Probing the Electroweak Symmetry Breaking in the TeV Region

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- Why study Electroweak Symmetry Breaking Sector
- Standard Model Higgs mechanism
- Electroweak Chiral Lagrangian
- Equivalence Theorem
- Effective-W approximation
- No Lose theorem
- Event structure of Signal and Backgrounds
- phenomenology

- Why Studying Electroweak Symmetry Breaking Sector
  - — Up to now, All experimental Data*
    Agree with Standard Model
  - — However, we still don't know either
    the dynamics of Symmetry Breaking
    or
    the origin of fermion masses

We know that
the W-bosons gain mass by "eating"
the Goldstone Bosons.

But, we don't know whether it is due
to the
Standard Model Higgs Mechanism
or
Something else, e.g., technicolor model, ...

$\Longrightarrow$ It's important to probe EWSB
by studying the interactions
among the Goldston Bosons

(* α_s, R_b, R_c, high-p_T jets, zoo-events ...)

- Standard Model Higgs Mechanism

In Standard Model, the mass of vector bosons (W^+, W^- and Z^0) is generated from Higgs mechanism.

This amounts to introducing a colorless complex doublet

$$\Phi = \begin{pmatrix} \dfrac{\sigma + i\phi^0}{\sqrt{2}} \\ i\,\phi^- \end{pmatrix}$$

The scalar sector of the SM lagrangian is

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2$$

with

$$D_\mu = \partial_\mu - i g_1 \frac{Y}{2} B_\mu - i g_2 \frac{\tau^j}{2} W_\mu^j ,$$

$j = 1, 2, 3$

$Y = -1$

$Tr(\tau^j \tau^k) = 2\delta_{jk}$

$\mathcal{L}_\Phi$ is $SU(2)_L \times U(1)_Y$ invariant.

$$\begin{pmatrix} Z_0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}$$

θ_W is the weak mixing angle. $\left(\tan\theta_W = \frac{g_1}{g_2}\right)$

The minimum of the potential energy occurs at v, we then expand σ around v,

Spontaneous Symmetry Breaking,

$$\sigma = v + H \qquad (v \simeq 246\ \text{GeV})$$

The vector boson masses are

$$M_W = \frac{1}{2} g v, \quad M_Z = \frac{g v}{2 \cos\theta_W} \qquad (g \equiv g_2)$$

The Goldstone Bosons (ϕ^+, ϕ^-, ϕ^0) are "eaten" by the vector bosons (W^+, W^-, Z^0), which are massive, and have three polarization states.

This is how the Longitudinal W's are generated. (They are the Goldstone Bosons.)

Since we are interested in Probing the Electroweak Symmetry Breaking by studying the interaction among the Goldstone Bosons, which are "equivalent" to the Longitudinal W's in the TeV region, (Equivalence Theorem) we will ignore the hypercharge $U(1)_Y$ interaction, and neglect Yukawa interactions.

$\Rightarrow$ We will only consider the coset space

$$\frac{SU(2)_L \times SU(2)_R}{SU(2)_V} \qquad \left(\begin{array}{c}\text{Custodial} \\ \text{Symmetry}\end{array}\right)$$

(Later, we shall discuss the full gauged effective theories.)

- Consider ungauged $\mathcal{L}_\Phi$

$$\Phi \equiv \begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix} \equiv \begin{pmatrix} \frac{\sigma + i\phi^0}{\sqrt{2}} \\ i\phi^- \end{pmatrix} \quad \text{in } \underline{2} \text{ rep. of } SU(2)$$

Since $SU(2)$ group has a Complex conjugate representation, we can construct a $\underline{2}^*$ rep.

$$\tilde{\Phi} \equiv i\tau_2 \Phi^* = \begin{pmatrix} -i\phi^+ \\ \frac{-\sigma + i\phi^0}{\sqrt{2}} \end{pmatrix}$$

— Define

$$M = \sqrt{2}\left(\Phi, -\tilde{\Phi}\right) = \underbrace{\sigma}_{SU(2)_V \text{ singlet}} + i\,\tau^j \underbrace{\phi^j}_{SU(2)_V \text{ triplet}} \qquad j=1,2,3 \quad (\phi^3 \equiv \phi^0)$$

M manifests the $SU(2)_L \times SU(2)_R$ global symmetry

$$M(x) \longrightarrow M'(x) = L\, M(x)\, R^+ , \qquad L = e^{-i\vec{\epsilon}_L \cdot \frac{\vec{\tau}}{2}}, \quad R = e^{-i\vec{\epsilon}_R \cdot \frac{\vec{\tau}}{2}}$$

When $L = R$, then $\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}$ is invariant.

This is the $SU(2)_V$ symmetry after spontaneous symmetry breaking.

— The ungauged

$$\mathcal{L}_\Phi = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2$$

can be rewritten as

$$\mathcal{L}_M = \frac{1}{4} \mathrm{Tr}\left(\partial_\mu M^\dagger \partial^\mu M \right) - \lambda \left[\frac{1}{4} \mathrm{Tr}\left(M^\dagger M - v^2 \right) \right]^2$$

Before the symmetry breaking, M is $SU(2)_L \times SU(2)_R$ invariant. When the symmetry is broken spontaneously, the symmetry is reduced to $SU(2)_V$, which gives one $SU(2)_V$ singlet and one $SU(2)_V$ triplet.

We can perform the "polar redefinition of the fields" to write

$$\underbrace{M}_{SU(2)_L \times SU(2)_R} \equiv \underbrace{\rho}_{SU(2)_V \text{ singlet}} \; \underbrace{\Sigma}_{SU(2)_V \text{ triplet}} , \qquad \Sigma = e^{\frac{i \tau^a \pi^a}{v}}, \quad \rho = v + h$$

Then $\mathcal{L}_\Phi = \mathcal{L}_M =$

$$\mathcal{L}_\Sigma = \frac{1}{2} (\partial_\mu \rho)(\partial^\mu \rho) + \frac{1}{4} \rho^2 \mathrm{Tr}\left(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma \right) - \lambda \left(\frac{\rho^2}{2} - \frac{v^2}{2} \right)^2$$

— When the symmetry is Spontaneously broken from $SU(2)_L \times SU(2)_R$ to $SU(2)_V$

$$M = \begin{pmatrix} \sigma + i\phi^0 & i\sqrt{2}\phi^+ \\ i\sqrt{2}\phi^- & \sigma - i\phi^0 \end{pmatrix} \xrightarrow[\sigma = v+H]{} \begin{pmatrix} v+H & 0 \\ 0 & v+H \end{pmatrix} + i \begin{pmatrix} \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -\phi^0 \end{pmatrix}$$

$$\rho\Sigma = \rho e^{i\frac{\tau^a \pi^a}{v}} \xrightarrow[\rho = v+h]{} \begin{pmatrix} v+h & 0 \\ 0 & v+h \end{pmatrix} + i \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$
$$+ i\,\frac{h}{v} \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} + \cdots$$

$\Longrightarrow$ (h, π^a) are a non-linear function of (H, ϕ^a)

But,

$$h = H\,(1 + \cdots)$$
$$\pi^a = \phi^a\,(1 + \cdots)$$

This field redefinition does not shift the origin of the field. Therefore, the "S-matrix" will be the same.

($\mathcal{L}_\Phi$, $\mathcal{L}_M$ and $\mathcal{L}_\Sigma$ all give the same S-matrix.)

— In the heavy Higgs mass limit, $m_H \to \infty$,

$$\rho \to v \qquad (v = 246 \text{ GeV})$$

then

$$\mathcal{L}_\Sigma \to \frac{1}{4} v^2 \, Tr\left(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right), \qquad \Sigma = e^{\frac{i\tau^a \phi^a}{v}}$$

This is the Low Energy Theorem Lagrangian.

In the low energy, $E < 4\pi v$, the interactions among the Goldston Bosons ϕ^a can be obtained from this effective Lagrangian, which is called as

non-linearly realized chiral Lagrangian.

• Why nonlinearly realized Chiral Lagrangian ?

In the low energy world, we want to construct a Lagrangian $\mathcal{L}$ that is invariant in the symmetry group H (e.g. $U(1)_{em}$)

If this low energy Lagrangian $\mathcal{L}$ arises from the Spontaneous Symmetry Breaking of a larger Symmetry group G (e.g. $SU(2)_L \times U(1)_Y$) in the high energy world, then we want to construct $\mathcal{L}$ so that it's also invariant in G.

This can be done if the Goldstone bosons are included.

$$\begin{array}{ccc} & \mathcal{L} & (G\text{-invariant}) \\ \text{Symmetry is lifted up.} & \Uparrow & \text{Goldstone Bosons} \quad (G/H) \\ & \mathcal{L} & (H\text{-invariant}) \end{array}$$

• Electroweak Chiral Lagrangian

– Consider theories $\mathcal{L}$ basedon coset space $\frac{G}{H}$. these theories describe the interactions of the Goldstone Bosons that arise by spontaneously breaking a compact, connected, semisimple Liegroup G down to a continuous subgroup H.

(C Callan, S. Coleman, J. Wess, B. Zumino)

In our case, $\frac{G}{H} = \frac{SU(2)_L \times SU(2)_R}{SU(2)_V}$.

Define the matrix field

$$\xi = e^{\frac{i\phi^a \tau^a}{2v}}$$

(τ^a is in G but not in H)
(ϕ^a is Goldstone Boson)

Under the transformation in G,

$$\xi \to \xi' = L \xi U^{\dagger}, \qquad U = U(\xi, L, R)$$

non-linear function

($L \in SU(2)_L$, $R \in SU(2)_R$, $U \in SU(2)_V$)

(When $L=R$, then $U=L=R$ and the transformation linearizes.
$\Rightarrow \phi^a$ transform as a triplet under $SU(2)_{V=L+R}$)

(Assume the underlying dynamics conserve Parity) QCD-like int.

- Under Parity transformation

$$\xi \longrightarrow \xi^{+} \qquad (\text{in } L\text{-}R)$$

$$L \longrightarrow R$$

$$U^{+} \longrightarrow U^{+} \qquad (\text{in } L+R)$$

then

$$\xi^{+} \longrightarrow R\,\xi^{+} U^{+}$$

or

$$\xi \longrightarrow U \xi R^{+}$$

Hence

$$\xi \longrightarrow \xi' = L \xi U^{+} = U \xi R^{+}$$

Define

$$\Sigma \equiv \xi\xi = e^{\frac{i\phi^a \tau^a}{v}}$$

then

$$\Sigma \longrightarrow \Sigma' = L \Sigma R^{+}$$

$$\Longrightarrow \quad Tr\left(\partial_\mu \Sigma^{+} \partial^\mu \Sigma\right) \quad \text{is invariant.}$$

From this, one can build the Low Energy Theorem Lagrangian

$$\mathcal{L}_{LET} = \frac{v^2}{4} Tr\left(\partial_\mu \Sigma^{+} \partial^\mu \Sigma\right)$$

- Ungauged Electroweak Chiral Lagrangian on $\frac{SU(2)_L \times SU(2)_R}{SU(2)_V}$ coset space

Question:

If a SM Higgs Boson does not exist, what are the other possibilities for the spontaneous Symmetry Breaking?

