The gauge fixing term of W^0 and A.

1. In R_ξ-gauge, it is

$$\mathcal{L}_{g.f} = -\frac{1}{2\alpha_0}\left(-\partial W^0 + \alpha_0 M_0 \phi^0\right)^2 - \frac{1}{2\alpha_A}\left(-\partial A\right)^2.$$

As discussed before, in order to absorb the mixing of Z^0-ϕ^0 and A-ϕ^0, we should introduce the free parameters $\mathcal{M}_0$ and $\mathcal{M}_A$, so that

$$\mathcal{L}_{gf} = \frac{-1}{2\alpha_0}\left(-\partial W^0 + \alpha_0 \mathcal{M}_0 \phi^0\right)^2 - \frac{1}{2\alpha_A}\left(-\partial A + \alpha_A \mathcal{M}_A \phi^0\right)^2. \quad \text{------ } ①$$

At tree level, $\mathcal{M}_0 = M_0$ and $\mathcal{M}_A = 0$.

2. The idea of dealing with the mixing of W^0 and A is to treat them simultaneously as a column matrix $\begin{pmatrix} W^0 \\ A \end{pmatrix}$.

Similar to the previous discussion, we define

$$\begin{pmatrix} W^0 \\ A \end{pmatrix}_{bare} = \begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{12}}{2} \\ \frac{a^{21}}{2} & 1+\frac{a^{22}}{2} \end{pmatrix} \begin{pmatrix} W^0 \\ A \end{pmatrix}_{ren.} \equiv Z_N^{1/2} N_{ren},$$

and

$$\phi^0_{bare} = Z_{\phi^0}^{1/2} \phi^0_{ren}.$$

1) First, let us rewrite ① as a matrix format:

$$\left\{(-\partial W^0 - \partial A) + \phi^0 (\mathcal{M}_0\ \mathcal{M}_A)\begin{pmatrix} \alpha_0 & 0 \\ 0 & \alpha_A \end{pmatrix}\right\} \cdot \begin{pmatrix} \frac{-1}{2\alpha_0} & 0 \\ 0 & \frac{-1}{2\alpha_A} \end{pmatrix} \cdot \left\{\begin{pmatrix} -\partial W^0 \\ -\partial A \end{pmatrix} + \begin{pmatrix} \alpha_0 & 0 \\ 0 & \alpha_A \end{pmatrix}\begin{pmatrix} \mathcal{M}_0 \\ \mathcal{M}_A \end{pmatrix}\phi^0\right\}$$

$$\equiv \frac{-1}{2}\left\{-\partial N^T + \phi^0 \mathcal{M}^T \alpha^T\right\} \cdot \alpha^{-1} \cdot \left\{-\partial N + \alpha \mathcal{M} \phi^0\right\},$$

* $\alpha^T = \alpha$, "T" means transpose.

with $$\mathcal{A} = \begin{pmatrix} \alpha_0 & 0 \\ 0 & \alpha_A \end{pmatrix}, \qquad \mathcal{A}^{-1} = \frac{1}{\alpha_0 \alpha_A} \begin{pmatrix} \alpha_A & 0 \\ 0 & \alpha_0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\alpha_0} & 0 \\ 0 & \frac{1}{\alpha_A} \end{pmatrix},$$

and $$\mathcal{M} = \begin{pmatrix} m_0 \\ m_A \end{pmatrix}.$$

2) Define

$$\mathcal{A}_{bare} = Z_N^{1/2} \cdot \mathcal{A}_{ren} \cdot \left(Z_N^{1/2}\right)^T = \mathcal{A}_{bare}^T,$$ (i.e. Both $\mathcal{A}_{bare}$ and $\mathcal{A}_{ren}$ are symmetric.)

and $$\mathcal{M}_{bare} = \left(Z_N^{-1/2}\right)^T \mathcal{M}_{ren} Z_{\phi^0}^{-1/2},$$

then $$\mathcal{A}_{bare}^{-1} = \left(Z_N^{-1/2}\right)^T \mathcal{A}_{ren}^{-1} Z_N^{-1/2},$$

and $$\mathcal{M}_{bare}^T = Z_{\phi^0}^{-1/2} \mathcal{M}_{ren}^T Z_N^{-1/2}.$$

Hence

$$\begin{aligned}
\mathcal{L}_{gf} = -\frac{1}{2} &\left\{ -\partial\left(N_{ren}^T \left(Z_N^{1/2}\right)^T\right) + Z_{\phi^0}^{1/2} \phi^0_{ren} Z_{\phi^0}^{-1/2} \mathcal{M}_{ren}^T Z_N^{-1/2} Z_N^{1/2} \mathcal{A}_{ren} \left(Z_N^{1/2}\right)^T \right\} \cdot \\
&\left(Z_N^{-1/2}\right)^T \mathcal{A}_{ren}^{-1} Z_N^{-1/2} \cdot \\
&\left\{ -\partial\left(Z_N^{1/2} N_{ren}\right) + Z_N^{1/2} \mathcal{A}_{ren} \left(Z_N^{1/2}\right)^T \left(Z_N^{-1/2}\right)^T \mathcal{M}_{ren} Z_{\phi^0}^{-1/2} Z_{\phi^0}^{1/2} \phi^0_{ren} \right\} \\
= -\frac{1}{2} &\left\{ -\partial N_{ren}^T + \phi^0_{ren} \mathcal{M}_{ren}^T \mathcal{A}_{ren} \right\} \mathcal{A}_{ren}^{-1} \left\{ -\partial N_{ren} + \mathcal{A}_{ren} \mathcal{M}_{ren} \phi^0_{ren} \right\}.
\end{aligned}$$

Therefore, the gauge fixing term can be expressed in terms of renormalized quantities.

Notice that $\mathcal{A}_{ren}$ is made to be symmetric.

