Renormalization Procedures:

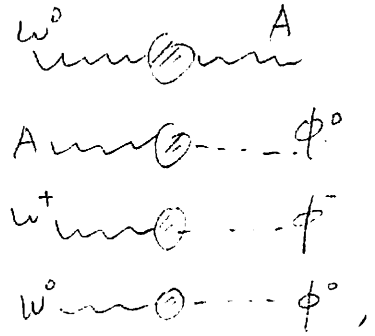
1. After we have done the one-loop calculations of the selfenergies, we are going to find out the renormalization procedures for the equivalence theorem.

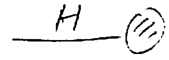
2 Recall that in R_ξ-gauge, the gauge fixing term is

$$\mathcal{L}_{gf} = -\frac{1}{\alpha_w}(-\partial w^+ + \alpha_w M \phi^+)(-\partial w^- + \alpha_w M \phi^-)$$
$$- \frac{1}{2\alpha_0}(-\partial w^0 + \alpha_0 M_0 \phi^0)^2 - \frac{1}{2\alpha_A}(-\partial A)^2 .$$

in tree level.

At the one loop level, there are mixing generated, e.g.



and the tadpole 

All of these "coupling terms" are vanished in the tree level. Therefore we should invent some terms, or mechanism into the one loop effective Lagrangian to absorb these divergences.

1) For the mixing of W^+-ϕ^- and W^0-ϕ^0, we can simply replace M by $\mathcal{M}_+$, and M_0 by $\mathcal{M}_0$ in $\mathcal{L}_{gf}$.

ie

$$-\frac{1}{\alpha_w}(-\partial w^+ + \alpha_w \mathcal{M}_+ \phi^+)(-\partial w^- + \alpha_w \mathcal{M}_- \phi^-) - \frac{1}{2\alpha_0}(-\partial w^0 + \alpha_0 \mathcal{M}_0 \phi^0)^2,$$

with $\mathcal{M}_- = (\mathcal{M}_+)^*$.

Also $\quad \mathcal{M}_+ = M_1 \quad$ and $\quad \mathcal{M}_0 = M_0 \quad$ at tree level

Notice that $\mathcal{M}_+$ and $\mathcal{M}_0$ are non-local terms, which will absorb the mixing between W and ϕ.

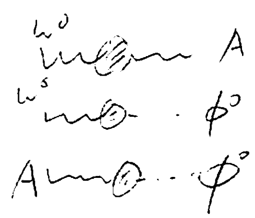
2) The tadpole H should be set to be zero to fix the parameter $\beta \equiv (\mu + \lambda f^2) = \mu + \frac{\lambda}{2} v^2$.
So that the self energies of ϕ^+ and ϕ^0, and H will be corrected due to the vanishing of tadpole. (Its physical meaning is that the vacuum expectation is also got shifted, therefore we should redefine the Higgs field so that $\langle H \rangle = 0$ at vacuum.)

3) The mixing between the photon A and unphysical Higgs ϕ^0 may be absorbed by introducing the gauge fixing term as

$$\frac{-1}{2\alpha_A}\left(-\partial A + \chi_A \mathcal{M}_A \phi^0\right)^2,$$

with $\quad \mathcal{M}_A = 0 \quad$ at tree level.

4) The mixing between W^0 and A is slightly more complicated. Because the mixing



are all coupled.

We should rerotate the fields W^0 and A, so that the mixing between W^0 and A is gone, while the mixing of W^0-ϕ^0 and A-ϕ^0 may be taken cared of using 1) and 3).

5) Recall that, at tree level,

$$\begin{pmatrix} W^0_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} B^3_\mu \\ C^0_\mu \end{pmatrix}.$$

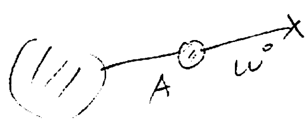
At one loop, one should rotate

$$\begin{pmatrix} W^0 \\ A \end{pmatrix} \rightarrow \begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{12}}{2} \\ \frac{a^{21}}{2} & 1+\frac{a^{22}}{2} \end{pmatrix} \begin{pmatrix} W^0 \\ A \end{pmatrix},$$

so that photon will remain massless.

This is going to be messy, but it can be done.

