Holographic Peierls metal insulator transition

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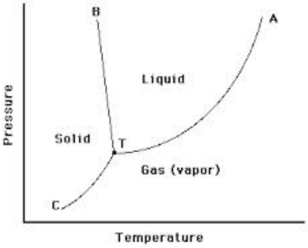
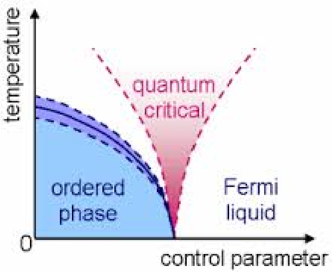
1. Introduction and motivation
2. Metal-insulator transition in CMP
3. Numerics for differential equations
4. Holographic model of charge density waves
5. Optical conductivity of holographic CDW
6. Conclusion and outlook

- The physical world is partially unified by remarkable RG flow in QFT
  - High Energy Physics: IR→UV(Reductionism)
  - Condensed Matter Physics: UV→IR(Emergence)
    - Thermal Phase Transition
    - Quantum Phase Transition





- Another seemingly distinct part is gravitation, which is understood as geometry by general relativity

Remarkably, with AdS/CFT correspondence, general relativity can also geometrize renormalization flow in particular when the quantum field theory is strongly coupled, namely

$$GR = RG.$$

In this sense, the world is further unified by AdS/CFT duality. This talk will focus on its particular application to condensed matter physics by general relativity.

Metal-insulator transition by holographic charge density waves

Conductivity

- Metal

$$\frac{dp}{dt} = qE - \frac{p}{\tau} \Rightarrow p(\omega) = \frac{q\tau E(\omega)}{1 - i\omega\tau} \Rightarrow$$
$$j(\omega) = nqv(\omega) = \frac{nqp(\omega)}{m} = \frac{nq^2\tau}{m(1-i\omega\tau)} E(\omega) \Rightarrow$$
$$\sigma(\omega) = \frac{nq^2\tau}{m(1-i\omega\tau)} = \frac{K\tau}{1-i\omega\tau},$$

where τ comes mainly from the interaction with impurities, defects, and phonons as well as the interaction between electrons. So

$$\sigma_{dc} = \sigma_0 - aT^2 - bT^5$$

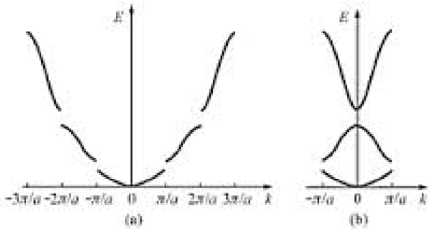
at low temperature.

- Insulator

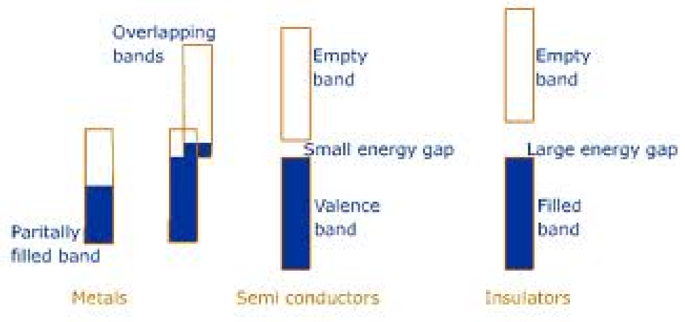
$$\sigma_{dc} = 0$$

at zero temperature and increases with temperature in the low temperature regime.

