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CISK–Kelvin Waves, CISK–Rossby Waves and Low–Frequency Oscillation*

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A baroclinic semi–geostrophic model, including the wave–CISK mechanism, was established to analyze the low–frequency oscillation (LFO) in the tropical atmosphere. In this paper, we not only considered the effect of CISK mechanism which was demonstrated as $N^2 \eta w_g$ in the thermodynamic equation, but also considered the variation of vertical velocity w_g at the top of boundary layer with latitude. Under those considerations, the analytic solutions of equations which can describe the LFO in the tropical atmosphere are obtained, and the Kelvin waves and Rossby waves excited by the CISK mechanism, CISK–Kelvin waves and CISK–Rossby waves are discussed. It is shown that the wave–CISK mechanism is very important to the LFO in the tropical atmosphere. In this treatise we expanded a series of the eigenfunction $S_m^{-3/2}(x)$ which was named as the Sonine polynomial. Of our results, we not only obtained the relation between the propagating velocities and the convective condensational heating parameter η, but also demonstrated the coupling effects for the upper and lower atmosphere in the tropics. It depends on the different values of η in the upper and lower atmosphere where the CISK–Kelvin waves and CISK–Rossby waves propagate eastward or westward, and these waves may be unstable.

Key words: wave–CISK mechanism; low–frequency oscillation; convective condensational heating parameter; CISK–Kelvin wave; CISK–Rossby wave.

1. INTRODUCTION

In the early 1970s, Madden and Julian[1] discovered first by using the spectrum analysis method on the 10 years observational data in Canton Island that the zonal wind and surface pressure are characteristic of the periodic oscillation with about 40–50 days. In their further research[2], they proved that in the all tropics there were the periodic oscillations with 40–50 days which propagate eastward and have the disturbance character of the zonal wavenumber 1.

As is well known, the main area is ocean in the tropics. In this area, there is sufficient supply of moisture, the atmosphere is usually the conditional unstable and the cumulus convection which plays a very important role in the tropical atmosphere is quite active. The re-

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searches on the tropical atmosphere must consider adequately these facts. Yamasaki[3] introduced first the mechanism of conditional unstable stratification into a linear two-dimensional model to analyze some waves in the tropical atmosphere. Hayashi[4] explained several phenomena including the LFO by the same method. And this method have been summarized as so-called the wave-CISK theory by Lindzen[5]. From this theory, when we assumed that the ratio of convective condensational heating is proportional to the vertical velocity, through some procedure of waves in the tropical atmosphere, the CISK could be motivated and then led to the unstable development of waves, the same that the Ekman pumping factor could do.

In the later 1980s, the researchers paid much attention to study the periodic LFO with 30-60 days. They not only studied it as a periodic phenomena in the atmosphere, but also researched further its genesis mechanism and dynamical structure etc., that is to say, they considered it as an atmospheric "substance".

Some theoretical and numerical investigations on the wave-CISK mechanism are developed by Li[6], Hayashi and Sumi[7], Lau and Peng[8], Miyakara[9], Chang and Lim[10] and Liu and Wang[11]. Their results showed that the LFO can basically be regarded as the Kelvin waves or Rossby waves responded to the condensational heating caused by the large-scale convergence lifting in low latitudes.

2. BASIC EQUATIONS

By considering a Boussinesq fluid with the stable stratification and applying the equatorial β-plane approximation, the basic equations of the baroclinic semi-geostrophic model including the wave-CISK mechanism can be written as follows:

$$\frac{\partial u}{\partial t} - \beta_0 yv = -\frac{\partial \varphi'}{\partial x}, \tag{1}$$

$$\beta_0 yu = -\frac{\partial \varphi'}{\partial y}, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \varphi'}{\partial z}\right) + N^2 w = \eta N^2 w_B . \tag{4}$$

where N is the Brunt-Vaisala frequency, β_0 is the Rossby parameter assumed to be constant, w_B is the vertical velocity at the top of boundary layer and η means the non-dimensional parameter of latent heat which is assumed to depend only on height z and non-zero only when $w_B > 0$. The term $\eta N^2 w_B$ on the right-hand side of Eq.(4) represents the condensational heat in the CISK mechanism. Other notations are standard.

Liu[12] demonstrated that Eqs.(1)-(4) can filter out the high-frequency inertia-gravity waves and retain the Rossby waves with large wavelength ($k \to 0$) and the Kelvin waves at low latitudes. The latter corresponds to the case of $v = 0$ in Eqs.(1)-(3).

