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(,   100871)

Legendre   Jacobi   ,   ,   .
,   Klein- Gordon   Boussinesq   mKdV   ,   .
,   .

: Jacobi   ,   ,   ,

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1.

,Jacobi
[1-4] ,   Jacobi
[1-4]
.   ,   Legendre

Jacobi

.

,   Legendre

$$(\phi) = \int_0^\phi \frac{1}{\sqrt{1 - m^2 \sin^2}} d , \qquad (1)$$

$0 < m < 1$.

$y = \sin$, $= \sin\phi$ Jacobi

$$( ) = \int_0^{=\sin\phi} \frac{1}{\sqrt{(1 - y^2)(1 - m^2 y^2)}} d y \qquad \mathrm{sn}^{-1} , \qquad (2)$$

$$= \mathrm{sn} \quad = \sin\phi. \qquad (3)$$

$y = \cos$, $= \cos\phi$,   (1)   Jacobi

$$( ) = \int_{=\cos\phi}^1 \frac{1}{\sqrt{(1 - y^2)(m^2 + m^2 y^2)}} d y$$

cn^{-1} ,   (4)

$$= \mathrm{cn} \quad = \cos\phi, \qquad (5)$$

m , $m^2 + m^2 = 1$.

,   (1)

$$\frac{\mathrm{d}\phi}{\mathrm{d}} = \sqrt{1 - m^2 \sin^2 \phi}, \qquad (6)$$

$$\sin\phi = \mathrm{sn} \ , \cos\phi = \mathrm{cn} \ . \qquad (7)$$

,   Legendre

$$(\phi) = \int_0^\phi \frac{1}{\sqrt{m^2 - \sin^2}} d \ . \qquad (8)$$

$y = \cos$, $= \cos\phi$,   (8)   Jaco-
bi

$$( ) = \int_{=\cos\phi}^1 \frac{1}{\sqrt{(1 - y^2)(y^2 - m^2)}} d y \qquad \mathrm{dn}^{-1} , \qquad (9)$$

$$= \mathrm{dn} \quad = \cos\phi. \qquad (10)$$

,   (8)

$$\frac{\mathrm{d}\phi}{\mathrm{d}} = \sqrt{m^2 - \sin^2 \phi}, \qquad (11)$$

$$\cos\phi = \mathrm{dn} \ , \sin\phi = m\mathrm{sn} \ . \qquad (12)$$

(6)   (11)   ,

,   . ,

,   Klein-
Gordon   Boussinesq   mKdV

.

2.　　　　Klein-Gordon

Klein-Gordon

$$u_{tt} - c_0^2 u_{xx} + u - u^3 = 0, \qquad (13)$$

$$u = u(\xi), \quad \xi = k(x - ct), \qquad (14)$$

k　c　　　　　　　　　　　　.

$$k^2(c^2 - c_0^2)\frac{d^2 u}{d\xi^2} + u - u^3 = 0. \qquad (15)$$

$u(\xi)$　　　　　　　　　　　　　　　　:

$$u(\xi) = \sum_{j=1}^{n} \sin^{j-1}\xi\,(a_j\sin\xi + b_j\cos\xi) + a_0, \qquad (16)$$

(6)　:

$$\frac{d\xi}{d\xi} = \sqrt{1 - m^2\sin^2\xi}, \qquad (17)$$

$$\frac{d^2\xi}{d\xi^2} = -m^2\sin\xi\cos\xi. \qquad (18)$$

,　(16)　　,　　　　n
(13)　　　　　　　　　　　　　　　　　,
　　　　　　,　　　　　　　　[1—4].
,　　　　$n = 1$,　　,　　(13)

$$u = a_0 + a_1\sin\xi + b_1\cos\xi. \qquad (19)$$

$$\frac{d^2 u}{d\xi^2} = (a_1\cos\xi - b_1\sin\xi)\frac{d^2\xi}{d\xi^2}$$
$$- (a_1\sin\xi + b_1\cos\xi)\left(\frac{d\xi}{d\xi}\right)^2$$
$$= -(1 + m^2)a_1\sin\xi - b_1\cos\xi$$
$$+ 2m^2 b_1\cos\xi\sin^2\xi + 2m^2 a_1\sin^3\xi. \qquad (20)$$