3. The bare Lagrangian we start with is

$$-\frac{1}{4}\left(\partial_\mu W^0_\nu - \partial_\nu W^0_\mu\right)^2 - \frac{1}{2}M_0^2 W^0_\mu W^0_\mu - \frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2$$
$$- \frac{1}{2\alpha_0}\left(-\partial_\mu W^0_\mu + \alpha_0 M_0 \phi^0\right)^2 - \frac{1}{2\alpha_A}\left(-\partial_\mu A_\mu\right)^2$$
$$= \frac{1}{2} W^0_\mu \partial_\nu \partial_\nu W^0_\mu - \frac{1}{2} W^0_\mu \partial_\mu \partial_\nu W^0_\nu + \frac{1}{2\alpha_0} W^0_\mu \partial_\mu \partial_\nu W^0_\nu - \frac{1}{2} W^0_\mu M_0^2 W^0_\mu$$
$$+ \frac{1}{2} A_\mu \partial_\nu \partial_\nu A_\mu - \frac{1}{2} A_\mu \partial_\mu \partial_\nu A_\nu + \frac{1}{2\alpha_A} A_\mu \partial_\mu \partial_\nu A_\nu$$
$$+ M_0 \phi^0 \partial W^0 - \frac{1}{2}\alpha_0 M_0^2 \phi_0^2 \qquad \text{———} \quad (2)$$

We would like to find out how to determine $Z_N^{1/2}$ through the one-loop self-energy results.

1) Recall that

$$\begin{pmatrix} W^0 \\ A \end{pmatrix} \rightarrow \begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{12}}{2} \\ \frac{a_{21}}{2} & 1+\frac{a_{22}}{2} \end{pmatrix} \begin{pmatrix} W^0 \\ A \end{pmatrix} \equiv Z_N^{1/2} \cdot N.$$

(1) The W^0 part of (2) becomes, not including ϕ^0,

$$\frac{1}{2}\left[\left(1+\frac{a^{11}}{2}\right)W^0_\mu + \frac{a^{12}}{2}A_\mu\right]\partial_\nu\partial_\nu\left[\left(1+\frac{a^{11}}{2}\right)W^0_\mu + \frac{a^{12}}{2}A_\mu\right]$$
$$-\frac{1}{2}\left(1\right)\left[\left(1+\frac{a^{11}}{2}\right)W^0_\mu + \frac{a^{12}}{2}A_\mu\right]\partial_\mu\partial_\nu\left[\left(1+\frac{a^{11}}{2}\right)W^0_\nu + \frac{a^{12}}{2}A_\nu\right]$$
$$-\frac{1}{2}\left[\left(1+\frac{a^{11}}{2}\right)W^0_\mu + \frac{a^{12}}{2}A_\mu\right] M_0^2 \left[\left(1+\frac{a^{11}}{2}\right)W^0_\mu + \frac{a^{12}}{2}A_\mu\right]$$
$$= \frac{1}{2}\left\{(1+a^{11}) W^0_\mu \partial_\nu\partial_\nu W^0_\mu + \frac{a^{12}}{2} A_\mu \partial_\nu\partial_\nu W^0_\mu + \frac{a^{12}}{2} W^0_\mu \partial_\nu\partial_\nu A_\mu\right\}$$
$$-\frac{1}{2}\left(1\right)\left\{(1+a^{11}) W^0_\mu \partial_\mu\partial_\nu W^0_\nu + \frac{a^{12}}{2} A_\mu \partial_\mu\partial_\nu W^0_\nu + \frac{a^{12}}{2} W^0_\mu \partial_\mu\partial_\nu A_\nu\right\}$$
$$-\frac{1}{2}M_0^2\left\{(1+a^{11}) W^0_\mu W^0_\mu + \frac{a^{12}}{2} A_\mu W^0_\mu + \frac{a^{12}}{2} W^0_\mu A_\mu\right\}(1+\delta M_0^2)$$

* δM_0^2 is defined through δM^2 and δC_θ^2, ie

$$M_0^2 = \frac{M^2}{C_\theta^2} \longrightarrow M_0^2(1+\delta M_0^2) = \frac{M^2(1+\delta M^2)}{C_\theta^2(1+\delta C_\theta^2)} = \frac{M^2}{C_\theta^2}\left(1+\delta M^2 - \delta C_\theta^2\right)$$

* $\frac{1}{\alpha}$ is chosen so that the gauge fixing term is invariant under renormalization

(2) The A part of ② becomes.

$$\frac{1}{2}\left[\left(1+\frac{a^{22}}{2}\right)A_\mu + \frac{a^{21}}{2}W^0_\mu\right]\partial_\nu\partial_\nu\left[\left(1+\frac{a^{22}}{2}\right)A_\mu + \frac{a^{21}}{2}W^0_\mu\right] + \cdots\cdots$$

$$= \frac{1}{2}\left\{(1+a^{22})A_\mu\partial_\nu\partial_\nu A_\mu + \frac{a^{21}}{2}W^0_\mu\partial_\nu\partial_\nu A_\mu + \frac{a^{21}}{2}A_\mu\partial_\nu\partial_\nu W^0_\mu\right\}$$

$$-\frac{1}{2}(\quad 1 \quad)\left\{(1+a^{22})A_\mu\partial_\mu\partial_\nu A_\nu + \frac{a^{21}}{2}W^0_\mu\partial_\mu\partial_\nu A_\nu + \frac{a^{21}}{2}A_\mu\partial_\mu\partial_\nu W^0_\nu\right\}.$$

2) The interactions due to the counterterms are, $(2\pi)^4 i$ is suppressed, as follows:

(1) W^0-W^0 :

$$\frac{a^{11}}{2}W^0_\mu\partial_\nu\partial_\nu W^0_\mu - \frac{a^{11}}{2}(\quad 1\quad)W^0_\mu\partial_\mu\partial_\nu W^0_\nu - \frac{a^{11}}{2}W^0_\mu M_0^2 W^0_\mu - \frac{1}{2}\delta M_0^2\, W^0_\mu M_0^2 W^0_\mu$$

$$\longrightarrow\ a^{11}(ik)^2\delta_{\mu\nu} - a^{11}(\quad 1\quad)(ik_\mu)(ik_\nu) - a^{11}M_0^2\delta_{\mu\nu} - \delta M_0^2\cdot M_0^2\delta_{\mu\nu}$$

$$= -k^2 a^{11}\delta_{\mu\nu} + a^{11}(\quad 1\quad)k_\mu k_\nu - M_0^2(\delta M_0^2 + a^{11})\delta_{\mu\nu}$$

$$= \delta_{\mu\nu}\left\{-k^2a^{11} - M_0^2(\delta M_0^2 + a^{11})\right\} + \frac{k_\mu k_\nu}{k^2}\left\{\ k^2 a^{11}\right\}.$$

* a^{11} is just like δZ_W .

