Twist-3 factorization for q_T integrated Drell-Yan processes

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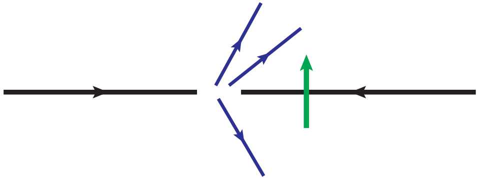
Outline

1. Introduction to SSA;
2. Collinear expansion;
3. Tree level calculation;
4. Summary and Outlook.

Single transverse Spin Asymmetry(SSA)

- Hadroproduction: $P_A(S_\perp) + P_B \to h + X$;
  L.Bland,1410.1140
- Semi-inclusive DIS: $e + P(S_\perp) \to e' + h + X$;
  Hermes Collaboration,0906.3918; Compass Collaboration,1205.5121;1205.5122
- Polarized Drell-Yan(DY): $P_A(S_\perp) + P_B \to \gamma^* + X \to l^+ l^- + X$.
  Prepared...



Explanation for SSA

Factorization:

$$\sigma = H(x_a, x_b, z_h) \otimes f_A(x_a) \otimes f_B(x_b) \otimes D_h(z_h) + \mathcal{O}(\frac{\Lambda_{QCD}}{Q}), \qquad (1)$$

Q: the hard scale in the hard scattering, $\sqrt{s}$ or $P_{h\perp}$. **PDFs**:

$$\int \frac{d\xi^-}{4\pi} e^{i\xi^- xP^+} \langle P, s|\bar{\psi}(0)\gamma^\mu \psi(\xi^-)|P, s\rangle = f_1(x) l^\mu,$$

$$\int \frac{d\xi^-}{4\pi} e^{i\xi^- xP^+} \langle P, s|\bar{\psi}(0)\gamma^\mu \gamma_5 \psi(\xi^-)|P, s\rangle = g_1(x) S^+ l^\mu + \frac{M}{P^+} g_T(x) S_\perp^\mu,$$

$$\int \frac{d\xi^-}{4\pi} e^{i\xi^- xP^+} \langle P, s|\bar{\psi}(0)\psi(\xi^-)|P, s\rangle = \frac{M}{P^+} e(x),$$

$$\int \frac{d\xi^-}{4\pi} e^{i\xi^- xP^+} \langle P, s|\bar{\psi}(0)\sigma^{\mu\nu} i\gamma_5 \psi(\xi^-)|P, s\rangle = h_1(x) S_\perp^{(\mu} l^{\nu)} + \frac{M}{P^+} h_L(x) S^+ l^{(\mu} n^{\nu)}. \qquad (2)$$

Light-cone coordinates: $a^\mu = a^+ l^\mu + a^- n^\mu + a_\perp^\mu$, $n \cdot l = 1$, $n^2 = l^2 = 0$.

Twist-3 factorization

- Twist-2 factorization: failed to explain SSA;
- Reason: Massless QCD conserves the helicity of fermion;
- Twist-3 factorization: $g_T(x)$?

However, $g_T(x)$ is not renormalizable, thus not proper to be used to factorize SSA. X.Ji&J.Osborne0102026
More general twist-3 distribution functions(Qiu-Sterman matrix elements) are required, J.Qiu&G.Sterman,PRL67(1991)2264;NPB378(1992)52

$$\begin{aligned} T_F(x_1,x_2)\tilde{s}_\perp^\mu =& g_s \int \frac{dy_1 dy_2}{4\pi} e^{i(-x_1y_1+x_2y_2)P^+} \langle Ps|\bar{\psi}(y_1 n)\gamma^+ G_\perp^{+\mu}(0)\psi(y_2 n)|Ps\rangle \\ T_F^\sigma(x_1,x_2) =& g_s \int \frac{dy_1 dy_2}{4\pi} e^{i(-x_1y_1+x_2y_2)P^+} \langle Ps|\bar{\psi}(y_1 n) i\gamma_{\perp\mu}\gamma^+ G_\perp^{+\mu}(0)\psi(y_2 n)|Ps\rangle . \end{aligned} \tag{3}$$

