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Long-term memory in climate variability: A new look based on fractional integral techniques

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[1] Long-term memory (LTM) in climate variability is studied by means of fractional integral techniques. We establish a new model, Fractional Integral Statistical Model (FISM), with which one can reproduce LTM of any given time series of climatic variability successfully. By using FISM, we further propose a new variable, Memory Kernel, based on which a clear picture of how the historical states maintain their impacts on the states in far future is drawn quantitatively. We find any climatic variability time series with LTM can be decomposed into two components: the weather-scale (or more accurately, smaller-scale) excitation and the cumulative memory component. By analyzing these two components, we reach an interpretation of climate memory in the end of the paper, that is, smaller time scale excitation pushes the present climate regime to begin to change, while slower response subsystems, such as the ocean, usually "remember" the forcing first, and then exhibit the influence slowly on a larger time scale. Since LTM is ubiquitous in climate, our findings in this paper may suggest a new perspective on the research of climate predictability.

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1. Introduction

[2] Ever since the middle of last century when the well known "Hurst Phenomenon" was revealed [*Hurst*, 1951], it has been widely recognized that the climate variability exhibits scale invariance, or more precisely, long-term memory (LTM) [*Koscielny-Bunde et al.*, 1998; *Malamud and Turcotte*, 1999; *Syroka and Toumi*, 2001]. As the name implies, long-term memory (or persistence) normally means the present states of a system may have long-term influences on the states in far future. One can simply use the autocorrelation function $C(s)$ to describe this scaling behavior. If the autocorrelation function $C(s)$ of a given time series decays as a power law, $C(s) \sim s^{-\gamma}$, $0 < \gamma < 1$, the typical correlation time according to $s_x = \frac{1}{C(0)}\int_0^\infty C(s)ds$ is not integrable, and we can further confirm this time series as long-term correlated [*Koscielny-Bunde et al.*, 1998]. By using a more advanced method named as detrended fluctuation analysis (DFA) [*Peng et al.*, 1994; *Kantelhardt et al.*, 2001], recent researches on extensive examples ranging from temperatures [*Eichner et al.*, 2003; *Monetti et al.*, 2003; *Király et al.*, 2006], relative humidities [*Chen et al.*, 2007], and river runoff [*Kantelhardt et al.*, 2006; *Koscielny-Bunde et al.*, 2006] to wind fields [*Govindan and Kantz*, 2004; *Feng et al.*, 2009], atmospheric general circulations [*Vyushin and Kushner*, 2009; *Vyushin et al.*, 2009], total ozone anomalies [*Varotsos and Kirk-Davidoff*, 2006; *Vyushin et al.*, 2007], etc., indicate that LTM is ubiquitous in climate. Therefore, understanding climate memory has become one of the principal goals of climatology.

[3] During the past few years, as [*Fraedrich et al.*, 2009] summarized, researches were mainly focused on (i) the detection of LTM in different climatic variables [*Király and Jánosi*, 2005; *Yuan et al.*, 2010], (ii) its influence on the internal stochastic trends and clustering of extremes [*Bunde et al.*, 2005; *Rybski et al.*, 2006; *Lennartz and Bunde*, 2009; *Franzke*, 2011], and (iii) its reproducibility by climate models [*Bunde et al.*, 2001; *Govindan et al.*, 2002; *Fraedrich and Blender*, 2003; *Vyushin et al.*, 2004; *Rybski et al.*, 2008]. From these previous works, people have gradually learned (i) how to diagnose the existence of LTM, (ii) how to apply LTM to the understanding of climate variability, and (iii) how to improve climate models with the reproducibility of LTM as the test criterion. However, based on these researches, our understanding on climate memory is still not enough. Simple questions such as why there is LTM in climate variability, or how the present states maintain their impacts on the states in far future, are not easy to be answered. Since compared with fully developed dynamical climate model, statistical model may show us a much simpler but clearer physical picture, it is therefore