Eliminating u from Eqs. (1) and (2) we have

$$\beta_0^2 y^2 v = -\left(\frac{\partial^2}{\partial t \partial y} - \beta_0 y \frac{\partial}{\partial x}\right)\varphi' \tag{5}$$

Differentiating Eq. (5) with respect to y leads to

$$\beta_0^2 y^2 \frac{\partial v}{\partial y} + 2\beta_0^2 yv = -\left(\frac{\partial^3}{\partial t \partial y^2} - \beta_0 y \frac{\partial^2}{\partial x \partial y} - \beta_0 \frac{\partial}{\partial x}\right)\varphi'. \tag{6}$$

Replacing $\partial\varphi'/\partial y$ on right-hand side of Eq. (6) by $-\beta_0 yu$ in terms of Eq. (2), we then obtain

$$\beta_0^2 y^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\beta_0^2 yv=-\frac{\partial^3\varphi'}{\partial t\partial y^2}+\beta_0\frac{\partial\varphi'}{\partial x}. \tag{7}$$

Replacing v in the second term on the left-hand side of Eq. (7) by φ' in terms of Eq. (5) yields

$$\beta_0^2 y^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=-\frac{\partial^3\varphi'}{\partial t\partial y^2}+\frac{2}{y}\frac{\partial^2\varphi'}{\partial t\partial y}-\beta_0\frac{\partial\varphi'}{\partial x}. \tag{8}$$

Utilizing Eq. (3) and replacing $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$ by $-\frac{\partial w}{\partial z}$, we have

$$\beta_0^2 y^3\frac{\partial w}{\partial z}=\left(y\frac{\partial}{\partial y}-2\right)\frac{\partial^2\varphi'}{\partial t\partial y}-\beta_0 y\frac{\partial\varphi'}{\partial x}. \tag{9}$$

On applying the operator $\frac{\partial^2}{\partial y\partial z}$ to Eq. (9) and in terms of Eq. (4) we then obtain

$$\mathcal{L}w=F, \tag{10}$$

where

$$\mathcal{L}\equiv\frac{\partial}{\partial t}\left\{N^2\left(y\frac{\partial^2}{\partial y^2}-2\frac{\partial}{\partial y}\right)+\beta_0^2 y^3\frac{\partial^2}{\partial z^2}\right\}-N^2\beta_0 y\frac{\partial}{\partial x} \tag{11}$$

and

$$F\equiv\eta N^2 y\frac{\partial^3 w_B}{\partial t\partial y^2}-2\eta N^2\frac{\partial^2 w_B}{\partial t\partial y}-\eta N^2\beta_0 y\frac{\partial w_B}{\partial x}. \tag{12}$$

The boundary conditions of Eq. (10) are given as

$$w|_{z=0,H}=0 \tag{13}$$

and

$$w|_{y\to\pm\infty}=0. \tag{14}$$

Eq.(10) is the basic equation which can be used to discuss the wave-CISK mechanism.

3. ANALYTIC SOLUTION WITH $\eta=0$

When $\eta=0$, Eq. (10) can be reduced to

$$\mathcal{L}w=0. \tag{15}$$

which is a homogeneous equation in w. It shows that the vertical velocity at the top of the boundary layer is descendent.

Considering Eq. (15) and boundary condition Eq. (13) and applying the normal mode method, we set

$$w=W(y)e^{i(kx+nz-\omega t)}, \tag{16}$$

where k and n are the wavenumbers in x and z directions respectively, and ω the angular frequency. If L is defined as the wavelength in x direction, we have

$$k=2\pi/L, \tag{17}$$

and from boundary condition Eq. (13) n may be written as

$$n = \pi / H. \tag{18}$$

Substituting Eq. (16) into Eq.(15) and boundary condition Eq. (14), we have

$$y^2 \frac{\mathrm{d}^2 W}{\mathrm{d}y^2} - 2y\frac{\mathrm{d}W}{\mathrm{d}y} - \left(\frac{1}{L_1^4} y^4 + \frac{\beta_0 k}{w} y^2 \right) W = 0, \tag{19a}$$

$$W|_{y \to \pm \infty} = 0, \tag{19b}$$

where L_1 is the Rossby radius of deformation in low latitudes. i.e.

$$L_1 \equiv \sqrt{\frac{c_1}{\beta_0}}, \qquad c_1 = \frac{N}{n} = \frac{NH}{\pi}. \tag{20}$$

Then, by setting

$$\xi_1 = \left(\frac{y}{L_1} \right)^2, \qquad W = \xi^{1/4} \hat{W}. \tag{21}$$

Eq. (19) can be simplified to find the following eigenvalue solution of the Whittaker equation

$$\frac{\mathrm{d}^2 \hat{W}}{\mathrm{d}\xi_1^2} + \left(-\frac{1}{4} + \frac{L_1}{\xi_1} + \frac{(1/4) - \mu^2}{\xi_1^2} \right) \hat{W} = 0, \tag{22a}$$

$$\hat{W}|_{\xi_1 \to \infty} = 0, \tag{22b}$$

where

$$l_1 = -\frac{kN}{4n\omega} = -\frac{kc_1}{4\omega}, \qquad \mu^2 = \frac{9}{16}. \tag{23}$$

