Homework #4

(Due Friday, Dec 5, 2014)

Please refer to PDG if you are not familiar with particles shown below.

1. Please read the 5th chapter in the textbook and review the properties of four fundamental interactions we have observed so far in our mother Nature. Please also compare the interaction strengths of the four interactions.

2. The quark model gives a theory of the magnetic moments of the octet baryons. If a quark were an elementary Dirac fermion, its magnetic moment would be

$$\vec{\mu}_q = g_q \frac{Q_q e}{2m_q} \vec{S}_q$$

where Q_q is the quark charge (2/3 or -1/3), m_q is the quark mass, $\vec{S}_q$ is the quark spin, and $g_q = 2$. We could then model the baryon magnetic moment as the sum of three quark magnetic moments,

$$\vec{\mu}_B = \sum_i \vec{\mu}_{qi}, \qquad i = 1, 2, 3$$

(a) Write explicitly the wave functions of the 8 octet baryon state with $S_3 = +1/2$. The wave function for the proton was given in the lecture. As was note there, it is trivial to write the wave functions for all of these states except Λ^0 and Σ^0. Find the wave function of Σ^0 by applying I_- to the wave function for Σ^+. Find the wave function for Λ^0 by noting that it must be orthogonal to the wave function fro Σ^0.

(b) Evaluate the matrix elements of the operator $\vec{\mu}_B$ in these eight states. Express the result in terms of the nuclear magneton

$$\mu_N = \frac{e\hbar}{2m_p} = \frac{e}{2m_p}$$

and the mass ratio m_q/m_p, m_s/m_p, treating the masses of the u and d quarks as equal and equal to m_q.

(c) Evaluate the baryon magnetic moments numerically by choosing $m_q = 300$ MeV, $m_s = 450$ MeV. Compare your simple quark model results with those measured value listed in the Particle Data Group in the unit of μ_N. (Note: there is no Σ^0 value in PDG.)

3. We have known the charged particle would interact with photon through the electromagnetism interaction. However, we did not observe the muon-lepton decaying into an electron plus a photon, i.e. $\mu^- \to e^-\gamma$, even though the decay is allowed by the law of energy-momentum conservation. Consider the electromagnetic interaction between the electron and muon as follows:

$$\mathcal{L} = e\bar{\psi}_\mu \gamma^\rho \psi_e A_\rho + h.c.$$

Why is it not allowed? A few experimental groups over the world are working hard to measure $\mu^- \to e^-\gamma$. It involves the so-called flavor changing neutral current (FCNC) interaction. If the interaction shown above is not allow, could you write an allowed interaction which could generate $\mu^- \to e^-\gamma$ process? Hint: you may start from the current conservation.

4. Consider the ϕ^4 scalar interaction given in the QFT class,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi + \mu_\phi^2 \phi^* \phi + \lambda \left(\phi^* \phi\right)^2 ,$$

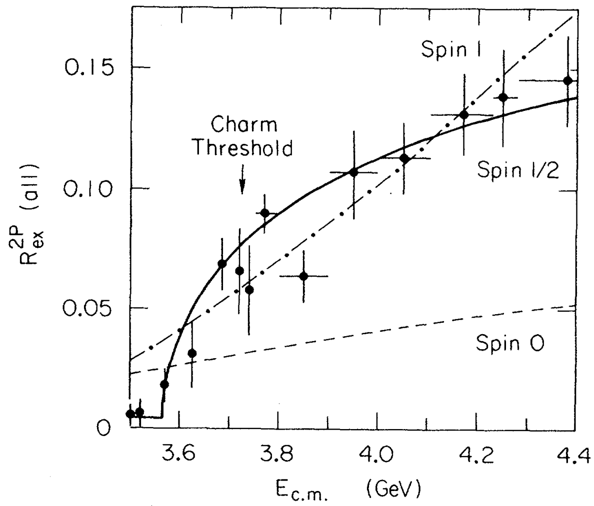
where μ_ϕ and λ are real parameters. Assuming the Lagrangian is invariant under the $U(1)_{em}$, please write the gauge invariant form of the kinetic and interaction terms.

5. The τ-lepton was first discovered in the collision of e^+e^- at SLAC. In the year of 1978, Bacino et al measured the threshold behavior of $\tau^+\tau^-$ production in e^+e^- annihilation. Their results clearly confirmed the tau-lepton is a fermion with mass 1782^{+7}_{-7} MeV; see Physical Review Letters 41 (1978) 13.

   (a) Please write the scattering amplitudes of $e^+e^- \to \gamma^* \to \tau^+\tau^-$ for spin-0 and spin-$\frac{1}{2}$ tau particles, respectively.

   (b) Use CalcHEP to calculate the cross section of $e^+e^- \to \gamma^* \to X^+X^-$ for various c.m. energy and different spin of X-particle. For simplicity, $m_X = 1.7$ GeV and $E_{\text{c.m.}}(\text{GeV}) \sim 3.5 - 5.0$. It would be better if you can derive it analytically, but there is no bonus point.

   Note that the production cross section of a pair of fermions exhibits a behavior as $\beta(3-\beta^2)$ around the threshold ($E_{c.m.} \sim 2m_f$) where β denotes the velocity of the fermion. The method of threshold scan is used quite often to search for new resonances today.



6. Write the gauge invariant kinetic terms in the Standard Model, $SU(2) \times U(1)_Y$, for ν_e, e^-, u and d fermions. It should include the kinetic terms and interaction terms describing charged and neutral currents. (I already did all of these in class. I simply want you to repeat the derivation once in your life.)

Textbook: "Particles and Fundamental Interactions: An Introduction to Particle Physics", by Braibant, Giacomeeli and Spurio.