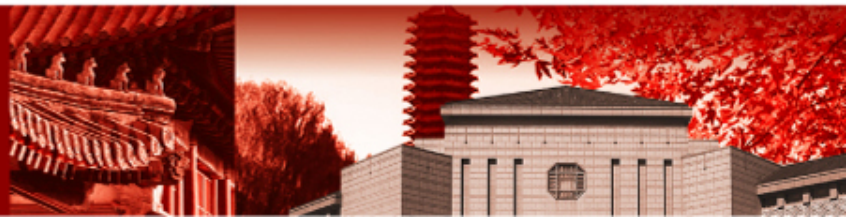
作替换

$$\omega^2 - \omega_{pe}^2 \Rightarrow \omega(\omega + i\nu_e) - \omega_{pe}^2$$

$$D(\omega, k) = \omega(\omega + i\nu_s) - k^2 c^2 - \omega_{pe}^2$$



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## (8) 受激Raman散射的阈值

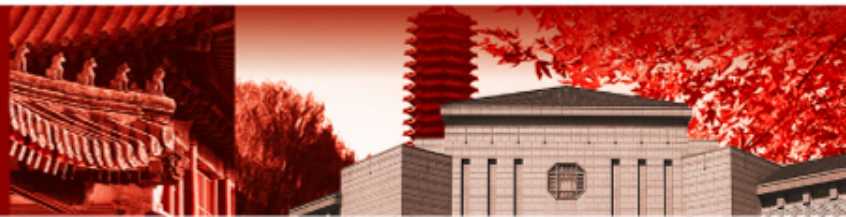
可得到同样的色散关系

$$\omega^2 - \omega_{ek}^2 + i\omega\nu_e = \frac{\omega_{pe}^2 k^2 \nu_{os}^2}{4} \left[ \frac{1}{D(\vec{k} - \vec{k}_0, \omega - \omega_0)} + \frac{1}{D(\vec{k} + \vec{k}_0, \omega + \omega_0)} \right].$$

$$D(k, \omega) = \omega^2 - c^2 k^2 - \omega_{pe}^2 + i\omega\nu_s.$$



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## (8) 受激Raman散射的阈值

### 讨论:

(1) 后向和侧向散射:  $D(k - k_0, \omega - \omega_0) = (\omega - \omega_0)^2 - (\vec{k} - \vec{k}_0)^2 c^2 - \omega_{pe}^2$

只保留下移光波, 忽略非共振的上移光波, 取  $\omega = \omega_{ek} + i\gamma$ ,

当  $(\omega_{ek} - \omega_0)^2 - (\vec{k} - \vec{k}_0)^2 c^2 - \omega_{pe}^2 = 0$ ,

选择 $k$ 以得到最大增长率, 那么可以得到

$$\gamma_0 = \frac{kv_{osc}}{4} \left[ \frac{\omega_{pe}^2}{\omega_{ek}(\omega_0 - \omega_{ek})} \right]^{1/2}$$

$$\left(\gamma + \frac{v_e}{2}\right)\left(\gamma + \frac{v_s}{2}\right) = \gamma_0^2.$$

$v_e, v_s$ : 电子等离子体波和散射光波的能量阻尼率

$\gamma_0$ : 没有阻尼时的增长率

$$k = k_0 + \frac{\omega_0}{c} \sqrt{1 - \frac{2\omega_{pe}}{\omega_0}}$$

振幅

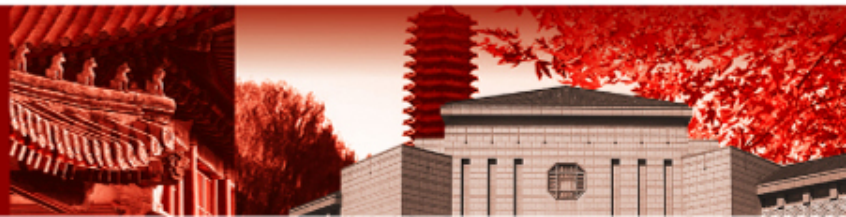
$$\gamma_e = v_e / 2, \gamma_s = v_s / 2$$

$$(\gamma + \gamma_e)(\gamma + \gamma_s) = \gamma_0^2.$$

$$\gamma_s = \frac{\omega_{pe}^2}{\omega_0^2} \frac{v_{ei}}{2}, \gamma_e = \frac{v_{ei}}{2} \quad (\text{只考虑碰撞阻尼})$$



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## (8) 受激Raman散射的阈值

由于阻尼引起的阈值条件是  $\gamma_0 \geq \sqrt{\gamma_s \gamma_e}$ .

例子：对于  $\omega_{pe} / \omega_0 \ll 1/2$ , 考虑后向散射，假设只有碰撞阻尼

$$\gamma = \frac{kv_{osc}}{4} \left[ \frac{\omega_{pe}^2}{\omega_{ek}(\omega_0 - \omega_{ek})} \right]^{1/2}$$

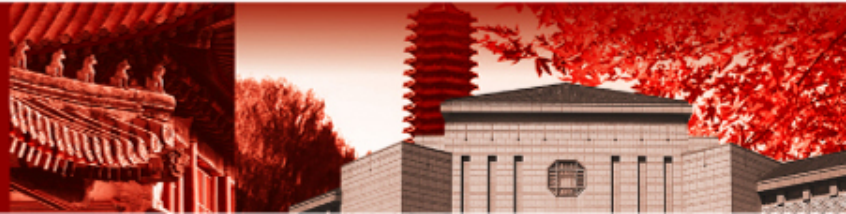
$$\left( \frac{v_{osc}}{c} \right)^2 > \frac{\omega_{pe}^2}{\omega_0^2} \frac{v_{ei}^2}{\omega_0 \omega_{pe}} \quad \text{碰撞频率}$$