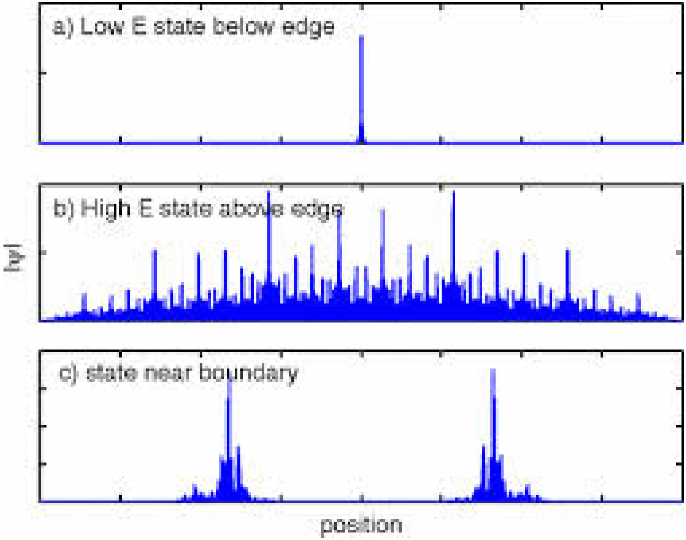
Band structure of electrons in a periodic potential with the period a



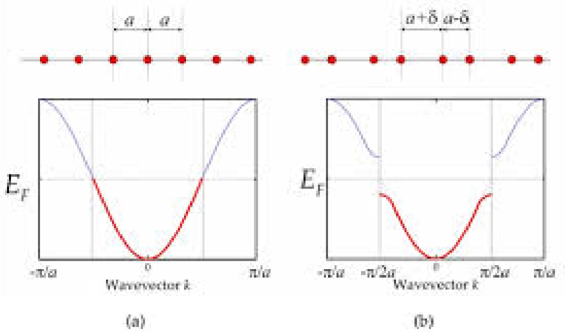
Bloch-Wilson insulator by band



Anderson insulator by disorder

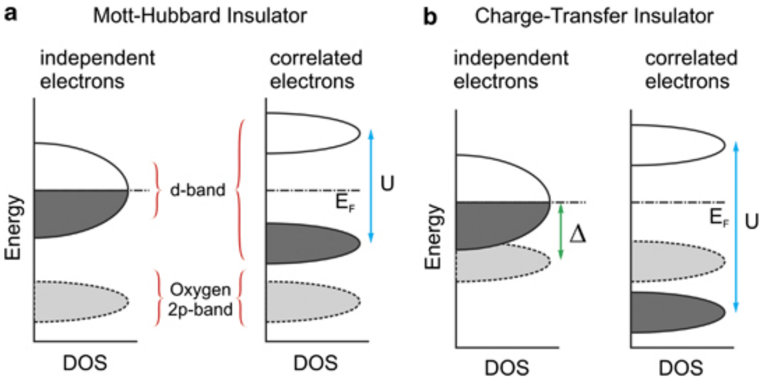


Peierls insulator by phonon



- Gapped single-particle excitation due to the opened gap at Fermi surface
- Gapless collective mode due to the spontaneous breaking of translation invariance by charge density waves, pinned

Mott insulator by the repulsive coulomb force between electrons



Pseudo-spectral method

By expanding the solution in terms of some sort of spectral functions, plugging it into eoms, and validating eoms at some grid points, the differential equations are replaced by a set of algebraic equations.

- The resultant solution thus has an analytical expression
- The numerical error goes like $\propto e^{-N}$ with N the number of grid points

complemented by three caveats

- The resultant algebraic equations are generically non-linear, so here comes Newton-Raphson method
- It proves convenient to employ Einstein-DeTurck method to get the stationary solutions to Einstein equation
- It turns out to be extremely time consuming to apply it in the time direction, if not impossible. Instead the finite difference methods such as Runge-Kutta or Crank-Nicolson method are often adopted

Action of model [Donos and Gauntlett,arXiv:1303.4398]

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[R+\frac{1}{L^2}-\frac{1}{4}t(\Phi)F^{\mu\nu}F_{\mu\nu}-\frac{1}{4}G^{\mu\nu}G_{\mu\nu}$$
$$-\frac{1}{2}(\partial_\mu\Phi\partial^\mu\Phi+m^2\Phi^2)-\frac{1}{2}u(\Phi)F^{\mu\nu}G_{\mu\nu}],$$

where $F=\mathrm{d}A$, $G=\mathrm{d}B$, $t(\Phi)=1-\frac{\beta}{2}L^2\Phi^2$, and $u(\Phi)=\frac{\gamma}{\sqrt{2}}L\Phi$.
For simplicity below we shall set $l^2=6L^2=\frac{1}{4}$, $m^2=-\frac{2}{l^2}=-8$, $\beta=-138$ and $\gamma=17.1$.