Polarized DY: $q_\perp$-integrated

Feature: There is only one hard scale $Q^2 = (l_1 + l_2)^2$.

$$P_A(S_\perp) + P_B \to \gamma^*(q) + X \to l(k_1)\bar{l}(k_2) + X, \tag{4}$$

where $P_{A,B}$ are along z-axis, spin vector is perpendicular to P_A. In light-cone coordinates,

$$P_A^\mu = P_A^+ l^\mu + \frac{M_A^2}{2P_A^+} \simeq P_A^+ l^\mu, \quad P_B^\mu = P_B^- n^\mu + \frac{M_B^2}{2P_B^-} \simeq P_B^- n^\mu, \tag{5}$$

where for any four vector $a^\mu = a^+ l^\mu + a^- n^\mu + a_\perp^\mu$, $n \cdot l = 1$, $n^2 = l^2 = 0$.

$$\begin{aligned} \frac{d\sigma(S_\perp)}{dQ^2 d\Omega} =& \frac{\alpha^2}{4SQ^4} \int d^4 q \delta(q^2 - Q^2) L_{\mu\nu} W^{\mu\nu} \\ =& \frac{\alpha^2}{8Q^4} \int dx dy \delta(xyS - Q^2) \int d^2 q_\perp L_{\mu\nu} W^{\mu\nu}, \end{aligned} \tag{6}$$

where $x = \frac{q^2}{2P_A \cdot q} \simeq \frac{q^+}{P_A^+}$, $y = \frac{q^2}{2P_B \cdot q} \simeq \frac{q^-}{P_B^-}$.

Hadronic tensor:

$$W^{\mu\nu} = \int \frac{d^4x}{(2\pi)^4} e^{iq\cdot x} \langle P_A S_A, P_B | j^\mu(x) j^\nu(0) | P_A S_A, P_B \rangle,$$
$$j^\mu = \bar{\psi}\gamma^\mu\psi. \tag{7}$$

Leptonic tensor:

$$L^{\mu\nu} = 2(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu} Q^2). \tag{8}$$

The cross section is defined in Collins-Soper(CS) frame, which is the rest frame of lepton pair.
The angle of one lepton is defined as:

$$k_1^\mu = \frac{Q}{2}(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

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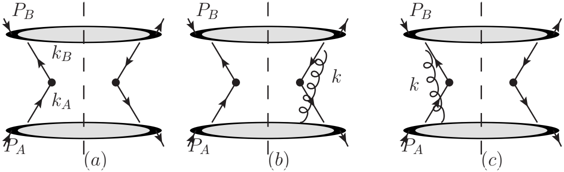
Twist-3 factorization



Three lowest order diagrams contributing to SSA. The dot means QED interaction. The bubbles represent all possible Feynman diagrams.

Basic formula:

$$\frac{d\sigma(S_\perp)}{dQ^2 d\Omega} = H_1 \otimes \bar{q} \otimes T_F + H_2 \otimes T_F^\sigma \otimes h_1 \tag{9}$$

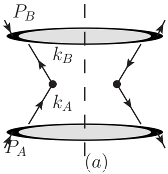
Lowest order diagrams

Basic assumption: All twist-3 contribution comes from collinear expansion.

- All partons are collinear, either to P_A or to P_B;
- Expansion in the transverse momenta of partons, including fermions and gluons.

Note: One can explicitly *check* this assumption *order by order*.

Example: Born diagram



$$W_a^{\mu\nu} = \int d^4k_A d^4k_B \delta^4(k_A + k_B - q)\gamma^{\mu}_{mi}\gamma^{\nu}_{jn} \int \frac{d^4\xi_a}{(2\pi)^4} e^{i\xi_a\cdot k_A} \langle P_A S_A|\bar{q}_j(0)q_i(\xi_a)|P_A S_A\rangle$$
$$\int \frac{d^4\xi_b}{(2\pi)^4} e^{i\xi_b\cdot k_B} \langle P_B|q_n(0)\bar{q}_m(\xi_b)|P_B\rangle, \quad (10)$$