If setting

$$\hat{W} = e^{-\xi_1/2} \xi_1^{\mu - 1/2} Z. \tag{24}$$

Eq.(22) can be transformed into the following eigenvalue solution of the Kummer equation (i.e. the confluent hypergeometric equation)

$$\xi_1 \frac{\mathrm{d}^2 Z}{\mathrm{d}\xi_1^2} + (2\mu + 1 - \xi_1)\frac{\mathrm{d}Z}{\mathrm{d}\xi_1} - \left(\mu + \frac{1}{2} - l_1 \right) Z = 0. \tag{25a}$$

$$Z|_{\xi_1 \to \infty} = O(\xi_1^m). \tag{25b}$$

The eigenvalues of Eq.(25) are

$$\mu + \frac{1}{2} - l_1 = -m. \qquad (m = 0, 1, 2 \cdots) \tag{26}$$

The corresponding eigenfunctions are

$$Z = A_m S_m^{2\mu}(\xi_1) = A_m \frac{(2\mu + 1)_m}{m!} F(-m, 2\mu + 1, \xi_1), \qquad (m = 0, 1, 2, \cdots) \tag{27}$$

where A_m is the arbitrary constant, and $(2\mu + 1)_m$ is the Gauss symbol defined as

$$(2\mu + 1)_m = (2\mu + 1)(2\mu + 2) \cdots (2\mu + m) = \frac{\Gamma(2\mu + 1 + m)}{\Gamma(2\mu + 1)}, \tag{28a}$$

$$(2\mu + 1)_0 = 1, \tag{28b}$$

while $S_m^{2\mu}(\xi)$ is known as the Sonine polynomial, and $F(-m, 2\mu + 1, \xi)$ as the Kummer funcion (i.e. the confluent hypergeometric function).

If taking $\mu = 3/4$ from Eq. (23), Eq. (26) may be reduced to

$$-\frac{1}{4} + \frac{kc_1}{4\omega} = -m, \qquad (m = 0,1,2,\cdots) \tag{29}$$

and from Eq. (27), we have

$$Z = A_m S_m^{-1/2}(\xi_1) = A_m \frac{(1/2)_m}{m!} F\left(-m, -\frac{1}{2}, \xi_1\right). \tag{30}$$

From Eq. (29) we find that the angular frequencies are

$$\omega = \frac{kc_1}{-4m+1}. \qquad (m = 0,1,2,\cdots) \tag{31}$$

Substituting Eq. (30) into Eqs. (24) and (21), we obtain

$$\begin{aligned} W_m(y) &= A_m e^{-\xi_1/2} \cdot S_m^{-3/2}(\xi_1) = A_m \frac{(-1/2)_m}{m!} e^{-\xi_1/2} F\left(-m, -\frac{1}{2}, \xi_1\right) \\ &= A_m e^{-y^2/(2L_1^2)} \cdot S_m^{-3/2}\left(\frac{y^2}{L_1^2}\right) = A_m \frac{(-1/2)_m}{m!} e^{-y^2/(2L_1^2)} F\left(-m, \frac{1}{2}, \frac{y^2}{2L_1^2}\right). \end{aligned} \tag{32}$$

From Eq. (31) we can see that when $m = 0$ it becomes

$$\omega = kc_1, \tag{33}$$

which is obviously the eastward propagating equatorial Kelvin waves; and when $m \neq 0$ then $\omega < 0$ in Eq. (31), which denotes the westward propagating equatorial Rossby waves.

From Eq. (31) we obtain the oscillatory period

$$T \equiv \frac{2\pi}{|\omega|} = \frac{|-4m+1|}{C_1} L. \tag{34}$$

By taking $c_1 = 30\ \mathrm{m \cdot s^{-1}}$, the values of T are given in the table below:

T \ L m	6.0×10^6 m	1.2×10^7 m
0	2.4 day	4.8 day
1	7 day	14 day
2	16 day	32 day

Evidently, there are many discrepancies between the results on condition that $\eta = 0$ and the observational LFO either for the propagating direction of waves or for the values of periods. Therefore, the Rossby waves and Kelvin waves without the condensational heating are not certainly the forcing of the LFO.

4. ANALYTIC SOLUTION WITH $\eta \neq 0$

When $\eta \neq 0$, Eq.(10) contains the CISK mechanism. First, applying the normal mode method, we set

$$w = W(y,z)e^{i(kx-\omega t)}, \tag{35a}$$

$$w_B = W_B(y)e^{i(kx-\omega t)}. \tag{35b}$$

Substituting Eq. (35) into Eq. (10) yields

$$\omega N^2\left(y\frac{\partial^2 W}{\partial y^2} - 2\frac{\partial W}{\partial y}\right) + \omega\beta_0^2 y^2\frac{\partial^2 W}{\partial z^2} - N^2 k\beta_0 yW$$
$$= \omega N^2\eta\left(y\frac{\mathrm{d}^2 W_B}{\mathrm{d}y^2} - 2\frac{\mathrm{d}W_B}{\mathrm{d}y}\right) - N^2 k\beta_0 \eta y W_B . \qquad (36)$$

and the boundary condition (14) in terms of (35a) is reduced to

$$W|_{y\to\pm\infty} = 0. \qquad (37)$$

(36) is a partial differential equation in W with respect to y and z. To find the solution we use a simple 3-layer model which is established by Takahashi[13] as shown in Fig.1. In this model. Δz is the interval in z direction and η varies with z as illustrated in Fig.2.