$$u^3 = (a_0^3 + 3a_0 b_1^2) + 3(a_0^2 + b_1^2)a_1\sin\xi$$
$$+ (3a_0^2 + b_1^2)b_1\cos\xi + 6a_0 a_1 b_1\cos\xi\sin\xi$$
$$+ 3a_0(a_1^2 - b_1^2)\sin^2\xi$$
$$+ (3a_1^2 - b_1^2)b_1\cos\xi\sin^2\xi$$
$$+ (a_1^2 - 3b_1^2)a_1\sin^3\xi. \qquad (21)$$

,(19)　　　　(15)

$$[a_0 - (a_0^3 + 3a_0 b_1^2)]$$
$$+ [-k^2(c^2 - c_0^2)(1 + m^2)a_1$$
$$+ a_1 - 3(a_0^2 + b_1^2)a_1]\sin\xi$$
$$+ [-k^2(c^2 - c_0^2)b_1 + b_1 - (3a_0^2 + b_1^2)b_1]\cos\xi$$
$$+ [-6a_0 a_1 b_1]\cos\xi\sin\xi$$

$$+ [-3a_0(a_1^2 - b_1^2)]\sin^2\xi + [2k^2(c^2 - c_0^2)m^2 b_1$$
$$- (3a_1^2 - b_1^2)b_1]\cos\xi\sin^2\xi$$
$$+ [2k^2(c^2 - c_0^2)m^2 a_1 - (a_1^2 - 3b_1^2)a_1]\sin^3\xi = 0. \qquad (22)$$

　　　　　　　　　　　　,

$$a_0 - (a_0^3 + 3a_0 b_1^2) = 0, \qquad (23)$$
$$-k^2(c^2 - c_0^2)(1 + m^2)a_1 + a_1$$
$$- 3(a_0^2 + b_1^2)a_1 = 0, \qquad (24)$$
$$-k^2(c^2 - c_0^2)b_1 + b_1 - (3a_0^2 + b_1^2)b_1 = 0, \qquad (25)$$
$$-6a_0 a_1 b_1 = 0, \qquad (26)$$
$$-3a_0(a_1^2 - b_1^2) = 0, \qquad (27)$$
$$2k^2(c^2 - c_0^2)m^2 b_1 - (3a_1^2 - b_1^2)b_1 = 0, \qquad (28)$$
$$2k^2(c^2 - c_0^2)m^2 a_1 - (a_1^2 - 3b_1^2)a_1 = 0. \qquad (29)$$

(23)—(29)　　　　　　　　　　　　:

1) $a_1 \neq 0, b_1 = 0$:

$$a_0 = 0, a_1 = \pm\sqrt{\frac{2m^2}{(1 + m^2)}},$$
$$k^2 = \frac{1}{(1 + m^2)(c^2 - c_0^2)}, \qquad (30)$$

2) $b_1 \neq 0, a_1 = 0$:

$$a_0 = 0, b_1 = \pm\sqrt{\frac{2m^2}{(2m^2 - 1)}},$$
$$k^2 = \frac{1}{(1 - 2m^2)(c^2 - c_0^2)}. \qquad (31)$$

(6),(7),(17)　(19)　　　　　(13)　　　　　　　:

$$u_1 = a_1\sin\xi = \pm\sqrt{\frac{2m^2}{(1 + m^2)}}$$
$$\times sn\left[\pm\sqrt{\frac{1}{(1 + m^2)(c^2 - c_0^2)}}(x - ct)\right], \qquad (32)$$

$$u_2 = b_1\cos\xi = \pm\sqrt{\frac{2m^2}{(2m^2 - 1)}}$$
$$\times cn\left[\pm\sqrt{\frac{1}{(1 - 2m^2)(c^2 - c_0^2)}}(x - ct)\right]. \qquad (33)$$

,　　　　　　(13)　　　　　　:

.