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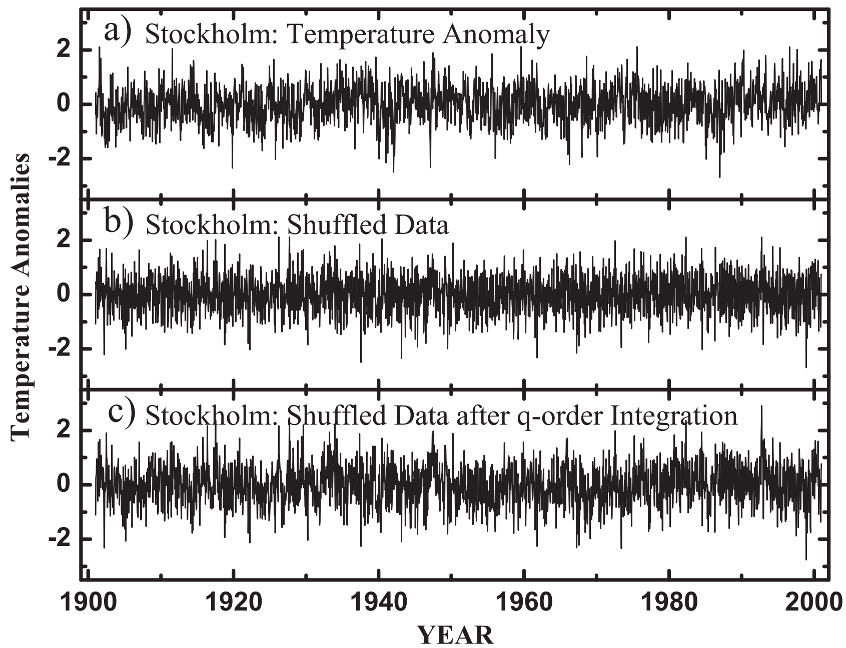

Figure 1. Time series we use for analysis in this study. (a) The temperature anomalies in Stockholm over the past century. (b) The random data shuffled from the temperature anomalies. (c) The new generated time series, which is obtained after a q-order (q = 0.24) integration from the shuffled data.

essential to establish a statistical model which can reproduce LTM appropriately and further be suitable for a better understanding of climate memory.

[4] In this study, based on fractional integral techniques [*Mandelbrot and Van Ness*, 1968], we establish a new statistical model, Fractional Integral Statistical Model (FISM). Compared with the well-known autoregressive model [*Caballero et al.*, 2002; *Vyushin et al.*, 2012], FISM can make a better simulation of LTM due to the fractional integral procedures (refer to section 3) and further be appropriate in the interpretation of climate memory. In this paper, we propose a new variable from FISM, *memory kernel* (MK), with which a clear picture of how the historical information affects the states in future is drawn (refer to section 4). In order to make a vivid interpretation of LTM, we further find that any given time series of climatic variability can be ultimately decomposed into two components. One is the cumulative memory component (CMC), and the other is the random excitations by weather-scale disturbances (or weather-scale excitations (WSE)). CMC is determined from the historical information. The stronger LTM is, the larger CMC will be, while WSE contains intrinsic dynamical characteristics. Different WSEs indicate that the considered time series may be governed by different dynamical mechanisms. By using a recently introduced method, detrended cross-correlation analysis (DCCA) [*Podobnik and Stanley*, 2008; *Podobnik et al.*, 2009], a brief discussion on WSE will be made at the end of section 4.

[5] The rest of this paper is organized as follows. In section 2, we will make a brief introduction of the data and the method (DFA, DCCA) we use for analysis. How to establish the Fractional Integral Statistical Model (FISM) will also be discussed in this section, along with a test of whether the model can reproduce LTM in any arbitrary time series of climatic variability. Based on FISM, a detailed study on climate memory is made in section 3, where we show how the present states maintain their impacts on the states in far future *quantitatively*, and further make a new interpretation of LTM by decomposing a given time series into two components (CMC+WSE). In section 4, we make a discussion and conclude this paper.

2. Data and Methodology

2.1. Data

[6] In order to demonstrate the usability of the proposed FISM, we have made extensive examinations on temperature records from different weather stations. Since the outputs are quite similar with each other, in this paper we only take the analysis of the daily temperature records observed in Stockholm as an example for illustration. The records are obtained from "Royal Netherlands Meteorological Institute Climate Explore" (http://climexp.knmi.nl/) and ranged from 1901 to 2000. Before our analysis, we first standardize the data by (a) averaging the daily data over 2 weeks to remove the short-term correlations due to general weather regimes, and (b) removing the seasonal trend through subtracting the annual cycle [*Koscielny-Bunde et al.*, 1998], as $\tau_i = T_i - \langle T_i \rangle$, where T_i is the biweekly temperatures and τ_i is the temperature anomalies that we use for analysis, see Figure 1a.