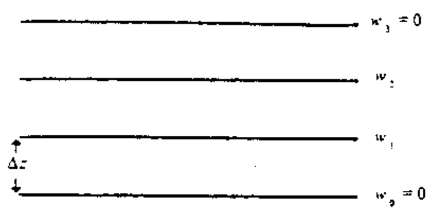

FIGURE 1. Schematic illustration of a 3-layer model.

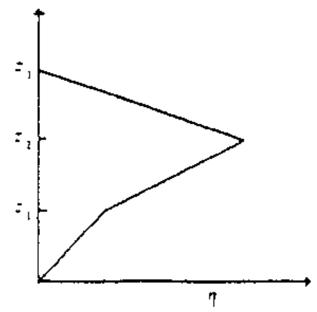

FIGURE 2. Schematic diagram showing the variation of η with z in the 3-layer model.

Next, let Eq.(36) be written on the first and second layer in the 3-layer model and the differentiation with respect to z is replaced by the difference. that is to say.

$$\left(\frac{\partial^2 W}{\partial z^2}\right)_1 = \frac{W_2 - 2W_1}{(\Delta z)^2}, \qquad \left(\frac{\partial^2 W}{\partial z^2}\right)_2 = \frac{W_1 - 2W_2}{(\Delta z)^2}. \qquad (38)$$

and we set

$$W_B = bW_1 \qquad (0 < b \leqslant 1). \qquad (39)$$

where b is a non-dimensional parameter. We then have

$$(1 - b\eta_1)\left[y^2\frac{\mathrm{d}^2}{\mathrm{d}y^2} - 2y\frac{\mathrm{d}}{\mathrm{d}y} - \left(\frac{y^4}{L_2^4} + \frac{k\beta_0}{\omega}y^2\right)\right]W_1 - \frac{y^4}{L_2^4}b\eta_1 W_1 + \frac{y^4}{2L_2^4}W_2 = 0 \qquad (40a)$$

$$b\eta_2\left[y^2\frac{\mathrm{d}^2}{\mathrm{d}y^2} - 2y\frac{\mathrm{d}}{\mathrm{d}y} - \left(\frac{y^4}{L_2^4} + \frac{k\beta_0}{\omega}y^2\right)\right]W_1 + b\left(\eta_2 - \frac{1}{2}\right)\frac{y^4}{L_4 2} W_1$$
$$- \left[y^2\frac{\mathrm{d}^2}{\mathrm{d}y^2} - 2y\frac{\mathrm{d}}{\mathrm{d}y} - \left(\frac{y^4}{L_2^4} + \frac{k\beta_0}{\omega}y^2\right)\right]W_2 = 0, \qquad (40b)$$

where

$$L_2 = \sqrt{\frac{c_2}{\beta_0}}, \qquad c_2 = \sqrt{\frac{1}{2}}N\Delta z. \qquad (41)$$

By setting

$$\xi_2 = \left(\frac{y}{L_2}\right)^2, \qquad W_{1,2} = \xi_2^{1/4}\hat{W}_{1,2}, \tag{42}$$

Eqs.(40) can be transformed into the following equations in $\hat{W}_1$ and $\hat{W}_2$

$$(1 - b\eta_1)\left\{\frac{d^2}{d\xi_2^2} + \left[-\frac{1}{4} + \frac{l_2}{\xi_2} + \frac{(1/4) - \mu^2}{\xi_2^2}\right]\right\}\hat{W}_1 - \frac{1}{4}b\eta_1\hat{W}_1 - \frac{1}{8}\hat{W}_2 = 0. \tag{43a}$$

$$b\eta_2\left\{\frac{d^2}{d\xi_2^2} + \left[-\frac{1}{4} + \frac{l_2}{\xi_2} + \frac{(1/4) - \mu^2}{\xi_2^2}\right]\right\}\hat{W}_1 + \frac{1}{4}b\left(\eta_2 - \frac{1}{2}\right)\hat{W}_1$$
$$-\left\{\frac{d^2}{d\xi_2^2} + \left[-\frac{1}{4} + \frac{l_2}{\xi_2} + \frac{(1/4) - \mu^2}{\xi_2^2}\right]\right\}\hat{W}_2 = 0. \tag{43b}$$

where

$$l_2 = -\frac{kc_2}{4\omega}, \qquad \mu^2 = \frac{9}{16}. \tag{44}$$

By setting again

$$\hat{W}_{1,2} = e^{-\xi_2/2}\xi_2^{\mu+(1/2)}Z_{1,2}. \tag{45}$$

Eqs.(43) can be reduced to the equations with the Kummer operator

$$(1 - b\eta_1)\left[\xi_2\frac{d^2}{d\xi_2^2} + (2\mu + 1 - \xi_2)\frac{d}{d\xi_2} - \left(\mu + \frac{1}{2} - l_2\right)\right]Z_1 - \frac{1}{4}b\eta_1\xi_2 Z_1$$
$$-\frac{1}{8}\xi_2 Z_2 = 0 \tag{46a}$$