该式表示的阈值功率是相当低的，一般等离子体波的Landau阻尼需要包含进去。

对于前向散射，假设只有碰撞阻尼

$$\gamma = \frac{\omega_{pe}^2}{2\sqrt{2}\omega_0} \frac{v_{os}}{c}$$

$$\left( \frac{v_{osc}}{c} \right)^2 > 2v_{ei}^2$$

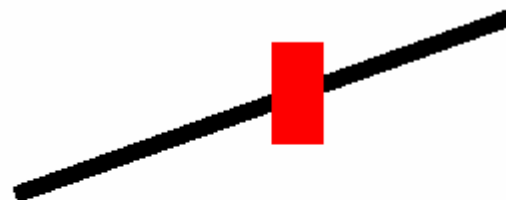


## (8) 受激Raman散射的阈值

等离子体的空间非均匀使得三波共振相互作用的区域受到限制。



均匀等离子体的共振区

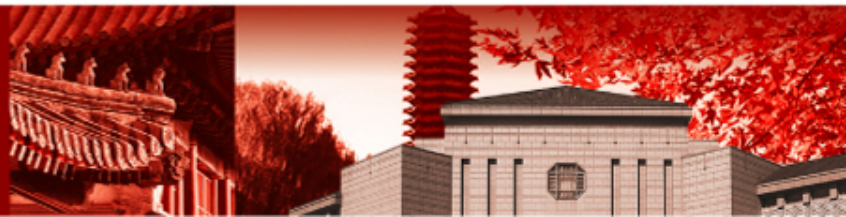


非均匀等离子体的共振区

图 4.1 共振示意图



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# (8) 考虑线性密度梯度等离子体中受激Raman散射的阈值

实际上，阈值强度通常由等离子体密度梯度确定，而不是由阻尼确定。

非均匀等离子体，定义  $k = k_1(z) - k_2(z) - k_s(\eta)$

在 $k=0$ 的某一点，三波发生耦合，但是离开该点，失配就发生。当相位漂移发展到一定尺度时，共振耦合被破坏。

利用条件  $\int_0^{l_{INT}} k dz \approx \frac{1}{2}$  可用来估计相互作用区域的大小。

对于匹配点  $k = k(0) + k'z$  作Taylor展开给出  $l_{INT} \sim \frac{1}{\sqrt{k'}}$

波的能量传出这个相互作用区引入一个近似的有效阻尼率

这里 $v_{gi}$ 是沿着密度梯度方向，第 $i$ 个波的群速度分量。

将这些阻尼率代入阈值条件  $\gamma_0 \geq \sqrt{\gamma_s \gamma_e}$  可以给出

**Rosenbluth criterion**

$$\frac{\gamma^2}{|k' v_{g1} v_{g2}|} \geq 1.$$

$$\frac{v_{gi}}{l_{INT}}$$



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## (8) 受激Raman散射的阈值

一个例子:

考虑 $n_e \ll n_{cr}/4$ 区域的反向受激Raman散射, 因为电子等离子体波的波数对密度的依赖要比横波敏感的得多,

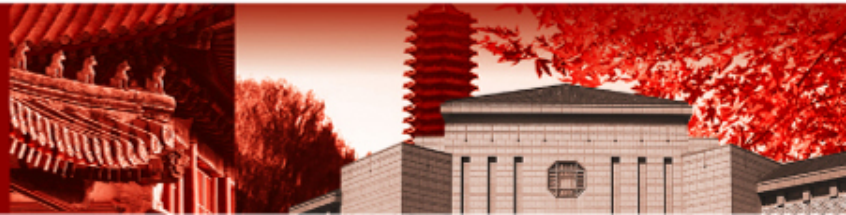
$$k' \approx \frac{\partial k}{\partial x} \text{ 和 } |v_{gp} k'| \approx \frac{\partial \omega_{pe}}{\partial x}$$

忽略温度梯度并假设密度标长 $L = n / (\frac{\partial n}{\partial x})$ 的局域线性密度变化, 则

$$|v_{gp} k'| \approx \frac{\partial \omega_{pe}}{\partial x} \approx \frac{\omega_{pe}}{2L}$$



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## (8) 受激Raman散射的阈值

对于侧向和后向的受激Raman散射，最大增长率是

$$\gamma = \frac{kv_{osc}}{4} \left[ \frac{\omega_{pe}^2}{\omega_{ek}(\omega_0 - \omega_{ek})} \right]^{1/2},$$

在稀薄等离子体中

$$2\omega_{pe} \ll \omega_0, k \sim 2k_0,$$

$$\Rightarrow \gamma^2 \approx \frac{k_0 v_{osc}^2 \omega_{pe}}{4c}.$$

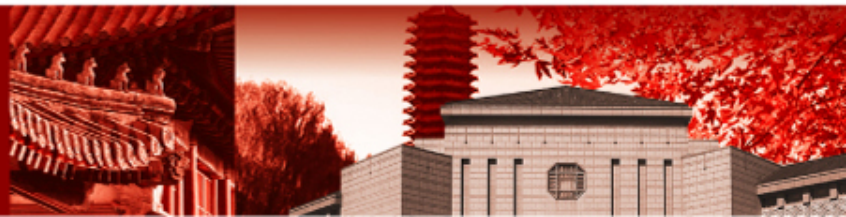
就可得到由密度梯度引起的阈值条件

$$\frac{\gamma^2}{|k' v_{g1} v_{g2}|} \geq 1.$$

$$\left( \frac{v_{os}}{c} \right)^2 \approx \frac{2}{k_0 L}$$



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## (8) 受激Raman散射的阈值

对于前向受激Raman散射，最大增长率是

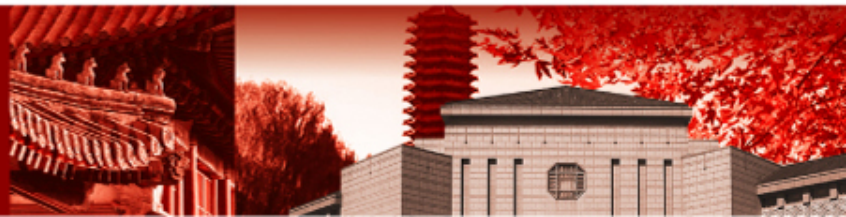
$$\gamma = \frac{\omega_{pe}^2}{2\sqrt{2}\omega_0} \frac{v_{os}}{c}$$

$$\frac{\partial\omega_{pe}}{\partial n_e} \approx \frac{\omega_{pe}}{2n_e}$$

$$\frac{\gamma^2}{|k'v_{g1}v_{g2}|} \geq 1.$$

$$|v_{gp}k'| \approx \frac{\partial\omega_{pe}}{\partial x} \approx \frac{\omega_{pe}}{2L}$$

$$\left(\frac{v_{os}}{c}\right)^2 \approx \frac{4\omega_0}{\omega_{pe}k_0L}$$



## (9) 受激Raman散射的非线性特点

前面讨论了**SRS**的线性理论

非线性:

电子加热

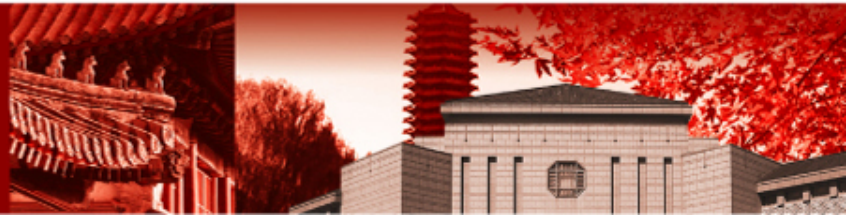
与受激**Brillouin**散射的竞争

相对论效应

自生磁场



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# (9) 讨论：受激Raman散射中超热电子的产生

(a)  $T_{hot} = m_e v_p^2 / 2$  依赖于密度和温度

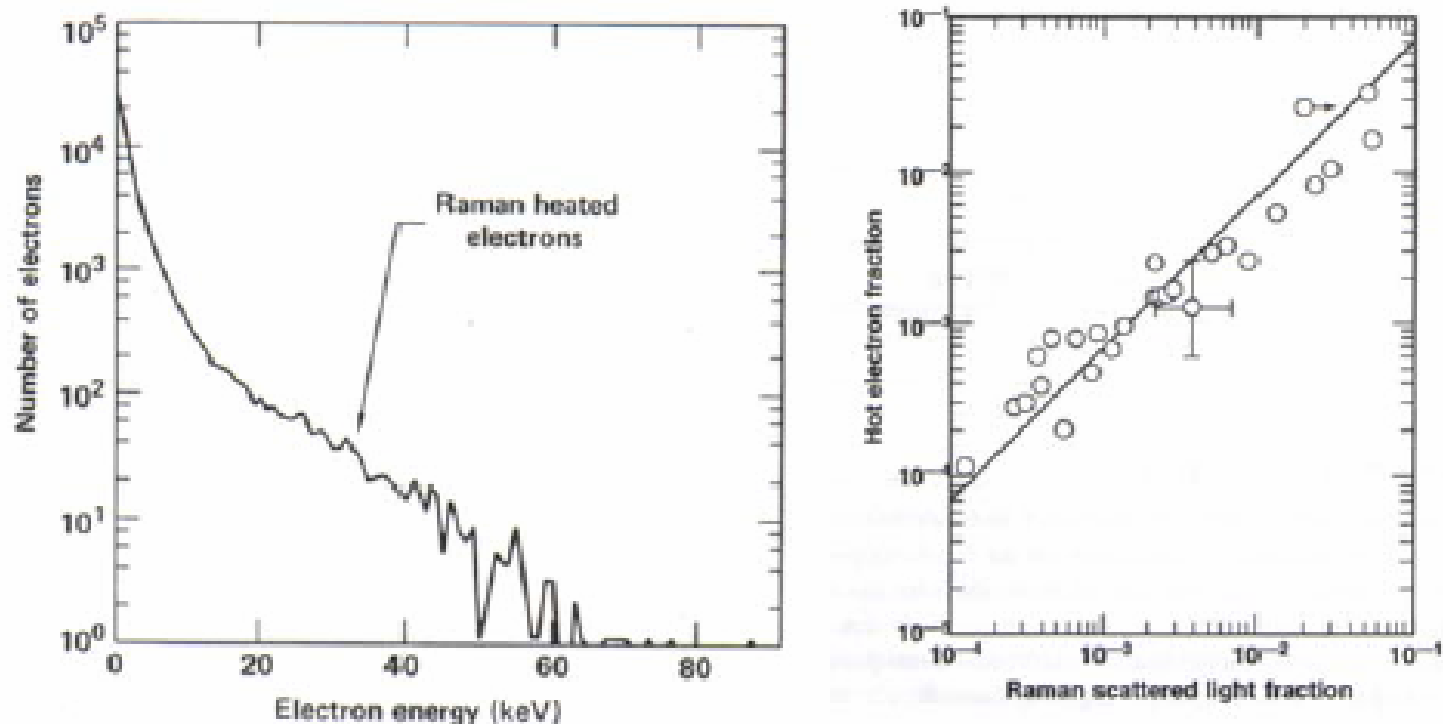
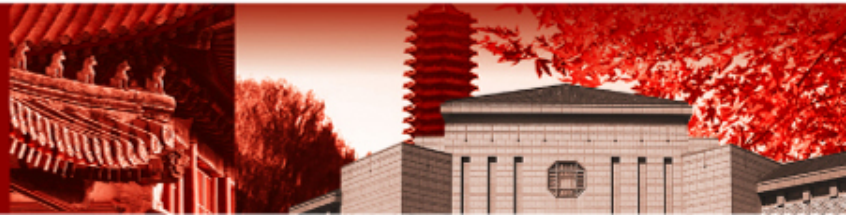


FIG. 11.10. (a) Hot electron distribution function obtained from a computer simulation of Raman back-scattering (Estabrook *et al.* 1980); (b) fraction of hot electrons inferred from bremsstrahlung X-rays plotted versus measured fraction of Raman scattered light (Drake *et al.* 1984).



# (10) 讨论：稀薄等离子体中的前向和后向受激 Raman 散射比较

十分稀薄等离子体中

$$\omega_{pe} / \omega_0 \ll 1/2,$$

前向散射

$K$ 很小

$$\gamma = \frac{\omega_{pe}^2}{2\sqrt{2}\omega_0} \frac{v_{osc}}{c}$$

$$\frac{\gamma}{\omega_0} = \frac{1}{2\sqrt{2}} \frac{v_{osc}}{c} \frac{n_e}{n_c}$$