Notice that the rerotation is an one loop correction, ie $O(g^2)$, therefore up to one-loop calculation, we still use the tree Lagrangian vertices doing calculation, the only difference will be how to convert a Green's function to a S-matrix. Because after we rerotate the fields W^0 and A, there is no mixing between W^0 and A and more, ie the diagrams



vanish.

For W^+

3. The bare Lagrangian is

$$-\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - M^2 W_\mu^+ W_\mu^- - \frac{1}{\alpha}(-\partial_\mu W_\mu^+ + \alpha m_+ \phi^+)(-\partial_\nu W_\nu^- + \alpha m_- \phi^-)$$

$$= W_\mu^+ \partial_\nu \partial_\nu W_\mu^- - W_\mu^+ \partial_\mu \partial_\nu W_\nu^- + \frac{1}{\alpha} W_\mu^+ \partial_\mu \partial_\nu W_\nu^- - W_\mu^+ M^2 W_\mu^-$$
$$+ m_+ \phi^+ \partial_\mu W_\mu^- + m_- \phi^- \partial_\nu W_\nu^+ - \alpha m_+ m_- \phi^+ \phi^-$$

1) Recall that

$$m_+ = M + \tilde{m}_+ ,$$

and $$m_- = M + \tilde{m}_- = (m_+)^* .$$

So $\tilde{m}_- = (\tilde{m}_+)^*$.

Also $\tilde{m}_+ = \tilde{m}_- = 0$, at tree level. Therefore $\tilde{m}_+$ and $\tilde{m}_-$ are of the order of $O(g^2)$, they are at the same order as δZ_ϕ, δZ_W etc.

To restore the tree Lagrangian in R_ξ-gauge, we should express

$$m_+ \phi^+ \partial W^- + m_- \phi^- \partial W^+ = M \phi^+ \partial W^- + M \phi^- \partial W^+$$
$$+ \tilde{m}_+ \phi^+ \partial W^- + \tilde{m}_- \phi^- \partial W^+ .$$

(*) Note. In fact, this is equivalent to say that:

We start from the R_ξ-gauge, i.e.

$$-\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - M^2 W^+ W^- - \frac{1}{\alpha}(-\partial W^+ + \alpha M \phi^+)(-\partial W^- + \alpha M \phi^-).$$

After the one loop calculation, we get the mixing between ϕ and W, so we introduce the counterterm

$$\tilde{m}_+ \phi^+ \partial W^- \quad \text{and} \quad \tilde{m}_- \phi^- \partial W^+$$

to absorb the infinities. And $\alpha \tilde{m}_+ \tilde{m}_- \phi^+ \phi^-$ also has to be introduced.

2) After we express the bare quantities in terms of renormalized quantities, we have

$$\alpha \to (1+\delta\alpha)\alpha$$
$$\omega^+ \to (1+\tfrac{1}{2}\delta Z_\omega)\omega^+ \equiv Z_\omega^{1/2}\omega^+$$
$$\omega^- \to (1+\tfrac{1}{2}\delta Z_\omega)\omega^- \equiv Z_\omega^{1/2}\omega^-$$

Thus

$$\frac{1}{\alpha}\omega_\mu^+\partial_\mu\partial_\nu\omega_\nu^- \to \frac{1}{\alpha(1+\delta\alpha)}(1+\tfrac{\delta Z_\omega}{2})(1+\tfrac{1}{2}\delta Z_\omega)\omega_\mu^+\partial_\mu\partial_\nu\omega_\nu^-$$
$$= \frac{1}{\alpha}(1+\delta Z_\omega-\delta\alpha)\,\omega_\mu^+\partial_\mu\partial_\nu\omega_\nu^-.$$

We require that

$$1+\delta Z_\omega-\delta\alpha = 1,$$

ie

$$\frac{1}{\alpha}\omega_\mu^+\partial_\mu\partial_\nu\omega_\nu^- \to \frac{1}{\alpha}\omega_\mu^+\partial_\mu\partial_\nu\omega_\nu^-,$$

