

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

## Part VIII

# Interaction with Solids

# Short pulse vs. long pulse interactions

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Traditional interaction physics (ICF – ns lasers):

- Collisional heating and creation of long scale-length plasmas
- Laser reflected at critical density surface
- Fast (keV) particles produced at 'high' intensities ( $10^{16} \text{ Wcm}^{-2}$ )

Femtosecond pulses

- Pulse length  $<$  ion motion (hydrodynamic) timescale
- Huge intensity range  $10^7$
- No single interaction model possible

# Typical interaction scenario: I. Creation of critical surface

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Field ionization over the first few laser cycles rapidly creates a surface plasma layer with a density many times the critical density  $n_c$ .

$$\omega^2 = \frac{4\pi e^2 n_c}{m}, \quad (135)$$

where  $e$  and  $m$  are the electron charge and mass respectively.

In practical units:

$$n_c \simeq 1.1 \times 10^{21} \left( \frac{\lambda}{\mu\text{m}} \right) \text{ cm}^{-3}. \quad (136)$$

## Interaction scenario: II. Ionization degree

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Example: aluminium has 3 valence electrons; 6 more can be created for a few hundred eV.

The electron density is given by:

$$n_e = Z^* n_i = \frac{Z^* N_A \rho}{A}. \quad (137)$$

effective ion charge:  $Z^* = 9$

atomic number:  $A = 26$

Avogadro number:  $N_A = 6.02 \times 10^{23}$

mass density:  $\rho = \rho_{\text{solid}} = 1.9 \text{ g cm}^{-3}$

electron density:  $n_e = 4 \times 10^{23} \text{ cm}^{-3}$

density contrast (1  $\mu\text{m}$ ):  $n_e/n_c = 400$

## Interaction scenario: III. Heating

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Target is heated via electron-ion collisions to 10s or 100s of eV depending on the laser intensity.

The plasma pressure created during heating causes ion blow-off (ablation) at the sound speed:

$$\begin{aligned}c_s &= \left( \frac{Z^* k_B T_e}{m_i} \right)^{1/2} \\ &\simeq 3.1 \times 10^7 \left( \frac{T_e}{\text{keV}} \right)^{1/2} \left( \frac{Z^*}{A} \right)^{1/2} \text{ cm s}^{-1},\end{aligned}\quad (138)$$

where  $k_B$  is the Boltzmann constant,  $T_e$  the electron temperature and  $m_i$  the ion mass.

# Interaction scenario: IV. Expansion

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Because of ion ablation, density profile formed is *exponential* with scale-length:

$$\begin{aligned} L &= c_s \tau_L \\ &\simeq 3 \left( \frac{T_e}{\text{keV}} \right)^{1/2} \left( \frac{Z^*}{A} \right)^{1/2} \tau_{\text{fs}} \text{\AA}. \end{aligned} \quad (139)$$

Eg: 100 fs Ti:sapphire pulse heats the target to a few hundred eV  $\rightarrow$  plasma with scale-length  $L/\lambda = 0.01\text{--}0.1$ . (cf: 100-1000 for ICF plasmas).

# Why is ionization important?

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

## Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

For multi-electron atoms, ionization degree,  $Z^*$ , needed for basic plasma properties like the electron density, equation of state, transport coefficients.

- 1 High density, optically thick plasmas: radiative and absorptive processes balanced – local thermal equilibrium (LTE) reached.
- 2 Short pulses: optically thin plasmas (radiation escapes!), which span many orders of magnitude in density and temperature all at once.