后向散射

$k \sim 2k_0$

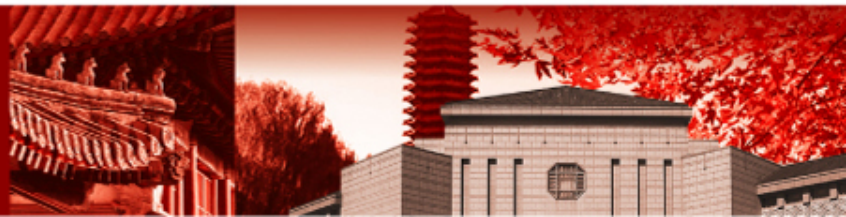
$$\gamma = \frac{kv_{osc}}{4} \left[ \frac{\omega_{pe}^2}{\omega_{ek}(\omega_0 - \omega_{ek})} \right]^{1/2} \Rightarrow \left( \frac{v_{osc}}{c} \right)^2 > \frac{\omega_{pe}^2}{\omega_0^2} \frac{v_{ei}^2}{\omega_0 \omega_{pe}}$$

$$\gamma \approx \frac{kv_{osc}}{4} \left( \frac{\omega_{pe}}{\omega_0} \right)^{1/2} \approx \frac{v_{osc}}{2c} (\omega_0 \omega_{pe})^{1/2}$$

$$\frac{\gamma}{\omega_0} \approx \frac{v_{osc}}{2c} \left( \frac{n_e}{n_c} \right)^{1/4}$$



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## (10) 讨论：稀薄等离子体中的受激Raman散射

$$\left(\frac{\gamma}{\omega_0}\right)^{BRS} > \left(\frac{\gamma}{\omega_0}\right)^{FRS}$$

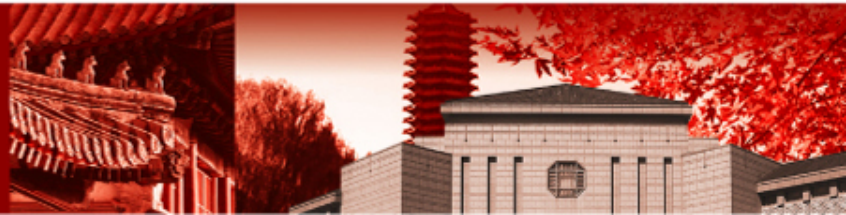
$$\frac{\gamma}{\omega} = -\sqrt{\frac{\pi}{8}} \frac{\omega_{pe}^2 \omega}{|k^3| v_e^3} \exp\left(-\frac{\omega^2}{2k^2 v_e^2}\right), \quad \omega^2 = \omega_{pe}^2 + 3k^2 v_e^2.$$

当 $\omega/k \leq v_e$ 即 $k\lambda_{De} \geq 0.4$ 时，Landau阻尼才很大。

在稀薄等离子体中，虽然BRS的增长率比FRS增长率高，但由于BRS的静电波矢大而受到强烈的Landau阻尼。



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## (10) 讨论：稀薄等离子体中的受激Raman散射

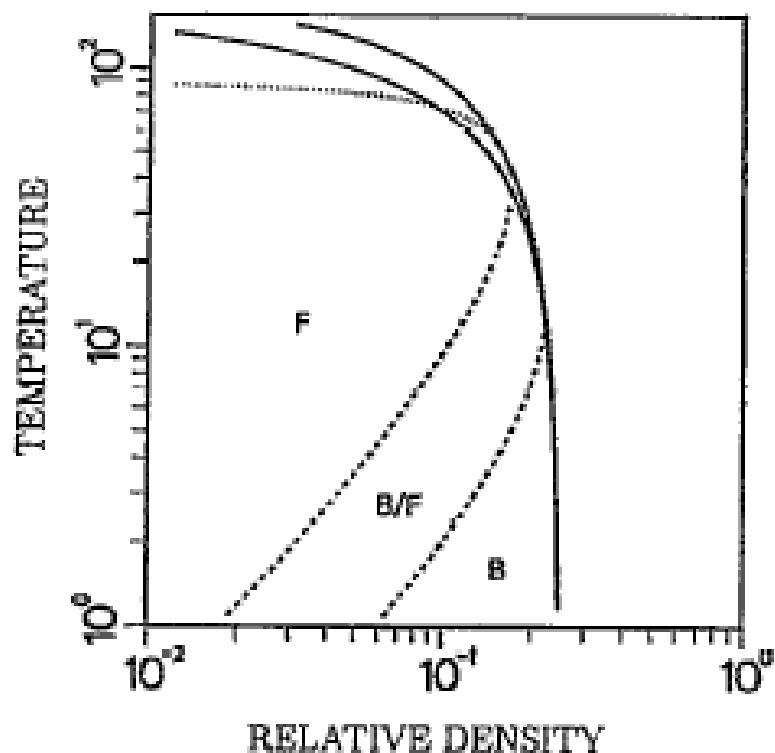
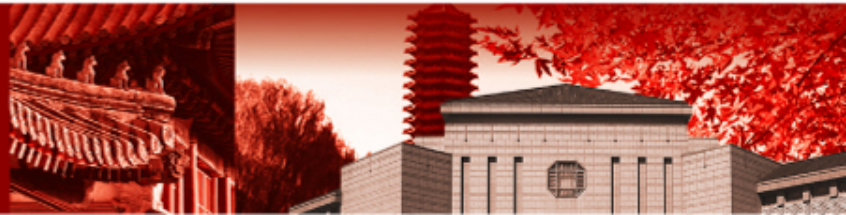
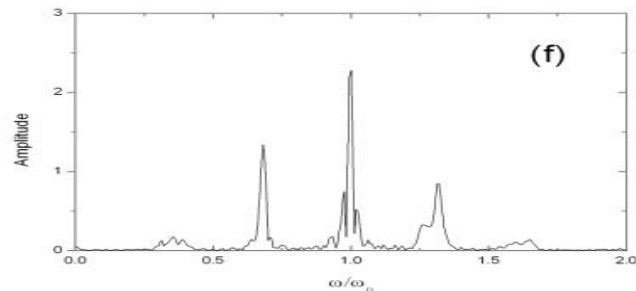
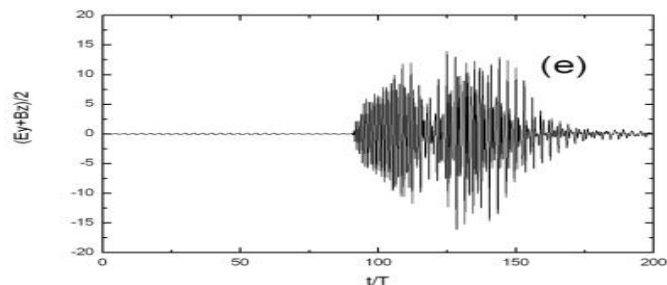
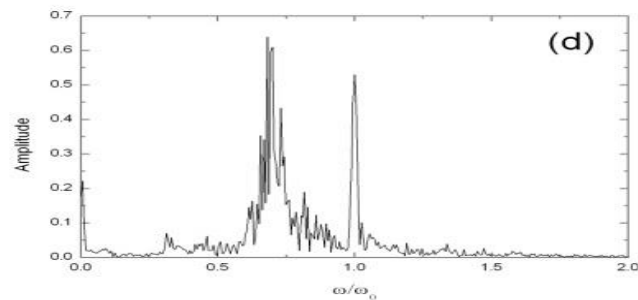
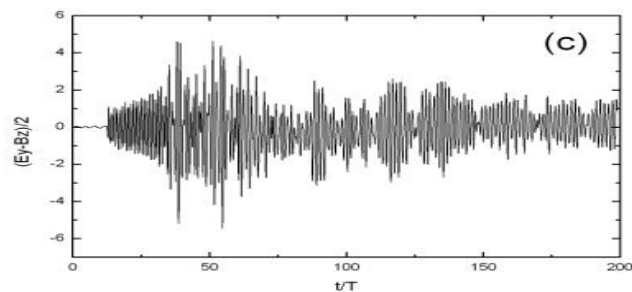
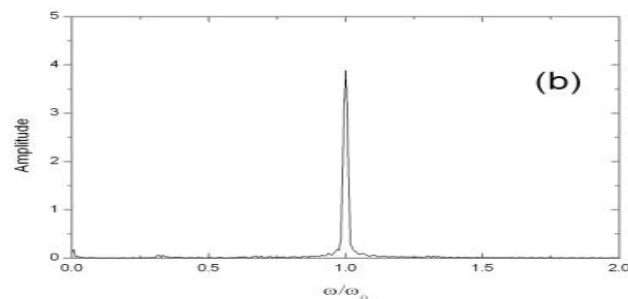
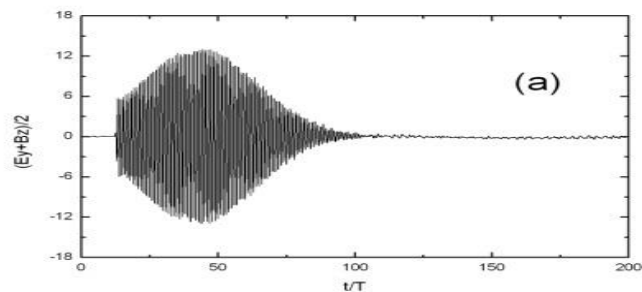