2.2. Methodology

2.2.1. DFA

[7] DFA has been widely used recently in the diagnosing of LTM. In this study, we employed the second-order, DFA-2, for our analysis [*Kantelhardt et al.*, 2001]. In DFA-2, one considers the cumulated sum (profile) $Y_i = \sum_{j=1}^{i} x_j$ of the record of interest $\{x_j\}$. One divides the profile into nonoverlapping windows of size s. In each window ν, the

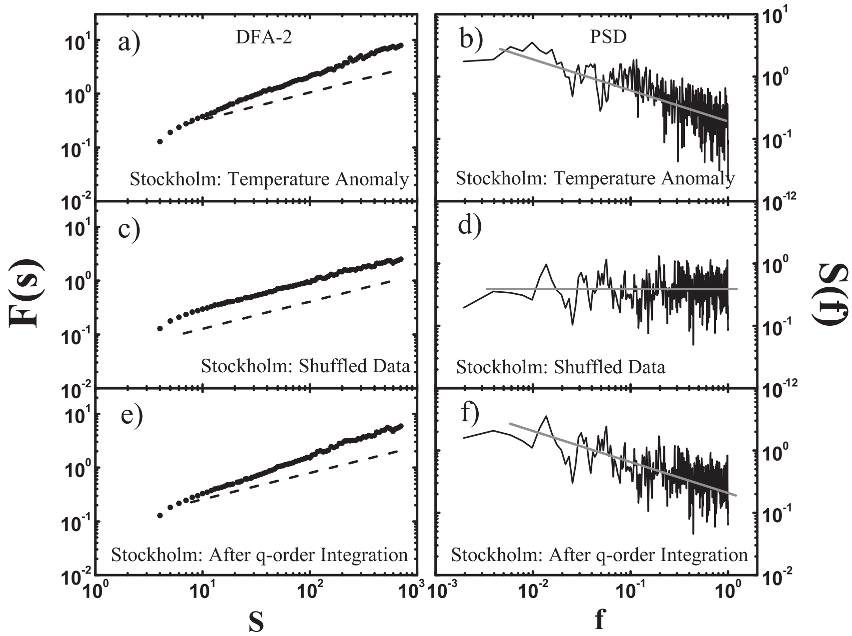

Figure 2. DFA-2 and power spectral density results of the three time series in Figure 1. (a and b) The results for temperature anomalies in Stockholm, one can find remarkable LTM property, with the DFA exponent $\alpha = 0.74$ and power spectral density exponent $\beta = 0.48$. (c and d) The results for the shuffled data, one can see the LTM is destroyed and the data are characterized as white noise. (e and f) The results for the new integrated data, one can find the LTM is reproduced. The dashed lines in Figures 2a, 2c, and 2e have slope of 0.5.

best quadratic fit of the profile and the standard deviation of the profile around this fit are determined. Then we average the result over all windows ν to obtain the mean fluctuation function $F(s)$. One is interested in the dependence of $F(s)$ on s. For the case of LTM, $F(s)$ increases by a power law, $F(s) \sim s^{\alpha}$, with the exponent $\alpha > 0.5$. While when $\alpha = 0.5$, the considered record shows no correlations, which can be determined as white noise. By estimating α, one therefore can tell whether the record is characterized by LTM [*Kantelhardt et al.*, 2001]. Furthermore, it is worth to note that $F(s)$ has close relations with the power spectral density $S(f)$. When $F(s)$ increases by a power law, $S(f)$ decays with f also by a power law, $S(f) \sim f^{-\beta}$, and the exponent β has relations with α as $\alpha = (1+\beta)/2$ [*Talkner and Weber*, 2000], as shown in Figures 2a and 2b.

2.2.2. DCCA

[8] DCCA is a generalization of DFA, but based on covariance. This method is designed to investigate long-term cross correlations between two different simultaneously recorded time series. The procedures are roughly the same as DFA, except when calculating the fluctuation functions. In DCCA, one calculates the covariance between two considered time series in each window ν, and the mean fluctuation function $F_{\mathrm{DCCA}}(s)$ is ultimately obtained by averaging the covariance over all windows. If $F_{\mathrm{DCCA}}(s)$ increases with s by a power law, the two considered time series are believed to be long-term cross-correlated. Whereas, if $F_{\mathrm{DCCA}}(s)$ remains zero at different time scales s, the two time series are not closely related. For more details of this method, please refer to *Podobnik and Stanley* [2008].

2.2.3. FISM

[9] To the end of this section, we will show how to establish the Fractional Integral Statistical Model (FISM). In 1976, ever since the concept of stochastic climate model was first introduced by *Hasselmann* [1976], one begins to believe that the slow change of climate can be explained as the integral response to continuous random excitation by short period "weather-scale" disturbances (weather-noise), as shown below,

$$\frac{dx}{dt} = \varepsilon(t), \tag{1}$$

where ε represents the "weather-scale" excitations, and x stands for "climate-scale" (slow) variability. In this classical paper, the climate-weather system was summarized in terms of the Brownian motion analogy. However, it has been noticed that models based on Brownian motion sometimes cannot provide satisfactory simulations of many natural time series, especially when modeling time series with self similarity properties or LTM. Therefore, we need to employ the fractional integral techniques, as discussed in *Mandelbrot and Van Ness* [1968].