$$b\eta_2\left[\xi_2\frac{d^2}{d\xi_2^2} + (2\mu - 1 - \xi_2)\frac{d}{d\xi_2} - \left(\mu + \frac{1}{2} - l_2\right)\right]Z_1 + \frac{1}{4}b\left(\eta_2 - \frac{1}{2}\right)\xi_2 Z_1$$
$$-\left[\xi_2\frac{d^2}{d\xi_2^2} + (2\mu - 1 - \xi_2)\frac{d}{d\xi_2} - \left(\mu + \frac{1}{2} - l_2\right)\right]Z_2 = 0. \tag{46b}$$

Taking $\mu = 3/4$ and expanding Z_1 and Z_2 in Sonine series by written

$$Z_1 = \sum_{m=0}^{\infty} A_m S_m^{-3/2}(\xi_2), \qquad Z_2 = \sum_{m=0}^{\infty} B_m S_m^{-3/2}(\xi_2). \tag{47}$$

where A_m and B_m are the expanding coefficients.

Substituting (47) into (46) and noting the recurrence relations of $S_m^{2\mu}(\xi)$

$$\xi\frac{d^2 S_m^{2\mu}}{d\xi^2} + (2\mu + 1 - \xi)\frac{dS_m^{2\mu}}{d\xi} = -mS_m^{2\mu}(\xi), \tag{48a}$$

$$\xi S_m^{2\mu}(\xi) = -(m + 1)S_{m+1}^{2\mu}(\xi) + (2\mu + 2m + 1)S_m^{2\mu}(\xi) - (m + 2\mu)S_{m-1}^{2\mu}(\xi). \tag{48b}$$

we then obtain the following equations:

$$(1 - b\eta_1)\sum_{m=0}^{\infty} A_m\left(\frac{1}{4} + l_2 - m\right)S_m^{-3/2} + \frac{1}{4}b\eta_2\sum_{m=0}^{\infty} A_m\left[-(m + 1)S_{m+1}^{-3/2} + \left(2m\right.\right.$$
$$\left.\left.-\frac{1}{2}\right)S_m^{-3/2} + \left(\frac{3}{2} - m\right)S_m^{-3/2}\right] + \frac{1}{8}\sum_{m=0}^{\infty} B_m\left[-(m + 1)S_{m+1}^{-3/2} + \left(2m - \frac{1}{2}\right)S_m^{-3/2}\right.$$

$$+\left(\frac{3}{2}-m\right)S_{m-1}^{-3/2}\Big]=0, \tag{49a}$$

$$b\eta_2\sum_{m=0}^{\infty}A_m\left(\frac{1}{4}+l_2-m\right)S_m^{-3/2}+\frac{1}{4}b\eta_2\sum_{m=0}^{\infty}A_m\Big[-(m+1)S_{m+1}^{-3/2}+\left(2m-\frac{1}{2}\right)S_m^{-3/2}$$

$$+\left(\frac{3}{2}-m\right)S_{m-1}^{-3/2}\Big]-\sum_{m=0}^{\infty}B_m\left(\frac{1}{4}-l_2-m\right)S_m^{-3/2}=0. \tag{49b}$$

By applying the orthogonality relation of $S_m^{2\mu}(\xi)$

$$\int_0^{\infty}\xi^{2\mu}e^{-\xi}S_n^{2\mu}(\xi)d\xi=\frac{\Gamma(2\mu+m+1)}{m!}\,\delta_{mn} \tag{50}$$

to Eqs. (49), we can obtain

$$\Big[(1-b\eta_1)\left(l_2+\frac{1}{4}-m\right)-\frac{1}{4}b\eta_1\left(2m-\frac{1}{2}\right)\Big]A_m+\frac{1}{8}\left(2m-\frac{1}{2}\right)B_m=0. \tag{51a}$$

$$\Big[b\eta_2\left(l_2+\frac{1}{4}-m\right)+\frac{1}{4}b\eta_2\left(2m-\frac{1}{2}\right)\Big]A_m+\left(l_2+\frac{1}{4}-m\right)B_m=0. \tag{51b}$$

The existence of nontrivial solutions demands that the determinant of the coefficients of A_m and B_m in Eq. (51) vanishes, which leads to

$$(1-b\eta_1)\left(l_2+\frac{1}{4}-m\right)^2-\frac{1}{4}b\left(\eta_1-\frac{1}{2}\eta_2\right)\left(2m-\frac{1}{2}\right)\left(l_2-\frac{1}{4}-m\right)$$
$$+\frac{1}{32}b\left(\eta_2-\frac{1}{2}\right)\left(2m-\frac{1}{2}\right)^2=0. \tag{52}$$

As $l_2=-kc_2/4\omega$, Eq. (52) is the quadratic equation with ω. From this we can obtain the dependence of ω on m, η_1 and η_2. By analyzing Eq. (52) we can see that when $b\eta_1\geqslant 1$ the imaginary part ω_i of angular frequency, $\omega=\omega_r-i\omega_i$, vanishes; this case corresponds to the stable propagating waves. When $0<b\eta_1<1$, ω_i is nonzero, which corresponds to the unstable propagating waves. In order to clearly show these we will discuss them under two conditions: $b\eta_1=1$ ($b=0.8$) and $\eta_1=1$.