This can be systematically studied by writing down an Effective Lagrangian (in contrast to a Fundamental Theory) using the Chiral Lagrangian approach.

$$\left\{ \begin{array}{ll} \text{Scalar} & \text{Resonance} \\ \text{Vector} & \text{Resonance} \\ \text{No} & \text{Resonance} \end{array} \right.$$

- Custodial Symmetry

- Before $\mathcal{L}_\Phi$ is gauged, it has the global symmetry $O(4)$, which is isomorphic to $SU(2)_L \times SU(2)_R$.
  The "additional" symmetry $SU(2)_R$ is the custodial symmetry.
  (Here, we assume fermions in one doublet have the same masses.)

- When $\mathcal{L}_\Phi$ is gauged by $SU(2)_L$, and the symmetry is broken spontaneously, i.e. $\sigma = v + H$, then $SU(2)_L$ is broken, but there is a vector-like symmetry $SU(2)_V$ remained, i.e. the coset space is $\dfrac{SU(2)_L \times SU(2)_R}{SU(2)_V}$

- When the hypercharge, i.e. $U(1)_Y$ interaction is included, or when the Yukawa interaction is included for fermions with different masses in one doublet, then this $SU(2)_V$ symmetry is broken. The coset space is $\dfrac{SU(2)_L \times U(1)_Y}{U(1)_{e.m.}}$

— The "additional" symmetry $SU(2)_R$ is the custordial symmetry which guarantee the tree level relation

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

Note: The custordial symmetry is broken by

(i) hypercharge : $g' \neq 0$

(ii) mass splitting in fermion doublet :

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \& \quad m_u \neq m_d$$

• How about in a Gauge theory ?

In Standard Model, the gauge group is $SU(2)_L \times U(1)_Y$, spontaneously broken to $U(1)_{e.m.}$ of electromagnetism.

The minimum global symmetry is

$$G = SU(2) \times U(1)$$
$$H = U(1)$$

Define $$\Sigma = e^{\frac{i\phi^a \tau^a}{v}}$$

under G transformation

$$\Sigma \longrightarrow \underbrace{e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}}}_{L} \Sigma \underbrace{e^{-i\frac{y\tau_3}{2}}}_{R}$$ (short-hand notation)

Define $$B_{\mu\nu} = \frac{1}{2}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)\tau_3$$

$$W_{\mu\nu} = \frac{1}{2}\left(\partial_\mu W_\nu - \partial_\nu W_\mu - \frac{i}{2} g\, [W_\mu, W_\nu]_-\right)$$

$$W_\mu \equiv W_\mu^a \tau^a$$

Covariant derivative

$$D_\mu \Sigma \equiv \partial_\mu \Sigma + \frac{i}{2} g W_\mu \Sigma - \frac{i}{2} g' B_\mu \Sigma \tau_3$$

Under G transformation

$$\Sigma \to L \Sigma R^+$$
$$D_\mu \Sigma \to L (D_\mu \Sigma) R^+$$
$$W_{\mu\nu} \to L W_{\mu\nu} L^+$$
$$B_{\mu\nu} \to R B_{\mu\nu} R^+ \qquad (\tau_3 \to R \tau_3 R^+)$$

with

$$\left(\frac{-i}{2} g W_\mu\right) \to L \left(\frac{-i}{2} g W_\mu + \partial_\mu\right) L^+$$
$$\left(\frac{-i}{2} g' B_\mu \tau_3\right) \to R \left(\frac{-i}{2} g' B_\mu \tau_3 + \partial_\mu\right) R^+$$

— With these, we can now construct Electroweak Chiral Lagrangian

1). The lowest order term contains two derivatives,

$$\mathcal{L}^{(2)} = \frac{v^2}{4} \mathrm{Tr}\left(D^\mu \Sigma^\dagger D_\mu \Sigma \right)$$

In the Unitary Gauge, $\Sigma = 1$,

$$\mathcal{L}^{(2)} = \frac{v^2}{4}\left(\frac{g^2}{2} W_\mu^a W_a^\mu + \frac{g'^2}{2} B_\mu B^\mu - g g' W_3^\mu B_\mu \right)$$

This gives

$W^\pm$ mass $\quad M_W = \frac{1}{2} g v$

Z $\quad M_Z = \frac{M_W}{\cos\theta_W}$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

Also, a massless photon

[$SU(2)_L \times U(1)_Y$ gauge invariance is non-linearly realized.]

If Electroweak Symmetry Breaking (EWSB) is driven by Strong interactions at TeV energies, the Electroweak Chiral Lagrangian (EWCL) provides the most economical description of Electroweak physics below this scale.

Deviation from the Standard Model (SM), but no Higgs scalar, can be parametried in terms of a low energy expansion.

— Here, we are considering the most difficult scenario that no new light particles exist below the TeV scale.
If this is not the case, then search for new light particles

The EWCL is constructed to respect the spontaneously broken $SU(2)_L \times U(1)_Y$ gauge symmetry,

using the dimensionless unitary unimodular matrix field

$$U = e^{\frac{i\pi^a \tau^a}{f}}$$

τ^a: Pauli matrices

π^a: Goldston Boson fields $a = 1, 2, 3$

f: vacuum expectation value (~ 246 GeV) characterizes the symmetry breaking scale.

$$\mathcal{L}_{eff} = \sum_n \ell_n \frac{f^{4+a_n-D_{a_n}}}{\Lambda^{a_n}} \cdot \mathcal{O}_n(W_{\mu\nu}, B_{\mu\nu}, D_\mu U, U, F, \bar{F})$$

$$= \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_F$$

$\mathcal{L}_G$: Gauge boson kinetic terms

$\mathcal{L}_S$: scalar boson int. terms (Containing GB self-int. & gaugeboson - GB int.)

$\mathcal{L}_F$: fermion int. terms

The bosonic part of EWCL :

$$
\begin{aligned}
\mathcal{L}_G &= -\tfrac{1}{2}\mathrm{Tr}(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} \ , \\
&\boxed{\mathcal{L}_S = \mathcal{L}^{(2)} + \mathcal{L}^{(2)\prime} + \sum_{n=1}^{14}\mathcal{L}_n \ ,} \\
\mathcal{L}^{(2)} &= \tfrac{f_\pi^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger (D^\mu U)] \ , \\
\mathcal{L}^{(2)\prime} &= \ell_0 (\tfrac{f_\pi}{\Lambda})^2 \, \tfrac{f_\pi^2}{4}[\mathrm{Tr}(\mathcal{T}\mathcal{V}_\mu)]^2 \ , \\
\mathcal{L}_1 &= \ell_1 (\tfrac{f_\pi}{\Lambda})^2 \, \tfrac{gg'}{2} B_{\mu\nu}\mathrm{Tr}(\mathcal{T}\mathbf{W}^{\mu\nu}) \ , \\
\mathcal{L}_2 &= \ell_2 (\tfrac{f_\pi}{\Lambda})^2 \, \tfrac{ig'}{2} B_{\mu\nu}\mathrm{Tr}(\mathcal{T}[\mathcal{V}^\mu, \mathcal{V}^\nu]) \ , \\
\mathcal{L}_3 &= \ell_3 (\tfrac{f_\pi}{\Lambda})^2 \, ig\mathrm{Tr}(\mathbf{W}_{\mu\nu}[\mathcal{V}^\mu, \mathcal{V}^\nu]) \ , \\
\mathcal{L}_4 &= \ell_4 (\tfrac{f_\pi}{\Lambda})^2 [\mathrm{Tr}(\mathcal{V}_\mu \mathcal{V}_\nu)]^2 \ , \\
\mathcal{L}_5 &= \ell_5 (\tfrac{f_\pi}{\Lambda})^2 [\mathrm{Tr}(\mathcal{V}_\mu \mathcal{V}^\mu)]^2 \ , \\
\mathcal{L}_6 &= \ell_6 (\tfrac{f_\pi}{\Lambda})^2 [\mathrm{Tr}(\mathcal{V}_\mu \mathcal{V}_\nu)]\mathrm{Tr}(\mathcal{T}\mathcal{V}^\mu)\mathrm{Tr}(\mathcal{T}\mathcal{V}^\nu) \ , \\
\mathcal{L}_7 &= \ell_7 (\tfrac{f_\pi}{\Lambda})^2 [\mathrm{Tr}(\mathcal{V}_\mu \mathcal{V}^\mu)]\mathrm{Tr}(\mathcal{T}\mathcal{V}_\nu)\mathrm{Tr}(\mathcal{T}\mathcal{V}^\nu) \ , \\
\mathcal{L}_8 &= \ell_8 (\tfrac{f_\pi}{\Lambda})^2 \, \tfrac{g^2}{4}[\mathrm{Tr}(\mathcal{T}\mathbf{W}_{\mu\nu})]^2 \ , \\
\mathcal{L}_9 &= \ell_9 (\tfrac{f_\pi}{\Lambda})^2 \, \tfrac{ig}{2}\mathrm{Tr}(\mathcal{T}\mathbf{W}_{\mu\nu})\mathrm{Tr}(\mathcal{T}[\mathcal{V}^\mu, \mathcal{V}^\nu]) \ , \\
\mathcal{L}_{10} &= \ell_{10} (\tfrac{f_\pi}{\Lambda})^2 \tfrac{1}{2}[\mathrm{Tr}(\mathcal{T}\mathcal{V}^\mu)\mathrm{Tr}(\mathcal{T}\mathcal{V}^\nu)]^2 \ , \\
\mathcal{L}_{11} &= \ell_{11} (\tfrac{f_\pi}{\Lambda})^2 \, g\epsilon^{\mu\nu\rho\lambda}\mathrm{Tr}(\mathcal{T}\mathcal{V}_\mu)\mathrm{Tr}(\mathcal{V}_\nu \mathbf{W}_{\rho\lambda}) \ , \\
\mathcal{L}_{12} &= \ell_{12} (\tfrac{f_\pi}{\Lambda})^2 \, 2g\mathrm{Tr}(\mathcal{T}\mathcal{V}_\mu)\mathrm{Tr}(\mathcal{V}_\nu \mathbf{W}^{\mu\nu}) \ , \\
\mathcal{L}_{13} &= \ell_{13} (\tfrac{f_\pi}{\Lambda})^2 \, \tfrac{gg'}{4}\epsilon^{\mu\nu\rho\lambda} B_{\mu\nu}\mathrm{Tr}(\mathcal{T}\mathbf{W}_{\rho\lambda}) \ , \\
\mathcal{L}_{14} &= \ell_{14} (\tfrac{f_\pi}{\Lambda})^2 \, \tfrac{g^2}{8}\epsilon^{\mu\nu\rho\lambda}\mathrm{Tr}(\mathcal{T}\mathbf{W}_{\mu\nu})\mathrm{Tr}(\mathcal{T}\mathbf{W}_{\rho\lambda}) \ ,
\end{aligned}
\tag{3.6}
$$

— gauge boson kinetic terms ($\mathcal{L}_G$)

— universal dim-2 ($\mathcal{L}^{(2)}$)

← $\Delta\rho$ (T) ($\mathcal{L}^{(2)\prime}$)

← S ($\mathcal{L}_1$)

← U ($\mathcal{L}_8$)

custodial $SU(2)_c$ inv. ($\mathcal{L}_4$, $\mathcal{L}_5$)

$SU(2)_c$ non-inv. ($\mathcal{L}_6$, $\mathcal{L}_7$, $\mathcal{L}_{10}$)

(quartic couplings) Pure scalar int. after taking $g, g' \to 0$ ($\mathcal{L}_4$–$\mathcal{L}_7$, $\mathcal{L}_{10}$)

$\cancel{P}$, CP ($\mathcal{L}_{11}$)

CP violating ($\mathcal{L}_{12}$–$\mathcal{L}_{14}$)

$(f_\pi = f)$

$$D_\mu U = \partial_\mu U + ig\,\underline{W_\mu}\,U - ig'\,U\,\underline{B_\mu} \ ,$$

$$\underline{W_\mu} = W_\mu^a \frac{\tau^a}{2}$$

$$\underline{B_\mu} = B_\mu \frac{\tau^3}{2}$$

$$W_{\mu\nu} = \partial_\mu \underline{W_\nu} - \partial_\nu \underline{W_\mu} - ig[\underline{W_\mu}, \underline{W_\nu}] \ ,$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\mathcal{V}_\mu = (D_\mu U)U^\dagger \ , \quad \mathcal{T} = U\tau_3 U^\dagger \ , \quad U = e^{\frac{i\pi^a\tau^a}{f}}$$

In the non-decoupling scenario, $\ell_n \sim \mathcal{O}(1)$

The cutoff scale

$$\Lambda = \min\left(M_{SB} ,\ 4\pi f \right)$$

mass of the lightest new particle

~ 3 TeV

$$\left(\left(\frac{f}{\Lambda}\right)^2 \sim \frac{1}{16\pi^2} \sim 0.6\% \text{ for } \Lambda = \Lambda_0 \equiv 4\pi f. \right)$$

To distinguish models of (strong) underlying dynamics, one needs to measure these ℓ's.

