How to include Z^0-propagator in $e^+e^- \to \mu^+\mu^-$

1. Define the bare mass of Z^0 to be M_0, which is the bare mass appearing in the Lagrangian.
Then the full propagator of Z^0 is

$$\Delta_{\mu\nu}(k) = \frac{\delta_{\mu\nu}}{k^2 + M_0^2 - \tilde{\pi}(k^2)},$$

where we neglect $k_\mu k_\nu$ terms which would give contributions proportional to external lepton masses.

1) Setting $k^2 = -s$, the scalar part of the propagators becomes

$$\Delta(s) = \frac{1}{-s + M_0^2 - \tilde{\pi}(-s)}$$

2) Let us define

$$\bar{s} = M_R^2$$

as the complex valued squared momentum at which the dominator vanishes, i.e.

$$\bar{s} = M_0^2 - \tilde{\pi}(-s),$$

or

$$M_R^2 = M_0^2 - \tilde{\pi}(-M_R^2)$$

3) Introducing the physical quantities

$$M_R^2 \equiv M_Z^2 - i M_Z \Gamma_Z$$

$$M_Z^2 = M_0^2 - Re\left(\tilde{\Pi}(-M_R^2)\right)$$
$$M_Z \Gamma_Z = Im\left(\tilde{\Pi}(-M_R^2)\right)$$

Then

$$\begin{aligned} -M_Z^2 + M_R^2 &= -\left(M_0^2 - Re\,\tilde{\Pi}(-M_R^2)\right) + \left(M_0^2 - \tilde{\Pi}(-M_R^2)\right) \\ &= Re\,\tilde{\Pi}(-M_R^2) + (-1)\left[Re\,\tilde{\Pi}(-M_R^2) + i\, Im\left(\tilde{\Pi}(-M_R^2)\right)\right] \\ &= -i\, Im\,\tilde{\Pi}(-M_R^2) \\ &= -i M_Z \Gamma_Z \qquad \approx \mathcal{O}(g^2)\cdot M_Z^2 \end{aligned}$$

4) The quantity

$$\begin{aligned} \Delta(s)^{-1} &= -s + M_0^2 - \tilde{\Pi}(-s) \\ &= -s + \left(M_R^2 + \tilde{\Pi}(-M_R^2)\right) - \tilde{\Pi}(-s) \\ &= \left(-s + M_R^2\right) - \left(\tilde{\Pi}(-s) - \tilde{\Pi}(-M_R^2)\right) \\ &= \left(-s + M_R^2\right)\left\{1 - \frac{1}{-s + M_R^2}\left(\tilde{\Pi}(-s) - \tilde{\Pi}(-M_R^2)\right)\right\}. \end{aligned}$$

2. If we are working at an ~~euro~~ energy off Z_0 resonance, then we can forget the imaginary part of M_R^2, and write

$$\begin{aligned} M_R^2 &= M_0^2 - \tilde{\Pi}(-M_R^2) \\ &= M_Z^2 - i M_Z \Gamma_Z \longrightarrow M_Z^2. \end{aligned}$$

Therefore

$$\Delta(s) = \frac{1}{-s+M_Z^2}\left\{1 + \frac{1}{-s+M_Z^2}\left[\tilde{\Pi}(-s) - \tilde{\Pi}(-M_Z^2)\right]_{Re}\right\}.$$

Re ← real part

<u>Note</u>: ① Since $\tilde{\Pi}(-s)$ is of the order $\mathcal{O}(g^2)$, therefore

$$\tilde{\Pi}(-M_R^2) \simeq \tilde{\Pi}(-M_Z^2) + \mathcal{O}(g^4)$$

② Since the physical Z^0 mass, M_Z, and its tree level mass $M_Z^{(t)}$ is very close (~ 3 GeV). Thus we can further approximate the above eq. and get

But $|M_Z^2 - M_Z^{t\,2}| \sim \mathcal{O}(g^2)$ not negligible

$$\Delta(s) \simeq \frac{1}{-s+M_Z^{(t)2}}\left\{1 + \frac{1}{-s+M_Z^{(t)2}}\left[\tilde{\Pi}(-s) - \tilde{\Pi}(-M_Z^{(t)2})\right]_{Re}\right\},$$

Re ← real part.

for the region of $|s - M_Z^2| \geq$ (few $M_Z \Gamma_Z$).