[10] In our study, we generalize the relations between climate-scale variability and weather-scale excitations by means of fractional integral. As shown in equation (2),

$$\frac{d^q x}{dt^q} = \varepsilon(t), \tag{2}$$

where q is the integral order and can be a fraction. By using the Riemann-Lioville fractional integral formula, equation (2) can be written in a more explicit way, as shown below,

$$x(t) = \frac{d^{-q}\varepsilon}{dt^{-q}} = \frac{1}{\Gamma(q)}\int_0^t \frac{\varepsilon(u)}{(t-u)^{1-q}}\mathrm{d}u, \tag{3}$$

where t stands for the present time point, u stands for a historical time point, $t-u$ represents the distance between the historical time point u and the present time t, and $\Gamma(q)$ denotes the Gamma function. One can simulate the climate-scale variability from weather-scale excitations $\varepsilon(u)$ by adjusting the integral-order q. When $q = 1$, equation (3) can be degenerated into equation (1).

[11] It is easy to prove that the integral-order q has one to one close relation with the DFA exponent α, or the power spectral density exponent β, as shown in equations (4) and (5). Since ε represents the weather-scale excitations, we can consider it as fast changing (white) noise. Therefore, its power spectral density follows equation (4),

$$\begin{cases} \varepsilon(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty} \hat{\varepsilon}(f)e^{ift}\mathrm{d}f \\ S_\varepsilon(f) = |\hat{\varepsilon}(f)|^2 \sim f^{-\beta_\varepsilon} = f^0, \end{cases} \tag{4}$$

where $S_\varepsilon(f)$ denotes the power spectral density of $\varepsilon(t)$, f is the frequency, and β_ε stands for the power spectral density exponent, which (according to the above conditions) should equal zero. After q-order integration, the power spectral density of $x(t)$, which represents the climate-scale variability, can be derived into equation (5),

$$\begin{cases} x(t) = \dfrac{d^{-q}\varepsilon}{dt^{-q}} = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty} \hat{\varepsilon}(f)(if)^{-q}e^{ift}\mathrm{d}f \\ S_x(f) = |\hat{\varepsilon}(f)(if)^{-q}|^2 \sim f^{-(\beta_\varepsilon+2q)} = f^{-2q}, \end{cases} \tag{5}$$

where we can find the power spectral density $S_x(f)$ is related with the frequency f as $S_x(f) \sim f^{-(\beta_\varepsilon+2q)}$. The exponent β_ε in equation (4) is changed into $\beta_x = \beta_\varepsilon + 2q$ in equation (5). Since $\beta_\varepsilon = 0$, β_x and q are thus related as $\beta_x = 2q$. Recall the relations between β and the DFA exponent α (as introduced in section 2.2.1), we further have relations among q, β, and α, as below,

$$2q = \beta = 2\alpha - 1. \tag{6}$$

For a given time series, the three exponents α, β, and q can be determined uniquely, and perform as indicators of LTM. Normally, people prefer to apply DFA to calculate the exponent α, because this method is less influenced by the statistical uncertainty due to the multi-windows detrending and averaging procedure [*Talkner and Weber*, 2000]. While for the integral-order q, one may use it to generate surrogate data with given LTM (DFA exponent α, or power spectral density exponent β), by setting an appropriate q-value in equation (3). In other words, we can use this Fractional Integral Statistical Model (FISM) to simulate the LTM, as shown in Figures 1 and 2.

[12] In Figure 1, we take the mean temperature in Stockholm as an example. Figure 1a shows the biweekly temperature anomalies in Stockholm over the past century. One can find it is long-term correlated, as shown in Figure 2a: the DFA exponents fitted from the log-log coordinates is $\alpha = 0.74$, larger than 0.5. Meanwhile, the power spectral density analysis suggests the same findings with $\beta = 0.48$, as shown in Figure 2b. If we shuffle the temperature anomalies randomly, as shown in Figure 1b, the LTM will be destroyed and the shuffled data are characterized as white noise, with $\alpha = 0.5$ and $\beta = 0$, as shown in Figures 2c and 2d. According to equations (3) and (6), by applying a q-order ($q = 0.24$) integration to this shuffled data, one could theoretically rebuild the LTM, which is also obtained experimentally, as shown in Figures 1c, 2e, and 2f. Therefore, FISM succeeds in simulating LTM, and the three exponents do have close relations as equation (6). However, it is worth to note that, by setting an appropriate q-value, FISM only succeeds in rebuilding time series with given LTM, but cannot reproduce the temporal variations in details. This is because the white noise we use (such as the shuffled data shown in Figure 1b) is arbitrary. To simulate the given time series in details, one needs to apply FISM to specific white noise. We will make a brief discussion on this issue later, before that, we would like to make a novel interpretation of LTM by using FISM first, as shown in the next section.