Background I

$$ds^2 = \frac{1}{z^2}\left(-(1-z)f(z)dt^2 + \frac{dz^2}{(1-z)f(z)} + dx^2 + dy^2\right)$$

with

$$f(z) = 4(1+z+z^2-\frac{z^3\mu_1^2}{16}), \quad A_t = \mu_1(1-z), \quad B = 0, \quad \Phi = 0.$$

Note that near the AdS boundary($z = 0$)

$$ds^2 \to \frac{1}{4z^2}[dz^2 + 16dt^2 + 4(dx^2 + dy^2)],$$

which gives rise to

$$(t_b, x_b, y_b) = \alpha(4t, 2x, 2y)$$

with α the globle scaling factor associated with the redefinition of the radial coordinate as $z' = \alpha z$. Thus

$$\mu = \frac{\mu_1}{4\alpha}, \qquad \frac{T}{\mu} = \frac{48-\mu_1^2}{16\pi\mu_1}.$$

Background II

$$ds^2 = \frac{1}{z^2}[-(1-z)f(z)Qdt^2 + \frac{Sdz^2}{(1-z)f(z)} + Vdy^2 + P(dx + z^2 U dz)^2],$$
$$A = \mu_1(1-z)\psi dt, \quad B = (1-z)\chi dt, \quad \Phi = z\phi,$$

where the eight variables involved in the ansatz are functions of x and z.

$$Q[x,0] = S[x,0] = P[x,0] = V[x,0] = \psi[x,0] = 1,$$
$$U[x,0] = \chi[x,0] = \phi[x,0] = 0.$$

The striped solution shows up at $T_c = 0.078\mu$ and $k_c = 0.325\mu$. The onset of CDW can be read off explicitly from the component of the gauge field B_t by holography as

$$B_t = -\rho(x)z + O(z^2),$$
$$\rho(x) = \rho_0 + \rho_1 \cos[k_c x] + ... + \rho_n \cos[nk_c x] + ...$$

Background II

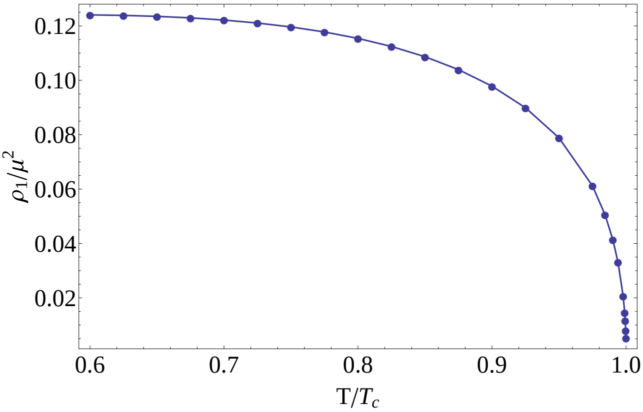


Figure: The first mode of CDW as a function of temperature.

Background II

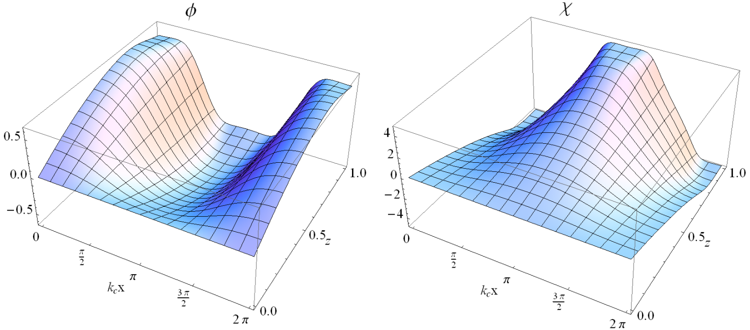


Figure: Solutions of the scalar field and the time component of the gauge field χ for $T = 0.8T_c$.