$m \to 1$,　sn　tanh,　cn　sech,　　,

$$u_3 = \pm\sqrt{\ }\tanh\left[\pm\sqrt{\frac{1}{2(c^2 - c_0^2)}}(x - ct)\right], \qquad (34)$$

$$u_4 = \pm \sqrt{\frac{2}{\ }}\,\text{sech}\left[\pm \sqrt{-\frac{\ }{(c^2 - c_0^2)}}\,(x - ct)\right]$$
$$(35)$$

.

(17)   (11) ,

$$\frac{\mathrm{d}\ }{\mathrm{d}\ } = \sqrt{m^2 - \sin^2\ }\,, \qquad (36)$$

$$\frac{\mathrm{d}^2\ }{\mathrm{d}\ ^2} = -\sin\ \cos\ . \qquad (37)$$

(19) , , (19)
(15) ,   (36) ,   (13)
:

3) $b_1 \quad 0 , a_1 = 0$:

$$a_0 = 0, b_1 = \pm \sqrt{\frac{2}{(2 - m^2)}}\,,$$

$$k^2 = \frac{\ }{(m^2 - 2)(c^2 - c_0^2)}. \qquad (38)$$

(11) ,(12) ,(36)   (19)   (13)

$$u_5 = b_1 \cos\ = \pm \sqrt{\frac{2}{(2 - m^2)}}$$
$$\times \mathrm{dn}\left[\pm \sqrt{\frac{\ }{(m^2 - 2)(c^2 - c_0^2)}}\,(x - ct)\right]. \quad (39)$$

,   (13)   :

.

$m \quad 1 \quad \mathrm{dn} \quad \text{sech}\ ,\ ,$
(35) .

3. Boussinesq

Boussinesq
$$u_{tt} - c_0^2 u_{xx} - \ u_{xxxx} - \ (u^2)_{xx} = 0, \qquad (40)$$

, (14)   (40)
$$(c^2 - c_0^2)\frac{\mathrm{d}^2 u}{\mathrm{d}\ ^2} - \ k^2 \frac{\mathrm{d}^4 u}{\mathrm{d}\ ^4} - \ \frac{\mathrm{d}^2 u^2}{\mathrm{d}\ ^2} = 0, \quad (41)$$

$$(c^2 - c_0^2)\,u - \ u^2 - \ k^2 \frac{\mathrm{d}^2 u}{\mathrm{d}\ ^2} = A, \qquad (42)$$

A . (16)   (41)
(40)
$$u = a_0 + a_1 \sin\ + b_1 \cos\ + a_2 \sin^2\ + b_2 \cos\ \sin\ .$$
$$(43)$$

,   (36) ((17)
). (43)   (42)

$$[(c^2 - c_0^2)\,a_0 - \ (a_0^2 + b_1^2) - 2\ k^2 m^2 a_2 - A\,]$$
$$+ [(c^2 - c_0^2)\,a_1 - \ (2 a_0 a_1 + 2 b_1 b_2)$$
$$+ \ k^2(1 + m^2)\,a_1\,]\sin\ + [(c^2 - c_0^2)\,b_1$$
$$- 2\ a_0 b_1 + \ k^2 m^2 b_1\,]\cos\ + [(c^2 - c_0^2)\,b_2$$
$$- \ (2 a_0 b_2 + 2 a_1 b_1) + \ k^2(1 + 4 m^2)\,b_2\,]\cos\ \sin\ $$
$$+ [(c^2 - c_0^2)\,a_2 - \ (2 a_0 a_2 + a_1^2 - b_1^2 + b_2^2)$$
$$+ 4\ k^2(1 + m^2)\,a_2\,]\sin^2\ + [-\ (2 a_1 b_2 + 2 a_2 b_1)$$
$$- 2\ k^2 b_1\,]\cos\ \sin^2\ + [-\ (2 a_1 a_2 - 2 b_2 b_1)$$
$$- 2\ k^2 a_1\,]\sin^3\ + [-2\ a_2 b_2$$
$$- 6\ k^2 b_2\,]\cos\ \sin^3\ + [-\ (a_2^2 - b_2^2)$$
$$- 6\ k^2 a_2\,]\sin^4\ = 0. \qquad (44)$$