3. If we are working at (or near by) the resonance of Z^0, then we have to do something else.

1) First, because near the Z^0 resonance,

$$(-s + M_R^2) \longrightarrow 0.$$

Therefore, we can expand the Z^0 self energy as

$$\tilde{\Pi}(-s) = \tilde{\Pi}(-M_R^2) + (-s+M_R^2)\,\tilde{\Pi}'(-M_R^2),$$

with

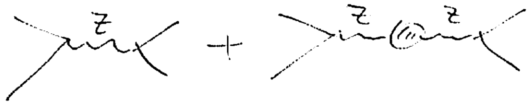
$$\tilde{\Pi}'(-M_R^2) = \left.\frac{\partial \tilde{\Pi}}{\partial k^2}\right|_{k^2 = -M_R^2}.$$

* Another way to derive

$$\Delta(s) = \frac{1}{-s+M_Z^2}\left\{1 + \frac{1}{-s+M_Z^2}\left[\tilde{\Pi}(-s) + \tilde{\Pi}(-M_Z^2)\right]\right\}$$

is using the identity

$$M_Z^2 = M_0^2 - \mathrm{Re}\,\tilde{\Pi}(-M_Z^2) + O(g^4)$$

Then  gives

$$\frac{1}{-s+M_0^2} + \frac{1}{-s+M_0^2}\tilde{\Pi}(-s)\frac{1}{-s+M_0^2} \equiv \mathcal{P}$$

And $$\frac{1}{-s+M_0^2} = \frac{1}{-s+M_Z^2+\mathrm{Re}\,\tilde{\Pi}(-M_Z^2)} = \frac{1}{-s+M_Z^2}\left\{1 - \frac{\mathrm{Re}\,\tilde{\Pi}(-M_Z^2)}{-s+M_Z^2}\right\}$$

$$\frac{1}{-s+M_0^2}\tilde{\Pi}(-s)\frac{1}{-s+M_0^2} \simeq \frac{1}{-s+M_Z^2}\tilde{\Pi}(-s)\frac{1}{-s+M_Z^2}$$

If off resonance, then neglecting imaginary part, and get

$$\mathcal{P} = \frac{1}{-s+M_Z^2}\left\{1 + \frac{1}{-s+M_Z^2}\mathrm{Re}\left[\tilde{\Pi}(-s) - \tilde{\Pi}(-M_Z^2)\right]\right\}.$$

2) From 4) of 1, one get

$$\Delta(s)^{-1} = (-s+M_R^2)\left\{1 - \frac{1}{-s+M_R^2}\left(\tilde{\pi}(-s) - \tilde{\pi}(-M_R^2)\right)\right\}$$

$$= (-s+M_R^2)\left\{1 - \tilde{\pi}'(-M_R^2)\right\}.$$

Since

$$\lim_{+s\to M_Z^2}(-s+M_R^2) = -M_Z^2 + M_R^2$$

$$= -i\, M_Z \Gamma_Z ,$$

and

$$\tilde{\pi}'(-M_R^2) \simeq \tilde{\pi}'(-M_Z^2) + O(g^4) .$$

Thus near Z^0 resonance

$$\Delta(s) = \lim_{s\to M_Z^2} \frac{1+\tilde{\pi}'(-M_R^2)}{-s+M_R^2}$$

$$= \frac{1+\tilde{\pi}'(-M_Z^2)}{-i\, M_Z \Gamma_Z} ,$$

where we have neglect terms $O(g^4)$ relative to the leading term.

<u>Note</u>. A full first-order calculation around the resonance requires the inclusion of the one-loop correction to the Z width Γ_Z.

This is because in the resonance region, i.e., for

$$|E - M_Z| \leq \Gamma_Z ,$$

the counting of powers of coupling constants is different than it is away from the resonance, where all one-loop diagrams represent an $O(g^2)$ correction relative to the Born diagrams.

(We shall come back to this subject later.)

4. Next, we give a general result for $\Delta(s)$:

$$\Delta(s)^{-1} = -s + M_0^2 - \tilde{\Pi}(-s)$$
$$= (-s+M_R^2)\left\{1 - \frac{1}{-s+M_R^2}\left(\tilde{\Pi}(-s) - \tilde{\Pi}(-M_R^2)\right)\right\}$$