(ie remains in the same gauge)

then

$$\delta\alpha = \delta Z_\omega,$$

or

$$\alpha \to (1+\delta\alpha)\alpha = Z_\omega\alpha$$

3) Consider the gauge fixing term

$$-\frac{1}{\alpha}(-\partial\omega^+ + \alpha m_+\phi^+)(-\partial\omega^- + \alpha m_-\phi^-),$$

then

$$\text{if}\left\{\begin{array}{l}\omega \to Z_\omega^{1/2}\omega\\ \phi \to Z_\phi^{1/2}\phi\\ \alpha \to Z_\omega\alpha\\ m_+ \to Z_\omega^{-1/2}Z_\phi^{-1/2}m_+\\ m_- \to Z_\omega^{-1/2}Z_\phi^{-1/2}m_-\end{array}\right\},\ \text{then}\left\{\begin{array}{l}\frac{1}{\alpha}(\partial\omega^+)(\partial\omega^-) \to \frac{1}{\alpha}(\partial\omega^+)(\partial\omega^-)\\ m_+\phi^+\partial\omega^- \to m_+\phi^+\partial\omega^-\\ m_-\phi^-\partial\omega^+ \to m_-\phi^-\partial\omega^+\\ \alpha m_+m_-\phi^+\phi^- \to \alpha m_+m_-\phi^+\phi^-\end{array}\right\}$$

Thus the gauge fixing term can be expressed in terms of renormalized quantities.

Let us reconsider carefully about the full propagator of W's after including counterterm and one loop correction.

1) Its Lagrangian is:

$$Z_W W_\nu^+ \partial_\mu \partial_\mu W_\nu^- - Z_W W_\mu^+ \partial_\mu \partial_\nu W_\nu^- + \frac{1}{\alpha} W_\mu^+ \partial_\mu \partial_\nu W_\nu^- - Z_W W_\mu^+ M^2(1+\delta M^2) W_\mu^-$$

(1) Bare propagator comes from

$$W_\nu^+ \partial_\mu \partial_\mu W_\nu^- - W_\mu^+ \partial_\mu \partial_\nu W_\nu^- + \frac{1}{\alpha} W_\mu^+ \partial_\mu \partial_\nu W_\nu^- - W_\mu^+ M^2 W_\mu^-$$

$$\longrightarrow \quad -\left\{ (ik)^2 \delta_{\mu\nu} - M^2 \delta_{\mu\nu} - \left(1-\frac{1}{\alpha}\right)(ik_\mu)(ik_\nu) \right\}^{-1} \qquad \left(\frac{1}{(2\pi)^4 i} \text{ is suppressed} \right)$$

$$= \left\{ k^2 \delta_{\mu\nu} + M^2 \delta_{\mu\nu} - \left(1-\frac{1}{\alpha}\right) k_\mu k_\nu \right\}^{-1}$$

$$= \left\{ (k^2+M^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \left(\frac{k^2}{\alpha} + M^2\right) \frac{k_\mu k_\nu}{k^2} \right\}^{-1}$$

$$= \frac{1}{k^2+M^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{\alpha}{k^2+\alpha M^2} \frac{k_\mu k_\nu}{k^2}$$

(2) Interaction terms due to the counterterms:

$$\delta Z_W W_\nu^+ \partial_\mu \partial_\mu W_\nu^- - \delta Z_W W_\mu^+ \partial_\mu \partial_\nu W_\nu^- - \delta Z_W W_\mu^+ M^2 W_\mu^- - \delta M^2 M^2 W_\mu^+ W_\mu^-$$

$$\longrightarrow \quad \delta Z_W (ik)^2 \delta_{\mu\nu} - \delta Z_W (ik_\mu)(ik_\nu) - \delta Z_W M^2 \delta_{\mu\nu} - \delta M^2 M^2 \delta_{\mu\nu} \qquad \left((2\pi)^4 i \text{ is suppressed} \right)$$

$$= -k^2 \delta Z_W \delta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} k^2 \delta Z_W - M^2(\delta Z_W + \delta M^2)\delta_{\mu\nu}$$

$$= -k^2 \delta Z_W \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - M^2(\delta Z_W + \delta M^2)\delta_{\mu\nu}.$$

(3) The one loop correction for W^+W^- self-energy, divided by $(2\pi)^4 i$, is

$$\Sigma^{W^+W^-} = \Delta \left\{ \delta_{\mu\nu}\left[\frac{4}{3}g^2 + \frac{M_\phi^2}{2} + \frac{3m_u^2}{2} + \frac{3m_d^2}{2}\right] + \frac{k_\mu k_\nu}{k^2}\left[-\frac{4}{3}g^2\right] \right\},$$

$$= \frac{4}{3} g^2 \Delta \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \delta_{\mu\nu} \Delta \left[\frac{M_\phi^2}{2} + \frac{3m_u^2}{2} + \frac{3m_d^2}{2}\right]$$

where $\Delta \equiv \frac{\pi^2}{(2\pi)^4}\left[\frac{-2}{n-4} - \gamma_E - \ln\pi\right]$ in n-dim regularization scheme.