Turn on the fluctuations of the form $e^{-i\omega t}$ on top of the background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, A_\mu = \bar{A}_\mu + a_\mu, B_\mu = \bar{B}_\mu + b_\mu . \Phi = \bar{\Phi} + \varphi.$$

Solve the linear perturbation equations together with de Donder gauge and Lorentz gauge condition

$$\bar{\nabla}^\mu \hat{h}_{\mu\nu} = 0, \qquad \bar{\nabla}^\mu a_\mu = 0, \qquad \bar{\nabla}^\mu b_\mu = 0$$

where $\hat{h}_{\mu\nu} = h_{\mu\nu} - h\bar{g}_{\mu\nu}/2$ is the trace-reversed metric perturbation. In particular, $b_x = (1 + j_x(x)z + ...)e^{-i\omega t}$ can be solved with the ingoing boundary condition at the horizon and the following consistent boundary condition at the AdS boundary

$$b_x(x,0) = 1, \quad a_x(x,0) = \frac{\partial_z \chi(x,0)}{\mu_1(1 - \partial_z \psi(x,0))}, others(x,0) = 0.$$

Then the homogeneous part of conductivity is given by holography as

$$\sigma(\omega/\mu) = \frac{4j_x^{(0)}}{i\omega}.$$

- Pinned collective mode
- Gapped single-particle excitation

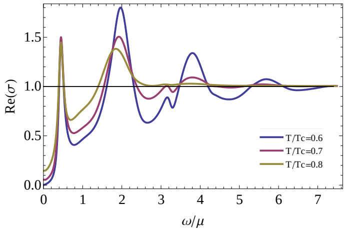


Figure: The optical conductivity for CDW, where the black horizontal line denotes the corresponding optical conductivity for AdS-RN black hole associated with the second gauge field.

$$\frac{d^2x}{dt^2} = \frac{qE}{m} - \frac{1}{\tau}\frac{dx}{dt} - \omega_0^2 x \Rightarrow$$
$$\sigma_{CDW}(\omega) = \frac{K\tau}{1 - i\omega\tau(1 - \omega_0^2/\omega^2)}$$

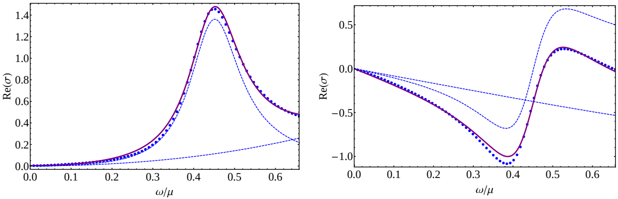


Figure: The fit of optical conductivity with two Lorentz oscillators in the low frequency regime for $T = 0.6T_c$. The contributions from the individual oscillator are also plotted with dashed lines.

The magnitude of single particle gap is estimated as

$$2\Delta/T_c \approx 20.51$$

by locating the position of the second minimum in the imaginary part of the conductivity, which is obviously much larger than the mean-field BCS value

$$2\Delta/T_c \approx 3.52,$$

but comparable to the values for some CDW materials such as the single crystalline TbTe3 compound whose gap is given by

$$2\Delta/T_c \approx 15.80.$$

Conclusion

- The first construction of a gravity dual to CDW.
- The first calculation of optical conductivity for holographic CDW, where the two fundamental features of CDW are reproduced.
- The first implementation of Peierls metal-insulator transition by a gravity dual.
- The comparability of holographic gap with real CDW materials suggests a promising window for one to understand CDW by holography.

Outlook

Holographic Anderson localization?

Thanks for your attention!