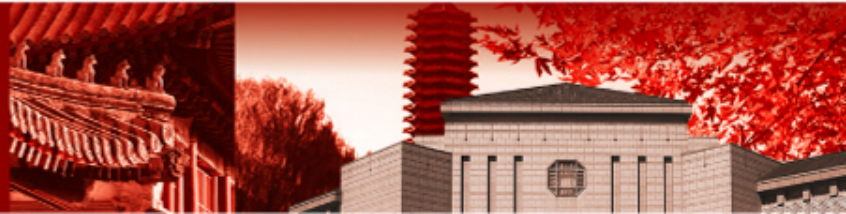
FIG. 1. Parameter regions for SRS-B and SRS-F in the  $(n_e, T_e)$  plane. Under the solid lines SRS is possible. The dashed curves correspond to  $k_B \lambda_D = 0.3$  and  $k_B \lambda_D = 0.8$ . SRS-B dominates in region "B," SRS-F dominates in region "F" and both processes coexist in region "B/F." Temperature is given in keV and relative density in  $n_e/n_{cr}$ .



# (10) 讨论：稀薄等离子体中的受激Raman散射



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## (12) 受激Raman散射的应用 – 由Raman光谱分析晕区等离子体状态 (electron temperature)

由Raman谱的红移推测晕区电子温度 $T_e$ ，但由于影响红移的因素较多，误差较大。近年来用的较多的是由Raman光谱的短波截断来确定等离子体电子温度。

### Landau阻尼:

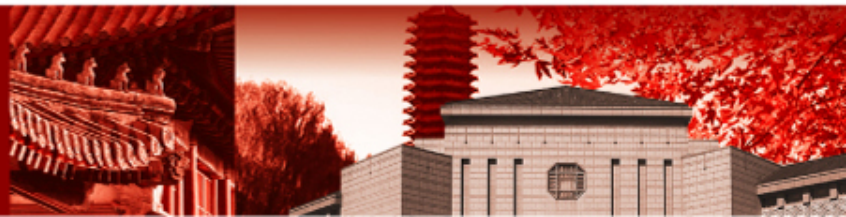
对于热速度为 $v_e$ 的Maxwell速度分布，有

$$\frac{\gamma}{\omega} = -\sqrt{\frac{\pi}{8}} \frac{\omega_{pe}^2 \omega}{|k^3| v_e^3} \exp\left(-\frac{\omega^2}{2k^2 v_e^2}\right), \quad \omega^2 = \omega_{pe}^2 + 3k^2 v_e^2.$$

当 $\omega/k \leq 3v_e$ 即 $k\lambda_{De} \geq 0.4$ 时，Landau阻尼才很大。



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# (12) 受激Raman散射的应用 – 由Raman光谱分析 晕区等离子体状态 (electron temperature)

等离子体波长越短，Landau阻尼愈大，到某个波长的Landau阻尼可使其更短波长的Langmuir波不存在，于是Raman散射也停止了。

由理论和实验可知，在截断处的 $k\lambda_{De} \approx 0.26-0.36$ ，再测出截断波长，即可估算出 $T_e$ 。

由频率匹配条件

$$1 - \lambda_0 / \lambda_s = \omega_{ek} / \omega_0$$

Langmuir波色散关系

$$\omega_{ek} = \omega_{pe} (1 + 3k^2 \lambda_{De}^2)^{1/2}$$

电磁波色散关系

$$\omega_{0,s}^2 = \omega_{pe}^2 + k_{0,s}^2 c^2$$



$$k\lambda_{De} = \left\{ \frac{1}{3} \left[ \frac{n_{cr}}{n} \left( 1 - \frac{\lambda_0}{\lambda_s} \right)^2 - 1 \right] \right\}^{1/2}$$