By setting the GB-fields to zeros ("going to Unitary gauge"), one finds

$$\left.\begin{aligned} S &= \frac{-1}{\pi}\, \ell_1 \cdot \left(\frac{\Lambda_0}{\Lambda}\right)^2 \\ \alpha T &= \frac{1}{8\pi^2}\, \ell_0 \cdot \left(\frac{\Lambda_0}{\Lambda}\right)^2 \\ U &= \frac{-1}{\pi}\, \ell_8 \cdot \left(\frac{\Lambda_0}{\Lambda}\right)^2 \end{aligned}\right\}$$

well measured by low energy data (including LEP/SLC)

$$\left(\Delta\rho_{new} = \Delta\rho - \Delta\rho_{SM} = \alpha T, \text{ for } \rho = 1 + \Delta\rho \right)$$

Equivalence Theorem

Ward Identity

$$\left(-\partial W + M_W \phi\right) \Big| \begin{matrix}\text{physical} \\ \text{state}\end{matrix} \Big\rangle = 0$$

(At tree level)

Longitudinal polarization vector of W boson

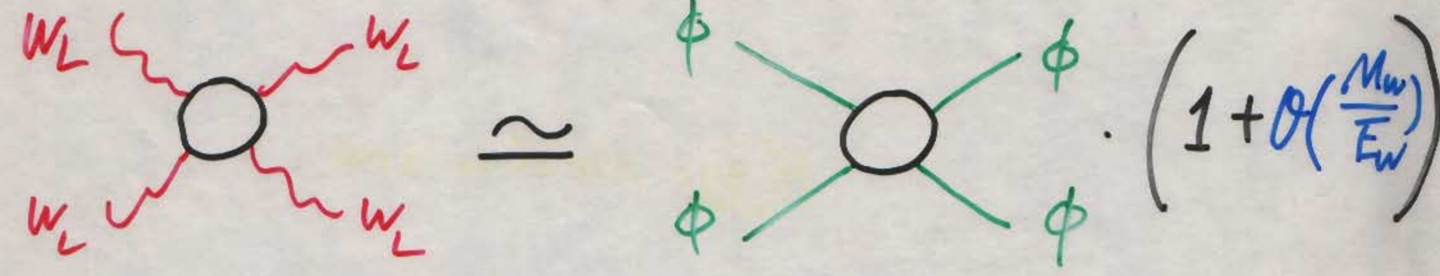
$$\varepsilon_L(k) = \frac{k}{M_W} + \mathcal{O}\left(\frac{M_W}{E_W}\right)$$

If $M_W \ll E_W$, then

$$(\text{Longitudinal } W) \simeq (\text{Goldston Boson } \phi)$$

(A proof to all order,
Thanks to H.-J. He, Y.-P. Kuang and X. Li
I will talk more about this later.)

Similarly, as $M_W \ll E_W$ (in TeV region)



$\Longrightarrow$ To probe the Electroweak Symmetry Breaking Sector, one needs to study Interactions among Goldstone Bosons, which is equivalent to study Interactions among Longitudinal W-Bosons.

Question:

How to prepare longitudinal W-boson beams?

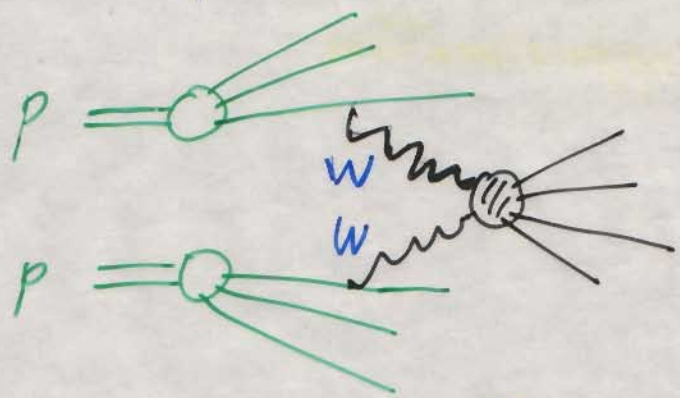
• WW fusion (Prepare W-Beams)

We will adopt the Effective-W approximation to do the calculation.

(ref. to Kane, Repko and Rolnick
Dawson
...)

(1) Why ?

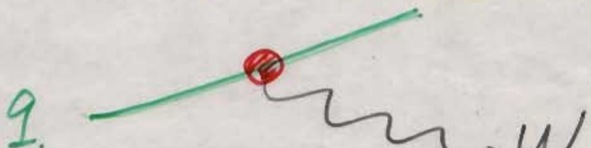
Consider the process



• We would like to have one scheme to do the calculation, even if the subprocess

becomes strongly interacting.

In other words, the W's emitted from quarks can be treated exactally the same, no matter whether WW interaction is perturbative or not.

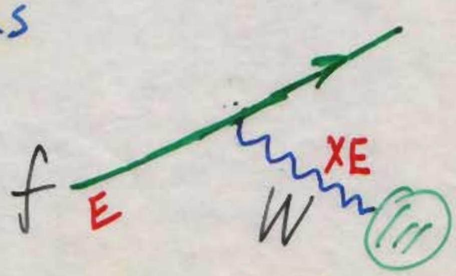


- Simplify the calculation..
- The calculation may be further simplified using the Equivalence Theorem

$$\left(W_L W_L \to W_L W_L \simeq \phi\phi \to \phi\phi \right)$$

(2) What is it?

Similar to the effective photon approximation for the distribution of photons in an electron, The effective-W approximation gives the structure functions of transverse and longitudinal W in a fermion f with the momentum fraction x as



$$F^{f}_{W_T}(x) = \frac{\alpha}{8\pi \sin^2\theta_W} C^{f}_{W} \frac{1+(1-x)^2}{x} \ln\left(\frac{p_T^2}{m_W^2}\right).$$

$$F^{f}_{W_L}(x) = \frac{\alpha}{4\pi \sin^2\theta_W} C^{f}_{W} \frac{1-x}{x}.$$

and

$$C^{f}_{W^\pm} = 1,$$

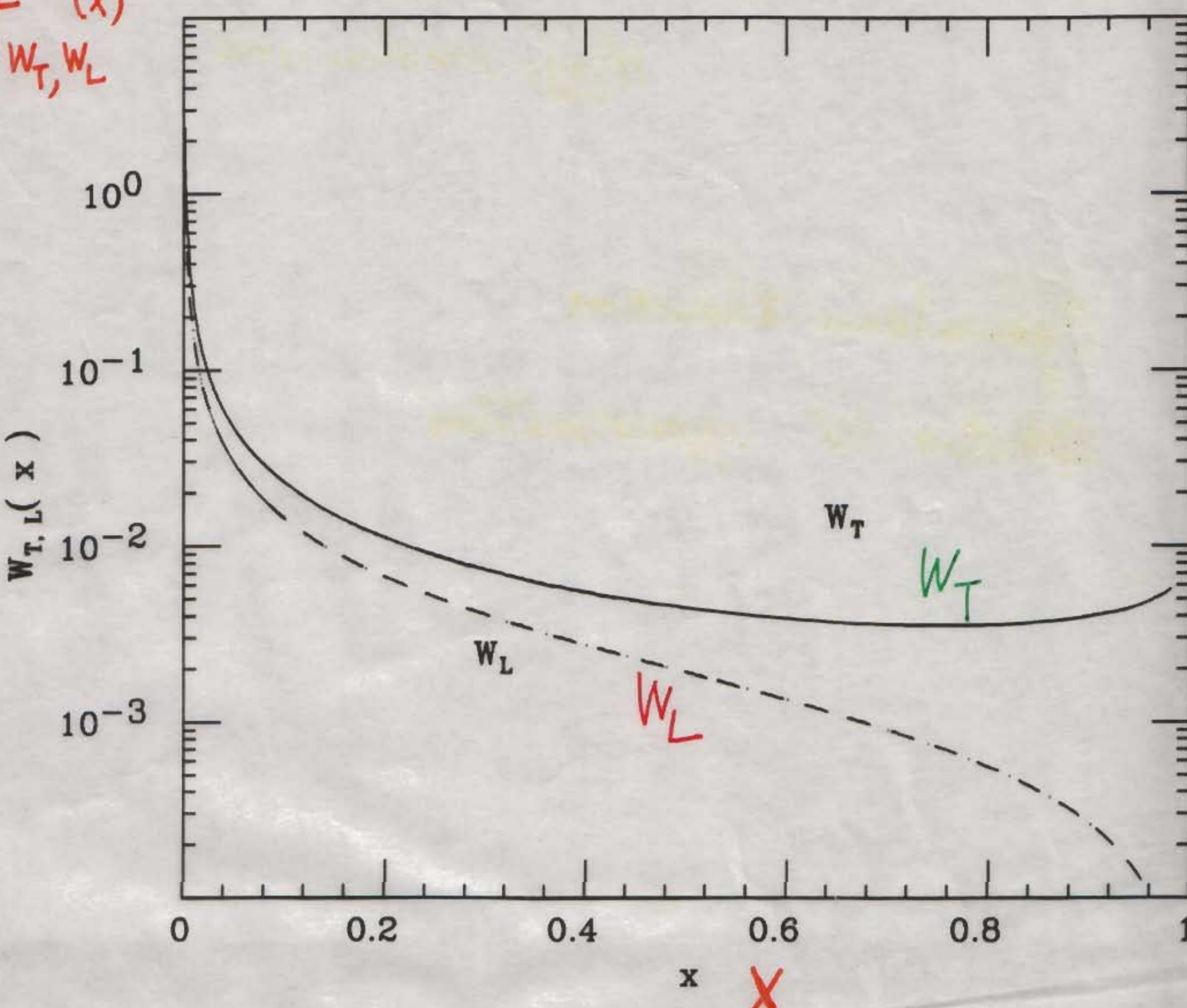
$$C^{f}_{Z^0} = \frac{1}{\cos^2\theta_W}\left[\left(T^{f}_{3L} - 2Q_f \sin^2\theta_W\right)^2 + \left(T^{f}_{3L}\right)^2\right].$$

- W's emitted from a fermion f should be treated as radiation.