(1) $b\eta_1=1$ ($b=0.8$, $\eta_1=1.25$)

From Eq. (52) we can obtain

$$l_2-\frac{1}{4}-m=\frac{1}{10}\cdot\frac{\eta_2-(1/2)}{1-(2\eta_2/5)}\left(2m-\frac{1}{2}\right). \tag{53}$$

so that

$$\omega=-\frac{2(5-2\eta_2)}{(4m-1)(9-2\eta_2)}kc_2. \qquad (m=0,1,2,\cdots) \tag{54}$$

When $m=0$, Eq. (54) reduces to

$$\omega=\frac{2(5-2\eta_2)}{9-2\eta_2}kc_2, \tag{55}$$

which represents the CISK–Kelvin waves. From Eq. (55) we can see that when $\eta_2<5/2$ or $\eta_2>9/2$, the CISK–Kelvin waves propagate eastward ($\omega>0$); when $5/2<\eta_2<9/2$, the CISK–Kelvin waves propagate westward ($\omega<0$). That is to say, when the convective condensation heating in the middle and upper troposphere is rather weak or strong, the

CISK–Kelvin waves propagate eastward; and when the convective condensation heating in the middle and upper troposphere is moderate, the CISK–Kelvin waves propagate westward.

From Eq. (55) we can obtain the oscillatory period of CISK–Kelvin waves given by

$$T_0 = \frac{2\pi}{|\omega|} = \left|\frac{9 - 2\eta_2}{2(5 - 2\eta_2)}\right| \frac{L}{c_2}. \tag{56}$$

As shown in Fig.3, if taking $L = 2.0 \times 10^7$ m and $c_2 = 2.4\ \mathrm{m \cdot s^{-1}}$ we can find that when $2 \leqslant \eta_2 < 5/2$, the CISK–Kelvin waves propagate eastward with $T_0 = 30\text{–}60$ days; when $5/2 \leqslant \eta_2 < 5.5/2$, the CISK–Kelvin waves propagate westward with $T_0 = 30\text{–}60$ days.

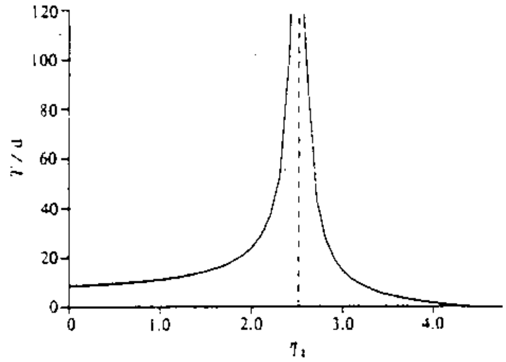

FIGURE 3. The variation of period of CISK–Kelvin waves with the convective condensational heating parameter η_2 for $L = 2.0 \times 10^7$ m.

When $m \neq 0$, Eq. (54) represents the CISK–Rossby waves. And when $\eta_2 < 2/5$ or $\eta_2 < 2/9$, the CISK–Rossby waves propagate westward ($\omega < 0$); when $2/5 < \eta_2 < 2/9$ the CISK–Rossby waves propagate eastward ($\omega > 0$). As an example, setting $m = 1$ Eq. (54) is written as

$$\omega = -\frac{2(5 - 2\eta_2)}{3(9 - 2\eta_2)} k c_2. \tag{57}$$

and the corresponding period is given by

$$T_1 = \left|\frac{3(9 - 2\eta_2)}{2(5 - 2\eta_2)}\right| \frac{L}{c_2}. \tag{58}$$

Similar to Fig.3, by setting $L = 2.0 \times 10^7$ m and $c_2 = 24\ \mathrm{m \cdot s^{-1}}$, we see that when $1.25 < \eta_2 \leqslant 2$, the CISK–Rossby waves propagate westward with $T_1 = 30\text{–}60$ days; when $3 < \eta_2 < 6.5/2$, the CISK–Rossby waves propagate eastward with $T_1 = 30\text{–}60$ days.

(2) $\eta_1 = 1$

From Eq. (52) we can obtain

$$l_2 + \frac{1}{4} - m = \frac{1}{2}\left[\left(1 - \frac{1}{2}\eta_2\right)\left(2m - \frac{1}{2}\right) \pm \frac{1}{2}\left|2m - \frac{1}{2}\right|\sqrt{(\eta_2 - 1)(\eta_2 - 5)}\right], \tag{59}$$

so that

$$\omega = -\frac{1}{2\left[(4 - \eta_2)\left(-\frac{1}{4} + m\right) \pm \left|m - \frac{1}{4}\right|\right]\sqrt{(\eta_2 - 1)(\eta_2 - 5)}} k c_2. \tag{60}$$

When $m=0$, Eq. (60) becomes

$$\omega = \frac{2}{(4-\eta_2) \pm \sqrt{(\eta_2 - 1)(\eta_2 - 5)}} kc_2, \tag{61}$$

which represents the CISK-Kelvin waves, but it is not the same as that in Eq. (55). Here ω may be a complex number, so we can obtain the CISK-Kelvin waves.