* Consider the terms

$$m_+ \phi^+ \partial w^-$$

$$\to m_+' \sqrt{Z_\phi}\phi^+ \sqrt{Z_w}\partial w^-$$

$\Rightarrow$ require $m_+' = Z_\phi^{-\frac{1}{2}} \cdot Z_w^{-\frac{1}{2}} m_+$

Then from $\alpha m_+ m_- \phi^+ \phi^-$, we have

$$\to \alpha' Z_\phi^{-1/2} Z_w^{-1/2} m_+ Z_\phi^{-1/2} Z_w^{-1/2} m_- Z_\phi \phi^+ \phi^-$$

$$\to \alpha' Z_w^{-1} m_+ m_- \phi^+ \phi^-$$

$\Rightarrow$ require $\alpha' = Z_w \alpha$

This agrees with what we had

(4) To determine the counterterm, let us look at the infinite piece only, then we require

$$(2)+(3) = \text{finite},$$

therefore

$$\left.\begin{aligned} \delta Z_W &= \frac{4}{3}\Delta \\ M^2(\delta Z_W + \delta M^2) &= \Delta\left[\frac{m_e^2}{2} + \frac{3m_u^2}{2} + \frac{3m_d^2}{2}\right] \\ &= \Delta F \end{aligned}\right\} \Rightarrow \begin{aligned} M^2\delta M^2 &= \Delta F - M^2\delta Z_W \\ &= \Delta F - \frac{4}{3}\Delta M^2 \end{aligned}$$

(5) Suppose the finite part of $(2)+(3) = A\,\delta_{\mu\nu} + B\,\frac{k_\mu k_\nu}{k^2}$, let us find out W's propagator:

Now it is treated as

① Bare propagator: (from $W_\nu^+\partial_\mu\partial_\mu W_\nu^- - W_\mu^+ M^2 W_\mu^-$)

$$\frac{1}{k^2+M^2}$$

② Treat the gauge fixing term as an interaction term, then

$$-W_\mu^+\partial_\mu\partial_\nu W_\nu^- + \frac{1}{\alpha}W_\mu^+\partial_\mu\partial_\nu W_\nu^-$$

$$\begin{aligned} \longrightarrow\quad & -\left(1-\frac{1}{\alpha}\right)(ik_\mu)(ik_\nu) \\ =\ & \left(1-\frac{1}{\alpha}\right)k_\mu k_\nu \\ =\ & 0\cdot\delta_{\mu\nu} + k^2\left(1-\frac{1}{\alpha}\right)\frac{k_\mu k_\nu}{k^2} \end{aligned}$$

③ From the finite part of

$$(2)+(3) = A\,\delta_{\mu\nu} + B\,\frac{k_\mu k_\nu}{k^2}.$$

④ Hence W's propagator is

$$\frac{1}{(2\pi)^4 i}\left\{\frac{1}{k^2+M^2-A}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right) + \frac{1}{k^2+M^2-\left(A+B+k^2\left(1-\frac{1}{\alpha}\right)\right)}\left(\frac{k_\mu k_\nu}{k^2}\right)\right\}$$

$$= \frac{1}{(2\pi)^4 i}\left\{\frac{1}{k^2+M^2-A}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right) + \frac{\alpha}{k^2+\alpha M^2-\alpha(A+B)}\left(\frac{k_\mu k_\nu}{k^2}\right)\right\}$$

2) Now, let us go back to see how to include counterterm and gauge fixing term in the propagator using above analyses:

(1) From (2) of 1), we have the counterterm contribution

$$\left[-k^2\delta Z_W - M^2(\delta Z_W + \delta M^2)\right]\delta_{\mu\nu} + \left(k^2\delta Z_W\right)\frac{k_\mu k_\nu}{k^2}$$