# Local thermal equilibrium (LTE)

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Relative ion populations related by the *Saha-Boltzmann* equation:

$$\frac{n_e n_{Z+1}}{n_Z} = \frac{g_{Z+1}}{g_Z} \frac{2m^3}{h^3} \left( \frac{2\pi T_e}{m} \right)^{3/2} \exp(-\Delta E_Z / T_e), \quad (140)$$

where  $n_Z, n_{Z+1}$  are the ion densities corresponding to ionization states  $Z$  and  $Z + 1$ ;  $g_Z, g_{Z+1}$  are the respective statistical weights of these levels (taking electron degeneracy into account), and  $\Delta E_Z$  is the energy difference between the two states. This equation is subject to the constraints:

$$\sum n_Z = n_0; \quad \sum Z n_Z = n_e. \quad (141)$$



# Transient plasmas (non-LTE)

Interaction with

Solids:

Overdense

Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Typical situation for short ( $< 1\text{ps}$ ) pulses. Need time-dependent atomic *rate equations* in order to determine the charge distribution:

$$\frac{dn_Z}{dt} = n_e n_{Z-1} S(Z-1) - n_e n_Z [S(Z) + \alpha(Z)] + n_e n_{Z+1} \alpha(Z+1), \quad (142)$$

where  $S(Z)$  and  $\alpha(Z)$  are the ionization and recombination rates of the ion with charge state  $Z$ , respectively.

Recombination rate generally comprises a number of separate processes, such as radiative recombination, 3-body collisional recombination and dielectronic recombination.

Highly complex procedure for high  $Z$  – thousands of transitions.

# Collisional absorption

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization  
Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Distinguish long/short pulse regimes via density scale-length  $L/\lambda$ :

- Long pulse (ps–ns)  $\rightarrow L/\lambda \gg 1$  (eg 10–100). Laser light mainly absorbed in underdense region via *inverse bremsstrahlung*.
- Sub-picosecond timescale, low intensities ( $I < 10^{15} \text{ Wcm}^{-2}$ )  $\rightarrow L/\lambda \leq 0.1$ : standard IB formula invalid. Less 'room' for absorption, but higher densities  $\rightarrow$  higher collision rates.
- Short pulse, high intensities: nonlinear *collisionless* absorption – cf. metal optics

# Collisional absorption: Helmholtz equations

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Standard method for electromagnetic wave propagation in an inhomogeneous plasma – see books by Ginzburg, Kruer.

Start from Maxwell's equations with small field amplitudes and a non-relativistic fluid response including collisional damping:

$$m \frac{\partial \mathbf{v}}{\partial t} = -e(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}) - m\nu_{ei}\mathbf{v}, \quad (143)$$

where  $\nu_{ei}$  is the electron-ion collision frequency.

Physically arises from binary collisions, resulting in a frictional drag on the electron motion.

# Electron-ion collisional frequency

Spitzer-Härm

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization  
Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Collision rate:

$$\begin{aligned}\nu_{ei} &= \frac{4(2\pi)^{1/2}}{3} \frac{n_e Z e^4}{m^2 v_{te}^3} \ln \Lambda \\ &\simeq 2.91 \times 10^{-6} Z n_e T_e^{-3/2} \ln \Lambda \text{ s}^{-1}.\end{aligned}\quad (144)$$

$Z$  = number of free electrons per atom

$n_e$  = electron density in  $\text{cm}^{-3}$

$T_e$  = temperature in eV

$\ln \Lambda$  is the *Coulomb logarithm*, with usual limits,  $b_{\min}$  and  $b_{\max}$ , of the electron-ion scattering cross-section.

# Electron-ion collisional frequency: Coulomb logarithm

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario  
Ionization  
Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Limits are determined by the classical distance of closest approach and the Debye length respectively, so that:

$$\Lambda = \frac{b_{\max}}{b_{\min}} = \lambda_D \cdot \frac{k_B T_e}{Z e^2} = \frac{9 N_D}{Z}, \quad (145)$$

where

$$\lambda_D = \left( \frac{k_B T_e}{4\pi n_e e^2} \right)^{1/2} = \frac{v_{te}}{\omega_p}, \quad (146)$$

and

$$N_D = \frac{4\pi}{3} \lambda_D^3 n_e$$

is the number of particles in a Debye sphere.