(2) A-A :

$$\frac{a^{22}}{2}A_\mu\partial_\nu\partial_\nu A_\mu - \frac{a^{22}}{2}(\quad 1\quad)A_\mu\partial_\mu\partial_\nu A_\nu$$

$$\longrightarrow\ a^{22}(-ik)^2\delta_{\mu\nu} - a^{22}(\quad 1\quad)(ik_\mu)(ik_\nu)$$

$$= \delta_{\mu\nu}\left\{-k^2a^{22}\right\} + \frac{k_\mu k_\nu}{k^2}\left\{\ k^2a^{22}\right\}$$

(3) $\omega^0 - A$: (This is separated from $A - \omega^0$)

$$\frac{a^{12}}{4}\omega^0_\mu \partial_\nu \partial_\nu A_\mu - \frac{a^{12}}{4}(\ 1 \ )\omega^0_\mu \partial_\mu \partial_\nu A_\nu - \frac{a^{12}}{4} M_0^2 \omega^0_\mu A_\mu$$

$$+ \frac{a^{21}}{4}\omega^0_\mu \partial_\nu \partial_\nu A_\mu - \frac{a^{21}}{4}(\ 1 \ )\omega^0_\mu \partial_\mu \partial_\nu A_\nu$$

$$\longrightarrow \frac{1}{2}(a^{12}+a^{21})(ik)^2 \delta_{\mu\nu} - \frac{1}{2}\left\{a^{12}(\ 1 \ ) + a^{21}(\ 1 \ )\right\}(ik_\mu)(ik_\nu) - \frac{a^{12}}{2} M_0^2 \delta_{\mu\nu}$$

$$= \delta_{\mu\nu}\left\{ -k^2 \frac{1}{2}(a^{12}+a^{21}) - M_0^2 \frac{a^{12}}{2}\right\} + \frac{k_\mu k_\nu}{k^2}\left\{ k^2 \frac{1}{2}(a^{12}+a^{21})\right\}$$

(*) $A - \omega^0$ is the same as $\omega^0 - A$, so a factor of 2 is put in the Feynman rules.

3) Requiring the counterterm to cancell the infinities from one-loop self-energies, we can fix these counterterms. For instance, for $\omega^0 - \omega^0$, we require

$$\delta_{\mu\nu}\left\{-k^2 a^{11} - M_0^2(\delta M_0^2 + a^{11})\right\} + \frac{k_\mu k_\nu}{k^2}\left\{k^2 a^{11}\right\} + \frac{(2\pi)^4 i \, \Sigma^{\omega^0\omega^0}}{(2\pi)^4 i} = \text{finite},$$

which will determine a^{11}.

Notice that δM_0^2 is <u>not</u> an independent parameter, it is determined by δM^2 and δC_θ^2.

$*$ $\quad \omega^0_\mu \partial_\nu \partial_\nu A_\mu = A_\mu \partial_\nu \partial_\nu \omega^0_\mu$,

also $\quad \omega^0_\mu \partial_\mu \partial_\nu A_\nu = A_\nu \partial_\nu \partial_\mu \omega^0_\mu$.

Pf:

① $\omega^0_\mu \partial_\nu \partial_\nu A_\mu = -\partial_\nu \omega^0_\mu \partial_\nu A_\mu$
$= +(\partial_\nu \partial_\nu \omega^0_\mu) A_\mu = A_\mu \partial_\nu \partial_\nu \omega^0_\mu$

② $\omega^0_\mu \partial_\mu \partial_\nu A_\nu = -\partial_\mu \omega^0_\mu \partial_\nu A_\nu$
$= +(\partial_\mu \partial_\nu \omega^0_\mu) A_\nu = A_\nu \partial_\mu \partial_\nu \omega^0_\mu$
$= A_\nu \partial_\nu \partial_\mu \omega^0_\mu$.

$*$ Obviously, $\quad \omega^0_\mu A_\mu = A_\mu \omega^0_\mu$

$*$ Hence the Lagrangian we have to get the Feynman rules of ω^0-A transition is

$$\frac{a'^2}{2} \omega^0_\mu \partial_\nu \partial_\nu A_\mu - \frac{a'^2}{2} \omega^0_\mu \partial_\mu \partial_\nu A_\nu - \frac{a'^2}{2} M_0^2 \omega^0_\mu A_\mu$$
$$+ \frac{a^{21}}{2} \omega^0_\mu \partial_\nu \partial_\nu A_\mu - \frac{a^{21}}{2} \omega^0_\mu \partial_\mu \partial_\nu A_\nu .$$

4. Since

$$\mathcal{M}_{bare} \equiv (Z_N^{-1/2})^T \mathcal{M}_{ren} Z_{\phi^0}^{-1/2},$$

So

$$\mathcal{M}_{ren} = (Z_N^{1/2})^T \mathcal{M}_{bare} Z_{\phi}^{1/2}.$$

We should verify that $\mathcal{M}_{ren}$ is finite.

1) $$\mathcal{M}_{bare} = \begin{pmatrix} m_0 \\ m_A \end{pmatrix}_{bare} = \begin{pmatrix} M_0^{ren}(1+\frac{1}{2}\delta M_0^2) + \tilde{m}_0 \\ \tilde{m}_A \end{pmatrix}.$$

Hence

$$\mathcal{M}_{ren} = \begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{21}}{2} \\ \frac{a^{12}}{2} & 1+\frac{a^{22}}{2} \end{pmatrix} \begin{pmatrix} M_0^{ren}(1+\frac{1}{2}\delta M_0^2)+\tilde{m}_0 \\ \tilde{m}_A \end{pmatrix} (1+\frac{\delta Z_{\phi^0}}{2})$$

$$= \begin{pmatrix} M_0^{ren} + \left\{ \frac{1}{2} M_0^{ren}(a^{11} + \delta Z_{\phi^0} + \delta M_0^2) + \tilde{m}_0 \right\} \\ \frac{1}{2} a^{12} M_0^{ren} + \tilde{m}_A \end{pmatrix} \equiv \begin{pmatrix} m_0^{ren} \\ m_A^{ren} \end{pmatrix}$$

Note: Since $a^{21} \sim O(g^2)$ and $\tilde{m}_0 \sim O(g^2)$, so we neglect the term like $a^{21}\tilde{m}_0$, etc..