Example: Born diagram

$$W_a^{\mu\nu} = \int d^4 k_A d^4 k_B \delta^4(k_A + k_B - q)\gamma^\mu_{mi}\gamma^\nu_{jn} \int \frac{d^4\xi_a}{(2\pi)^4} e^{i\xi_a\cdot k_A}\langle P_A S_A|\bar{q}_j(0)q_i(\xi_a)|P_A S_A\rangle$$
$$\int \frac{d^4\xi_b}{(2\pi)^4} e^{i\xi_b\cdot k_B}\langle P_B|q_n(0)\bar{q}_m(\xi_b)|P_B\rangle, \tag{11}$$

- Twist-2 contribution comes ONLY from *collinear region*;
- Twist-3 contribution can be extracted from *collinear expansion* of hard part to the next to leading power($\mathcal{O}(\lambda)$);
- Collinear region:

$$k_A^\mu = (k_A^+, k_A^-, k_{A\perp}^\mu) \sim Q(1, \lambda^2, \lambda),$$
$$k_B^\mu = (k_B^+, k_B^-, k_{B\perp}^\mu) \sim Q(\lambda^2, 1, \lambda), \ \lambda = \frac{\Lambda_{QCD}}{Q}. \tag{12}$$

Collinear expansion: Born diagram

Collinear limit for antiquark:
$\delta^4(k_A + k_B - q) \simeq \delta(k_A^+ - q^+)\delta(k_B^- - q^-)\delta^2(k_{A\perp} - q_\perp)$.

$$\begin{aligned} W_a^{\mu\nu} &= \int d^4k_A \delta(k_A^+ - q^+)\delta(k_{A\perp} - q_\perp)\gamma^\mu_{mi}\gamma^\nu_{jn} \int \frac{d^4\xi_a}{(2\pi)^4} e^{i\xi_a \cdot k_A} \langle P_A S_A | \bar{q}_j(0) q_i(\xi_a) | P_A S_A \rangle \\ &\quad \int d^4k_B \delta(k_B^- - q^-) \int \frac{d^4\xi_b}{(2\pi)^4} e^{i\xi_b \cdot k_B} \langle P_B | q_n(0) \bar{q}_m(\xi_b) | P_B \rangle \\ &= -\frac{1}{2N_c} \bar{q}(y) \int d^4k_A \delta(k_A^+ - q^+)\delta(k_{A\perp} - q_\perp)\gamma^\mu_{mi}(\gamma^\nu\gamma^+)_{jn} \\ &\quad \int \frac{d^4\xi_a}{(2\pi)^4} e^{i\xi_a \cdot k_A} \langle P_A S_A | \bar{q}_j(0) q_i(\xi_a) | P_A S_A \rangle, \end{aligned} \tag{13}$$

where $\int \frac{d\xi_b^+}{2\pi} e^{i\xi_b^+ k_B^-} \langle P_B | q_n(0) \bar{q}_m(\xi_b) | P_B \rangle = -\frac{1}{2N_c}[\bar{q}(y)\gamma^+ + \cdots]$.

Collinear expansion: Born diagram

Collinear expansion for quark,

$$k_A^+ \sim \mathcal{O}(1), \;\; k_{A\perp} \sim \mathcal{O}(\lambda),$$
$$\delta(k_{A\perp} - q_\perp) = \delta(q_\perp) - \frac{\partial \delta(q_\perp)}{\partial q_\perp^\rho} k_{A\perp}^\rho + \mathcal{O}(\lambda). \tag{14}$$

Integrating over $k_{A\perp}$ and k_A^-, the twist-3 contribution of $\delta(q_\perp)$-term concerns two distributions, $\langle P, s|\bar{q}(0)(\gamma_\perp^\rho, \gamma_\perp^\rho \gamma_5)q(\xi^-)|P, s\rangle$. But,

- Using PT transformation, one can show

  $$\langle PS_A|\bar{q}(0)\gamma_\perp^\rho q(\xi^-)|PS_A\rangle = \langle P, -S_A|\bar{q}(0)\gamma_\perp^\rho q(\xi^-)|P, -S_A\rangle,$$

  which is spin independent;
- $\gamma_\perp^\rho \gamma_5$ corresponds to distribution $g_T(x)$, but γ_5 causes μ, ν to be antisymmetric.