# Wave equations

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

The relevant EM wave equations for  $\mathbf{E}$  and  $\mathbf{B}$  are obtained in the usual way by taking the curl of the Faraday and Ampère equations (60, 61) respectively, to give:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial \mathbf{J}}{\partial t} + \nabla(\nabla \cdot \mathbf{E}), \quad (147)$$

$$\nabla^2 \mathbf{B} - \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} = -\frac{4\pi}{c} \nabla \times \mathbf{J}. \quad (148)$$

The right-hand sides of each equation represent the *source* terms of the EM waves in the plasma.

# Linearized wave equations

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Assume that all field and fluid quantities have a harmonic time-dependence  $\exp(-i\omega t)$ , where  $\omega$  is the laser frequency:

$$f(\mathbf{x}, t) = f_0(\mathbf{x}) + f_1(\mathbf{x})e^{-i\omega t} + f_2(\mathbf{x})e^{-2i\omega t} + \dots,$$

which results in the following simplifications:

$$\begin{aligned}\frac{\partial}{\partial t} &\rightarrow -i\omega \\ n_e &\rightarrow n_0 + n_1 \\ \mathbf{J} &\rightarrow -en_0\mathbf{v}_1 \\ (\mathbf{E} + \mathbf{v} \times \mathbf{B}) &\rightarrow \mathbf{E}_1.\end{aligned}$$

# Ohm's Law

Inserting these approximations into the Lorentz equation (143) allows us to solve for  $\mathbf{v}_1$ , namely:

$$\mathbf{v}_1 = \frac{-i}{\omega + i\nu_{ei}} \frac{e\mathbf{E}_1}{m}.$$

This immediately gives us the induced plasma current

$$\mathbf{J}_1 = -en_o\mathbf{v}_1 = \sigma_e\mathbf{E}_1, \quad (149)$$

where  $\sigma_e$ , the *AC electrical conductivity*, is

$$\sigma_e = \frac{i\omega_p^2}{4\pi\omega(1 + i\tilde{\nu})}. \quad (150)$$

Note  $\tilde{\nu} = \nu_{ei}/\omega$ .



# General wave equation

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization  
Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Substituting expression (149) for  $\mathbf{J}_1$  into the RHS of the wave equation (147) for  $\mathbf{E}_1$  gives us a general expression for the electric field:

$$\nabla^2 \mathbf{E}_1 + \frac{\omega^2}{c^2} \mathbf{E}_1 = \frac{\omega_p^2}{c^2} \frac{\mathbf{E}_1}{1 + i\tilde{\nu}} + \nabla(\nabla \cdot \mathbf{E}_1). \quad (151)$$

# Dispersion relation

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

For a planar, transverse EM wave propagating in a uniform plasma we have  $\nabla \rightarrow i\mathbf{k}$ , and  $\mathbf{E}_1$  perpendicular to  $\mathbf{k}$ , so that  $\nabla \cdot \mathbf{E}_1 = 0$ . In this limit we recover the standard linear dispersion relation:

$$-k^2 + \frac{\omega^2}{c^2} \left( 1 - \frac{\omega_p^2}{\omega^2(1 + i\tilde{\nu})} \right) = 0. \quad (152)$$

# Dielectric constant

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

From this we identify the dielectric constant of the propagation medium

$$\varepsilon \equiv \frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2(1 + i\tilde{\nu})} = 1 + \frac{4\pi i\sigma_e}{\omega}.$$

Can be readily generalized to a non-uniform plasma by allowing permittivity  $\varepsilon(x)$  to vary in space.

# One-dimensional density gradient

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization  
Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Consider plasma density with a gradient in one direction, so that

$$\varepsilon(x) \equiv n^2(x) = 1 - \frac{n_0(x)/n_c}{(1 + i\tilde{\nu}(x))}, \quad (153)$$

where  $n(x)$  is the local refractive index,  $n_0$  the equilibrium electron density and  $n_c$  the critical density of the EM wave.