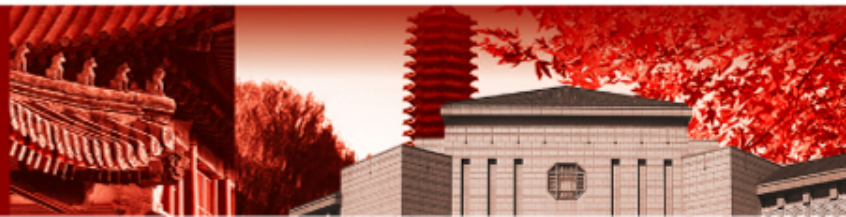


$$k = 2 \frac{\omega_0}{c} \left( 1 - \frac{n}{n_{cr}} \right)^{1/2} \left( 1 - \sqrt{\frac{n}{n_{cr}}} + \frac{1}{4} \frac{n}{n_{cr}} \right)^{1/2},$$

$$k_s = \frac{\omega_0}{c} \left[ 2 \left( 1 - \frac{n}{n_{cr}} \right)^{1/2} \left( 1 - \sqrt{\frac{n}{n_{cr}}} + \frac{1}{4} \frac{n}{n_{cr}} \right)^{1/2} - 1 \right].$$



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# (12) 受激Raman散射的应用 – 由Raman光谱分析 晕区等离子体状态 (electron temperature)

$$\lambda_0 / \lambda_s = k_s / k_0$$

$$k_s = \frac{\omega_0}{c} \frac{\lambda_0}{\lambda_s}$$

$$\rightarrow \frac{\lambda_0}{\lambda_s} = 2 \left(1 - \frac{n}{n_{cr}}\right)^{1/2} \left(1 - \sqrt{\frac{n}{n_{cr}}} + \frac{1}{4} \frac{n}{n_{cr}}\right)^{1/2} - 1,$$

$$T_e = \frac{c^2}{\omega_0^2} \frac{\pi e^2}{3k_B} \frac{n_{cr} \left(1 - \frac{\lambda_0}{\lambda_s}\right)^2 - n}{\left(1 - \frac{n}{n_{cr}}\right) \left(1 - \sqrt{\frac{n}{n_{cr}}} + \frac{1}{4} \frac{n}{n_{cr}}\right)} \approx 42.642 \frac{\left[-\frac{n}{n_{cr}} + \left(1 - \frac{\lambda_0}{\lambda_s}\right)^2\right]}{\left(1 - \frac{n}{n_{cr}}\right) \left(1 - \sqrt{\frac{n}{n_{cr}}} + \frac{1}{4} \frac{n}{n_{cr}}\right)} \text{ keV}$$

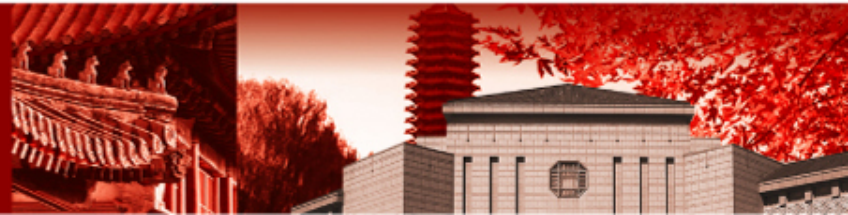
由Raman光谱得到Landau短波截断波长 $\lambda_s$ ;

⇒计算出短波截断处的电子密度;

⇒估算处电子温度。(注意: **underdense**区域近似是等温区, 所以估算出等离子体冕区温度。

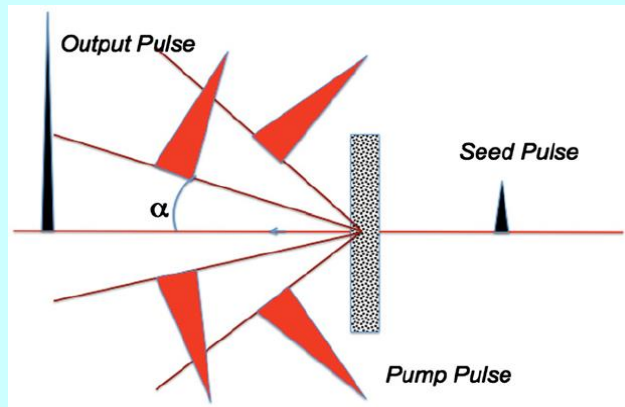
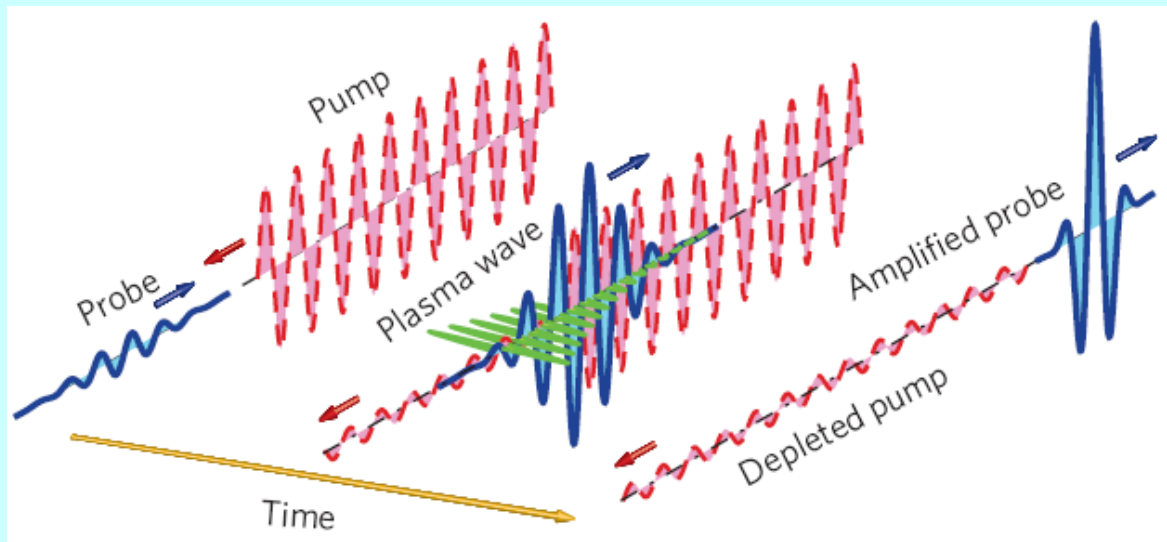