Since ~~the~~

$$\tilde{\Pi}(-s) - \tilde{\Pi}(-M_R^2) = \tilde{\Pi}(-s) - \tilde{\Pi}(-M_R^2) - (-s+M_R^2)\left.\frac{\partial\tilde{\Pi}}{\partial k^2}\right|_{k^2=-M_R^2}$$
$$+ (-s+M_R^2)\left.\frac{\partial\tilde{\Pi}}{\partial k^2}\right|_{k^2=-M_R^2},$$

so

$$\Delta(s)^{-1} = (-s+M_R^2)\left\{1 - \left.\frac{\partial\tilde{\Pi}}{\partial k^2}\right|_{k^2=-M_R^2} - \frac{1}{-s+M_R^2}\left[\tilde{\Pi}(-s) - \tilde{\Pi}(-M_R^2) - (-s+M_R^2)\left.\frac{\partial\tilde{\Pi}}{\partial k^2}\right|_{k^2=-M_R^2}\right]\right\}$$

Thus

$$\Delta(s) = \frac{1}{-s+M_R^2}\left\{1 + \tilde{\Pi}'(-M_R^2) + \left(\frac{-s+M_R^2}{M_Z^2}\right)\cdot Q(s, M_R^2)\right\},$$

with

$$Q(s, M_R^2) = \frac{M_Z^2}{(-s+M_R^2)^2}\left\{\tilde{\Pi}(-s) - \tilde{\Pi}(-M_R^2) - (-s+M_R^2)\,\tilde{\Pi}'(-M_R^2)\right\},$$

$$\tilde{\Pi}'(-M_R^2) = \left.\frac{\partial\tilde{\Pi}}{\partial k^2}\right|_{k^2=-M_R^2}.$$

<u>Note</u>: Both $\tilde{\Pi}'$ and Q are $O(g^2)$.

Since $M_R^2 - M_Z^2 = \mathcal{O}(g^2) \cdot M_Z$, therefore, we may replace

$$\tilde{\Pi}'(-M_R^2) \to \tilde{\Pi}'(-M_Z^2),$$

$$(-s+M_R^2)\,Q(s,M_R^2) \to (-s+M_Z^2)\,Q(s,M_Z^2),$$

with an error $\mathcal{O}(g^4)$.

Hence

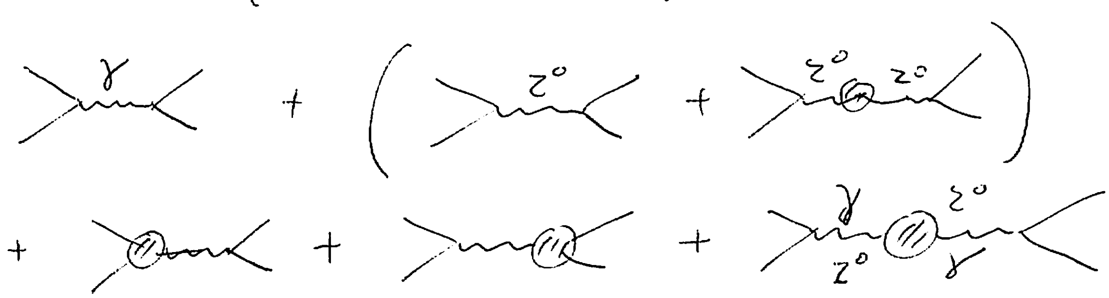
$$\Delta(s) = \frac{1}{-s+M_R^2}\left\{1 + \tilde{\Pi}'(-M_Z^2) + \left(\frac{-s+M_Z^2}{M_Z^2}\right) Q(s, M_Z^2)\right\},$$

with

$$Q(s,M_Z^2) = \frac{M_Z^2}{(-s+M_Z^2)^2}\left\{\tilde{\Pi}(-s) - \tilde{\Pi}(-M_Z^2) - (-s+M_Z^2)\,\tilde{\Pi}'(-M_Z^2)\right\}.$$

$$\tilde{\Pi}'(-M_Z^2) = \left.\frac{\partial \Pi}{\partial k^2}\right|_{k^2=-M_Z^2}; \qquad \left(\underset{\text{at resonance}}{M_R^2 = M_Z^2 - iM_Z\Gamma_Z}\right)$$

5. While doing $e^+e^- \to \mu^+\mu^-$ upto one loop, one should consider (near Z^0 resonance)



Its effective amplitude can be expressed as

$$\mathcal{M} = a \cdot \frac{1}{-s+M_Z^2 - iM_Z\Gamma_Z(E)} + b \cdot \frac{1}{(-s)}$$

photon (Born diagram)

(The detailed will be given later.)

Here, we used

$$M_R^2 = M_Z^2 - i M_Z \Gamma_Z(E),$$

Note. $M_R^2 = M_Z^2 - i M_Z \Gamma_Z$, at resonance.

Here $M_Z \Gamma_Z(E) = M_Z \Gamma_Z - \mathrm{Im}\,\tilde{\Pi}(-M_Z^2) + \mathrm{Im}\,\tilde{\Pi}(-s)$.