# Simplified wave equation: S-polarization

Interaction with  
Solids:  
Overdense  
Plasmas

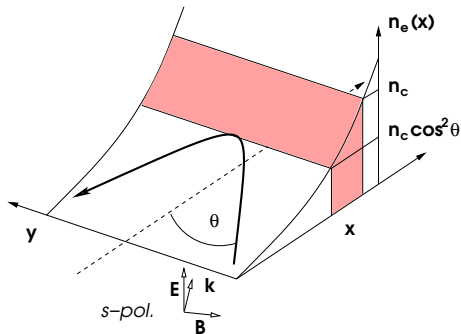
Short pulse  
interaction  
scenario  
ionization  
Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics



**Figure:** Geometry of plane-wave incident on a plasma density profile for s-polarized light (E-field in the z-direction).

# S-polarized wave

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Assume incident wave is at some fixed angle to the density gradient, polarized out of the propagation plane, see Fig. 13. In this case the wave has a *periodicity* in  $y$  given by:

$$\mathbf{E}_1 = (0, 0, E_z)e^{iky \sin \theta}.$$

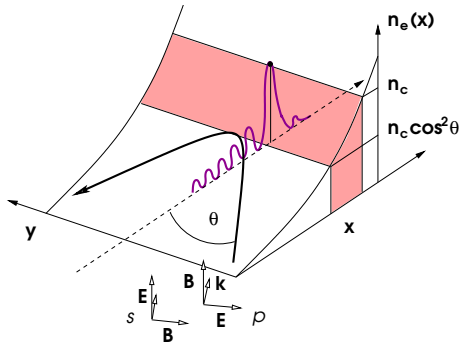
Thus  $\nabla = (\partial/\partial x, ik \sin \theta, 0)$ , so that  $\nabla \cdot \mathbf{E}_1 = 0$ . Making use of Eq. (153), the wave equation reduces to the Helmholtz equation for the electric field:

$$\frac{\partial^2 E_z}{\partial x^2} + k^2(\epsilon - \sin^2 \theta)E_z = 0. \quad (154)$$

# P-polarized wave

Consider now a  $p$ -polarized wave  $\mathbf{E}_1 = (E_x, E_y, 0)$ .

In this case,  $\nabla \cdot \mathbf{E}_1 \neq 0$ ; a component of the laser field lies along the density gradient.



Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario  
Ionization  
Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

# Helmholtz equation for $\mathbf{B}$

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

The equation for the electric field in this case is complicated because it contains both EM and ES components.

Easier to solve for  $B_z$  instead, and then obtain  $E$  from Ampère's law, which after substituting  $\mathbf{J}_1 = \sigma_e \mathbf{E}_1$ , becomes (?)

$$\nabla \times \mathbf{B}_1 = -\frac{i\omega\epsilon}{c} \mathbf{E}_1. \quad (155)$$

In an analogous fashion, Faraday's law can be written:

$$\nabla \times \mathbf{E}_1 = \frac{i\omega}{c} \mathbf{B}_1. \quad (156)$$



# Helmholtz equation for $\mathbf{B}$

As with the electric field, we substitute the expression for the current Eq. (149) into the magnetic field wave equation (148), and then use Eq. (155) and Eq. (156) to eliminate  $\mathbf{E}_1$ ; Eq. (153) to eliminate  $\sigma_e$ :

$$\nabla^2 \mathbf{B}_1 + \frac{\omega^2}{c^2} \mathbf{B}_1 + \frac{\nabla \epsilon}{\epsilon} \times (\nabla \times \mathbf{B}_1) = 0. \quad (157)$$

Applying the same oblique-incidence ansatz as before

$$\mathbf{B}_1 = (0, 0, B_z) e^{iky \sin \theta},$$

we get the Helmholtz equation for  $B$ :

$$\frac{\partial^2 B_z}{\partial x^2} - \frac{1}{\epsilon} \frac{\partial \epsilon}{\partial x} \frac{\partial B_z}{\partial x} + k^2 (\epsilon - \sin^2 \theta) B_z = 0. \quad (158)$$

# Numerical solution of Helmholtz equations

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization  
Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Helmholtz equations (154) and (158) are ordinary differential equations which can be solved numerically by standard matrix-inversion.