,

$$(c^2 - c_0^2)\,a_0 - \ (a_0^2 + b_1^2) - 2\ k^2 m^2 a_2 - A = 0,$$
$$(45)$$
$$(c^2 - c_0^2)\,a_1 - \ (2 a_0 a_1 + 2 b_1 b_2)$$
$$+ \ k^2(1 + m^2)\,a_1 = 0, \qquad (46)$$
$$(c^2 - c_0^2)\,b_1 - 2\ a_0 b_1 + \ k^2 m^2 b_1 = 0, \qquad (47)$$
$$(c^2 - c_0^2)\,b_2 - \ (2 a_0 b_2 + 2 a_1 b_1)$$
$$+ \ k^2(1 + 4 m^2)\,b_2 = 0, \qquad (48)$$
$$(c^2 - c_0^2)\,a_2 - \ (2 a_0 a_2 + a_1^2 - b_1^2 + b_2^2)$$
$$+ 4\ k^2(1 + m^2)\,a_2 = 0, \qquad (49)$$
$$- \ (2 a_1 b_2 + 2 a_2 b_1) - 2\ k^2 b_1 = 0, \qquad (50)$$
$$- \ (2 a_1 a_2 - 2 b_2 b_1) - 2\ k^2 a_1 = 0, \qquad (51)$$
$$- 2\ a_2 b_2 - 6\ k^2 b_2 = 0, \qquad (52)$$
$$- \ (a_2^2 - b_2^2) - 6\ k^2 a_2 = 0. \qquad (53)$$

(45) —(53)   (40)   :

1) $a_1 = b_1 = b_2 = 0$:
$$a_0 = \frac{c^2 - c_0^2}{2} + \frac{2\ k^2(1 + m^2)}{\ }\,,$$
$$_2 = -\frac{6\ k^2}{\ }. \qquad (54)$$

2) $a_1 = b_1 = 0 , b_2 \quad 0$:
$$a_0 = \frac{c^2 - c_0^2}{2} + \frac{k^2(1 + 4 m^2)}{2}$$
$$a_2 = -\frac{3\ k^2}{\ }\,, b_2 = \pm i\frac{3\ k^2}{\ }\,,$$
$$i = \sqrt{-1}. \qquad (55)$$

(11) ,(12) ,(36)   (43)   (40)
$$u_1 = a_0 + a_2 \sin^2\ = \frac{c^2 - c_0^2}{2}$$

$$+ \frac{2 k^2 (1 + m^2)}{\ } - \frac{6 k^2}{\ } m^2 \mathrm{sn}^2[\, k(x - ct)\,]$$

$$= \frac{c^2 - c_0^2}{2} + \frac{2 k^2 (1 - 2m^2)}{\ }$$

$$+ \frac{6 k^2}{\ } m^2 \mathrm{cn}^2[\, k(x - ct)\,]$$

$$= \frac{c^2 - c_0^2}{2} + \frac{2 k^2 (m^2 - 2)}{\ } + \frac{6 k^2}{\ } \mathrm{dn}^2[\, k(x - ct)\,],$$

(56)

$$u_2 = a_0 + a_2 \sin^2 + b_2 \cos \sin$$

$$= \frac{c^2 - c_0^2}{2} + \frac{k^2 (1 + 4m^2)}{2}$$

$$- \frac{3 k^2}{\ } m^2 \mathrm{sn}^2[\, k(x - ct)\,]$$

$$\pm \mathrm{i} \frac{3 k^2}{\ } m \mathrm{sn}[\, k(x - ct)\,] \mathrm{dn}[\, k(x - ct)\,]. \quad (57)$$

m 1   ,

$$u_3 = \frac{c^2 - c_0^2}{2} + \frac{4 k^2}{\ } - \frac{6 k^2}{\ } \tanh^2[\, k(x - ct)\,]$$

$$= \frac{c^2 - c_0^2}{2} - \frac{2 k^2}{\ } + \frac{6 k^2}{\ } \mathrm{sech}^2[\, k(x - ct)\,],$$