Note: ① At resonance $M_Z \Gamma_Z(M_Z) = M_Z \Gamma_Z + \mathcal{O}(g^4)$.

② For $E \neq M_Z$, but $|E - M_Z| \leq \Gamma_Z$, then

$$M_Z \Gamma_Z(E) - M_Z \Gamma_Z = -\mathrm{Im}\,\tilde{\Pi}(-M_Z^2) + \mathrm{Im}\,\tilde{\Pi}(-s) \sim \mathcal{O}(g^4)$$

③ Outside resonance region, $|E - M_Z| > \Gamma_Z$,

$$M_Z \Gamma_Z(E) = \mathrm{Im}\,\tilde{\Pi}(-s) \sim \mathcal{O}(g^2).$$

④ From ① and ②, one deduce, $M_Z \Gamma_Z$ should be calculated upto $\mathcal{O}(g^4)$. Therefore its width Γ_Z should also be calculated up to $\mathcal{O}(g^4)$.

⑤ In ③, away from resonance region, then

$$-s + M_Z^2 - i M_Z \Gamma_Z(E) \simeq (-s + M_Z^2) + \mathcal{O}(g^2).$$

In this case $\Gamma_Z(E)$ only has to be calculated up to tree level, ie $\mathcal{O}(g^2)$.

$e^+e^- \to \mu^+\mu^-$

(OFF Z^0-resonance).

1. If we take Mark II data $\sqrt{s} = 29$ GeV, then the contribution from the pure weak interaction term is not negligible compared with the QED and weak mixing term, then the total cross section should be:

$$R = 1 - 2\chi h_{VV} \cos\delta_R + \chi^2 \left(h_{VV}^2 + h_{AA}^2 + 2 h_{VA}^2 \right), \qquad (\because h_{VA} = h_{AV})$$

And the forward-backward asymmetry is

$$A = \frac{3}{4R}\left(-2\chi h_{AA} \cos\delta_R + 4\chi^2 h_{VV} h_{AA}\right)$$

Note: If $R = 1 - \Delta$, then $\frac{1}{R} = 1 + \Delta = 2 - (1-\Delta) = 2 - R$,

where $R = \dfrac{\sigma(e^+e^- \to \mu^+\mu^-)}{\sigma(e^+e^- \to \mu^+\mu^-)_{QED}}$.

In above eqs,

$$h_{VV} = \left(-\tfrac{1}{2} + 2 S_\theta^2\right)^2$$

$$h_{AA} = \left(-\tfrac{1}{2}\right)^2$$

$$\tan\delta_R = \frac{M_Z \Gamma_Z}{M_Z^2 - s}$$

$$\chi = \frac{G_F}{2\sqrt{2}\pi\alpha} \frac{s M_Z^2}{\left[(M_Z^2 - s)^2 + M_Z^2 \Gamma_Z^2\right]^{1/2}}$$

2. To convert the above notation into ours, we substitute

$$\chi h_{VV} \to \left[\frac{-g^2}{g^2 S_\theta^2}\right] \cdot \left(-\mathcal{E}_{VV}'\right) \doteq H_{VV} = H_{VV}^{(1)} + g^2 H_{VV}^{(2)},$$

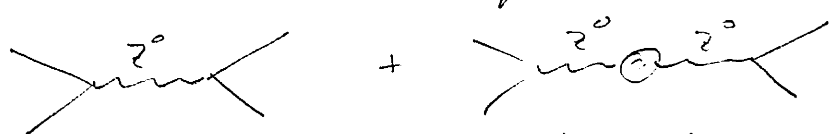
where $\mathcal{E}_{VV}'$ is defined as $\mathcal{E}_{VV}$ substracts the QED contribution

i.e. $\left(\dfrac{g^2 S_\theta^2}{-q^2}\right)$.