$P_T = 100\,GeV$

$F_{W_T, W_L}(x)$

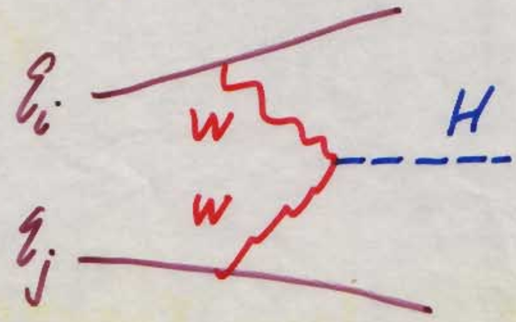
W_T- and W_L- Structure Function : Pper=100 GeV



$$F_{W_T}(x) = \frac{\alpha_w}{8\pi} \frac{1+(1-x)^2}{x} \ln\left(\frac{P_T^2}{m_w^2}\right), \qquad \alpha_w = \frac{\alpha}{\sin^2\theta_w}$$

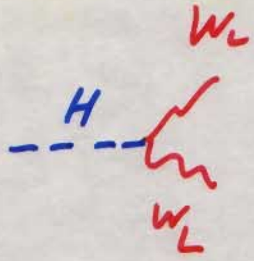
$$F_{W_L}(x) = \frac{\alpha_w}{4\pi} \frac{1-x}{x}$$

• For heavy Higgs, $m_H \gtrsim 800$ GeV, its production is dominated by Longitudinally polarized W's fusion. $(m_t \sim 150\,\text{GeV})$

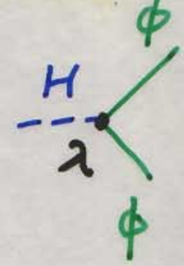


$$\underbrace{\Gamma(H\to W_L W_L)}_{\sim \frac{1}{m_H}\left(\frac{m_H^2}{v^2}\right)^2} \gg \underbrace{\Gamma(H\to W_T W_T)}_{\sim \frac{1}{m_H}(g)^2}$$

(From E.T.



$\simeq$

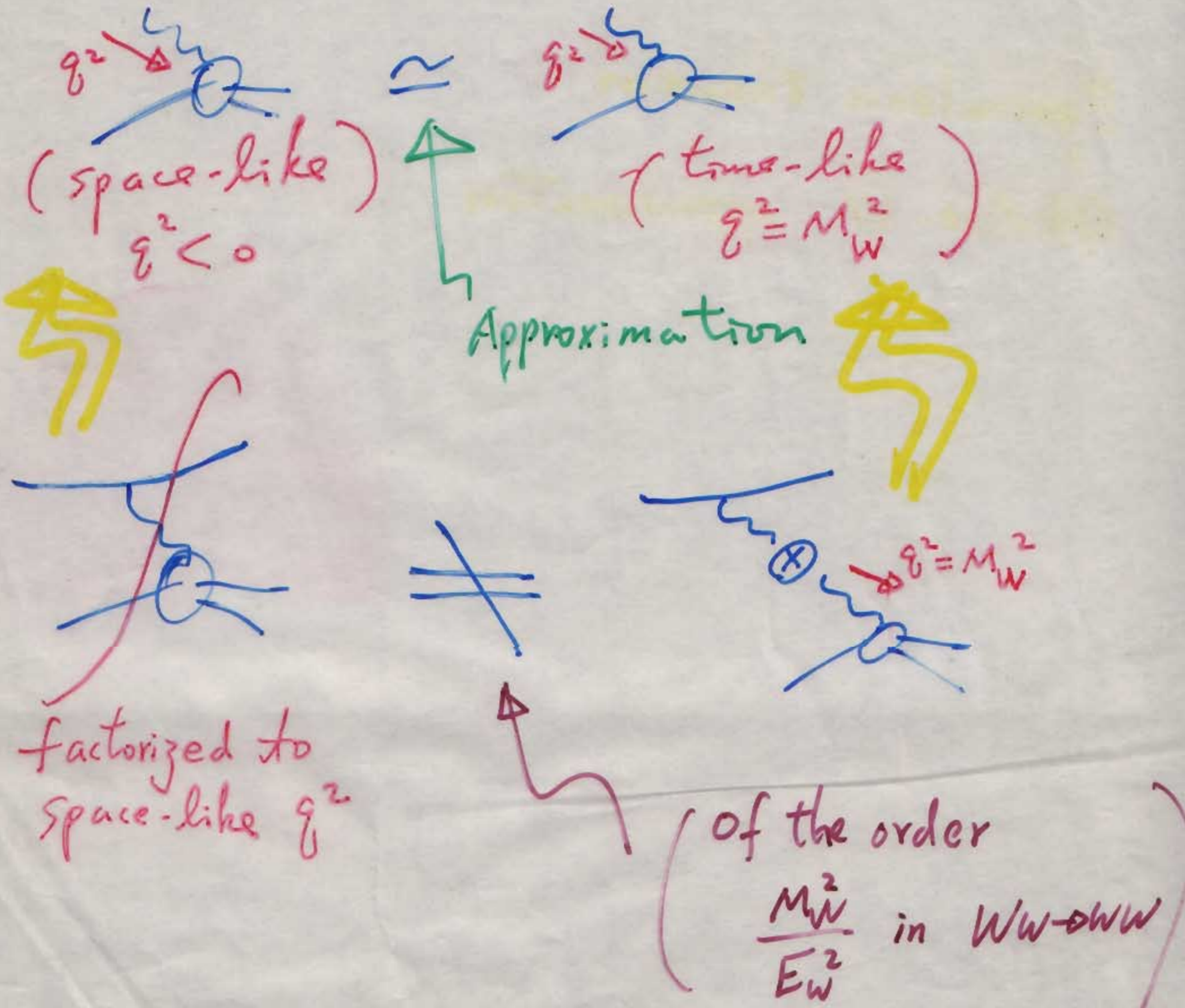


)

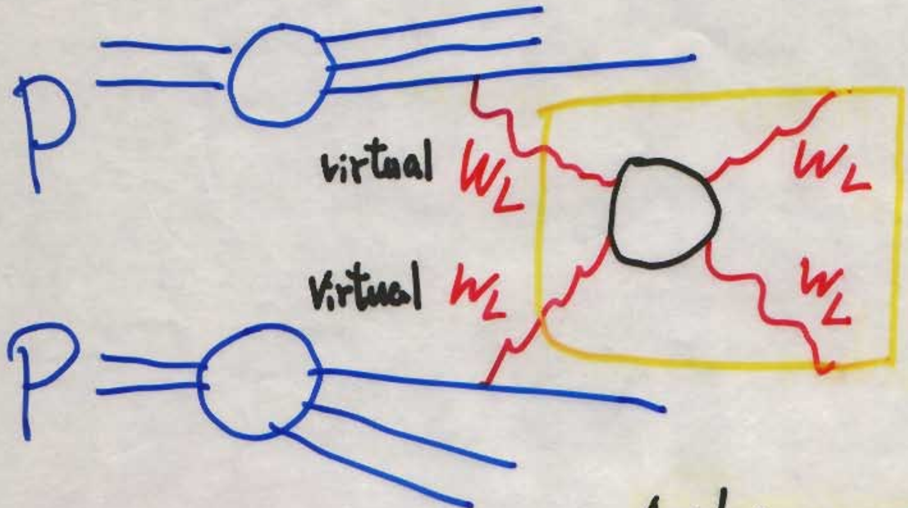
$$\sigma(q_i q_j \to q q H) \simeq \sum_{\lambda = L,T} \int_0^1 dx_1\, F^{q_i}_{W_\lambda}(x_1) \int_0^1 dx_2\, F^{q_j}_{W_\lambda}(x_2) \cdot \hat{\sigma}(W_\lambda W_\lambda \to H)$$

Effective-W Approximation

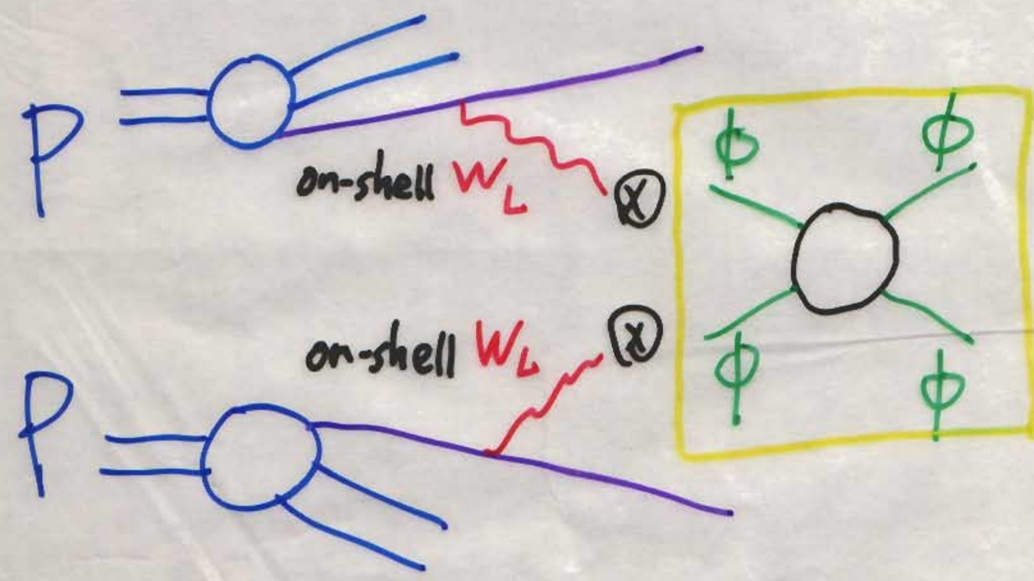
Note that when applying the effective-W approximation, assumption has been made about the dynamics of the hard part process:



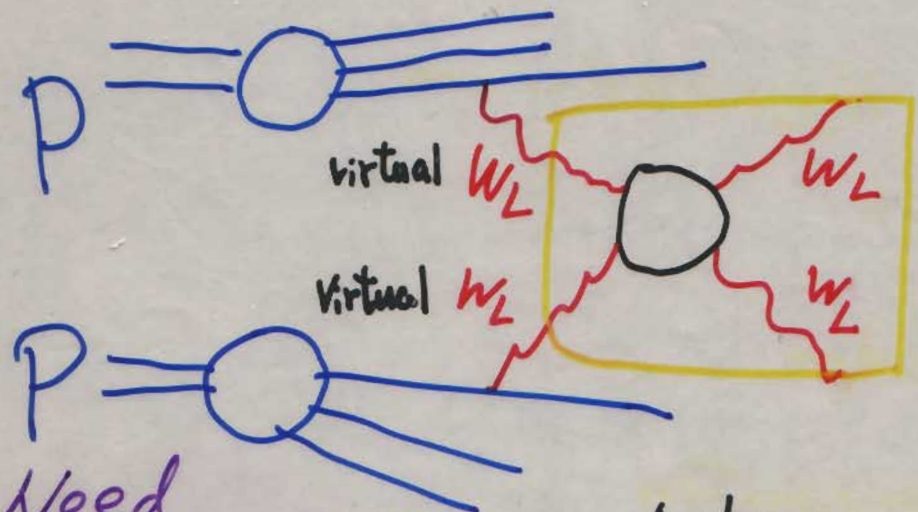
- Event signature of Signal



Applying Effective-W Approximation
and Equivalence Theorem



· Event signature of Signal

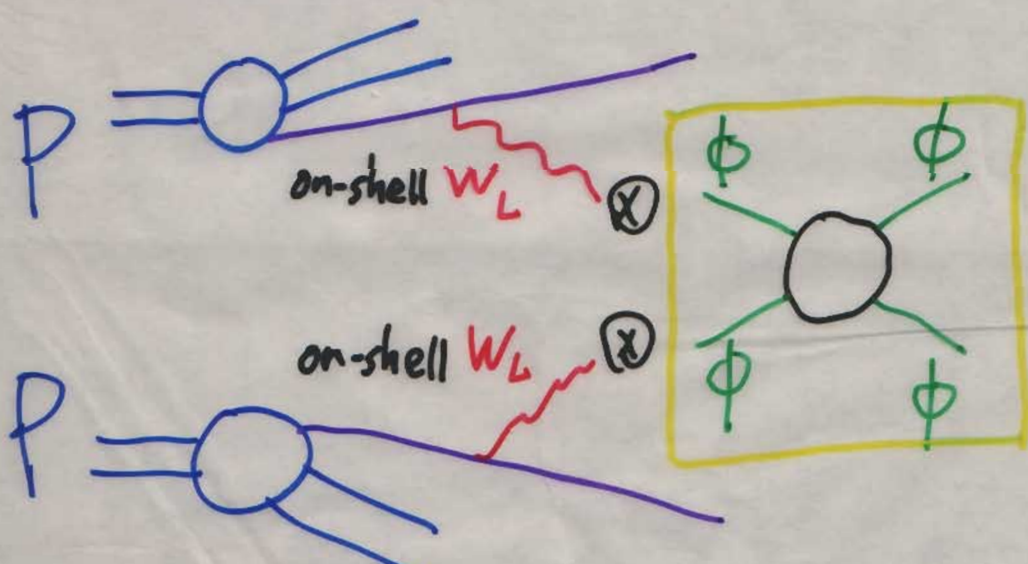


Δ 2 Tool

Need $E_W \gg M_W$ (TeV region)

$\Downarrow$

Applying Effective-W Approximation and Equivalence Theorem



Δ 2 Goal

Given the amplitude

$$T =$$

tree loop

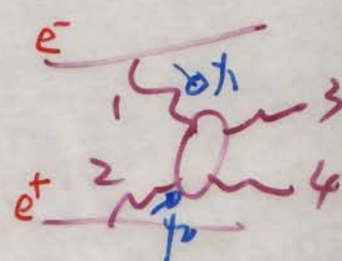
from $\mathcal{L}_G + \mathcal{L}^{(2)}$

tree from $\mathcal{L}^{(2)'} + \sum_{n=1}^{14} \mathcal{L}_n$

one can calculate rates

$$\sigma = \int dy_1\, F_{W_1}(y_1)\, dy_2\, F_{W_2}(y_2)\, \hat{\sigma}(W_1 W_2 \to W_3 W_4)$$

for LC

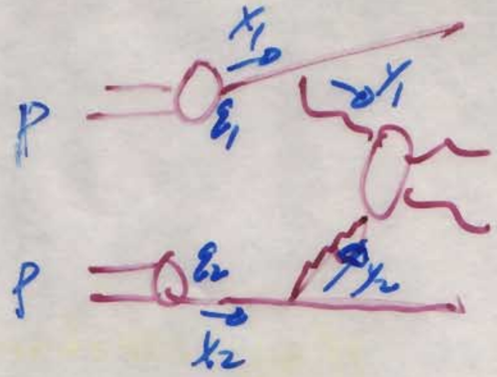


where

$$\hat{\sigma}(W_1 W_2 \to W_3 W_4) = \int d\Phi_2 \cdot \underbrace{|T|^2}$$

keep terms up to $\mathcal{O}\left(\frac{1}{\Lambda_0^2}, \frac{1}{\Lambda^2}\right)$.

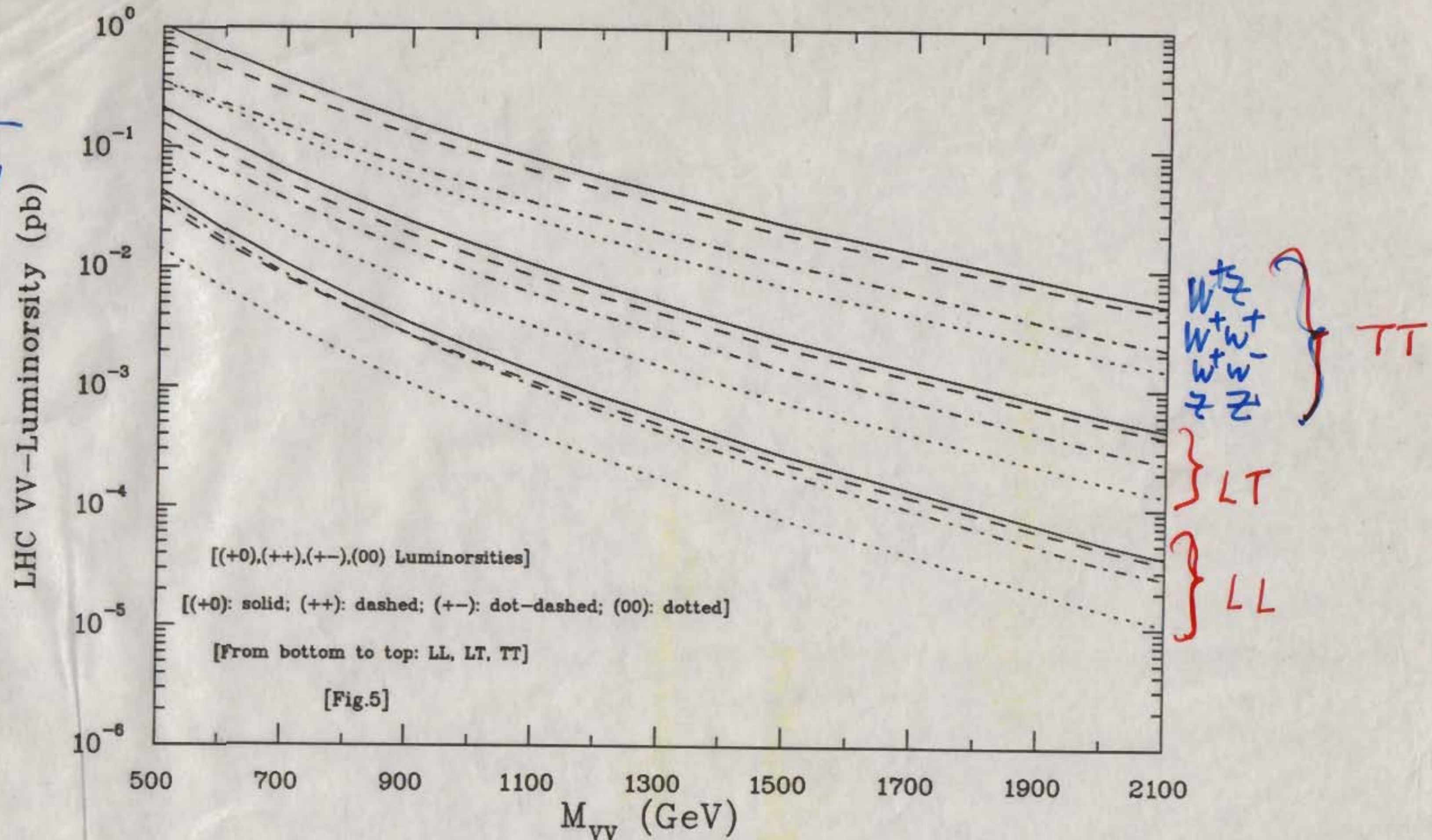
At LHC



$$\sigma = \int dx_1\, dx_2\, f_{\xi_1}(x_1)\, f_{\xi_2}(x_2) \cdot \int dy_1\, F_{W_1}(y_1)\, dy_2\, F_{W_2}(y_2) \cdot \hat{\sigma}(W_1 W_2 \to W_3 W_4)$$

$$\equiv \int dM_{WW}^2 \underbrace{\frac{d\mathcal{L}_{WW}}{dM_{WW}^2}}_{\text{effective-}W\text{ parton luminosity}} \hat{\sigma}(M_{WW}^2)$$

$$\frac{d\mathcal{L}_{WW}}{d M_{WW}^2}$$

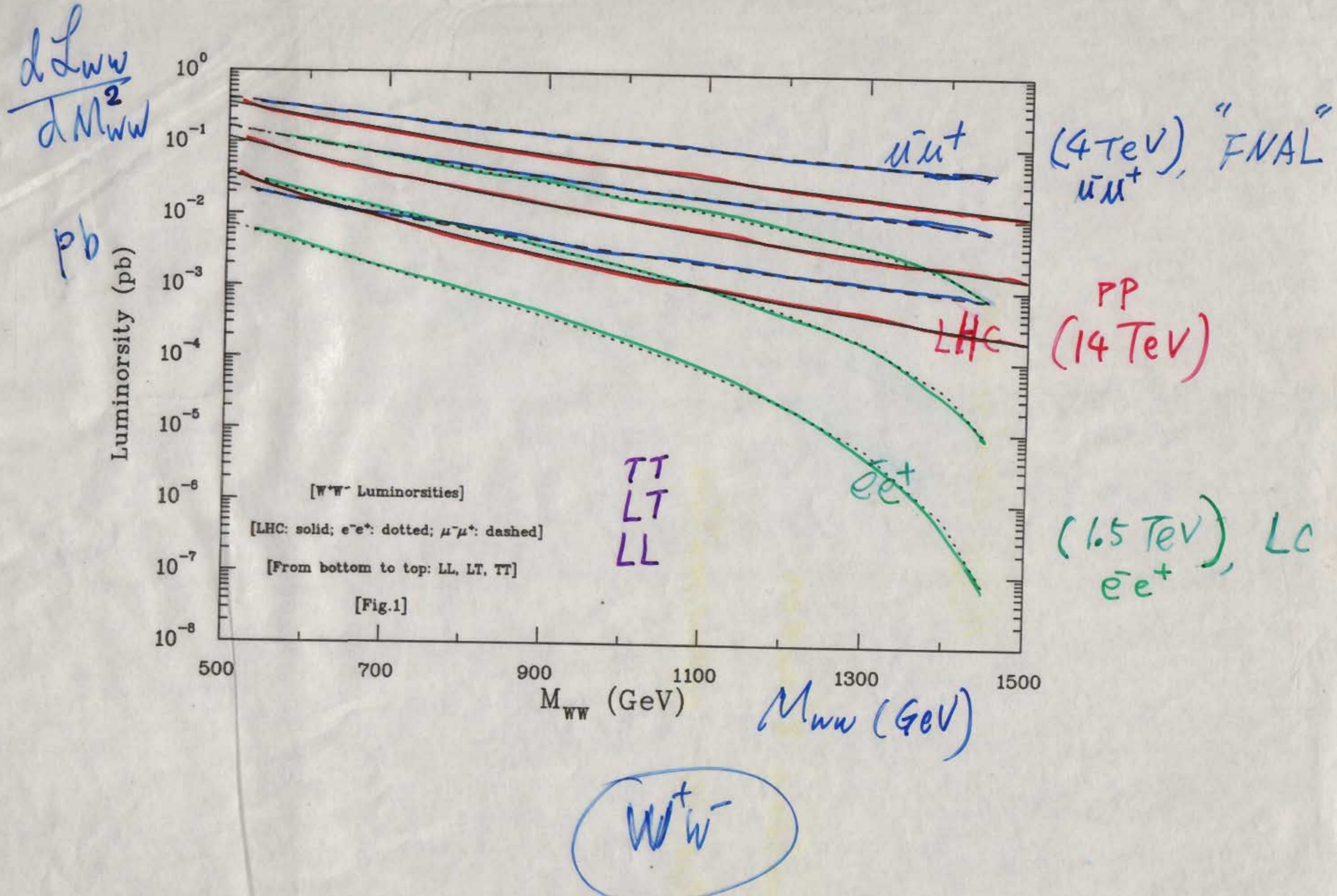


[Fig.5]

LHC

(PP, 14TeV)

$$\frac{\mathcal{L}(W^-W^-)}{\mathcal{L}(W^+W^+)} \sim \frac{1}{(3 \text{ to } 5)}$$


$\frac{d\mathcal{L}_{WW}}{dM_{WW}^2}$
pb
Luminorsity (pb)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
500
700
900
1100
1300
1500
M_{WW} (GeV)
M_{ww} (GeV)
$\mu^-\mu^+$
$(4\,TeV)$ $\mu^-\mu^+$, "FNAL"
pp (14 TeV)
LHC
e^-e^+
$(1.5\,TeV)$ e^-e^+, LC
TT
LT
LL
[W^+W^- Luminorsities]
[LHC: solid; e^-e^+: dotted; $\mu^-\mu^+$: dashed]
[From bottom to top: LL, LT, TT]
[Fig.1]
W^+W^-

- Phenomenology

Given the above tools, one can calculate signal and background rates for interesting processes at various colliders.