2) Consider ϕ^0 - propagator:
Its Lagrangian is

$$-\frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2}\phi^0 (m_0 \; m_A) \begin{pmatrix} \alpha_0 & 0 \\ 0 & \alpha_A \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha_0} & 0 \\ 0 & \frac{1}{\alpha_A} \end{pmatrix} \begin{pmatrix} \alpha_0 & 0 \\ 0 & \alpha_A \end{pmatrix} \begin{pmatrix} m_0 \\ m_A \end{pmatrix} \phi^0$$

$$= -\frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2}\alpha_0 m_0^2 \phi_0^2 - \frac{1}{2}\alpha_A m_A^2 \phi_0^2$$

(1) Decompose

$$m_0 = M_0 + \widetilde{m}_0 ,$$

and

$$m_A = \widetilde{m}_A .$$

Then we get

$$-\frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2}\alpha_0 M_0^2\phi_0^2 - \frac{1}{2}\alpha_0 M_0\left(2\cdot\widetilde{m}_0\right)\phi_0^2 - \frac{1}{2}\alpha_A\widetilde{m}_A^2\phi_0^2$$

$$\longrightarrow -\frac{1}{2}(1+\delta Z_{\phi^0})\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2}\alpha_0(1+\delta\alpha_0)M_0^2(1+\delta M_0^2)(1+\delta Z_{\phi^0})\phi_0^2$$
$$-\frac{1}{2}\alpha_0 M_0(2\widetilde{m}_0)\phi_0^2 - \frac{1}{2}\alpha_A\widetilde{m}_A^2\phi_0^2$$

$$= -\frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2}\alpha_0 M_0^2\phi_0^2$$
$$-\frac{1}{2}\delta Z_{\phi^0}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2}(\delta\alpha_0+\delta M_0^2+\delta Z_{\phi^0})\alpha_0 M_0^2\phi_0^2$$
$$-\frac{1}{2}\alpha_0 M_0(2\widetilde{m}_0)\phi_0^2 - \frac{1}{2}\alpha_A\widetilde{m}_A^2\phi_0^2 .$$

Therefore the counterterms are

$$\phi^0 \text{- - -}\times\text{- - -}\phi^0 \qquad -\delta Z_{\phi^0}k^2 - (\delta\alpha_0+\delta M_0^2+\delta Z_{\phi^0})\alpha_0 M_0^2 - 2\alpha_0 M_0\widetilde{m}_0$$

* Notice that the term $\alpha_A\widetilde{m}_A^2$ is a high order, $\mathcal{O}(g^4)$, term. Therefore it is neglected.

(2) The one loop result should also include the vacuum expectation value shift, i.e.

$$\Sigma^{\phi^0\phi^0}_{\text{true}} = \phi^0\text{- - -}\bigcirc\text{- - -}\phi^0 + \underset{-\beta}{\phi^0 \text{———}\times\text{———}\phi^0}$$

(3) Require that

$$\phi^0\text{- - -}\bigcirc\text{- - -}\phi^0 + \underset{-\beta}{\phi^0\text{———}\times}\ \phi^0 + \phi^0\text{- - -}\times\text{- - -}\phi^0 = \text{finite}.$$

14) α_0 has be chosen as

$$\underline{\alpha}_{bare} = \underline{Z}_N^{1/2}\,\underline{\alpha}_{ren}\,(\underline{Z}_N^{1/2})^T, \qquad \text{ie}$$

$$\begin{pmatrix} \alpha_0 & 0 \\ 0 & \alpha_A \end{pmatrix}_{bare} \equiv \begin{pmatrix} \alpha_0^{ren}(1+\delta\alpha_0) & 0 \\ 0 & \alpha_A^{ren}(1+\delta\alpha_A) \end{pmatrix}$$

$$= \begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{12}}{2} \\ \frac{a^{21}}{2} & 1+\frac{a^{22}}{2} \end{pmatrix}\begin{pmatrix} \alpha_0^{ren} & \alpha_{12}^{ren} \\ \alpha_{21}^{ren} & \alpha_A^{ren} \end{pmatrix}\begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{21}}{2} \\ \frac{a^{12}}{2} & 1+\frac{a^{22}}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_0^{ren}(1+\frac{a^{11}}{2})+\frac{a^{12}}{2}\alpha_{21}^{ren} & \alpha_{12}^{ren}(1+\frac{a^{11}}{2})+\alpha_A^{ren}\frac{a^{12}}{2} \\ \frac{a^{21}}{2}\alpha_0^{ren}+\alpha_{21}^{ren}(1+\frac{a^{22}}{2}) & \frac{a^{21}}{2}\alpha_{12}^{ren}+\alpha_A^{ren}(1+\frac{a^{22}}{2}) \end{pmatrix}\begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{21}}{2} \\ \frac{a^{12}}{2} & 1+\frac{a^{22}}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_0^{ren}(1+a^{11})+\frac{a^{12}}{2}\alpha_{21}^{ren}+\frac{a^{12}}{2}\alpha_{12}^{ren} & \alpha_0^{ren}\frac{a^{21}}{2}+\alpha_{12}^{ren}(1+\frac{a^{11}}{2}+\frac{a^{22}}{2})+\alpha_A^{ren}\frac{a^{12}}{2} \\ \frac{a^{21}}{2}\alpha_0^{ren}+\alpha_{21}^{ren}(1+\frac{a^{22}}{2}+\frac{a^{11}}{2})+\frac{a^{12}}{2}\alpha_A^{ren} & \alpha_{21}^{ren}\frac{a^{21}}{2}+\frac{a^{21}}{2}\alpha_{12}^{ren}+\alpha_A^{ren}(1+a^{22}) \end{pmatrix}$$

Therefore, we require

$$\frac{a^{21}}{2}\alpha_0^{ren} + \alpha_{12}(1+\frac{a^{11}}{2}+\frac{a^{22}}{2}) + \frac{a^{12}}{2}\alpha_A^{ren} = 0.$$