Similarly, we have the substitutions.

$$\not{x} h_{AA} \longrightarrow H_{AA}$$
$$\not{x} h_{VA} \longrightarrow H_{VA} = H_{AV}.$$

3. To consider the contributions from the two diagrams:



1) the complete propagator for off Z^0-resonance is

$$\frac{1}{-s+M_0^{(1)2}-\tilde{\Pi}(-s)}$$

$$= \left\{-s+M_{op}^2+\tilde{\Pi}(-M_{op}^2)-\tilde{\Pi}(-s)\right\}^{-1}; \qquad M_{op}^2 = M_0^{(1)2}-\tilde{\Pi}(-M_{op}^2)$$

physical Z^0-mass (The pole of the propagator).

$$= \frac{1}{-s+M_{op}^2}\left\{1+\frac{1}{-s+M_{op}^2}\mathrm{Re}\left[\tilde{\Pi}(-s)-\tilde{\Pi}(-M_{op}^2)\right]\right\}.$$

<u>Note</u>: Because $M_{op}^2 - M_0^{(0)2} \simeq \mathcal{O}(g^2)$, therefore the above eq is <u>not</u> equal to (up to $\mathcal{O}(g^2)$),

$$\frac{1}{-s+M_0^{(0)2}}\left\{1+\frac{1}{-s+M_0^{(0)2}}\mathrm{Re}\left[\tilde{\Pi}(-s)-\tilde{\Pi}(-M_0^{(0)2})\right]\right\}.$$

Since
$$\frac{1}{-s+M_{op}^2} = \frac{1}{-s+M_0^{(0)2}+(M_{op}^2-M_0^{(0)2})} = \frac{1}{-s+M_0^{(0)2}}\left[1-\frac{M_{op}^2-M_0^{(0)2}}{-s+M_0^{(0)2}}\right],$$

where $M_{op}^2 - M_0^{(0)2} \simeq \mathcal{O}(g^2)$.

2) If we parametrize it as

$$\frac{1}{p^2+M_0^{(1)2}} \qquad\qquad \frac{1}{p^2+M_0^{(1)2}}\,\tilde{\Pi}\,\frac{1}{p^2+M_0^{(1)2}}$$

then they give

$$\frac{1}{p^2+M_0^{(1)2}}\left[1+\tilde{\Pi}(p^2)\frac{1}{p^2+M_0^{(1)2}}\right].$$

But $\quad M_0^{(1)2} = M_0^{(0)2}\left(1+\delta_1-\delta_2-\delta_3+\frac{s_0^2}{c_0^2}\delta_3\right) \equiv M_0^{(0)2}(1+\Delta),$

therefore

$$\frac{1}{p^2+M_0^{(1)2}} = \frac{1}{p^2+M_0^{(1)2}+M_0^{(1)2}\Delta} = \frac{1}{p^2+M_0^{(1)2}}\,\frac{1}{1+\frac{M_0^{(0)2}\Delta}{p^2+M_0^{(1)2}}}$$

$$\simeq \frac{1}{p^2+M_0^{(1)2}}\left[1-\frac{M_0^{(0)2}\Delta}{p^2+M_0^{(1)2}}\right], \quad \text{for off-}Z^0\text{-resonance}$$

<u>Note</u>. We will use this parametrization in our calculations.

4. Numerical results:

1) Take
$$M^{(0)} = 0.0815 \quad (\text{In TeV unit})$$
$$g = 0.664$$
$$s^2 = 0.22\,; \qquad c^2 = 1-s^2 = 0.78$$
$$\frac{1}{f_\pi^2} = 0.0126651$$
$$M_0^{(1)} = \frac{M}{c} = 0.0923$$

For $\sqrt{s} = 0.029\,, \quad q^2 = -8.4\times10^{-4} \quad$ and $\quad \frac{1}{q^2+M_0^{(1)2}} = 130.3$

$\sqrt{s} = 0.060\,; \quad q^2 = -3.6\times10^{-3} \quad$ and $\quad \frac{1}{q^2+M_0^{(1)2}} = 203.4$

$e^+e^- \to \mu^+\mu^-$

(Near Z^0 resonance)

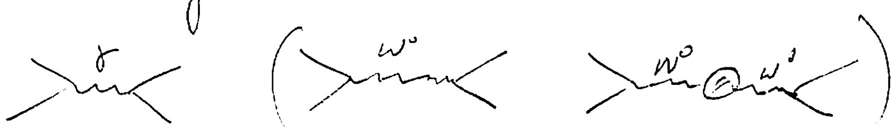
1. On Z^0 resonance, the dressed Z^0-propagator is

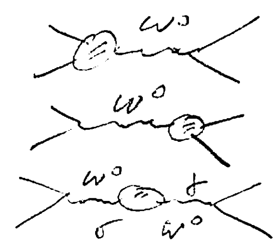
$$\Delta(s) = \lim_{s=M_Z^2} \frac{1+\tilde{\pi}'(-M_R^2)}{-s+M_R^2}, \qquad \tilde{\pi}'(p^2) \equiv \frac{\partial \tilde{\pi}(p^2)}{\partial p^2}$$

$$= \frac{1+\tilde{\pi}'(-M_Z^2)}{-iM_Z\Gamma_Z},$$

(Here M_Z is the physical Z^0 mass. Γ_Z is the Z^0-width)