[13] It is worth to note that, in this study, we actually have also made extensive examinations on records from other weather stations. Since the results are quite similar, we thus will not show all the detailed results in this paper.

3. A New Look on LTM

[14] Since Long-term memory is ubiquitous in climate, understanding the so-called climate memory has become one of the principal goals of climatology. According to the discussion above, it is well known that LTM normally means the historical states of a system can have long-term influences on the present states, and one can use the autocorrelation function $C(s)$, the power spectral density $S(f)$, or more conveniently the DFA results $F(s)$ to determine whether a considered time series is characterized by LTM or not. However, besides the diagnosis of LTM, how to describe this long-lasting influence *quantitatively*, or more accurately, how to interpret the so-called LTM, is still not well concerned. Since it has been proved that the statistical model, FISM, does reproduce the LTM (or the "Hurst Phenomenon") appropriately and flexibly. By using FISM, in this section, we aim to make a novel interpretation of LTM, including providing a physical picture, from which one can see clearly how the historical information affects the present states.

[15] We first need to reexamine equation (3) carefully and simplify it into a convolution form, as shown below.

$$x(t) = \frac{1}{\Gamma(q)}\int_0^t \frac{\varepsilon(u)}{(t-u)^{1-q}}\mathrm{d}u = K \otimes \Psi. \tag{7}$$

In this equation, similar to equation (3), time t can be considered as the present time, while time u stands for the historical time point. $(t-u)$ therefore represents the distance between the historical time point u and the present time t. In this way, $K \equiv \{k(t-u)\}$, $k(t-u) = \frac{1}{\Gamma(q)(t-u)^{1-q}}$ represents the integrating factor of every single step, and $\Psi \equiv \{\varepsilon(u)\}$ represents the weather-scale excitation at each historical time point. Theoretically, the integrating factor $k(t-u)$ in fact describes to what extent the historical weather-scale excitations $\varepsilon(u)$ may affect the present state $x(t)$. Therefore, we can further name the integrating factor $k(t-u)$ as *Memory Kernal* (MK for short), which is a new concept in our study. By calculating the convolution between K and Ψ, or more vividly, by taking the MK into account, the long-lasting influence of the weather-scale excitations can thus be well modeled and ultimately the slow change of climate can be estimated, as shown in equation (7).

[16] Obviously, compared with the weather-scale excitations $\varepsilon(u)$, MK plays a more important role since it controls

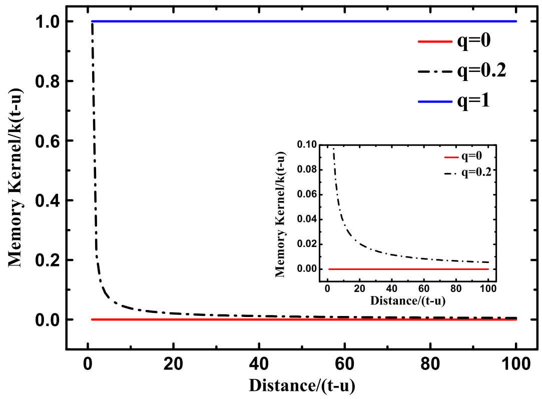

Figure 3. Memory Kernel (MK) with different integral-order q. The red line represents the case $q = 0$. We can find MK= 0 at all the distances, which means there is no memory in the system. The blue line represents the case $q = 1$. We can see MK= 0 at all distances, which means there the system is Brownian motion analogy. The black dash-dotted line represents the case $q = 0.2$. We can find MK decreases from 1 to 0 gradually. Even when the distance is far enough, as shown in the inserted diagram, MK never reaches 0. This case shows exactly the so-called LTM behavior.