Collinear expansion: Born diagram

Only derivative term *may* give twist-3 contribution,

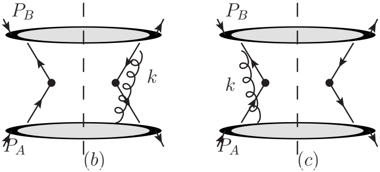
$$W^{\mu\nu} = \frac{i}{4N_c}\bar{q}(y)\int dk_A^+\delta(k_A^+ - q^+)Tr(\gamma^+\gamma^\mu\gamma^-\gamma^\nu)\frac{\partial\delta(q_\perp)}{\partial q_\perp^\rho}$$
$$\int \frac{d\xi_a^-}{4\pi}e^{i\xi_a^- k_A^+}\langle P_A S_A|\bar{q}(0)\gamma^+\partial_\perp^\rho q(\xi_a^-)|P_A S_A\rangle. \quad (15)$$

Once again, PT invariance leads to $\langle PS_A|\bar{q}(0)\gamma^+\partial_\perp^\rho q(\xi^-)|PS_A\rangle = \langle P, -S_A|\bar{q}(0)\gamma^+\partial_\perp^\rho q(\xi^-)|P, -S_A\rangle$, which is also spin independent.

Conclusion: Born diagram has no twist-3 contribution.

One gluon exchanged diagrams

Following the same procedure, one can calculate the twist-3 contribution of Fig.b,c,



The momentum of the gluon is k, and going **into** the EM vertex in both diagrams. The bubbles represent all possible Feynman diagrams.

One gluon exchanged diagrams

$$
\begin{aligned}
W_b^{\mu\nu} =& \frac{1}{N_c k_B^-} \bar{q}(y) \int \frac{dk^+}{k^+ + i\epsilon} \frac{i}{2} g_s \int \frac{d\xi_a^- d\xi^-}{(2\pi)^2} e^{i\xi^- k^+ + i\xi_a^- k_A^+} \\
& [k_B^- g_\perp^{\mu\nu} \frac{\partial \delta(q_\perp)}{\partial q_\perp^\rho} \langle \bar{q}(0)\gamma^+ G^+(\xi^-)(\partial_\perp^\rho q)(\xi_a^-)\rangle \\
& + \delta^2(q_\perp)\langle \bar{q}(0)\gamma^+ (\partial^+ G_\perp^\mu - \partial_\perp^\mu G^+)(\xi^-) l^\nu q(\xi_a^-)\rangle], \\
W_c^{\mu\nu} =& \frac{1}{N_c k_B^-} \bar{q}(y) \int \frac{dk^+}{k^+ + i\epsilon} \frac{i}{2} g_s \int \frac{d\xi_a^- d\xi^-}{(2\pi)^2} e^{i\xi^- k^+ + i\xi_a^- k_A^+} \\
& [-k_B^- g_\perp^{\mu\nu} \frac{\partial \delta(q_\perp)}{\partial q_\perp^\rho} \langle \bar{q}(0)\gamma^+ \partial_\perp^\rho (G^+(\xi^-) q(\xi_a^-))\rangle \\
& + \delta^2(q_\perp)\langle \bar{q}(0)\gamma^+ (\partial^+ G_\perp^\nu - \partial_\perp^\nu G^+)(\xi^-) l^\mu q(\xi_a^-)\rangle].
\end{aligned} \tag{16}
$$