2. The diagrams we will consider are





3. Near Z^0-resonance

$$\Delta(s) = \frac{1+\tilde{\pi}'(-M_Z^2)}{-s+M_Z^2-iM_Z\Gamma(E)},$$

with

$$M_Z\Gamma(E) = M_Z\Gamma_Z - \mathrm{Im}\,\tilde{\pi}(-M_Z^2) + \mathrm{Im}\,\tilde{\pi}(-s)$$

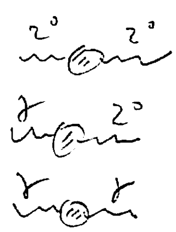
4. To get the cross section of $e^+e^- \to \mu^+\mu^-$ near Z^0 resonance, one had better get rid of the γ-Z^0 mixing first.

1) We should look at the transversal part of the propagators:

$$\mathbb{T} = \begin{pmatrix} \dfrac{1}{p^2+M_0^2-f_{11}} & \dfrac{f_{12}}{(p^2+M_0^2-f_{11})(p^2-f_{22})} \\ \mathbb{T}_{12} & \dfrac{1}{p^2-f_{22}} \end{pmatrix},$$

where

$$f_{11} = \tilde{\pi}(p^2) =$$
$$f_{12} =$$
$$f_{22} =$$



2) Physical w^0 is defined as the eigenstates of the matrix

$$\mathbb{W} = \lim_{p^2 = -M_R^2} (p^2 + M_R^2)\, \Pi^{-}$$

$$= \left.\begin{pmatrix} 1 & \frac{-f_{12}}{p^2 - f_{22}} \\ \mathbb{W}_{12} & 0 \end{pmatrix}\right|_{p^2 = -M_R^2}$$

(1) Its trace is $1 + 0 = 1$, therefore the two eigenvalues are 1 and 0.

(2) The physical w^0 state is defined as the eigenstate of $\mathbb{W}$, so that

$$\mathbb{W}\begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} = 1 \cdot \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix}.$$

Thus

$$\begin{pmatrix} 0 & \mathbb{W}_{12} \\ \mathbb{W}_{12} & -1 \end{pmatrix}\begin{pmatrix} C_{\theta_1} \\ S_{\theta_1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Or

$$\mathbb{W}_{12}\, C_{\theta_1} - S_{\theta_1} = 0 \implies \tan\theta_1 = \mathbb{W}_{12} = \left.\frac{f_{12}}{p^2 - f_{22}}\right|_{p^2 = -M_R^2}$$

__Note.__ $\tan\theta_1 \sim O(g^2) \sim \sin\theta_1$, and $\cos\theta_1 \sim O(1)$.

Since $f_{22} \sim O(g^2)$, therefore

$$\tan\theta_1 \sim \left.\frac{f_{12}}{p^2}\right|_{p^2 = -M_R^2} \sim \sin\theta_1$$

(3) The physical w^0_{phy} state is therefore

$$w^0_{phy} = w^0 \cdot \cos\theta_1 + A \sin\theta_1$$

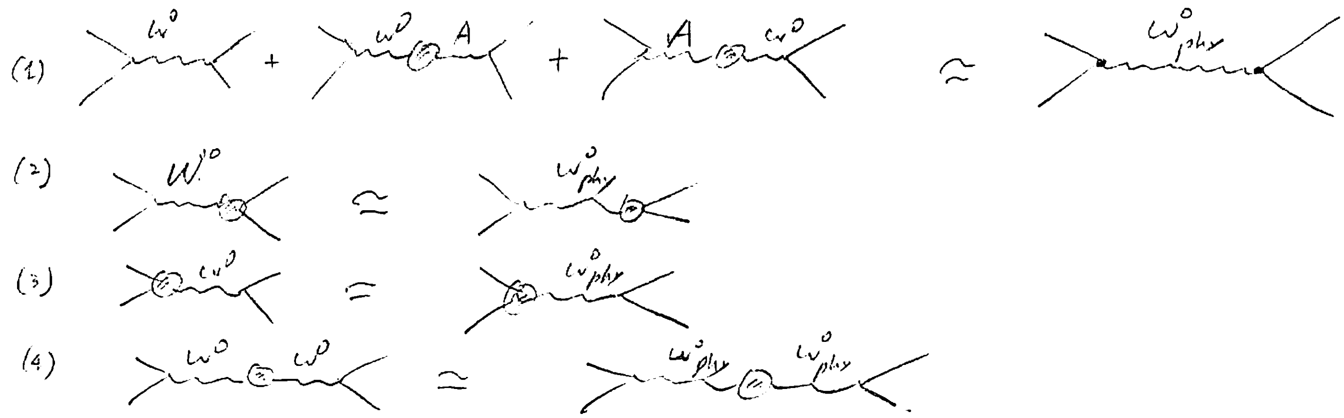
$$\simeq w^0 + \left.\frac{f_{12}}{p^2}\right|_{p^2 = -M_R^2} \cdot A$$