how the historical information $\varepsilon(u)$ affects the present states. Therefore, we will pay more attention to this new concept. From the definition, it is clear that MK is only determined by the integral-order q. With different q, the MK may behave differently. As shown in Figure 3, we provide the MK curves of three cases: $q = 0$ (red), $q = 0.2$ (black), and $q = 1$ (blue). For the case $q = 0$, $k(t-u) = 0$ at all the distances $(t-u)$. This means there is no memory in the system, and the time series $x(t)$ behaves as a white noise (corresponding to the case when the DFA exponent $\alpha = 0.5$ and $\beta = 0$), just like the fast changing weather-scale excitations ε. For the case $q = 1$, $k(t-u) = 1$ at all the distances $(t-u)$. This means the system is Brownian motion analogy (corresponding to the case when the DFA exponent $\alpha = 1.5$ and $\beta = 2$), and the FISM is degenerated into equation (1). While for the case $q = 0.2$, MK decreases from 1 to 0 gradually. It is worth to note that MK never reaches 0, even when the distance $(t-u)$ is far enough, as shown in the inserted diagram. Therefore, this case shows exactly the so-called LTM behavior (corresponding to the case when the DFA exponent $\alpha = 0.7$ and $\beta = 0.4$). Therefore, Figure 3 is a clear picture from which one can see how the historical information $\varepsilon(u)$ affects the present states $x(t)$.

[17] From the above discussion, besides the physical picture of MK, one may further realize that a present state $x(t)$ should consist of two components. As shown below,

$$x(t) = M(t) + \varepsilon(t), \tag{8}$$

where $\varepsilon(t)$ represents the short period weather-scale excitations (WSE) occurred at the present time, while $M(t)$ stands for the cumulative memory component (CMC) cumulated from the historical information: $\varepsilon(0), \varepsilon(\delta), \cdots, \varepsilon(t-\delta)$ (δ is the sampling intervals of the observed time series). By using FISM, in this study, we can successfully extract the cumulative memory component (CMC) of a given time series $\{x\}$, as shown below,

$$M(t) = \frac{1}{\Gamma(q)} \int_{u=0}^{t-\delta} \frac{\varepsilon(u)}{(t-u)^{1-q}} \mathrm{d}u, \tag{9}$$

where $M(t)$ is determined only by the historical information $\varepsilon(u)$ and the memory kernel (MK). Obviously, the stronger LTM is, the larger CMC should be.

[18] For detailed interpretation, in this section, we take two artificial time series as examples. Figures 4a and 4c show two different white noise series: "White Noise I" and "White Noise II". By making q-order integration to both white noise series—$q = 0.2$ for White Noise I, while $q = 0.4$ for White Noise II—one could generate two new time series characterized by LTM, as shown in Figures 4b and 4d. According to equation (6), the DFA exponent α of these two new generated time series theoretically should be 0.7 and 0.9, and we indeed confirmed these α values experimentally by applying DFA (results are not shown here).

[19] From the above discussion, it is clear that the new generated time series can be decomposed into two components. One is the white noise which we make q-order integration, corresponding to the second term, $\varepsilon(t)$, of equation (8). The other one is the cumulative component when making the integration, corresponding to the first term, $M(t)$, of equation (8). For $M(t)$, as the red curves in Figures 4b and 4d show, it can be determined from equation (9) and represents the cumulative memory component (CMC) *quantitatively*. The stronger LTM is, the more apparent CMC will be in the whole fluctuation (Figure 4d). While for $\varepsilon(t)$, one should note that they are not only random records. In fact, $\varepsilon(t)$ represents the random excitations of weather-scale disturbances. Temporal variation of $\varepsilon(t)$ contains intrinsic dynamical characteristics, and different $\varepsilon(t)$ may lead the climate regime to different directions.

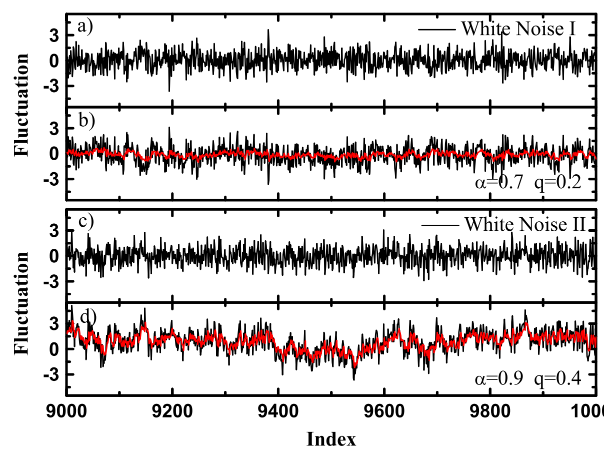

Figure 4. Examples on the understanding of the two components, WSE and CMC. (a and c) The two different white noise series: "White Noise I" and "White Noise II". (b and d) The two new integrated time series from Figures 4a ($q = 0.2$) and 4c ($q = 0.4$), respectively. The red curves are determined from equation (9), which represent the cumulative memory component (CMC). Obviously, the stronger LTM is, the more apparent CMC will be in the whole fluctuations.