Recall that $\underline{\alpha}_{ren}$ is defined to be symmetric, so $\alpha_{12} = \alpha_{21}$.
We then can solve α_{12} as

$$\alpha_{12}^{ren} = \frac{-1}{1+\frac{a^{11}}{2}+\frac{a^{22}}{2}}\left[\frac{a^{12}}{2}\alpha_A^{ren} + \frac{a^{21}}{2}\alpha_0^{ren}\right]$$

$$\underset{\text{neglect } O(g^4)\text{ terms}}{=\!=} -\left(\frac{a^{12}}{2}\alpha_A^{ren} + \frac{a^{21}}{2}\alpha_0^{ren}\right).$$

Hence

$$\tilde{\alpha}_{ren} = \begin{pmatrix} \alpha_0^{ren} & -\left(\frac{a^{12}}{2}\alpha_A^{ren} + \frac{a^{21}}{2}\alpha_0^{ren}\right) \\ -\left(\frac{a^{12}}{2}\alpha_A^{ren} + \frac{a^{21}}{2}\alpha_0^{ren}\right) & \alpha_A^{ren} \end{pmatrix},$$

and

$$\alpha_0^{ren}(1+\delta\alpha_0) = \alpha_0^{ren}(1+a^{11}) + \alpha_{12}\left(\frac{a^{12}}{2}+\frac{a^{12}}{2}\right)$$
$$= \alpha_0^{ren}(1+a^{11}), \qquad \delta\alpha_0 = a^{11}.$$

Note. $\alpha_{12}\, a^{12} \sim O(g^4)$ is neglected.

Similarly

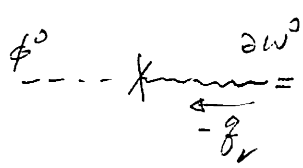
$$\alpha_A^{ren}(1+\delta\alpha_A) = \alpha_A^{ren}(1+a^{22}); \qquad \delta\alpha_A = a^{22}.$$

3) Consider the mixing terms:

They are

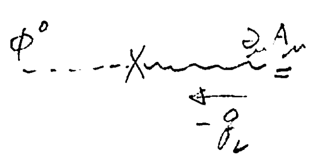
$$+ m_0 \phi^0 \partial_\mu W^0_\mu + m_A \phi^0 \partial_\mu A_\mu$$
$$= M_0 \phi^0 \partial_\mu W^0_\mu + \tilde{m}_0 \phi^0 \partial_\mu W^0_\mu + \tilde{m}_A \phi^0 \partial_\mu A_\mu .$$

The first term cancels with what we have from the Higgs sector. Therefore the counterterms are



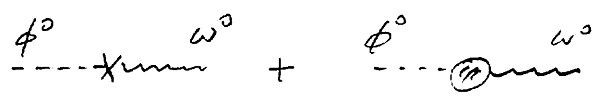
$= \tilde{m}_0 (-i q_\nu)$,

and



$= \tilde{m}_A (-i q_\nu)$.

where $\tilde{m}_0$ and $\tilde{m}_A$ are fixed by requiring that there are no mixings between ϕ^0 and ω^0, or ϕ^0 and A^0, i.e.



$= 0$,

and

$= 0$.

4) We now reexamine the renormalized quantity α_{ren}.
We found that there are a^{12} and a^{21} fooling around, which are infinite.
Therefore, something is wrong!
We therefore redefine our bare gauge fixing term, so that

$$\alpha_{bare} = \begin{pmatrix} \alpha_0 & \alpha_{12} \\ \alpha_{21} & \alpha_A \end{pmatrix},$$ which is <u>not</u> diagonal, and also

$\alpha_{12} = \alpha_{21}$.

(1) We will choose α_{bare} so that α_{ren} is diagonal, ie.

$$\alpha_{bare} = Z_N^{\frac{1}{2}} \alpha_{ren} (Z_N^{\frac{1}{2}})^T, \quad \text{and}$$

$$\begin{pmatrix} \alpha_0 & \alpha_{12} \\ \alpha_{21} & \alpha_A \end{pmatrix}_{bare} := \begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{12}}{2} \\ \frac{a^{21}}{2} & 1+\frac{a^{22}}{2} \end{pmatrix} \begin{pmatrix} \alpha_0^{ren} & 0 \\ 0 & \alpha_A^{ren} \end{pmatrix} \begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{21}}{2} \\ \frac{a^{12}}{2} & 1+\frac{a^{22}}{2} \end{pmatrix},$$

$$= \begin{pmatrix} \alpha_0^{ren}(1+\frac{a^{11}}{2}) & \frac{a^{12}}{2}\alpha_A^{ren} \\ \frac{a^{21}}{2}\alpha_0^{ren} & \alpha_A^{ren}(1+\frac{a^{22}}{2}) \end{pmatrix} \begin{pmatrix} 1+\frac{a^{11}}{2} & \frac{a^{21}}{2} \\ \frac{a^{12}}{2} & 1+\frac{a^{22}}{2} \end{pmatrix},$$

$$= \begin{pmatrix} \alpha_0^{ren}(1+a^{11}) & \frac{a^{21}}{2}\alpha_0^{ren} + \frac{a^{12}}{2}\alpha_A^{ren} \\ \frac{a^{21}}{2}\alpha_0^{ren} + \frac{a^{12}}{2}\alpha_A^{ren} & \alpha_A^{ren}(1+a^{22}) \end{pmatrix}.$$

Therefore,

$$\alpha_0 = \alpha_0^{ren}(1+a^{11}),$$
$$\alpha_A = \alpha_A^{ren}(1+a^{22}),$$
$$\alpha_{12} = \alpha_{21} = \frac{a^{21}}{2}\alpha_0^{ren} + \frac{a^{12}}{2}\alpha_A^{ren}.$$

(*) α_{12} is of the order of $O(g^2)$, so at tree level $\alpha_{12} = \alpha_{21} = 0$.