(2) From the formula of ④ of (5) of 1), we have W's propagator as

$$\frac{1}{(2\pi)^4 i}\left\{\frac{1}{k^2+M^2+k^2\delta Z_W+M^2(\delta Z_W+\delta M^2)}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\frac{1}{k^2+M^2-\left(-k^2\delta Z_W-M^2(\delta Z_W+\delta M^2)+k^2\delta Z_W+k^2\left(1-\frac{1}{\alpha}\right)\right)}\left(\frac{k_\mu k_\nu}{k^2}\right)\right\}.$$

$$=\frac{1}{(2\pi)^4 i}\left\{\frac{1}{Z_W\left(k^2+M^2(1+\delta M^2)\right)}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\frac{1}{k^2+M^2(1+\delta Z_W+\delta M^2)-k^2\left(1-\frac{1}{\alpha}\right)}\left(\frac{k_\mu k_\nu}{k^2}\right)\right\}$$

$$=\frac{1}{(2\pi)^4 i}\left\{\frac{1}{Z_W\left(k^2+M^2(1+\delta M^2)\right)}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\frac{\alpha}{k^2+\alpha M^2(1+\delta Z_W+\delta M^2)}\left(\frac{k_\mu k_\nu}{k^2}\right)\right\}$$

⊛ Note: The dressed propagator

$$\frac{1}{(2\pi)^4 i}\frac{\delta_{\mu\nu}}{k^2+M^2} \qquad \left[(2\pi)^4 i\,\Sigma_{\alpha\beta}\right]$$

$$=\frac{1}{(2\pi)^4 i}\left\{\frac{1}{k^2+M^2-\frac{A}{(2\pi)^4 i}}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\frac{1}{k^2+M^2-\frac{A+B}{(2\pi)^4 i}}\left(\frac{k_\mu k_\nu}{k^2}\right)\right\},$$

with the self-energy expressed as

$$\left[(2\pi)^4 i\,\Sigma_{\alpha\beta}\right]=A\,\delta_{\alpha\beta}+B\,\frac{k_\alpha k_\beta}{k^2}$$
$$=A\left(\delta_{\alpha\beta}-\frac{k_\alpha k_\beta}{k^2}\right)+(A+B)\frac{k_\alpha k_\beta}{k^2}$$

Consider the mixing terms.

1) Its Lagrangian is generated from

$$\frac{-1}{\alpha}\left(-\partial w^+ + \alpha m_+ \phi^+\right)\left(-\partial w^- + \alpha m_- \phi^-\right).$$

Recall that all of these quantities are bare quantities.
To see the difference between m_+ and M, let us express m_+ (m_-) as

$$m_+ = M + \tilde{m}_+$$

and.

$$m_- = M + \tilde{m}_- . \qquad \left(\tilde{m}_- = (\tilde{m}_+)^*\right)$$

The part of Lagrangian $M\phi^+\partial w^- + M\phi^-\partial w^+$ will be cancelled from the same terms with opposite sign from the Higgs sector.
Therefore, we are left with the terms

$$\tilde{m}_+ \phi^+ \partial w^- + \tilde{m}_- \phi^- \partial w^+$$

as the mixing of ϕ and w.

2) The renormalization procedures related to these two terms are

$$\phi^+ \to \left(1+\tfrac{1}{2}\delta Z_\phi\right)\phi^+$$
$$\phi^- \to \left(1+\tfrac{1}{2}\delta Z_\phi\right)\phi^-$$
$$w^+ \to \left(1+\tfrac{1}{2}\delta Z_w\right)w^+$$
$$w^- \to \left(1+\tfrac{1}{2}\delta Z_w\right)w^-$$
$$\tilde{m}_+ \to \left(1+\delta\tilde{m}_+\right)\tilde{m}_+$$
$$\tilde{m}_- \to \left(1+\delta\tilde{m}_-\right)\tilde{m}_- ,$$
$$\alpha \to (1+\delta\alpha)\,\alpha ,$$

$$\left(\begin{array}{l}\tilde{m}_- = (\tilde{m}_+)^* \\ \delta\tilde{m}_- = (\delta\tilde{m}_+)^*\end{array}\right)$$

Hence, we get

$$(1+\delta\tilde{m}_+)\tilde{m}_+\left(1+\tfrac{1}{2}\delta Z_\phi\right)\phi^+\left(1+\tfrac{1}{2}\delta Z_w\right)\partial w^-$$

$$= \underbrace{\tilde{m}_+}_{O(g^2)}\phi^+\partial w^-\Big(1+\tfrac{1}{2}\underbrace{\delta Z_\phi}_{O(g^2)}+\tfrac{1}{2}\underbrace{\delta Z_w}_{O(g^2)}+\underbrace{\delta\tilde{m}_+}_{O(g^2)}\Big) = \underbrace{\tilde{m}_+}_{O(g^2)}\phi^+\partial w^-.$$

