Renormalize the Yukawa Coupling (Y-1)

1) Consider

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu s_0\right)^2 - \frac{1}{2} m_{s_0}^2 s_0^2 + \bar{t}_0\left(i\not{\partial} - m_{t_0}\right)t_0 - y_0 \bar{t}_0 t_0 s_0 + \cdots$$

We first substitute those bare fields and couplings by

$$s_0 = Z_s^{1/2} s \qquad \left(m_{s_0}^2 = m^2 + \delta m^2 \right.$$

$$t_0 = Z_t^{1/2} t \qquad \left. m_{t_0} = m_t + \delta m_t \right)$$

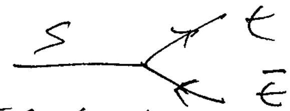
$$y_0 = y + \delta y \equiv y\left(1 + \frac{\delta y}{y}\right)$$

Hence,

$$\mathcal{L} = \frac{1}{2} Z_s \left(\partial_\mu s\right)^2 - \frac{1}{2}\left(m^2 + \delta m^2\right) Z_s\, s^2 + Z_t\, \bar{t}\left(i\not{\partial} - (m_t + \delta m_t)\right)t - (y + \delta y)\, Z_t Z_s^{1/2}\, \bar{t} t s$$

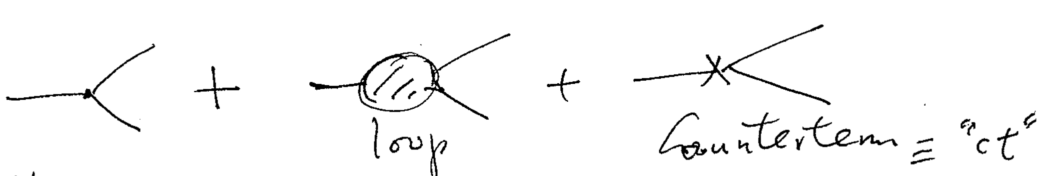
$$= \mathcal{L}_{\text{ren.}} + \mathcal{L}_{\text{counter-terms}}$$

2) Consider the case that the Yukawa coupling y is measured via the process



$\Rightarrow$ At tree level, its coupling is (y).

Beyond the tree level, we need to consider

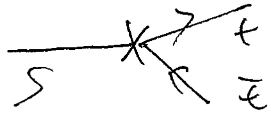


with the 1-P-I-diagrams.
(one-particle-irreducible)

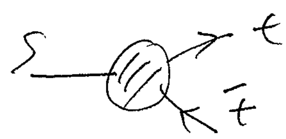
Define $Z_t = 1 + \delta Z_t$

$Z_s = 1 + \delta Z_s$

then



$= y\left(\frac{\delta y}{y} + \delta Z_t + \frac{1}{2}\delta Z_s\right)$



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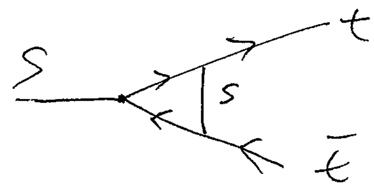
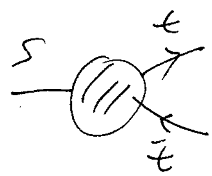


1-P-I diagrams

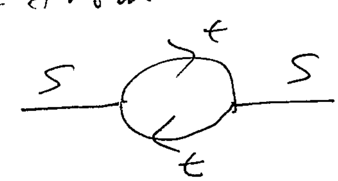
The renormalization procedure is such that the sum of the above two contributions becomes finite.

Hence

$(\delta y) =$  $- \left(\delta Z_t + \frac{1}{2}\delta Z_s\right)\cdot y$

where, δZ_t and δZ_s come from

and



3) Determine the running Yukawa Coupling.

Since $\frac{dy_0}{d\ln\mu} = 0$ (Bare coupling does not have μ-dependence.)

So, $\frac{d(y+\delta y)}{d\ln\mu} = 0 = \frac{dy}{d\ln\mu} + \frac{d(\delta y)}{d\ln\mu}$

$\Rightarrow \frac{dy}{d\ln\mu} = -\frac{d(\delta y)}{d\ln\mu}$, where $(\delta y) \sim \left(\frac{y^2}{16\pi^2}\right)\cdot y$ at one-loop order