4 principle parameters of interest:

- 1 scale-length
- 2 polarization
- 3 angle of incidence
- 4 collision frequency

Typically find characteristic angular dependence in *reflectivity* for a given value of  $kL$ .

# Solution: long density scale-lengths

Interaction with

Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization  
Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

Remark: wave modes are purely transverse: no coupling between an s-polarized EM wave and electrostatic modes (or Langmuir waves), since  $n_1 = \nabla \cdot \mathbf{E}_1 = 0$ .

If density gradient  $L/\lambda \gg 1$ , then  $\varepsilon(x)$  is slowly varying over a laser wavelength, i.e.  $L^{-1} \sim \varepsilon^{-1} \partial \varepsilon / \partial x \ll 1$ . then Eq. (154) can be solved via the Wentzel-Kramers-Brillouin (WKB) approximation.

→ Airy functions.

# Absorption in long-scale-length plasma for S-light

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario  
Ionization  
Collisional  
Absorption

**Helmholtz  
equations**

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

For s-light, the absorption coefficient in this limit is given for an exponential profile by (Kruer, 1988):

$$\eta_{\text{WKB}} = 1 - \exp\left(-\frac{8\nu_{ei}L}{3c} \cos^3 \theta\right). \quad (159)$$

# Resonance absorption: P-light with $kL \gg 1$

Ginzburg (1964)

Interaction with

Solids:

Overdense

Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

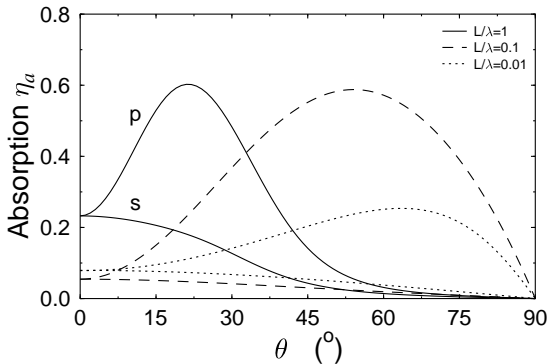
Hydrodynamics

For  $p$ -polarized light, EM wave drives plasma resonance at  $n_e = n_c$ , which may be collisionally damped, leading to a maximum absorption of about 60% at an *optimum angle of incidence* given by

$$\sin \theta_{\text{opt}} = 0.8 (kL)^{-1/3}. \quad (160)$$

# Absorption for S- and P-light at various density scale-lengths

Absorption fraction of both *s*- and *p*-light for three different scale-lengths:  $L/\lambda=1$  (solid curves),  $L/\lambda=0.1$  (dashed) and  $L/\lambda=0.01$  (dotted).



Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization  
Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

# Absorption in steep density profiles: skin effect

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

**Normal skin  
effect**

Heating and  
Thermal  
Conduction

Hydrodynamics

In the limit  $L \rightarrow 0$ , recover Fresnel-like absorption behavior of metal-optics. Consider first  $s$ -polarized light. Starting from the Helmholtz equation 154 for the electric field, we represent the density by a Heaviside step function:

$$n_0(x) = n_0 \Theta(x),$$

and for the time-being neglect collisions in the dielectric constant, so that Eq. (153) reduces to:

$$\varepsilon(x) = 1 - \frac{\omega_p^2}{\omega^2} \Theta(x).$$

# Vacuum field solution

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario  
Ionization  
Collisional  
Absorption  
Helmholtz  
equations

**Normal skin  
effect**

Heating and  
Thermal  
Conduction  
Hydrodynamics

In the vacuum region ( $x < 0$ ), the electric field thus has the solution

$$E_z = 2E_0 \sin(kx \cos \theta + \phi), \quad (161)$$

where  $k = \omega/c$ ,  $E_0$  is the amplitude of the laser field and  $\phi$  a phase factor still to be determined.