(4) Recall that: we only keep terms higher than $O(g^2)$.

thus

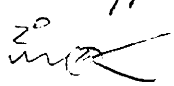
$$\cancel{w^0} \simeq \cancel{w^0_{phy}} + O(g^4).$$

3) Hence, in terms of physical field W^0_{phy} and A_{phy}, one get the Feynman diagrams are (near Z^0 resonance):



If we denote W^0_{phy} as Z^0, then near Z^0-resonance, we have to consider

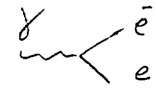
when the Z^0-propagator is given as $\Delta(s)$ in 3.

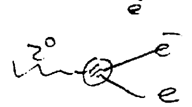
5. Since in our prescription (following Veltman's) the bare parameters are fixed through low energy data. therefore the wave function factors of Z^0 in  is also fixed at low energy, not on Z^0-shell.

Hence the effective amplitude will be

$$\mathcal{M} = \frac{g^{\gamma ee} \cdot g^{\gamma \bar{\mu}\mu}}{-s} + \frac{1+\tilde{\Pi}'(-M_Z^2)}{-s+M_Z^2 - iM_Z\Gamma_Z}\left\{ g^{Z\bar{e}e}\cdot g^{Z\bar{\mu}\mu} + V^{Z\bar{e}e} g^{Z\bar{\mu}\mu} + g^{Z\bar{e}e} V^{Z\bar{\mu}\mu} \right\},$$

with

$g^{Z\bar{e}e}$ =  , $g^{\gamma ee}$ = 

$V^{Z\bar{e}e}$ =  $\sim \mathcal{O}(g^2)$

$$\tilde{\Pi}'(-M_Z^2) = \frac{\partial}{\partial p^2}\tilde{\Pi}(p^2)\Big|_{p^2=-M_Z^2}$$

tree values

$$\mathcal{M} = \frac{g^{\gamma\bar{e}e}_{(0)}\, g^{\gamma\bar{\mu}\mu}_{(0)}}{-s} + \frac{1+\tilde{\Pi}'(-M_Z^2)}{-s+M_Z^2-iM_Z\Gamma_Z}\cdot g^{Z\bar{e}e}_{(1)}\, g^{Z\bar{\mu}\mu}_{(1)} \left[1+\frac{V^{Z\bar{e}e}}{g^{Z\bar{e}e}_{(0)}}\right]\left[1+\frac{V^{Z\bar{\mu}\mu}}{g^{Z\bar{\mu}\mu}_{(0)}}\right]$$

bare quantities.

6. Summary:

From above derivation, we learned that:
If we use Veltman's prescription to fix the parameter g, s_0 and M from low energy, then we can not take the advantage of using Marciano's prescription, so that the wavefunction renormalization factor $1+\tilde{\pi}'(-M_Z^2)$ will be cancelled out by defining a coupling through $Z\bar{e}e$ vertex, for instance.

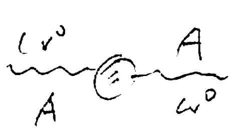
Also in Veltman's prescription, right on Z^0-mass shell, the denominator $iM_Z\Gamma_Z$ should be expressed in terms of the bare parameters $g^{(1)}$, $s_0^{(1)}$ and $M^{(1)}$ fixed from low energy data.
This will make the whole expression awkard.

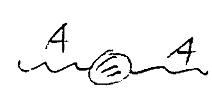
Hence, it is just <u>not</u> smart to do the Z^0-resonance physics using Veltman's prescription.

7. To check that the expression given in 5. is finite, we can consider one heavy fermion loop contribution, i.e. assume

$$V^{Z\bar{e}e} = V^{Z\bar{\mu}\mu} = 0$$

and the only non-vanished loop diagrams are

, 

and 

(We will assume a complete forth family.)

1) Recall that

$$M_Z^2 = M_0^{(1)2} - \tilde{\pi}(-M_Z^2)$$

and Γ_Z is calculated up to one-loop level.