(58)

$$u_4 = \frac{c^2 - c_0^2}{2} + \frac{5 k^2}{2} - \frac{3 k^2}{\ } \tanh^2[\, k(x - ct)\,]$$

$$\pm \mathrm{i} \frac{3 k^2}{\ } \tanh[\, k(x - ct)\,] \mathrm{sech}[\, k(x - ct)\,].$$

(59)

4.   mKdV

mKdV

$$u_t + {}_0 v_x + {}^{} u^2 u_x + {}^{} u_{xxx} = 0,$$
$$v_t + {}_1 (uv)_x + {}_2 v v_x = 0, \quad (60)$$

,

$$u = u(\ ), v = v(\ ), \quad = k(x - ct). \quad (61)$$

(61)   (60)

$$- cu + {}_0 v + \frac{}{3} u^3 + {}^{} k^2 \frac{\mathrm{d}^2 u}{\mathrm{d} {}^2} = C,$$

$$- cv + {}_1 uv + \frac{{}_2}{2} v^2 = C_2. \quad (62)$$

(60)

$$u(\ ) = \sum_{j=1}^{n_1} \sin^{j-1} (\ ) (a_j \sin + b_j \cos\ ) + a_0,$$

$$v(\ ) = \sum_{j=1}^{n_2} \sin^{j-1} (\ ) (A_j \sin + B_j \cos\ ) + A_0.$$

(63)

(63)   (62) $n_1 = n_2 = 1$,

$$u(\ ) = a_1 \sin + b_1 \cos + a_0,$$

$$v(\ ) = A_1 \sin + B_1 \cos + A_0. \quad (64)$$

(17)   , (64)   (62)

(60)   :

1) a_1 0 , $b_1 = 0$:

$$a_0 = 0, a_1 = \pm \sqrt{\frac{6 m^2 (c_{\ 2} + 2 {}_0 {}_1)}{(1 + m^2) {}_2}}, \quad A_0 = \frac{2c}{{}_2},$$

$$A_1 = - \frac{{}_1}{{}_2} a_1, B_1 = 0, k^2 = - \frac{c_{\ 2} + 2 {}_0 {}_1}{(1 + m^2) {}_2}.$$

(65)

2) b_1 0 , $a_1 = 0$:

$$a_0 = 0, b_1 = \pm \sqrt{\frac{6 m^2 (c_{\ 2} + 2 {}_0 {}_1)}{(2 m^2 - 1) {}_2}}, \quad A_0 = \frac{2c}{{}_2},$$

$$A_1 = 0, B_1 = - \frac{{}_1}{{}_2} b_1,$$

$$k^2 = \frac{c_{\ 2} + 2 {}_0 {}_1}{(2 m^2 - 1) {}_2}, \quad (66)$$

$$u_1 = a_1 \sin = \pm \sqrt{\frac{6 m^2 (c_{\ 2} + 2 {}_0 {}_1)}{(1 + m^2) {}_2}}$$

$$\times \mathrm{sn}\left[\pm \sqrt{- \frac{c_{\ 2} + 2 {}_0 {}_1}{(1 + m^2) {}_2}} (x - ct) \right],$$

$$v_1 = A_0 + A_1 \sin$$

$$= \frac{2c}{{}_2} \mp \frac{{}_1}{{}_2} \sqrt{\frac{6 m^2 (c_{\ 2} + 2 {}_0 {}_1)}{(1 + m^2) {}_2}}$$

$$\times \mathrm{sn}\left[\pm \sqrt{- \frac{c_{\ 2} + 2 {}_0 {}_1}{(1 + m^2) {}_2}} (x - ct) \right]; \quad (67)$$