When we perform the one-loop calculation, we assume in our tree Lagrangian, there is no mixing between $\phi^\pm$ and $w^\pm$, ie assume R_ξ-gauge and $\tilde{m}_+=\tilde{m}_-=0$. Therefore, we should treat the whole term $\tilde{m}_+\phi^+\partial w^-(1+\frac{1}{2}\delta Z_\phi+\frac{1}{2}\delta Z_w+\delta\tilde{m}_+)$ as the counterterm, which will absorb the infinity generated from the one loop calculation.

Thus the counterterms are

$$\phi^+ \text{- - -}\times\!\sim\!\sim\!\sim \partial w^-,\ \overleftarrow{-q_\nu} = \left[\tilde{m}_+\left(1+\tfrac{1}{2}\delta Z_\phi+\tfrac{1}{2}\delta Z_w+\delta\tilde{m}_+\right)\right](-iq_\nu) = \tilde{m}_+(-iq_\nu)$$

$$\partial w^+_\mu \sim\!\sim\!\sim\!\times\text{- - -}\phi^-,\ \overrightarrow{q} = \left[\tilde{m}_-\left(1+\tfrac{1}{2}\delta Z_\phi+\tfrac{1}{2}\delta Z_w+\delta\tilde{m}_-\right)\right](+iq_\mu) = \tilde{m}_-(+iq_\mu)$$

(*) <u>Note</u>. $\tilde{m}_+$ and $\tilde{m}_-$ are of the order of $O(g^2)$. They are at the same order as δZ_ϕ, δZ_w or $\delta\tilde{m}_+$. Thus we only keep $\tilde{m}_+$ and $\tilde{m}_-$ terms.

3) $$\phi^+ \text{- - -}\circledast\!\sim\!\sim\!\sim w^-,\ \overleftarrow{-q_\nu} = \Sigma^{\phi^+w^-} = (-iq_\nu)\left[\Delta\frac{1}{M}\left(\frac{m_e^2}{2}+\frac{3m_u^2}{2}+\frac{3m_d^2}{2}\right)\right]$$

$$w^+_\mu \sim\!\sim\!\sim\!\circledast\text{- - -}\phi^-,\ \overrightarrow{q_\mu} = \Sigma^{w^+\phi^-} = (+iq_\mu)\left[\Delta\frac{1}{M}\left(\frac{m_e^2}{2}+\frac{3m_u^2}{2}+\frac{3m_d^2}{2}\right)\right]$$

* These are the one loop calculation.

4) To eliminate these mixing, we require $\phi^+\text{- - -}\times\!\sim\!\sim w^- + \phi^+\text{- -}\circledast\!\sim\!\sim w^- = 0$,

i.e. $$\tilde{m}_+ = (\tilde{m}_-)^* = \frac{1}{M}\Delta F, \qquad F = \frac{m_e^2}{2}+\frac{3m_u^2}{2}+\frac{3m_d^2}{2}.$$

Consider the ϕ propagator

1) It's Lagrangian is generated from

$$-\partial\phi^{+}\partial\phi^{-} - \alpha\, m_{+} m_{-}\, \phi^{+}\phi^{-}$$

Recall that all of them are bare quantities

2) Doing the same analysis as in the previous section, we have

$$-\partial_\mu\phi^{+}\partial_\mu\phi^{-} - \alpha M^2 \phi^{+}\phi^{-} - \alpha M(\tilde{m}_{+} + \tilde{m}_{-})\phi^{+}\phi^{-}$$

where $\tilde{m}_{+} = \tilde{m}_{-} = 0$ at tree level.