In our case, since there is no V^{Zee} etc, Γ_Z is just the tree results.

2) $$M_0^{(1)} = \frac{M^{(1)}}{C_0^{(1)}} .$$

$M^{(1)}$ and $C_0^{(1)}$, in our case, are just some kind of function of self-energies evaluated at low energy; ie $q^2=0$.

3) Since all these self-energies can be expressed as

$$\tilde{\pi}(q^2) = A(q^2)\cdot q^2 + B(q^2) .$$

Therefore, $\tilde{\pi}(0) = B(0) \longrightarrow$ infinite

$$\tilde{\pi}'(-M_0^2) = \left.\frac{\partial \tilde{\pi}(q^2)}{\partial q^2}\right|_{q^2=-M_0^2} = A(-M_0^2) \longrightarrow \text{infinite}$$

Some how there is not any obvious reason that the quantity $\frac{1+\tilde{\pi}'(-M_Z^2)}{-iM_Z\Gamma_Z}$ should be finite ?

4) Let us use another language to say the same thing.

If we use the counterterm language, then instead of saying that using those three processes to fix the bare parameters, which are infinite, we say using those three processes to fix the counter terms, which are infinite.

(1) Now consider the self-energy of W^0,

$$W^0 \sim\!\!\bigcirc\!\!\sim W^0 = A_{W^0}(q^2)\delta_{\mu\nu} + B_{W^0}(q^2)\left(\frac{q_\mu q_\nu}{q^2}\right)$$

$$W^0 \sim\!\!\times\!\!\sim W^0 = \delta_{\mu\nu}\left[-\delta Z_{W^0} q^2 - M_0^2(\delta Z_{W^0} + \delta M_0^2)\right] + \frac{q_\mu q_\nu}{q^2}\left[q^2 \delta Z_{W^0}\right]$$

for

$$W^0_{bare} = \left(1 + \frac{1}{2}\delta Z_{W^0}\right) W^0_{ren.} + \left(\frac{1}{2}\delta Z_{W^0 A}\right) A_{ren}$$

(2) We found that:

As we use these three low energy processes to fix our counterterms, we already fix

$$\left(W^0 \sim\!\!\times\!\!\sim W^0\right)_{q^2=0} = \delta_{\mu\nu}\left[-M_0^2(\delta Z_{W^0} + \delta M_0^2)\right]$$

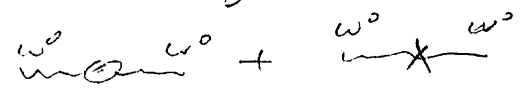
Therefore, both the mass and wavefunction counterterms of the W^0 have been fixed.

(3) If we write down the full propagator of W^0 in terms of counterterms, then it is

$$\frac{1}{(2\pi)^4 i}\left\{\frac{1}{q^2+M_0^2-A'}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \frac{\alpha}{q^2+\alpha M^2 - \alpha(A'+B')}\left(\frac{q_\mu q_\nu}{q^2}\right)\right\}$$

* α is the gauge parameter.

where $A' \delta_{\mu\nu} + B'\left(\frac{q_\mu q_\nu}{q^2}\right)$

$\equiv$ 

Let us concentrate on $\delta_{\mu\nu}$ piece, which corresponds the transversed part of this propagator (and both the mass and wave function renormalizations are fixed through this part).

Thus

$$q^2+M_0^2-A' = q^2+M_0^2-\left\{A_{\omega^0}(q^2) - \delta Z_{\omega^0}\, q^2 - M_0^2\left(\delta Z_{\omega^0}+\delta(M_0^2)\right)\right\},$$

<u>Note</u> Here M_0 is the renormalized quantity, ie $M_0^2 = M_0^{(0)\,2}$.

Hence

$q^2+M_0^2-A'$ is finite.

(4) Because we used low energy experiments to fix our counterterms therefore we use $q^2=0$ subtraction, <u>not</u> on-shell (or $q^2=-M_Z^2$) subtraction.

This answers our question:

Because $\underset{\text{on-shell subtraction}}{\tilde{\pi}'(-M_Z^2)} = \underset{q^2=0\text{ subtraction}}{\tilde{\pi}'(0)}$.

Although $1+\tilde{\pi}'(-M_Z^2)$ is not finite,
yet after we put in all the fixed bare parameters
we should have $\mathcal{M}$ being finite, ie

$\left[1+\tilde{\pi}(-M_Z^2)\right]\cdot\left\{g\cdot g + Vg + gV\right\}$ is finite.