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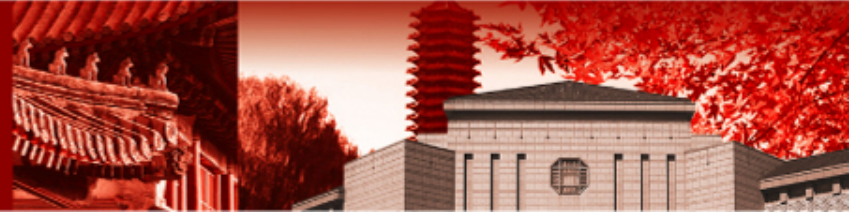
# (12) 受激Raman散射的应用 – 等离子体Raman放大 (Raman amplification in plasma)



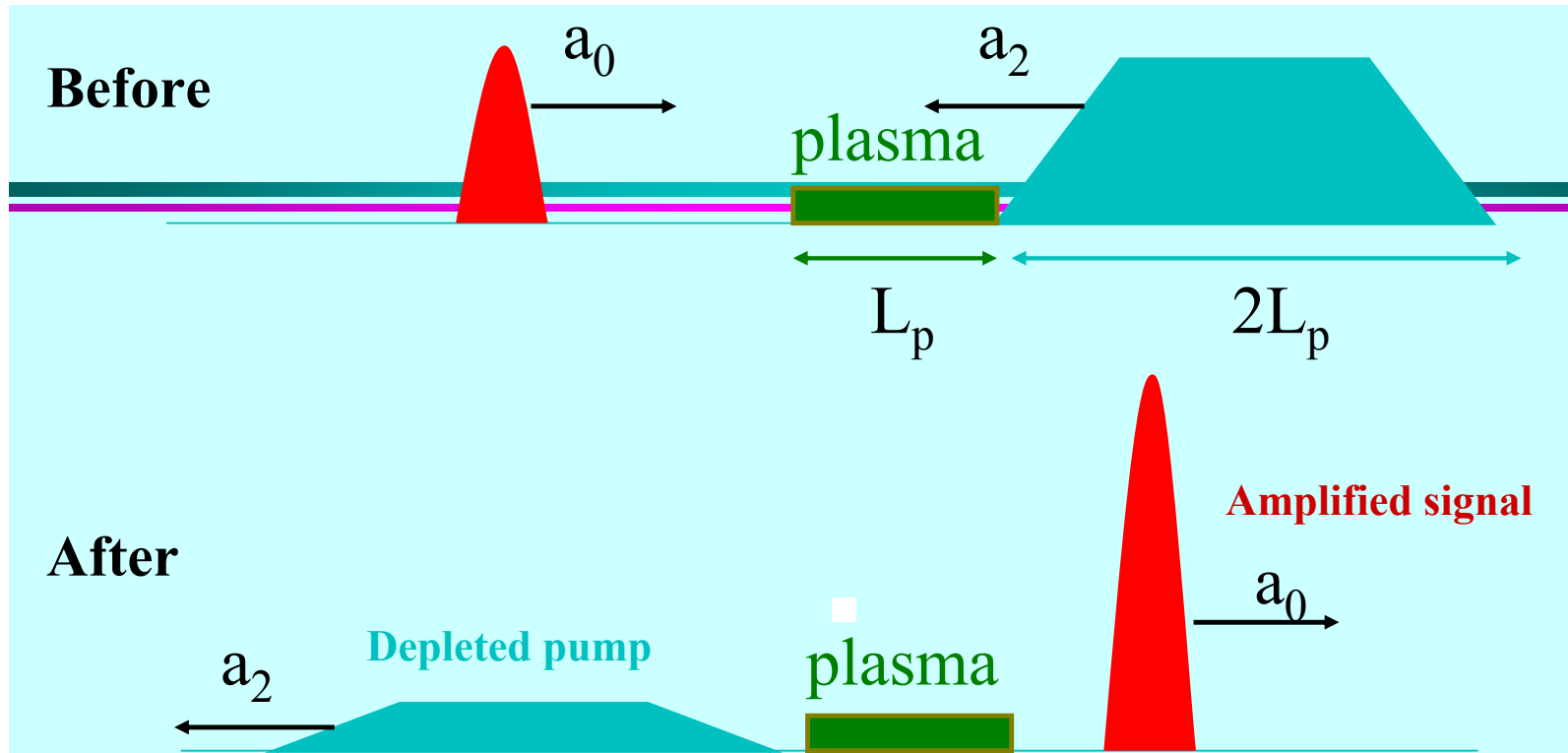
- G. Shvets et al., Phys. Rev. Lett. 81, 4879 (1998).  
B. Ersfeld and D. A. Jaroszynski, Phys. Rev. Lett. 95, 165002 (2005);  
R. M. G. M. Trines et al., Nature Physics, (2010);  
G.A. Mourou et al. Opt. Comm. 285 (2012) 720–724



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# (12) 受激Raman散射的应用 – 等离子体Raman放大 (Raman amplification in plasma)



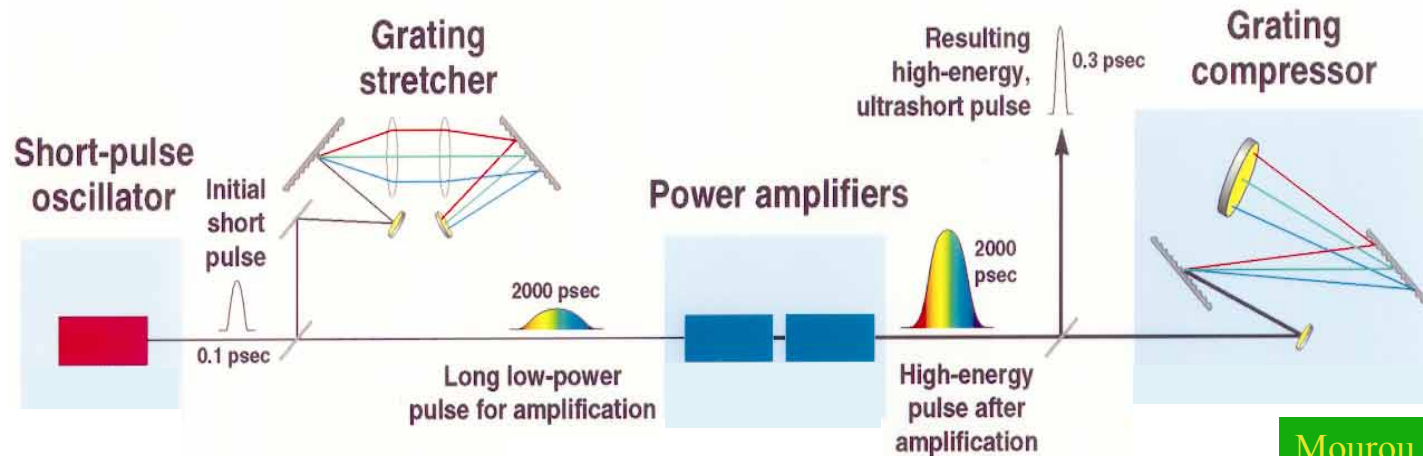
Short pulse absorbed energy of long pump while remaining short:  
pulse compression without chirping and gratings.