Much work was done for selecting kinematic cuts for suppressing backgrounds and enhansing signal-to-background ratio.

"Real" Work
(hard Calculation)
is needed to do phenomenology.

$\Rightarrow$ Rates & distributions
of
Signal & Backgrounds
are needed.

M. Chanowitz

- No Lose Theorem

— If Light Higgs Boson exists,
LHC can find it.

— If Light Higgs Boson does not exist,
the self-interactions of W_L's
become strong in the TeV region,
and can be observed in $W_L W_L$ two-body
interactions via leptonic decays
of W_L's.

In particular, $W^{\pm}W^{\pm}$ mode is favored
because the Standard Model (SM) background
is small.

Event rates per LHC year for $W_L W_L$ fusion signals (in Gold-plated mode)

Table 3: Event rates per LHC-year for $W_L W_L$ fusion signals from the different models, together with backgrounds, assuming $\sqrt{s} = 14$ TeV, an annual luminosity of 100 fb^{-1}, and $m_t = 175$ GeV. Cuts are listed in Table 1. Jet-vetoing and tagging efficiencies are listed in Table 2. The $W^\pm Z(M_T^{cut})$ row refers to the $W^\pm Z$ events with $0.8 < M_T(WZ) < 1.1$ TeV, optimized to search for a 1 TeV isovector signal.

	Bkgd.	SM	Scalar	O(2N)	Vec 1.0	Vec 2.5	LET-CG	LET-K	Delay-K
$ZZ(4\ell)$	0.7	9	4.6	4.0	1.4	1.3	1.5	1.4	1.1
$ZZ(2\ell 2\nu)$	1.8	29	17	14	4.7	4.4	5.0	4.5	3.6
W^+W^-	12	27	18	13	6.2	5.5	5.8	4.6	3.9
$W^\pm Z$	4.9	1.2	1.5	1.2	4.5	3.3	3.2	3.0	2.9
$W^\pm Z(M_T^{cut})$	0.82				2.3				
$W^\pm W^\pm$	3.7	5.6	7.0	5.8	12	11	13	13	8.4

Table 4: Number of years (if < 10) at LHC required for a 99% confidence level signal.

	Model							
Channel	SM	Scalar	O(2N)	Vec 1.0	Vec 2.5	LET CG	LET K	Delay K
$ZZ(4\ell)$	1.0	2.5	3.2					
$ZZ(2\ell 2\nu)$	0.5	0.75	1.0	3.7	4.2	3.5	4.0	5.7
W^+W^-	0.75	1.5	2.5	8.5		9.5		
$W^\pm Z$				7.5				
$W^\pm W^\pm$	4.5	3.0	4.2	1.5	1.5	1.2	1.2	2.2

$$\int \mathcal{L} = 100 \text{ fb}^{-1}/\text{year}$$
$$= 100 \times 10^3 \text{ pb}^{-1}/\text{year}$$

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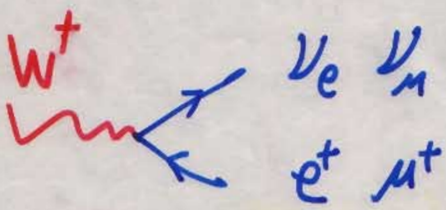
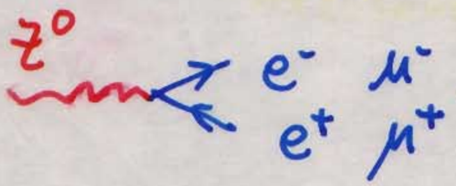
$$\int \mathcal{L} = 100\ \mathrm{fb}^{-1}/\mathrm{year}$$
$$= 100 \times 10^3\ \mathrm{pb}^{-1}/\mathrm{year}$$

"Gold-Plated" modes

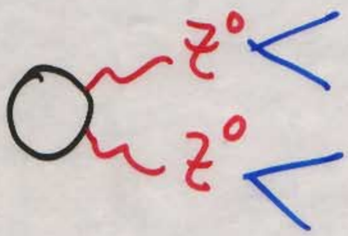
- Branching Ratio



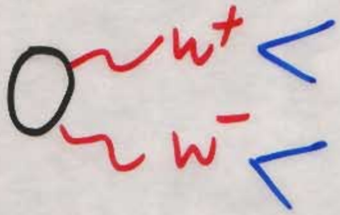
$\sim \frac{1}{9} \times 2$



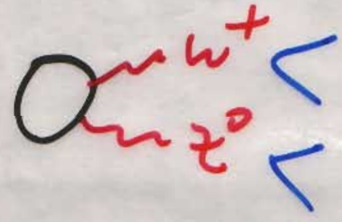
$\sim 3.3\% \times 2$



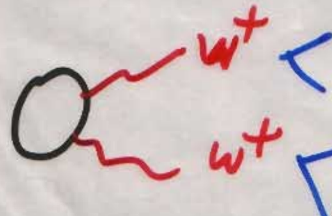
$\sim 0.45\%$



$\sim 4.7\%$



$\sim 1.5\%$

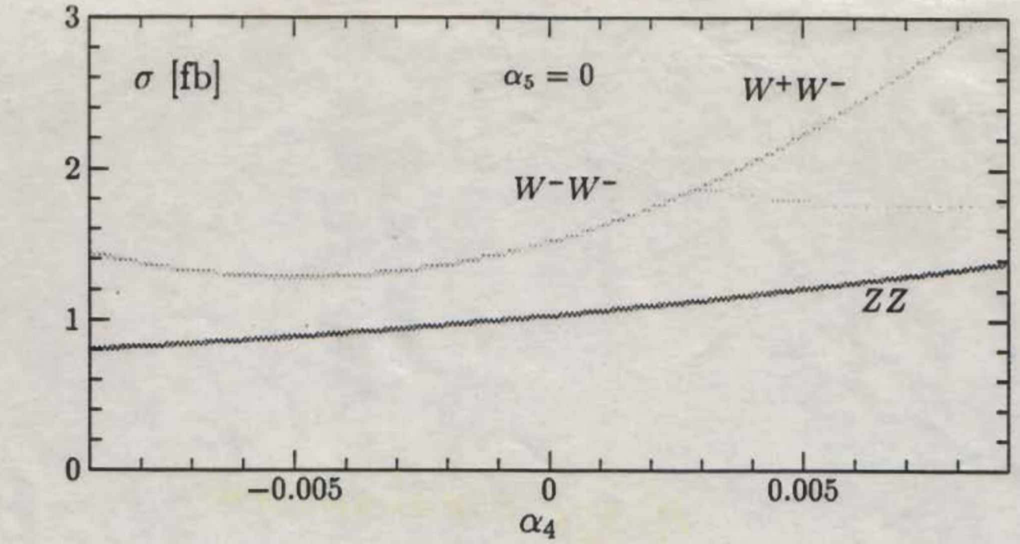


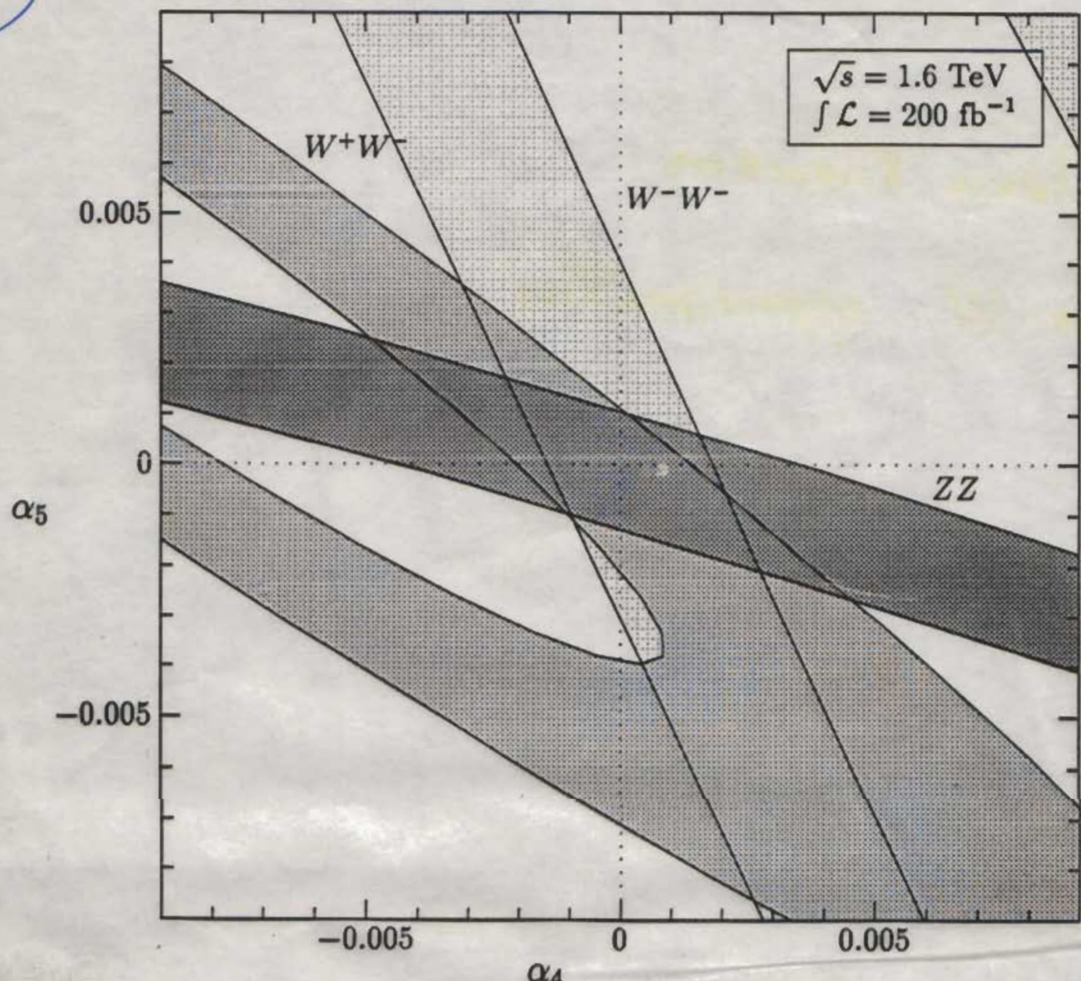
$\sim 4.7\%$

LC

(1.6 TeV $e^- e^+$)