(2) The inverse of α_{bare} is

$$\alpha_{bare}^{-1} = \begin{pmatrix} \alpha_0 & \alpha_{12} \\ \alpha_{21} & \alpha_A \end{pmatrix}^{-1}.$$

$$= \frac{1}{\Delta} \begin{pmatrix} \alpha_A & -\alpha_{12} \\ -\alpha_{21} & \alpha_0 \end{pmatrix}, \quad \Delta = \alpha_0 \alpha_A - \alpha_{12}^2 \underset{\text{up to order } O(g^2)}{\approx} \alpha_0 \alpha_A . \qquad (\alpha_{12} = \alpha_{21})$$

Hence the bare gauge fixing we started with is

$$\mathcal{L}_{gf} = \frac{-1}{2} \left\{ -\partial N^T + \phi^0 M^T \alpha^T \right\} \alpha^{-1} \left\{ -\partial N + \alpha M \phi^0 \right\}$$

$$= \frac{-1}{2} \left\{ (-\partial w^0 \;\; -\partial A) + \phi^0 (m_0 \;\; m_A) \begin{pmatrix} \alpha_0 & \alpha_{12} \\ \alpha_{21} & \alpha_A \end{pmatrix} \right\} \frac{1}{\Delta} \begin{pmatrix} \alpha_A & -\alpha_{12} \\ -\alpha_{21} & \alpha_0 \end{pmatrix} \cdot$$

$$\left\{ \begin{pmatrix} -\partial w^0 \\ -\partial A \end{pmatrix} + \begin{pmatrix} \alpha_0 & \alpha_{12} \\ \alpha_{21} & \alpha_A \end{pmatrix} \begin{pmatrix} m_0 \\ m_A \end{pmatrix} \phi^0 \right\}$$

$$= \frac{-1}{2} \left\{ (-\partial w^0 \;\; -\partial A) \frac{1}{\Delta} \begin{pmatrix} \alpha_A & -\alpha_{12} \\ -\alpha_{21} & \alpha_0 \end{pmatrix} + \phi^0 (m_0 \;\; m_A) \right\} \cdot \left\{ \begin{pmatrix} -\partial w^0 \\ -\partial A \end{pmatrix} + \begin{pmatrix} \alpha_0 & \alpha_{12} \\ \alpha_{21} & \alpha_A \end{pmatrix} \begin{pmatrix} m_0 \\ m_A \end{pmatrix} \phi^0 \right\}$$

$$= \frac{-1}{2} \left\{ \frac{1}{\Delta} \begin{pmatrix} -\alpha_A \partial w^0 + \alpha_{21} \partial A & \alpha_{12} \partial w^0 - \alpha_0 \partial A \end{pmatrix} \begin{pmatrix} -\partial w^0 \\ -\partial A \end{pmatrix} \right.$$

$$+ \phi^0 \left(- m_0 \partial w^0 - m_A \partial A \right)$$

$$+ (-\partial w^0 - \partial A) \begin{pmatrix} m_0 \\ m_A \end{pmatrix} \phi^0$$

$$\left. + \phi^0 \begin{pmatrix} \alpha_0 m_0 + \alpha_{21} m_A & \alpha_{12} m_0 + \alpha_A m_A \end{pmatrix} \begin{pmatrix} m_0 \\ m_A \end{pmatrix} \phi^0 \right\},$$

$$= \frac{-1}{2\alpha_0} (-\partial w^0 + \alpha_0 m_0 \phi^0)^2 - \frac{1}{2\alpha_A} (-\partial A + \alpha_A m_A \phi^0)^2 +$$

$$\frac{-1}{2} \left\{ \frac{-2\alpha_{12}}{\alpha_0 \alpha_A} \partial_\mu w^0_\mu \partial_\nu A_\nu + 2\alpha_{12} m_0 m_A \phi^{0 2} \right\}.$$

Where we have use the assumption $\alpha_{12} = \alpha_{21}$.

Notice that

$$m_0 = M + \tilde{m}_0 ,$$
$$m_A = \tilde{m}_A ,$$

and $\tilde{m}_0$, $\tilde{m}_A$ and α_{12} are of the order $O(g^2)$.

If we only keep the counterterms up to $O(g^2)$ in the Lagrangian, then the gauge fixing term is

$$\mathcal{L}_{g.f} = -\frac{1}{2\alpha_0}(-\partial W^0 + \alpha_0 m_0 \phi^0)^2 - \frac{1}{2\alpha_A}(-\partial A + \alpha_0 m_A \phi^0)^2 + \frac{\alpha_{12}}{\alpha_0 \alpha_A}\partial_\mu W^0_\mu \partial_\nu A_\nu .$$

(The term $\alpha_{12} m_A \sim O(g^4)$ is neglected.)

(3) Summary:

$$\mathcal{L}_{bare} = Z_N^{1/2} \mathcal{L}_{ren} (Z_N^{1/2})^T = \mathcal{L}_{bare}^T .$$

$$\begin{pmatrix} W^0 \\ A \end{pmatrix}_{bare} = Z_N^{1/2} \begin{pmatrix} W^0 \\ A \end{pmatrix}_{ren.}$$

$$\mathcal{L}_{ren} = \begin{pmatrix} \alpha_0^{ren} & 0 \\ 0 & \alpha_A^{ren} \end{pmatrix} ,$$

$$\mathcal{L}_{bare} = \begin{pmatrix} \alpha_0^{ren}(1+a^{11}) & \frac{a^{21}}{2}\alpha_0^{ren} + \frac{a^{12}}{2}\alpha_A^{ren} \\ \frac{a^{21}}{2}\alpha_0^{ren} + \frac{a^{12}}{2}\alpha_A^{ren} & \alpha_A^{ren}(1+a^{22}) \end{pmatrix} .$$

$$\mathcal{L}_{gf} = -\frac{1}{2\alpha_0}(-\partial W^0 + \alpha_0 m_0 \phi^0)^2 - \frac{1}{2\alpha_A}(-\partial A + \alpha_0 m_A \phi^0)^2 + \frac{\alpha_{12}}{\alpha_0 \alpha_A}\partial_\mu W^0_\mu \partial_\nu A_\nu .$$

Program for W^0 and A

Here we assume all the quantities from the loop calculation have been devided by $(2\pi)^4 i$.

1. 

$\equiv A_{W^0}\,\delta_{\mu\nu} + B_{W^0}\left(\frac{q_\mu q_\nu}{q^2}\right)$,

$$a^{11} = (A_{W^0})_{q^2}, \qquad \delta M_0^2 = (A_{W^0})_{q^2=0}\,\frac{1}{M_0^2} - a^{11}.$$

2. 