Now express it in terms of renormalized quantities:

$$-(1+\delta Z_\phi)\partial_\mu\phi^{+}\partial_\mu\phi^{-} - \alpha(1+\delta\alpha) M^2 (1+\delta M^2)(1+\delta Z_\phi)\phi^{+}\phi^{-}$$
$$- \alpha(1+\underset{O(g^2)}{\delta\alpha}) M(1+\tfrac{1}{2}\underset{O(g^2)}{\delta M^2})\, \underset{O(g^2)}{\tilde{m}_{+}}\, (1+\underset{O(g^2)}{\delta\tilde{m}_{+}})\, \underset{O(g^2)}{\tilde{m}_{-}}(1+\underset{O(g^2)}{\delta\tilde{m}_{-}})(1+\underset{O(g^2)}{\delta Z_\phi})\phi^{+}\phi^{-}$$

Notice that:

$\tilde{m}_{+}$ and $\tilde{m}_{-}$ are of the order of $O(g^2)$, therefore after the shift $\tilde{m}_{+} \to \tilde{m}_{+}(1+\frac{1}{2}\delta\tilde{m}_{+}) \simeq \tilde{m}_{+}$, so is $\tilde{m}_{-}$.

Consequently, we have the Lagrangian in terms of renormalized quantities as

$$-(1+\delta Z_\phi)\partial_\mu\phi^{+}\partial_\mu\phi^{-} - (1+\delta\alpha+\delta M^2+\delta Z_\phi)\,\alpha M^2\phi^{+}\phi^{-} - \alpha M(\tilde{m}_{+}+\tilde{m}_{-})\phi^{+}\phi^{-}$$

$$= \phi^{+}\partial_\mu\partial_\mu\phi^{-} - \alpha M^2\phi^{+}\phi^{-}$$
$$+ \delta Z_\phi \phi^{+}\partial_\mu\partial_\mu\phi^{-} - (\delta\alpha+\delta M^2+\delta Z_\phi)\,\alpha M^2\phi^{+}\phi^{-} - \alpha M(\tilde{m}_{+}+\tilde{m}_{-})\phi^{+}\phi^{-}$$

Thus the counterterms are

$$\phi^+ \text{- - -}\times\text{- - -} \phi^- \qquad \delta Z_\phi (ik)^2 - (\delta\alpha + \delta M^2 + \delta Z_\phi)\alpha M^2 - \alpha M(\widetilde{m}_+ + \widetilde{m}_-)$$

$$= -k^2 \delta Z_\phi - \alpha M^2(\delta\alpha + \delta M^2 + \delta Z_\phi) - \alpha M(\widetilde{m}_+ + \widetilde{m}_-).$$

3) The one loop results.

$$\text{- - -}\circ\text{- - -} \; + \; \underset{-\beta}{\text{———}\times\text{———}}$$

$$\Sigma^{\phi^+\phi^-}_{true} = \Delta \frac{1}{M^2} k^2 \left[\frac{m_e^2}{2} + \frac{3m_u^2}{2} + \frac{3m_d^2}{2} \right] = k^2 \frac{1}{M^2} \Delta F.$$

4) Require that $\text{- -}\times\text{- -} + \text{- -}\circ\text{- -} + \underset{-\beta}{\text{——}\times\text{——}}$ to be finite, we have

$$\delta Z_\phi = \frac{1}{M^2} \Delta F,$$

and

$$-\alpha M^2(\delta\alpha + \delta M^2 + \delta Z_\phi) - \alpha M(\widetilde{m}_+ + \widetilde{m}_-) = \text{finite}$$

(*) $$\phi^+ \text{- - -}\circ\text{- - -} \phi^- = (2\pi)^4 i \, \Sigma^{\phi^+\phi^-}$$

$$= \frac{2i\pi^2}{n-4} \frac{1}{M^2} \left\{ k^2 \left[\frac{m_e^2}{2} + \frac{3m_u^2}{2} + \frac{3m_d^2}{2} \right] + (m_e^4 + 3m_u^4 + 3m_d^4) \right\}.$$

The above self energy is not the true self energy we are going to deal with, for the vacuum expectation value also get shifted in order to have $\langle H \rangle = 0$ at one loop. Requiring the tadpole $H\text{—}\circ$ contribution to be zero fixes up the one loop value of $\beta = \mu + \lambda F^2 = \mu + \frac{\lambda}{2} v^2$.

$$H\text{—}\circ = 0 \Rightarrow \beta = -\frac{1}{(2\pi)^4 i} \beta', \quad \beta' = \frac{-1}{2M}(T_1 + T_2 + T_3 + T_4).$$

And $\phi^+ \text{- -}\times\text{- -} \phi^- = -(2\pi)^4 i \beta = \beta'$. Thus $(2\pi)^4 i \Sigma^{\phi^+\phi^-}_{true} = (2\pi)^4 i \left[\Sigma^{\phi^+\phi^-} - \beta \right]$