Using PT transformation, one can show $[\cdots]$ is **real**. Thus, in order to obtain a real $W^{\mu\nu}$, one should take the discontinuity of the amplitudes,

$$
\frac{1}{k^+ + i\epsilon} \to -i\pi\delta(k^+). \tag{17}
$$

Non-derivative part

In order to obtain a real $W^{\mu\nu}$, one should take the discontinuity of the amplitudes, *i.e.*,

$$\frac{1}{k^+ + i\epsilon} \to -i\pi\delta(k^+). \tag{18}$$

Then non-derivative part:

$$W^{\mu\nu}_{b+c} \supset \frac{\pi}{N_c k_B^-} \bar{q}(y)\delta(q_\perp)\frac{1}{2}g_s \lim_{k^+\to 0} \int \frac{d\xi_a^- d\xi^-}{(2\pi)^2} e^{i\xi_a^- k_A^+ + i\xi^- k^+} \\ \langle \bar{q}(0)\gamma^+(G_\perp^{+\mu} l^\nu + G_\perp^{+\nu} l^\mu)(\xi^-) q(\xi_a^-)\rangle, \tag{19}$$

which satisfies color gauge invariance automatically. But the derivative term is not the case.

Derivative term

Derivative part:

$$W^{\mu\nu}_{b+c} \supset -\frac{1}{N_c}\bar{q}(y)g^{\mu\nu}_{\perp}\frac{\partial\delta(q_{\perp})}{\partial q^{\rho}_{\perp}}\frac{1}{2}g_s\int dk^{+}\delta(k^{+})\int\frac{d\xi^{-}_{a}d\xi}{4\pi}e^{i\xi^{-}_{a}k^{+}_{A}+i\xi^{-}k^{+}}$$
$$\langle\bar{q}(0)\gamma^{+}(\partial^{\rho}_{\perp}G^{+})(\xi^{-})q(\xi^{-}_{a})\rangle. \quad (20)$$

Thanks to the factor $\delta(k^{+})$, one can write the corresponding distribution into gauge invariant form, $\partial^{\rho}_{\perp}G^{+} = -G^{+\rho}_{\perp} + \partial^{+}G^{\rho}_{\perp}$,

$$\delta(k^{+})\int d\xi^{-}e^{i\xi^{-}k^{+}}\partial^{+}G^{\rho}_{\perp}(\xi^{-}) = \delta(k^{+})\left(G^{\rho}_{\perp}(\infty^{-}) - G^{\rho}_{\perp}(-\infty^{-})\right) = 0. \quad (21)$$

Now the derivative term can be written into gauge invariant form through the replacement $\partial^{\rho}_{\perp}G^{+} \to -G^{+\rho}_{\perp}$.

Another way

One may ask how to recover the nonlinear term in field strength tensor, i.e., $G^+G^\rho_\perp$. It is difficult, because in derivative term, all coherent gluons are G^+-gluon. The derivative on the fermion field may be helpful, since $\partial^\rho_\perp\psi = D^\rho_\perp\psi - igG^\rho_\perp\psi$ and $D_\perp$-term does not contribute to SSA due to PT invariance.

However, there is a more elegant way. Remember that the number of G^+-gluon connecting hard part and the jet can be infinity. And all these gluons can be summed up into the gauge link. In this sense, Fig.a,b,c all are the same, and the derivative term is

$$W^{\mu\nu} = \frac{i}{N_c}\bar{q}(y)g_\perp^{\mu\nu}\frac{\partial\delta(q_\perp)}{\partial q^\rho_\perp}\int\frac{d\xi_a^-}{4\pi}e^{i\xi_a^- q^+}\langle\bar{q}(0)\gamma^+\partial^\rho_\perp\left(\mathcal{L}_n^\dagger(\xi_a^-)q(\xi_a^-)\right)\rangle, \tag{22}$$

where $\mathcal{L}_n(x) \equiv Pexp[-ig_s\int_{-\infty}^0 d\lambda G^+(x+\lambda n)]$.

Gauge link

A useful identity:

$$\partial_\perp^\rho \left(\mathcal{L}_n^\dagger q\right)(\xi^-) = \mathcal{L}_n^\dagger D_\perp^\rho q(\xi^-) - ig_s \int d\lambda \theta(-\lambda) \left(\mathcal{L}_n^\dagger G_\perp^{+\rho} \mathcal{L}_n\right)(\lambda n + \xi^-)\mathcal{L}_n^\dagger q(\xi^-), \tag{23}$$

which can be derived from EOM: $n \cdot D\mathcal{L}_n = 0$.
Ignoring $D_\perp$-term and using the definition of θ function