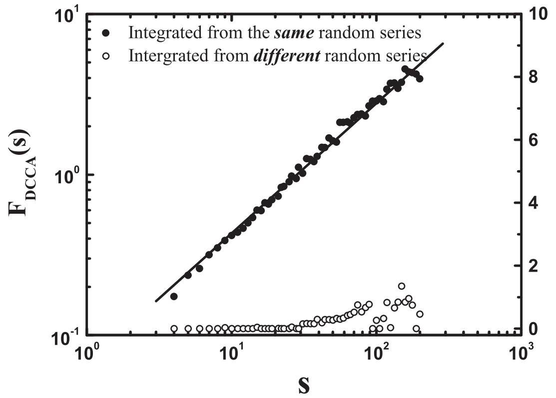

Figure 5. Open circles represent the DCCA results of the two time series in Figures 4b and 4d. Since the two time series are integrated from different types of white noise, the DCCA fluctuation function is around zero (see the right-hand side axis), which indicates that the two time series are not related. Solid circles represent the DCCA results of two time series, which are both integrated from "White Noise I". In this case, one can see that the two time series are indeed long-term cross-correlated.

Therefore, time series with different WSEs can be considered to be governed by different dynamical mechanisms. In the following, we will apply a recently introduced method, detrended cross-correlation analysis (DCCA), to demonstrate the importance of $\varepsilon(t)$.

[20] We first apply DCCA to the two long-term correlated time series in Figures 4b and 4d. Since these two time series are generated from two different white noise series (Figures 4a and 4c), according to the above analysis, the generated time series should be governed by different mechanisms, or in other words, there should be no cross correlations between them. As shown in Figure 5, we indeed find the two time series are not related, for the fluctuation function $F_{\mathrm{DCCA}}(s)$ are around zero at different time scales (open circles). However, if we use the same white noise to generate long-term correlated time series, the obtained time series should be long-term cross-correlated. For better demonstration, in this study we also generate another time series (not shown here) from "White Noise I", with the integral order $q = 0.4$. The DCCA results of the two time series (integrated from "White Noise I") are shown as the solid circles in Figure 5, where we find the two time series are indeed long-term cross-correlated. Or in other words, the two time series are governed by the same mechanisms (the same ε).

[21] Thus, for these two components, $\varepsilon(t)$ and $M(t)$, we have reasons to believe that, it is $\varepsilon(t)$ that triggers the change, while $M(t)$ that shows the way of changing, or response. In climatology, we then can make a novel interpretation of the climate memory (LTM) as the following: the weather-scale (or more accurately, the smaller-scale) excitations (WSE, $\varepsilon(t)$) push the present climate regime to begin to change, while slower response subsystems, such as the ocean, usually "remember" the forcing first, and then exhibit the influence slowly on a larger scale.

4. Discussion and Conclusion

[22] In this paper, we mainly focus on the understanding of long-term memory in climate variability. By means of fractional integral techniques, we establish a new statistical model, FISM, which can be used to reproduce the LTM of