# Evanescent field solution

Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization

Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

In the overdense region, the field is evanescent:

$$E_z = E(0) \exp(-x/l_s), \quad (162)$$

where

$$l_s = \frac{c}{\omega_p} \left( 1 - \frac{\omega^2}{\omega_p^2} \cos^2 \theta \right)^{-1/2}. \quad (163)$$

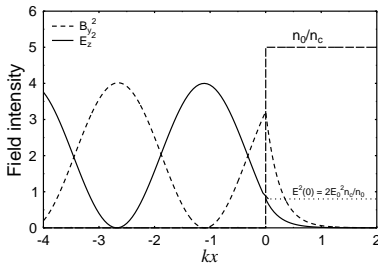
– *collisionless skin-depth*. In highly overdense limit,  $n_0/n_c \gg 1$ , we have  $l_s \simeq c/\omega_p$ .

# Phase matched overall solution

To complete our solution, we match up Eq. (161) and Eq. (162) together with their derivatives at the boundary  $x = 0$ . This gives:

$$E(0) = 2E_0 \frac{\omega}{\omega_p} \cos \theta$$

$$\tan \phi = -l_s \frac{\omega}{c} \cos \theta.$$



Interaction with  
Solids:  
Overdense  
Plasmas

Short pulse  
interaction  
scenario

Ionization  
Collisional  
Absorption

Helmholtz  
equations

Normal skin  
effect

Heating and  
Thermal  
Conduction

Hydrodynamics

# Reflectivity: Fresnel equations

Fresnel equations for light reflectivity on a conducting surface:

$$R_s = \left| \frac{\sin(\theta - \theta_t)}{\sin(\theta + \theta_t)} \right|^2, \quad \text{for } s\text{-light} \quad (164)$$

and

$$R_p = \left| \frac{\tan(\theta - \theta_t)}{\tan(\theta + \theta_t)} \right|^2, \quad \text{for } p\text{-light} \quad (165)$$

where  $\theta$  is the angle of incidence as before, and

$$\theta_t = \sin^{-1} \left\{ \frac{\sin \theta}{n} \right\}$$

is the generalized, complex angle of the transmitted light rays (from Snell's law).

# Reflectivity: example

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Plasmas

Short pulse  
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The refractive index  $n = \sqrt{\epsilon}$  can be obtained from Eq. (153) as before, setting the density equal to the solid density – *Drude model*.

## Example

Solid aluminium target:  $Z^* = 3$

$$n_e \simeq 2 \times 10^{23} \text{ cm}^{-3}$$

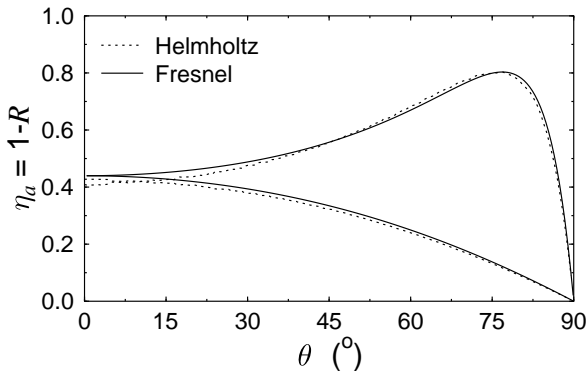
Ti-Sa laser:  $\lambda_L = 0.8 \mu\text{m}$

$$n_e/n_c \simeq 100.$$

Assume the plasma is initially heated to 120 eV, so that according to Eq. (144), we have  $\nu/\omega = 5$  at the maximum density.

# Reflectivity: example

The resulting absorption curves calculated from Eqs. (164) and (165) are shown in Fig. 7 along with numerical solution of the Helmholtz equations for an exponential profile with  $L/\lambda = 0.001$ .



# Thermal transport

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Energy transport equation for a collisional plasma (?)

$$\frac{\partial \epsilon}{\partial t} + \nabla \cdot (\mathbf{q} + \Phi_a) = 0, \quad (166)$$

where  $\epsilon$  is the energy density,  $\mathbf{q}$  is the heat flow and  $\Phi_a = \eta_a \Phi_L$  is the absorbed laser flux.