$$\theta(-\lambda) = \frac{-i}{2\pi} \int \frac{d\omega}{\omega + i\epsilon} e^{-i\omega\lambda}, \tag{24}$$

we have

$$W^{\mu\nu} \supset \frac{i}{N_c} \bar{q}(y) g_\perp^{\mu\nu} \frac{\partial \delta(q_\perp)}{\partial q_\perp^\rho} \int \frac{d\omega}{\omega + i\epsilon} \left[\frac{g_s}{2\pi} \int \frac{d\xi_a^-}{4\pi} e^{i\xi_a^- q^+} \int_{-\infty}^{\infty} d\lambda e^{i\lambda\omega} \right.$$
$$\left. \langle \bar{q}(0)\gamma^+ \left(\mathcal{L}_n^\dagger G_\perp^{+\rho}\mathcal{L}_n\right)(\lambda n + \xi_a^-)\mathcal{L}(\xi_a^-)_n^\dagger q(\xi_a^-))\rangle\right]. \tag{25}$$

The factor $[\cdots]$ is real from PT symmetry. Hence, we have to take the discontinuity of the other part.

$$W^{\mu\nu} \supset \frac{i}{N_c}\bar{q}(y)g_\perp^{\mu\nu}\frac{\partial\delta(q_\perp)}{\partial q_\perp^\rho}\int\frac{d\omega}{\omega+i\epsilon}\left[\frac{g_s}{2\pi}\int\frac{d\xi_a^-}{4\pi}e^{i\xi_a^- q^+}\int_{-\infty}^{\infty}d\lambda e^{i\lambda\omega}\right.$$
$$\left.\langle\bar{q}(0)\gamma^+\left(\mathcal{L}_n^\dagger G_\perp^{+\rho}\mathcal{L}_n\right)(\lambda n+\xi_a^-)\mathcal{L}(\xi_a^-)_n^\dagger q(\xi_a^-)\rangle\right]. \qquad (26)$$

The factor $[\cdots]$ is real from PT symmetry. Hence, we have to take the discontinuity of the other part.

$$W^{\mu\nu} \supset \frac{1}{N_c}\bar{q}(y)g_\perp^{\mu\nu}\frac{\partial\delta(q_\perp)}{\partial q_\perp^\rho}\frac{g_s}{2}\lim_{k^+\to 0}\int\frac{d\xi_a^- d\xi^-}{4\pi}e^{i\xi_a^- k_A^+ + i\xi^- k^+}$$
$$\langle\bar{q}(0)\gamma^+\left(\mathcal{L}_n^\dagger G_\perp^{+\rho}\mathcal{L}_n\right)(\xi^-)\mathcal{L}(\xi_a^-)_n^\dagger q(\xi_a^-)\rangle, \qquad (27)$$

which is gauge invariant, and there is no the noise of nonlinear term.

Final result

- Hadronic tensor:

$$W^{\mu\nu} = \frac{1}{2N_c}\bar{q}(y)T_F(x,x)\left[\frac{1}{q^-}(l^\mu \tilde{s}_\perp^\nu + l^\nu \tilde{s}_\perp^\mu)\delta^2(q_\perp) + g_\perp^{\mu\nu}\tilde{s}_\perp^\rho \frac{\partial \delta^2(q_\perp)}{\partial q_\perp^\rho}\right], \tag{28}$$

where $\tilde{s}_\perp^\mu \equiv \epsilon_\perp^{\mu\nu} s_\nu, \ \epsilon_\perp^{12} = 1.$

- EM gauge invariance: $\int d^2 q_\perp q_\mu W^{\mu\nu} F(q_\perp) = 0.$
- Cross section:

$$\frac{d\sigma}{dQ^2 d\Omega} = -\frac{\alpha^2 |S_\perp|}{8N_c Q^3} \sin 2\theta \sin(\phi - \phi_s) \int dx dy \delta(xyS - Q^2)\bar{q}(y)T_F(x,x). \tag{29}$$

Summary and Outlook

- Collinear expansion and color and EM gauge invariance;
- Tree level result;
- Generalize to Semi-inclusive DIS;
- One loop correction: very important examination of twist-3 factorization.