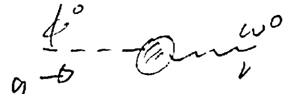
$\equiv A_A\,\delta_{\mu\nu} + B_A\left(\frac{q_\mu q_\nu}{q^2}\right)$,

$$a^{22} = (A_A)_{q^2}.$$

3. 

$\equiv A_{W^0A}\,\delta_{\mu\nu} + B_{W^0A}\left(\frac{q_\mu q_\nu}{q^2}\right)$,

$$a^{12} = \frac{2}{M_0^2}(A_{W^0A})_{q^2=0}, \qquad a^{21} = 2(A_{W^0A})_{q^2} - a^{12}.$$

4. 

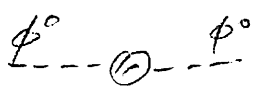
$\equiv (-iq_\nu)\,A_{\phi^0 W^0}$,

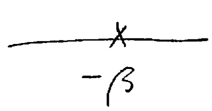
$$\tilde{m}_0 = -A_{\phi^0 W^0}$$

5 

$\equiv (-iq_\nu)\,A_{\phi^0 A}$,

$$\tilde{m}_A = -A_{\phi^0 A}$$

6. 

$+$ 

$\equiv A'_{\phi^0}$.

$$\delta Z_{\phi^0} = (A'_{\phi^0})_{q^2}$$

7. $$m_{ren} \equiv \begin{pmatrix} m_0^{ren} \\ m_A^{ren} \end{pmatrix} = \begin{pmatrix} M_0^{ren} + \left\{\frac{1}{2}M_0^{ren}(a^{11} + \delta Z_{\phi^0} + \delta M_0^2) + \tilde{m}_0\right\} \\ \frac{1}{2}a^{12}M_0^{ren} + \tilde{m}_A \end{pmatrix} \xRightarrow{\text{check}} \text{finite}$$

Consider the propagators of w^0 and A.

1. Through the previous discussion, we learned that: [$(2\pi)^4 i$ is suppressed.]

(1) $w^0 \sim\times\sim w^0 = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\left\{-k^2 a^{11} - M_0^2(\delta M_0^2 + a^{11})\right\} + \frac{k_\mu k_\nu}{k^2}\left\{-M_0^2(\delta M_0^2 + a^{11})\right\}$,

(2) $A \sim\times\sim A = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\left\{-k^2 a^{22}\right\}$.

(3) $w^0 \sim\times\sim A = A \sim\times\sim w^0$

$$= \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\left\{-k^2 \frac{1}{2}(a^{12} + a^{21}) - M_0^2 \frac{a^{12}}{2}\right\} + \frac{k_\mu k_\nu}{k^2}\left\{-M_0^2 \frac{a^{12}}{2}\right\},$$

Let us decompose

(4) $w^0 \sim\!\bigcirc\!\sim w^0 = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) f'_{11} + \frac{k_\mu k_\nu}{k^2} g'_{11}$.

(5) $w^0 \sim\!\bigcirc\!\sim A = A \sim\!\bigcirc\!\sim w^0$

$$= \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) f'_{12} + \frac{k_\mu k_\nu}{k^2} g'_{12}.$$

(6) $A \sim\!\bigcirc\!\sim A = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) f'_{22} + \frac{k_\mu k_\nu}{k^2} g'_{22}$. $\quad (g'_{22} = 0)$.

(7) $\phi^0 - - \bigcirc - - \phi^0 + \phi^0 \underset{-\beta}{\text{———}\times\text{———}} \phi^0 + \phi^0 - - \times - - \phi^0 \equiv A_{\phi^0}$.

2. To get the full propagator, we first sum up the counterterm and the one-loop corrections.

For instance, we assume

$$w^0 \sim\times\sim w^0 + w^0 \sim\!\bigcirc\!\sim w^0 = \left[\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) f_{11} + \frac{k_\mu k_\nu}{k^2} g_{11}\right] (2\pi)^4 i$$

$$= \left[f_{11}\, \delta_{\mu\nu} + (g_{11} - f_{11}) \frac{k_\mu k_\nu}{k^2}\right] \cdot (2\pi)^4 i \equiv (2\pi)^4 i \Sigma$$

Then its full propagator is

$$\frac{1}{(2\pi)^4 i}\left\{\frac{1}{k^2 + M_0^2 - f_{11}}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{1}{k^2 + M_0^2 - \left(g_{11} + k^2\left(1 - \frac{1}{\alpha_0}\right)\right)} \frac{k_\mu k_\nu}{k^2}\right\}$$

$$= \frac{1}{(2\pi)^4 i}\left\{\frac{1}{k^2 + M_0^2 - f_{11}}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{\alpha_0}{k^2 + \alpha_0 M_0^2 - \alpha_0 g_{11}} \frac{k_\mu k_\nu}{k^2}\right\}$$

Let S^{AA} be the propagator of A-A, then the propagator matrix

$$\mathbb{S} = \begin{pmatrix} S^{w^0 w^0} & S^{w^0 A} \\ S^{A w^0} & S^{AA} \end{pmatrix}$$
$$\equiv \frac{1}{(2\pi)^4 i} \left\{ \mathbb{T} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) + \mathbb{L} \frac{k_\mu k_\nu}{k^2} \right\}$$

Note that here we define

$$\sim\!\times\!\sim \;+\; \sim\!\bigcirc\!\sim \;\equiv\; (2\pi)^4 i \,\Sigma \, .$$

3. Because of the mixing, we should find the full propagators in a matrix form. Also we can consider the transversed and longitudinal parts separately.

1) To find out the physical poles of ω^0 and A, we should consider the transversed part of the propagators, for they are gauge-independent (except the wave-function renormalization, but it is irrelevent in finding the poles.)

(1) Define the bare propagators of ω^0 and A as

$$\Delta^1 = \frac{1}{k^2 + M_0^2} \, ,$$

$$\Delta^2 = \frac{1}{k^2} \, .$$

(2) The transversed parts of the propagators:

$$\delta_{ij} - f_{ij}\,\Delta^j = \begin{pmatrix} 1 - f_{11}\Delta^1 & -f_{12}\Delta^2 \\ -f_{12}\Delta^1 & 1 - f_{22}\Delta^2 \end{pmatrix} .$$

(Here j is not summed, and $f_{12} = f_{21}$.)