From Eq. (61) we see that when $\eta_2 < 1$ or $\eta_2 > 5$ ω is a real number. The cases when the convective condensation heating is either very weak or very strong happen rarely; we will not discuss them in this paper.

For $1 < \eta_2 < 5$, Eq. (61) can be written as

$$\omega^{(0)} = \omega_r^{(0)} + i\omega_i^{(0)}, \qquad (1 < \eta_2 < 5), \tag{62}$$

where

$$\omega_r^{(0)} = \frac{2(4-\eta_2)}{11-2\eta_2} kc_2, \qquad \omega_i^0 = \frac{2\sqrt{(\eta_2 - 1)(5-\eta_2)}}{11-2\eta_2} kc_2. \tag{63}$$

From Eq. (63) it is seen that on the one hand η_2 can vary in a wide range where $1 < \eta_2 < 4$, which implies that the CISK-Kelvin waves propagate eastward and are unstable; on the other hand η_2 can also vary in a narrow range where $4 < \eta_2 < 5$, $\omega_r^{(0)} < 0$ and $\omega_i^{(0)} \neq 0$ which implies that the CISK-Kelvin waves propagate westward and are also unstable.

From Eq. (63) the oscillatory period is given by

$$T_0 = \frac{2\pi}{|\omega_r^{(0)}|} = \left|\frac{11-2\eta_2}{2(4-\eta_2)}\right| \frac{L}{c_2}, \tag{64}$$

and it can be discussed similarly to Eq. (56).

When $m \neq 0$, Eq. (60) represents the CISK-Rossby waves. For example, in case of $m=1$, Eq. (60) then can be reduced to

$$\omega = -\frac{2/3}{(4-\eta_2) \pm \sqrt{(\eta_2 - 1)(\eta_2 - 5)}} kc_2. \tag{65}$$

For $1 < \eta_2 < 5$, it yields

$$\omega^{(1)} = \omega_r^{(1)} + i\omega_i^{(1)} \qquad (1 < \eta_2 < 5), \tag{66}$$

where

$$\omega_r^{(1)} = -\frac{2(4-\eta_2)}{11-2\eta_2} kc_2, \qquad \omega_i^{(1)} = \frac{2\sqrt{(\eta_2 - 1)(\eta_2 - 5)}}{3(11-2\eta_2)} kc_2. \tag{67}$$

From Eq. (65) we find that the oscillatory period is given by

$$T_1 = \left|\frac{3(11-2\eta_2)}{2(4-\eta_2)}\right| \frac{L}{c_2}. \tag{68}$$

Evidently, for $1 < \eta_2 < 4$ the CISK-Rossby waves propagate westward ($\omega_r^{(1)} < 0$) and are unstable. For $4 < \eta_2 < 5$ the CISK-Rossby waves propagate eastward ($\omega_r^{(1)} > 0$) and are also unstable.

Evidently, the solutions of Eqs.(1)-(4) can readily be obtained for a given convective condensation heating, and the w and u so obtained are illustrated in Fig.4 and Fig.5, respectively.

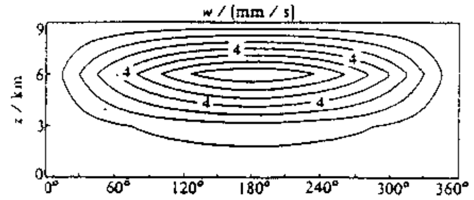

FIGURE 4. Longitude–height section of w over the equator. Contour interval is 10^{-3} m · s^{-1}.

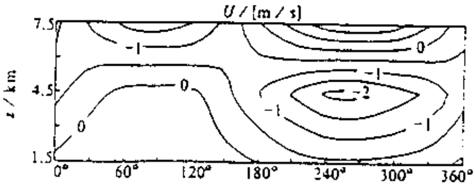

FIGURE 5. Longitude–height section of u over the equator. Contour interval is 0.5 m · s^{-1}.

For comparison, Fig.6 and Fig.7 represent the vertical structures of $\omega \equiv dp / dt$ and u obtained by Murakami and Nakazawa[14] from the observational data of the Northern Hemisphere in the summer of 1979. From these diagrams we see that the theoretical results so obtained are satisfactory.