$$u_2 = b_1 \cos = \pm \sqrt{\frac{6 m^2 (c_{\ 2} + 2 {}_0 {}_1)}{(2 m^2 - 1) {}_2}}$$

$$\times \mathrm{cn}\left[\pm \sqrt{\frac{c_{\ 2} + 2 {}_0 {}_1}{(2 m^2 - 1) {}_2}} (x - ct) \right],$$

$$v_2 = A_0 + B_1 \cos$$

$$= \frac{2c}{{}_2} \mp \frac{{}_1}{{}_2} \sqrt{\frac{6 m^2 (c_{\ 2} + 2 {}_0 {}_1)}{(2 m^2 - 1) {}_2}}$$

$$\times \mathrm{cn}\left[\pm \sqrt{\frac{c_{\ 2} + 2 {}_0 {}_1}{(2 m^2 - 1) {}_2}} (x - ct) \right]. \quad (68)$$

$$u_3 = \pm \sqrt[3]{\frac{3 (c_{\ 2} + 2 {}_0 {}_1)}{{}_2}}$$

$$\times \tanh\left[\pm \sqrt{- \frac{c_{\ 2} + 2 {}_0 {}_1}{2 {}_2}} (x - ct) \right],$$

$$v_3 = \frac{2c_2}{2} \mp \frac{2_1}{2} \sqrt{\frac{3(c_2 + 2_{01})}{2}}$$
$$\times \tanh\left[\pm\sqrt{-\frac{c_2 + 2_{01}}{2_2}}(x - ct)\right]; (69)$$

$$u_4 = \pm\sqrt{\frac{6(c_2 + 2_{01})}{2}}$$
$$\times \mathrm{sech}\left[\pm\sqrt{\frac{c_2 + 2_{01}}{2}}(x - ct)\right],$$

$$v_4 = \frac{2c_2}{2} \mp \frac{2_1}{2}\sqrt{\frac{6(c_2 + 2_{01})}{2}}$$
$$\times \mathrm{sech}\left[\pm\sqrt{\frac{c_2 + 2_{01}}{2}}(x - ct)\right]. \quad(70)$$

(36) ，(64) (62)
(60) ：
3) b_1 0 , $a_1 = 0$:
$$a_0 = 0, b_1 = \pm\sqrt{\frac{6(c_2 + 2_{01})}{(2 - m^2)_2}}, A_0 = \frac{2c_2}{2},$$
$$A_1 = 0, B_1 = -\frac{2_1}{a_2}b_1,$$
$$k^2 = \frac{c_2 + 2_{01}}{(2 - m^2)_2}, \quad(71)$$

$$u_5 = b_1\cos = \pm\sqrt{\frac{6(c_2 + 2_{01})}{(2 - m^2)_2}}$$
$$\times \mathrm{dn}\left[\pm\sqrt{\frac{c_2 + 2_{01}}{(2 - m^2)_2}}(x - ct)\right],$$

$$v_5 = A_0 + B_1\cos = \frac{2c_2}{2} \mp \frac{2_1}{2}\sqrt{\frac{6(c_2 + 2_{01})}{(2 - m^2)_2}}$$
$$\times \mathrm{dn}\left[\pm\sqrt{\frac{c_2 + 2_{01}}{(2 - m^2)_2}}(x - ct)\right], \quad(72)$$

5.

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， Jacobi
[1—4] ， [5—9] ，
[10—12] ， [13,14] ， [15,16] ，
sine-cosine [17] ．
[1—32] ．
Jacobi ，
， ，
．

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A new method to construct solutions to nonlinear wave equations [*]

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Abstract

From the definition of Legendre elliptic integration and Jacobi elliptic function, new transformations are obtained and applied to construct the exact solutions of nonlinear wave equations. The nonlinear Klein - Gordon equation, Boussinesq equation and the coupled mKdV equations are taken as three examples to illustrate the detailed steps in obtaining exact solutions. There new analytical solutions such as periodic solutions and soliton solutions are derived for these nonlinear evolution equation (or equations).