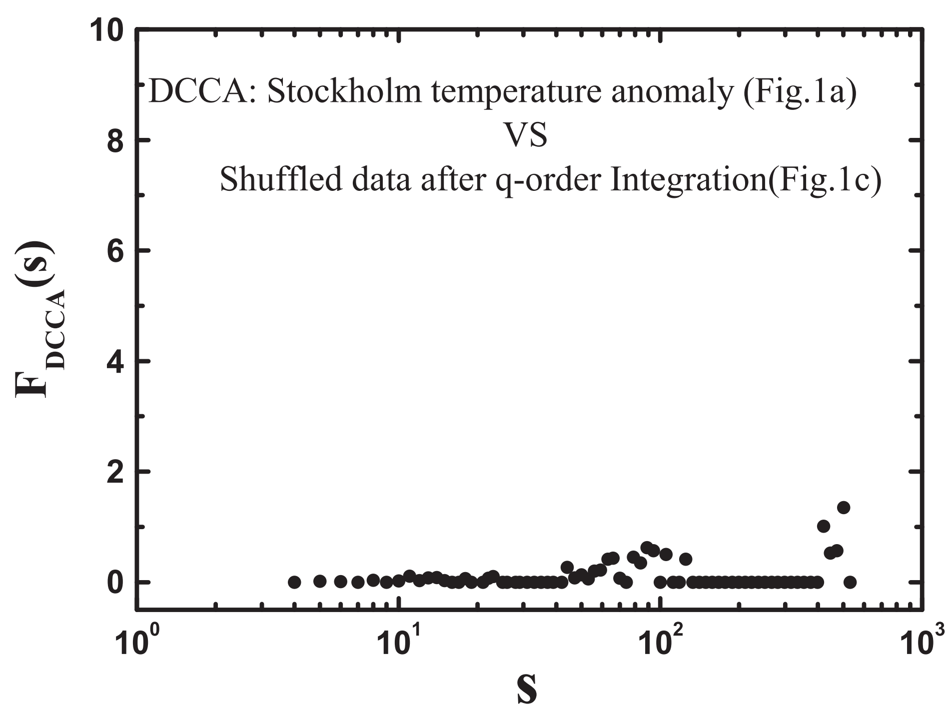

Figure 6. DCCA results of the two time series in Figures 1a and 1c. Although we can reproduce the LTM successfully, as shown in Figure 2, due to the difference of WSE, the two time series are still not related.

any given time series of climatic variability. From FISM, we further proposed a new concept, *Memory Kernel* (MK), with which one can describe *quantitatively* how the historical information affects the states in far future. By decomposing a given time series into two components: the weather-scale excitation (WSE) and the cumulative memory component (CMC), a novel interpretation of climate memory is made at the end of the paper.

[23] As for the cumulative memory component (CMC), it is determined only by the historical information and the memory strength. The stronger LTM is, the more apparent CMC will be in the whole fluctuations. While for the weather-scale excitations (WSE), it is worth to note that they are actually not only random records but also contain intrinsic dynamical characteristics. Different WSEs may trigger transitions of climatic regimes in different directions, therefore time series integrated from different WSEs may have no relations with each other. Such as the time series displayed in Figure 1, although the new integrated time series (Figure 1c) can reproduce the LTM of the original temperature anomalies (Figure 1a), since the time series is integrated from a randomly shuffled data (Figure 1b), its temporal variations will not be related with that of the original temperature anomalies (Figure 1a). As the DCCA result shows (Figure 6), the values of $F_{\mathrm{DCCA}}(s)$ are roughly around zero at different time scales, which means the two time series are governed by different mechanisms or different WSEs.

[24] Therefore, a new problem arises. Although we can rebuild time series with long-term memory by FISM and further extract the cumulative memory component (CMC), if the WSE is different from that of the given time series, our simulation from FISM cannot reproduce the temporal variations of the given data (as Figure 1). In contrast, if we can extract the WSE hidden in the given data, by using this specific WSE, we cannot only reproduce the LTM but also simulate the given data in details. In this way, we may improve our predicted skills of climate variability from a new point of view [*Zhu et al.*, 2010], by using the simple statistical model, FISM. Therefore, how to extract the WSE should be an important issue that needs to be discussed in a future research.

[25] However, even so, our work in this paper still shows a new way of understanding the climate memory. From recent works, we have known that LTM is ubiquitous in climate, and further realized that the strength of LTM may vary with time and space [*Fraedrich and Blender*, 2003; *Yuan and Fu*, 2013]. Thus, questions like how to understand the existence of LTM, can we use the concept of LTM to improve the climate predictability, have become the important issues needing to be addressed. We believe that the simple statistical model (FISM) proposed in this paper can be a useful tool in dealing with these issues, even though there are works still need to be done (as discussed in the previous paragraph).

[26] In the end, to better demonstrate the approach proposed in this paper, we would like to summarize several steps, following which one may analyze the long-term memory property of any given climatic system or time series, and further extract the cumulative memory component (CMC) more conveniently. Suppose we have a climatic time series, one needs to

[27] 1. Determine the strength of LTM by DFA and obtain the exponent α and q.

[28] 2. Calculate the memory kernel (MK) by using equation (7), where MK is defined as $k(t-u) = \frac{1}{\Gamma(q)(t-u)^{1-q}}$.

[29] 3. Extract the weather-scale excitations (WSE) $\varepsilon(t)$.

[30] 4. Calculate the cumulative memory component (CMC) by using equation (9).

[31] Obviously, the stronger LTM is, the more apparent CMC will be in the whole fluctuations, and the better predictability we may get. Of course, the key point of the above steps is to extract the WSE hidden in the considered data. It is crucial and determines whether we can simulate the temporal variations of the given data in details. To address this problem, a general idea is that we may obtain the WSE by deriving equation (7) reversely. For more details on how to extract WSE, a thorough discussion will be shown in a following paper, along with a further research on the climate predictability.

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