$l_{4,5}$





$$\alpha_i = \frac{l_i}{16\pi^2} \sim 6\times10^{-3} \cdot l_i$$

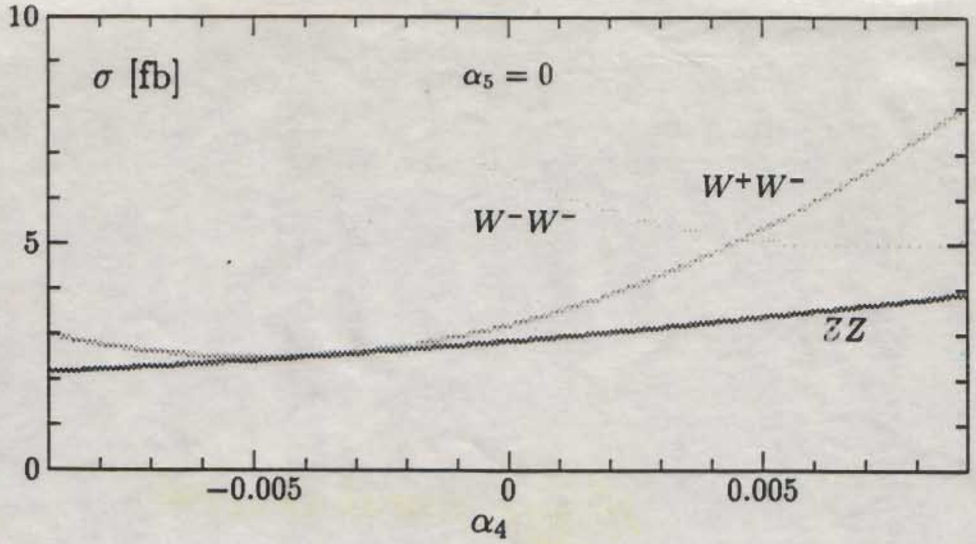
All VV modes need to be measured.

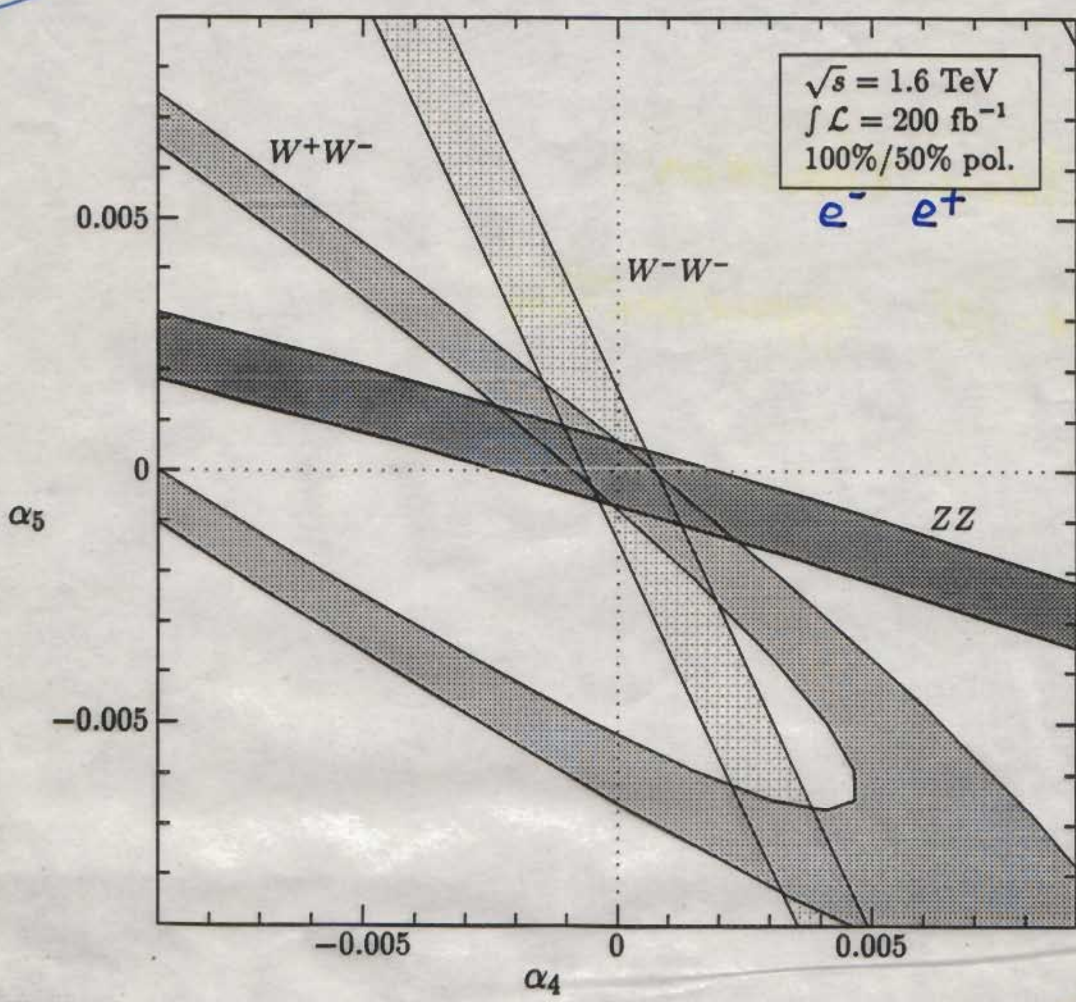
Polarized

LC

(1.6 TeV
e^-e^+)

$l_{4,5}$





$$\alpha_i = \frac{l_i}{16\pi^2} \sim 6\times10^{-3}\, l_i$$

All VV modes need to be measured.

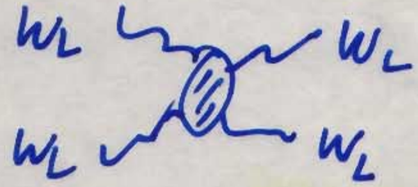
- Conclusion

To probe the Electroweak Symmetry Breaking needs to study the interactions of longitudinal W's in the TeV region.

Even if a light ($\lesssim$ 800 GeV) SM Higgs Boson is found, one still has to check that nothing new occurs in the TeV region.

Remarks on m_H bounds

1. Partial wave analysis (Unitarity Bound)



$$|a_\ell^I| < 1$$

$$\Rightarrow \quad m_H \lesssim 1 \text{ TeV}$$

2. Perturbative Expansion

ρ - Parameter shift $\qquad \rho \equiv \dfrac{M_W^2}{M_Z^2 \cos^2\theta_W}$

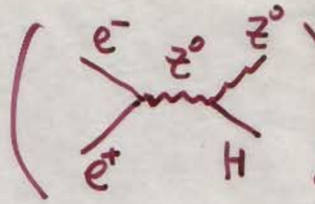
$$\rho \simeq 1 + \underbrace{c_1 \cdot g^2 \cdot \ln(m_H^2)}_{\substack{\uparrow \\ \text{Screening Theorem} \\ \text{(one loop)}}} + \underbrace{c_2 \, g^4 \frac{m_H^2}{M_W^2}}_{\text{(two loops)}}$$

Demand perturbative expansion,

$$\Rightarrow \quad m_H \lesssim 1 \quad \text{TeV}$$

· What do we know about m_H ?

1. LEP $m_H \gtrsim 107$ GeV



2. Theoretical bounds on m_H :

— Unitarity bound
— Validity of perturbative calculation
— Triviality of ϕ^4

Within a factor of 2
It's about $\lesssim 1$ TeV

Or, Renormalization Group Equation
(Dynamical Symmetry breaking)
to relate m_H and m_t .

3. Radiative Corrections difficult to Constrain m_H .
(Screening Theorem)

Any next-to-leading order contribution can only have $\ln(m_H^2)$ corrections, but no m_H^2 Corrections.

- Higgs at LHC

m_H

light $\quad M_Z \quad 2M_Z \quad$ Heavy $\quad$ 600 ~ 800 GeV $\quad$ TeV

Intermediate

Obese

LEP

LEP2 (~110 GeV)

$H \to ZZ \to 4\ell's$

$H \to ZZ \to \ell^{\pm}\ell^{\mp}\nu's$

$H \to ZZ \to \ell^{\pm}\ell^{\mp}jj$

$H \to W^+W^- \to \ell^{\pm}\nu j_1 j_2$

$H \to W^+W^- \to \ell^{\pm}\nu J$

$H \to ZZ^* \to 4\ell's$ ($\gtrsim 125$ GeV)