(3) The inverse of this matrix, up to $O(g^4)$,

$$\frac{1}{(1 - f_{11}\Delta^1)(1 - f_{22}\Delta^2)} \begin{pmatrix} 1 - f_{22}\Delta^2 & f_{12}\Delta^2 \\ f_{12}\Delta^1 & 1 - f_{11}\Delta^1 \end{pmatrix} = (\delta_{ij} - f_{ij}\Delta^j)^{-1} .$$

(4) The propagators (transversed part) are

$$\frac{1}{(2\pi)^4 i}\,\Delta^i \,(\delta_{ij} - f_{ij}\,\Delta^j)^{-1} \cdot \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \equiv \frac{1}{(2\pi)^4 i}\, T_{ij} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) .$$

① $\omega^0 - \omega^0$:

$$T_{11} = \Delta^1 \frac{1}{(1-f_{11}\Delta^1)(1-f_{22}\Delta^2)} (1 - f_{22}\Delta^2) = \Delta^1 \frac{1}{1-f_{11}\Delta^1}$$

$$= \frac{1}{k^2+M_0^2} \frac{1}{1 - f_{11}\frac{1}{k^2+M_0^2}} = \frac{1}{k^2+M_0^2 - f_{11}}$$

② $\omega^0 - A$:

$$T_{12} = \Delta^1 \frac{1}{(1-f_{11}\Delta^1)(1-f_{22}\Delta^2)} f_{12}\Delta^2$$

$$= \frac{1}{k^2+M_0^2} \frac{f_{12}\frac{1}{k^2}}{\left(1 - f_{11}\frac{1}{k^2+M_0^2}\right)\left(1 - f_{22}\frac{1}{k^2}\right)}$$

$$= \frac{f_{12}}{(k^2+M_0^2 - f_{11})(k^2 - f_{22})}$$

③ $A - \omega^0$:

$$T_{21} = \Delta^2 \frac{1}{(1-f_{11}\Delta^1)(1-f_{22}\Delta^2)} f_{12}\Delta^1 .$$

$$= \frac{1}{k^2} \frac{f_{12}\frac{1}{k^2+M_0^2}}{\left(1-f_{11}\frac{1}{k^2+M_0^2}\right)\left(1-f_{22}\frac{1}{k^2}\right)}$$

$$= \frac{f_{12}}{(k^2+M_0^2 - f_{11})(k^2 - f_{22})} = T_{12} .$$

④ $A - A$:

$$T_{22} = \Delta^2 \cdot \frac{1}{(1-f_{11}\Delta^1)(1-f_{22}\Delta^2)} (1-f_{11}\Delta^1) = \Delta^2 \frac{1}{1-f_{22}\Delta^2}$$

$$= \frac{1}{k^2 - f_{22}} .$$

2) To find out the longitudinal parts of propagators, we use the following procedures:

(1) The bare propagators of ω^0 and A are

$$\Delta^1 = \frac{1}{k^2+M_0^2}$$

$$\Delta^2 = \frac{1}{k^2}$$

(2) The longitudinal parts of the propagators:

$$L = \begin{pmatrix} 1-\left(g_{11}+k^2(1-\frac{1}{\alpha_0})\right)\Delta^1 & -g_{12}\Delta^2 \\ -g_{12}\Delta^1 & 1-\left(g_{22}+k^2(1-\frac{1}{\alpha_A})\right)\Delta^2 \end{pmatrix}, \quad g_{22}=0.$$

(3) The inverse of this matrix, up to $O(g^4)$,

$$L^{-1} = \frac{1}{\left[1-(g_{11}+k^2(1-\frac{1}{\alpha_0}))\Delta^1\right]\left[1-(g_{22}+k^2(1-\frac{1}{\alpha_A}))\Delta^2\right]} \begin{pmatrix} 1-(g_{22}+k^2(1-\frac{1}{\alpha_A}))\Delta^2 & g_{12}\Delta^2 \\ g_{12}\Delta^1 & 1-(g_{11}+k^2(1-\frac{1}{\alpha_0}))\Delta^1 \end{pmatrix}$$

(4) The longitudinal parts of the propagators are

$$\frac{1}{(2\pi)^4 i}\Delta^i L^{-1}\frac{k_\mu k_\nu}{k^2} \equiv \frac{1}{(2\pi)^4 i} L_{ij}\frac{k_\mu k_\nu}{k^2}.$$

① ω^0-ω^0:

$$L_{11} = \Delta^1 \cdot \frac{1}{1-\left(g_{11}+k^2(1-\frac{1}{\alpha_0})\right)\Delta^1}$$

$$= \frac{1}{k^2+M_0^2-\left(g_{11}+k^2(1-\frac{1}{\alpha_0})\right)} = \frac{\alpha_0}{k^2+\alpha_0 M_0^2-\alpha_0 g_{11}}$$

② $\omega^0 - A$:

$$L_{12} = \Delta^1 \cdot \frac{1}{\left[1-(g_{11}+k^2(1-\frac{1}{\alpha_0}))\Delta^1\right]\left[1-(g_{22}+k^2(1-\frac{1}{\alpha_A}))\Delta^2\right]} \quad g_{12}\Delta^2$$

$$= \frac{g_{12}}{\left[k^2+M_0^2-(g_{11}+k^2-\frac{k^2}{\alpha_0})\right]\left[k^2-(g_{22}+k^2-\frac{k^2}{\alpha_A})\right]}$$

$$= \frac{\alpha_0\alpha_A \; g_{12}}{(k^2+\alpha_0 M_0^2-\alpha_0 g_{11})(k^2-\alpha_A g_{22})}, \qquad (g_{22}=0)$$

③ $A - \omega^0$:

$$L_{21} = L_{12}.$$

④ $A - A$:

$$L_{22} = \Delta^2 \cdot \frac{1}{1-(g_{22}+k^2(1-\frac{1}{\alpha_A}))\Delta^2}$$

$$= \frac{\alpha_A}{k^2 - \alpha_A g_{22}}. \qquad (g_{22}=0).$$