Homework #3

(Due Friday, Nov 21, 2014)

If you are not familiar with particles shown below, please search for it in PDG.

1. Please find the non-relativistic limit of the following interactions

$$\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}, \quad \bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu},$$

and also discuss its properties under C, P and T transformation.

2. The cross section of $e^+e^- \to e^+e^-$ has a peak abut the ρ^0 mass, at $E_{cm} = 768$ MeV, with width 151.5 ± 1.2 MeV. The total cross section at the peak equals $\sigma_{\max} = (50 \pm 10)$ pb; ρ has spin 1.

   (a) Calculate the mean life of ρ^0 and the branching ratio

$$\mathrm{Br}\left(\frac{\rho^0 \to e^+e^-}{\rho^0 \to \pi^+\pi^-}\right),$$

   knowing that practically 100% of the ρ^0 width is due to the $\pi^+\pi^-$ mode.

   (b) Derive the expression for $\sigma(e^+e^- \to \pi^+\pi^-)$ about the resonance, as a function of energy.

   (c) Calculate the cross section of $\pi^+\pi^- \to e^+e^-$.

   (d) (**Bonus Point**) At approximately the same energy as above, the total cross section $e^+e^- \to 3\pi$ presents a peak with width 10 MeV; at the peak, $\sigma(e^+e^- \to 3\pi) = (1.7 \pm 0.1)\mu$b. Name the new resonance as ω. Find the superposition of ρ and ω corresponding to physical states with definite mass and mean life, making use of

$$\mathrm{Br}\left(\frac{\omega \to \pi^+\pi^-}{\omega \to \text{all}}\right) = (2.21 \pm 0.3) \times 10^{-2}.$$

   Hint: The mass of a resonance is $m_{\text{res}} + i\Gamma$. You can treat $m_\rho = m_\omega$ for simplicity and diagonalize the imaginary part of the mass.

3. The two most prominent resonances observed in pion-nucleon scattering are the Δ resonances at 1232 MeV and the N^* ("Roper") resonance at 1440 MeV. The Δ has isospin $I = 3/2$ and spin $S = 3/2$. The Roper has $I = 1/2$ and $S = 1/2$ and can be thought of as a radial excitation of the nucleon. The absolute rates of the reactions that form these resonances need to be computed from a dynamical strong interaction theory. However, the relative rates of different reactions producing the same resonances can be computed using isospin symmetry and Clebsch-Gordan coefficients. The quantum mechanical amplitude to produce a resonance of isospin I from initial states with isospins (I_1, I_1^3) and (I_2, I_2^3) is proportional to the Clebsch-Gordan coefficient

$$\left\langle I_1 I_2 I_1^3 I_2^3 | I_1 I_2 I I^3 \right\rangle$$

with $I^3 = I_1^3 + I_2^3$. The amplitude for the decay of a resonance to two particles of definite isospin is similarly proportional to the relevant Clebsch-Gordan coefficient. You can find a very readable table of Clebsch-Gordan coefficients for $SU(2)$ at the Particle Data Group web site, under Mathematical Tools.

(a) Using particle beams from accelerators, it is straightforward to produce the Δ and N^* resonances in reactions with the initial states

$$\pi^+ p, \quad \pi^+ n, \quad \pi^- p, \quad \pi^- n \ .$$

Compute the relative rates for production of the resonances in these four reactions. (Why is π^0 omitted here?)

(b) Each resonance can, in principle, decay to any of the 6 final states $(\pi^+, \pi^0, \pi^-) \times (p, n)$. Work out the relative rates of the 24 reactions $\pi N \to$ (resonance) $\to \pi N$ for each of the resonances Δ and N^*.

4. The ϕ meson ($M = 1019.412 \pm 0.008$ MeV, $\Gamma = 4.41 \pm 0.07$ MeV) decays in various modes with the corresponding branching ratio as follows:

$$\begin{array}{ll} K^+K^-, & 49.1 \pm 0.8\%, \\ K^0\bar{K}^0, & 34.4 \pm 0.7\%, \\ \pi^+\pi^0\pi^-, & 15.3 \pm 1.5\%, \\ \eta\gamma, & 1.28 \pm 0.6\%, \\ e^+e^-, & (3.1 \pm 0.1) \times 10^{-4}, \\ \mu^+\mu^-, & (2.5 \pm 0.31) \times 10^{-4}. \end{array}$$

(a) Compare the modulus of the decay amplitudes in the first two channels. Take into account corrections coming from phase space. $m_{K^+} = 493.646 \pm 0.009$ MeV and $m_{K^0} = m_{\bar{K}^0} = 497.671 \pm 0.031$ MeV.

(b) The decay mode $\phi \to \pi^0\pi^0\pi^0$ is not observed. What can we conclude on the isospin and angular momentum of the two-body $\pi^+\pi^-$ system participating in the 3π decay?

(c) Write down and calculate the resonance cross section for the processes of $e^+e^- \to \phi \to K^+K^-/K^0\bar{K}^0/e^+e^-/\mu^+\mu^-$.

(d) ϕ has a negative G-parity since it decays in 3π; use this fact to conclude that its isospin I is even. From the existence of K^+K^- decay mode deduce further that $I = 0$.

5. For each of the following reactions (a) establish whether it is allowed or not, (b) if it is not, give the reasons (there may be more than one), (c) establish the types of interaction that allow it:

$$\begin{array}{rcl} \pi^- + p \to \pi^0 + n & , & \pi^+ \to \mu^+ + \nu_\mu \\ \pi^+ \to \mu^+ + \bar{\nu}_\mu & , & \pi^0 \to \gamma\gamma \\ \pi^0 \to \gamma\gamma\gamma & , & e^+ + e^- \to \gamma \\ p + \bar{p} \to \Lambda + \Lambda & , & p + p \to \Sigma^+ + \pi^+ \\ n \to p + e^- & , & n \to p + \pi^- \\ \mu^+ \to e^+ + \gamma & , & e^- \to \nu_e + \gamma \\ p + p \to \Sigma^+ + K^+ & , & p + p \to p + \Sigma^+ + K^- \\ p \to e^+ + \nu_e & , & p + p \to \Lambda + \Sigma^+ \\ p + n \to \Lambda + \Sigma^+ & , & p + n \to \Xi^0 + p \\ p \to n + e^+ + \nu_e & , & n \to p + e^- + \nu_e \end{array}$$

6. The ρ^0 has spin 1. From the observation that the strong decay $\rho^0 \to \pi^+\pi^-$ exists but $\rho^0 \to \pi^0\pi^0$ does not, what information can be extracted about the ρ quantum numbers: J, P, C, G, I? Sate the three reasons forbidding the decay $\rho^0 \to \pi^0\pi^0$.

Note: We are going to have questions similar to the 5^{th} & 6^{th} questions in final exam.

Textbook: "Particles and Fundamental Interactions: An Introduction to Particle Physics", by Braibant, Giacomeeli and Spurio.