# Heating rate

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If penetration depth of the heat wave  $l_h < l_s = c/\omega_p$  (skin depth), then the thermal transport can be neglected.

Volume heated simultaneously:  $V \simeq l_s \pi \sigma^2$ .

Setting  $\epsilon = \frac{3}{2} n_e k_B T_e$  and  $\nabla \cdot \Phi_a \sim \Phi_a / l_s$ , have

$$\frac{dT_e}{dt} \simeq \frac{\Phi_a}{n_e l_s}, \quad (167)$$

or

$$\frac{d}{dt}(k_B T_e) \simeq 4 \frac{\Phi_a}{\text{Wcm}^{-2}} \left( \frac{n_e}{\text{cm}^{-3}} \right)^{-1} \left( \frac{l_s}{\text{cm}} \right)^{-1} \text{keV fs}^{-1}. \quad (168)$$

# Onset of transport: heat carried into target

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After a few femtoseconds, huge temperature gradients generated: heat is carried away from the surface into the colder target material according to Eq. (166). For ideal plasmas, we write

$$\epsilon = \frac{3}{2} n_e k_B T_e$$

as before, and

$$q(x) = -\kappa_e \frac{\partial T_e}{\partial x}, \quad (169)$$

which is the usual Spitzer-Härm heat-flow (??).



# Spitzer-Härm heat-flow

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Substituting for  $\epsilon$  and  $q$  in Eq. (166) and restricting ourselves to 1D by letting  $\nabla = (\partial/\partial x, 0, 0)$ , gives a *diffusion* equation for  $T_e$ :

$$\frac{3}{2} n_e k_B \frac{\partial T_e}{\partial t} = \frac{\partial}{\partial x} \left( \kappa_e \frac{\partial T_e}{\partial x} \right) + \frac{\partial \Phi_L}{\partial x}. \quad (170)$$

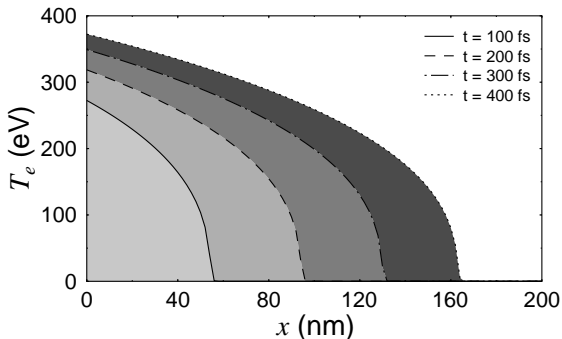
$\kappa_e$  is known as the Spitzer thermal conductivity and is given by:

$$\kappa_e = 32 \left( \frac{2}{\pi} \right)^{1/2} \frac{n_e}{\nu_0 m^{5/2}} T_e^{5/2}, \quad (171)$$

where

$$\nu_0 = \frac{2\pi n_e Z e^4 \log \Lambda}{m^2}.$$

# Nonlinear heat wave



**Figure:** Nonlinear heat-wave advancing into a semi-infinite, solid-density plasma. The curves are obtained from the numerical solution of the Spitzer heat flow equation for constant laser absorption at the target surface (left boundary).

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# Hydrodynamic plasma simulation

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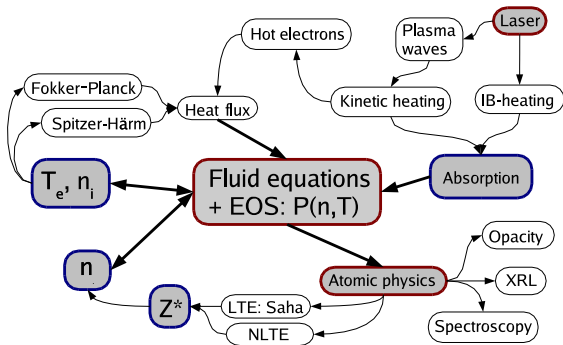


Figure: Building blocks of a hydrodynamic laser-plasma simulation model.