Applying the same 3–layer model, the phase velocities c_r and c_i obtained by Takahasi[14] are illustrated in Fig.8a and Fig.8b, respectively. From these diagrams we see that for $\eta_1 > 1$, the waves are stable ($c_i \doteq 0$) and propagating ($c_r \doteq 0$). Those results are consistent with ours. However, our research is more deepgoing and comprehensive than Takahashi's. He obtained only the Kelvin waves, but we obtained both the CISK–Kelvin waves and the CISK–Rossby waves. Furthermore, under the interaction of the convective

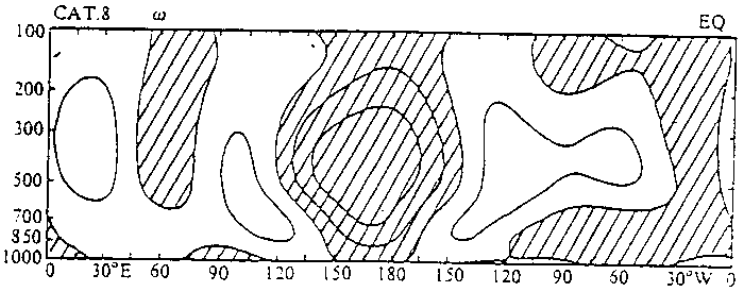

FIGURE 6. Longitude–height section of ω with a 30–60 day oscillation over the equator. Shaded regions correspond to areas where $\omega < 0$, i.e. descending motion. Contour interval is 5×10^{-5} hPa · s^{-1}.

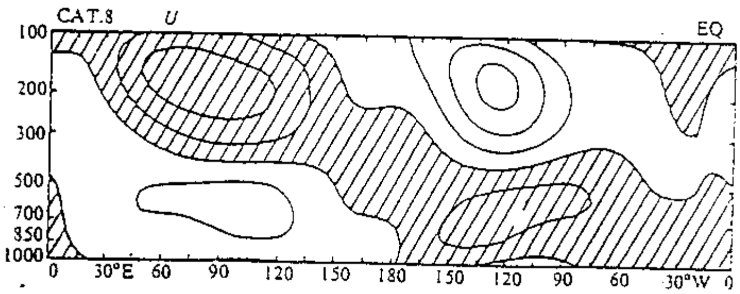

FIGURE 7. Longitude–height section of u with a 30–60 day oscillation over the equator. Shaded regions correspond to areas where $u < 0$, i.e. easterlies. Contour interval is 2.0 m · s^{-1}.

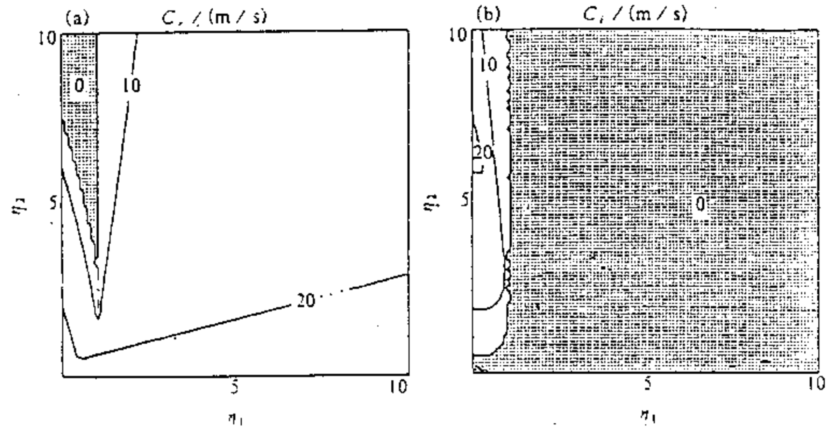

FIGURE 8. The variation of c_r and c_i with η_1 and η_2 by Takahashi[14]. (a) c_r; (b) c_i.

condensational heating between the lower and upper atmosphere, these waves may propagate either eastward or westward and may be unstable.

5. CONCLUSIONS

The baroclinic semi-geostrophic model including the wave-CISK mechanism can be theoretically used to discuss properly the LFO in the tropical atmosphere. It is shown that:

(1) By applying Takahashi's 3-layer model we can find the interaction between the upper and lower troposphere. The disposition of convective condensational heating in the lower and upper tropospheric atmosphere can affect not only the propagating velocities and periods of CISK-Kelvin waves and CISK-Rossby waves but also the stabilities of waves.

(2) When $\eta_1 = 1.25$ and $\eta_2 < 2.5$, the CISK-Kelvin waves propagate eastward slowly, and the CISK-Rossby waves propagate westward slowly. Hence, the main intraseasonal oscillation in the LFO is the CISK-Kelvin waves, but when $5/2 < \eta_2 < 9/2$, the CISK-Rossby waves propagate eastward slowly and the CISK-Kelvin waves propagate eastward slowly, and the main intraseasonal oscillation in the LFO is the CISK-Rossby waves.

(3) When $\eta_2 = 1$ and $1 < \eta_2 < 4$ the CISK-Kelvin waves and CISK-Rossby waves may propagate either eastward or westward, and they are both unstable.

Our above conclusions are consistent with the numerical results of some researchers such as Takahashi et al., and can be applied as a physical basis and mathematical model for the analyses of LFO.

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