Note:  $I_{\text{pulse}} \gg I_{\text{pump}}$ , yet energy flows from pump to probe



# (12) 受激Raman散射的应用 – 等离子体Raman放大 (Raman amplification in plasma)

Chirped Pulse Amplification: stretch, amplify, then recompress



Mourou,  
et al.

## Limitations of CPA

1. Thermal damage to expensive gratings: limits fluence to  $\text{J}/\text{cm}^2$   
e.g. fused-silica damage at  $2\text{--}4 \text{ J}/\text{cm}^2$  for  $1\text{--}10 \text{ ps}$ ;  $0.3 \text{ J}/\text{cm}^2$  at  $10 \text{ fs}$
2. Requires broad-bandwidth high-fluence amplifiers

$\text{GW}/\text{cm}^2$  in amplifier  $\rightarrow 10^3$  compression

$\text{J}/\text{cm}^2 < 10 \text{ ps}$



$\text{TW}/\text{cm}^2$



$10^3 \text{ cm}^2$  gratings

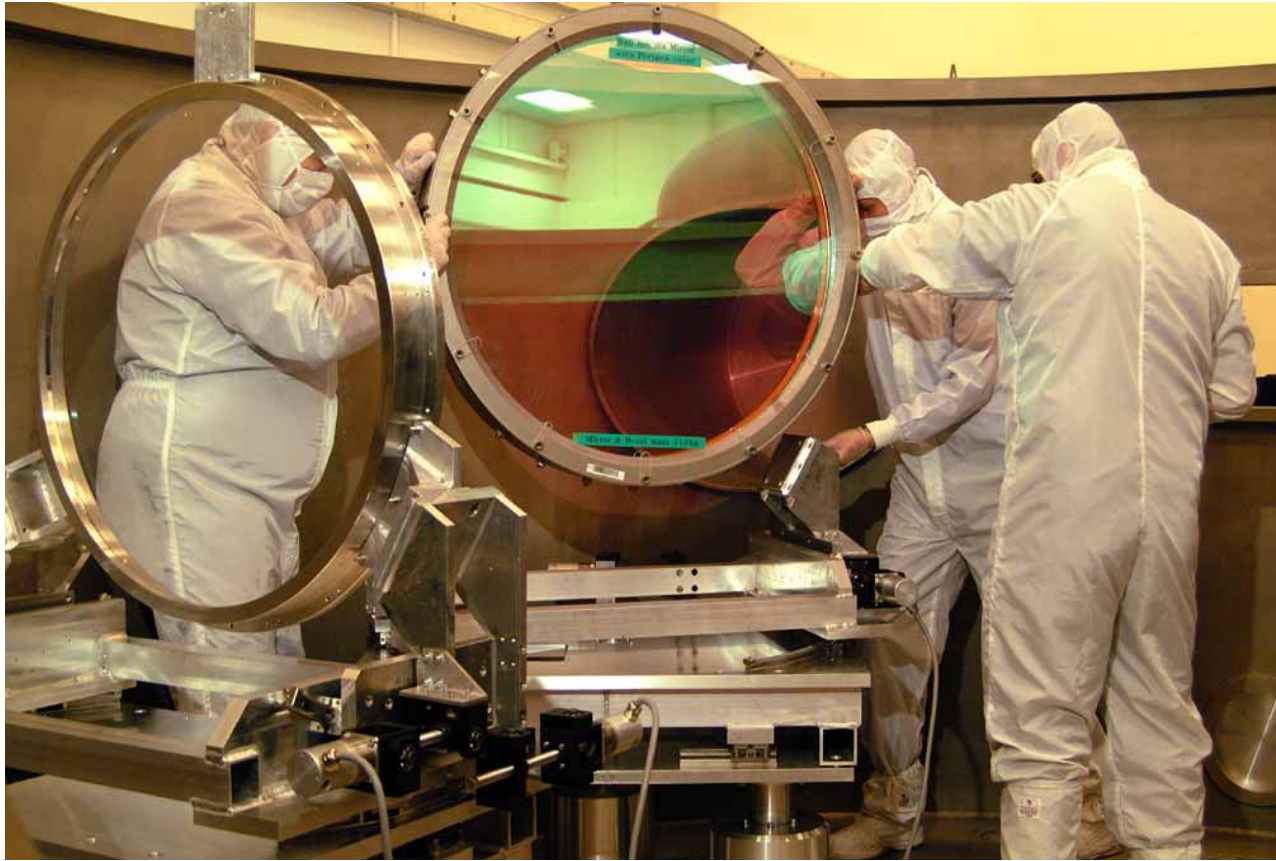


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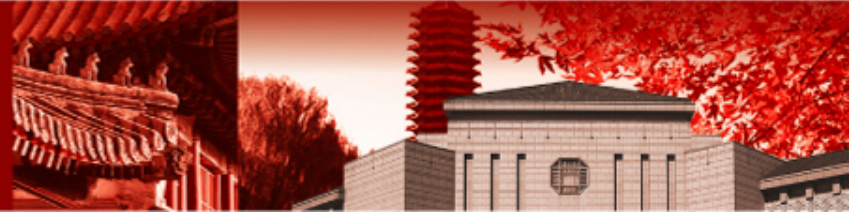


# (12) 受激Raman散射的应用 – 等离子体Raman放大 大 (Raman amplification in plasma) Gratings for Petawatt ( $10^{15}\text{W}$ ) Lasers

(Currently the maximum grating size is limited to about 1m)



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谢谢!



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