$H \to \gamma\gamma$

$W^{\pm}H \to \ell^{\pm}\nu\, b\bar{b}$

$W^{\pm}H \to \ell^{\pm}\nu\,\gamma\gamma$

$t\bar{t}H$
$\quad \hookrightarrow \gamma\gamma$

Unitarity Bounds

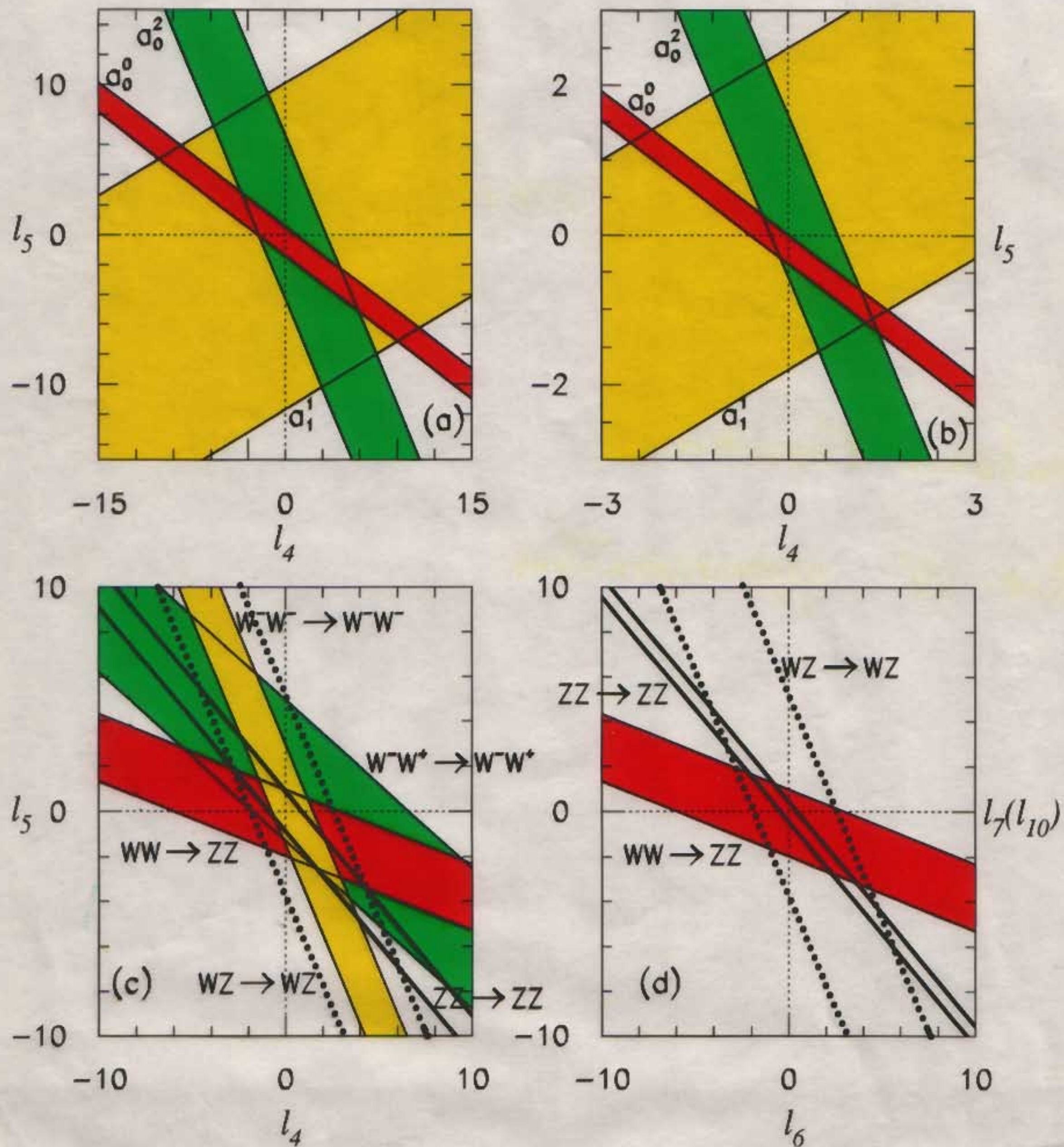


Figure 2: Unitarity bounds from $VV \to VV$ fusions at (a) 0.8 TeV, (b) 1.2 TeV, and (c,d) 1.0 TeV. Here (a) and (b) are plotted for the partial wave a_J^I with given weak isospin, while (c) and (d) display the S-wave bounds for all physical channels by including the $SU(2)_c$-violating parameters $\ell_{6,7,10}$ [in (d)].

unpolarized
e^-, e^+ beams

Polarized beams
e^- (90%)
e^+ (65%)

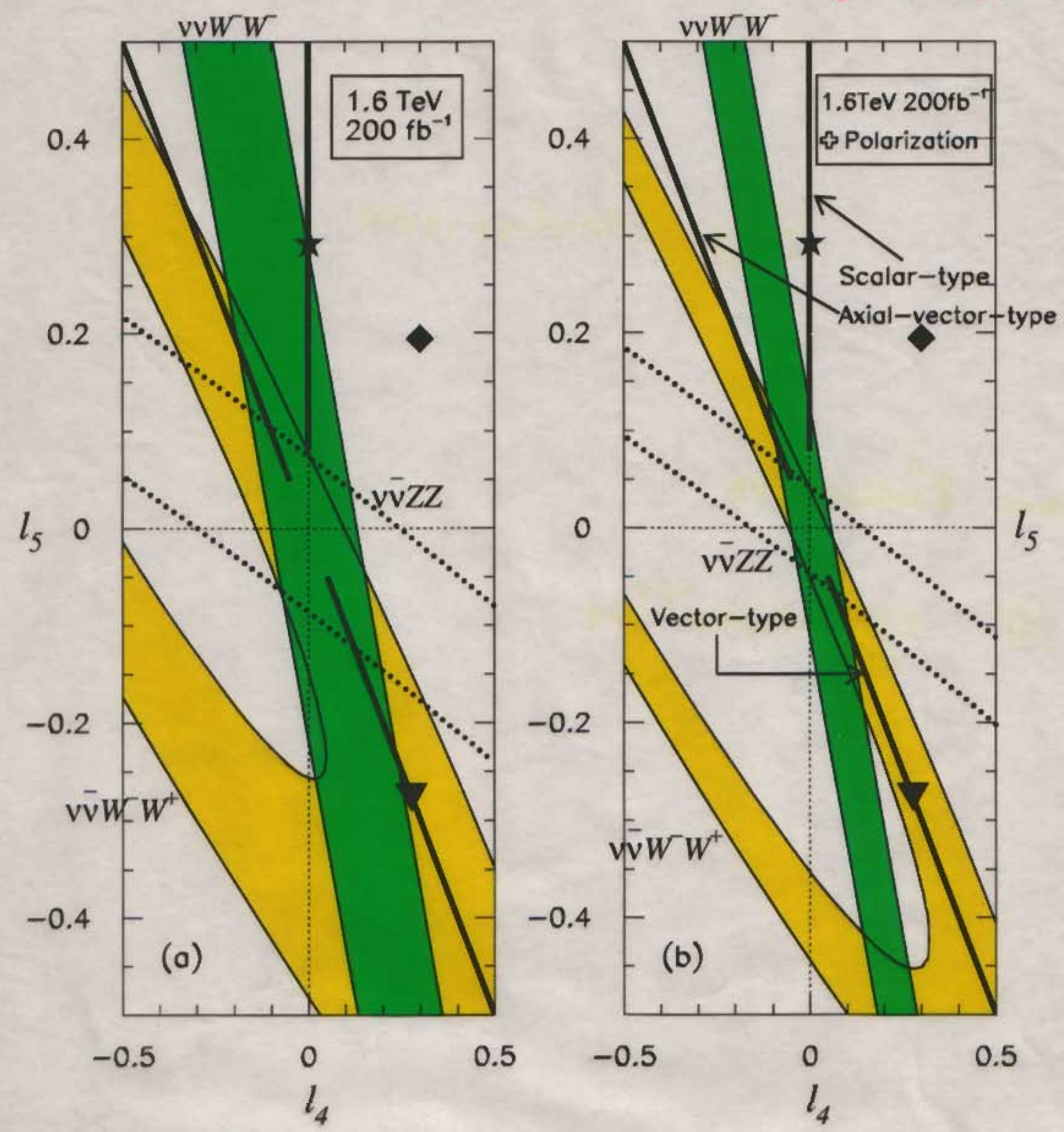


FIGURE 2. Determining the $SU(2)_c$-symmetric parameters ℓ_4-ℓ_5 at 1.6 TeV e^-e^+/e^-e^- LCs. Here the $\pm 1\sigma$ exclusion contours are displayed. (a). unpolarized case; (b). the case with 90%(65%) polarized $e^-(e^+)$ beam. Contributions from three types of resonance models (scalar, vector and axial-vector) to (ℓ_4, ℓ_5) are shown by the thick solid lines. The different points on these solid lines correspond to different values of their couplings to the weak gauge bosons. Note that for axial-vector-type, it is also possible to have $\ell_4 + \ell_5 = 0$ with $\ell_4 \geq 0$, i.e., similar to the vector-type case. This makes the discrimination more involved. Big-star: from a scalar; black-triangle-down: from a vector; black-lozenge: from mixed contributions of a heavy scalar and vector. (Here we typically set these heavy resonances around 2 TeV.)

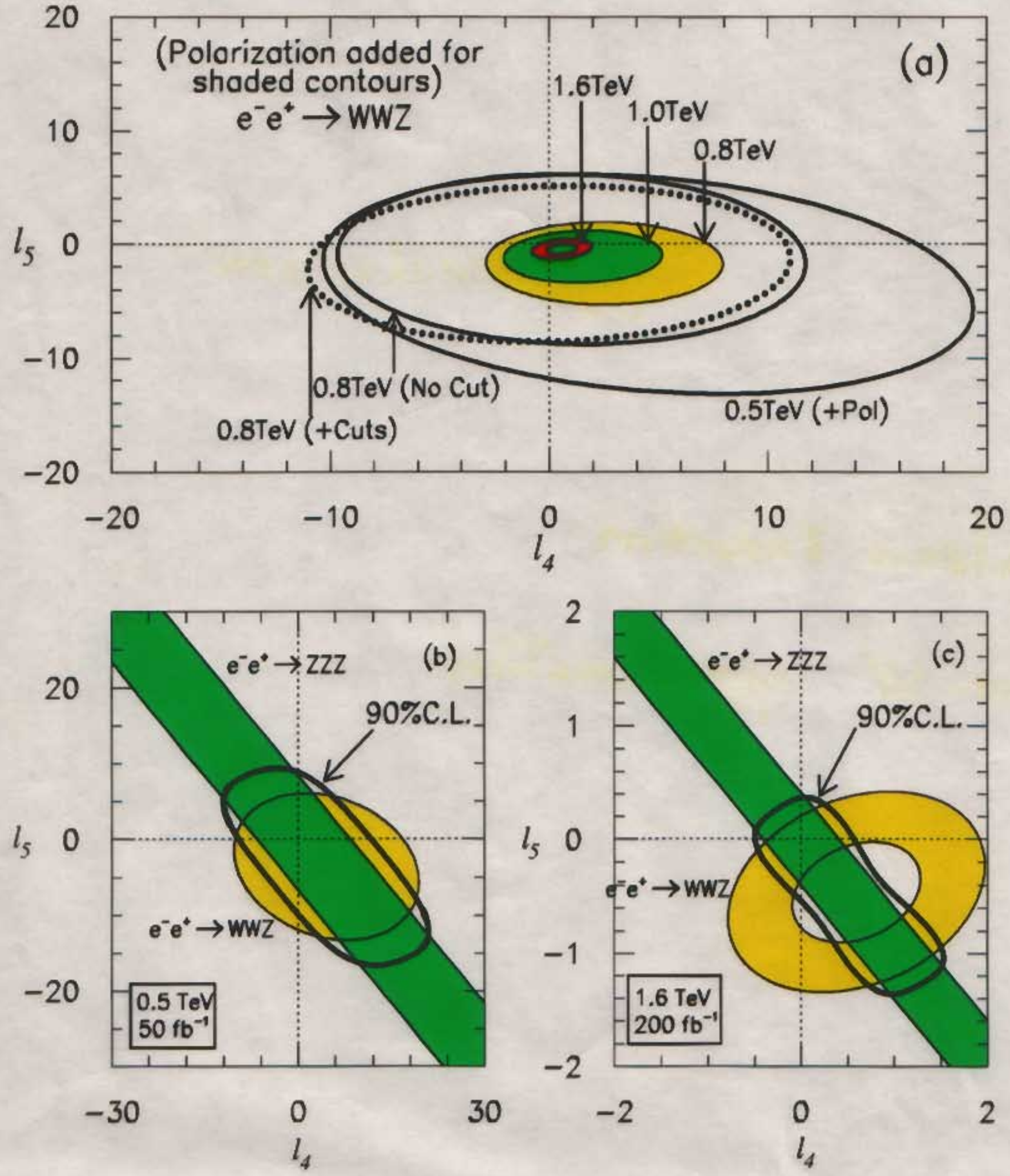


FIGURE 3. Probing ℓ_4-ℓ_5 via WWZ and ZZZ production processes. The roles of the polarization and the higher collider energy for $e^-e^+ \to WWZ$ are shown by the $\pm 1\sigma$ exclusion contours in (a). The integrated luminosities used here are 50 fb^{-1} (at 500 GeV), 100 fb^{-1} (at 800 GeV) and 200 fb^{-1} (at 1.0 and 1.6 TeV). In (b) and (c), the $\pm 1\sigma$ contours are displayed for ZZZ/WWZ final states at $\sqrt{s}$ =0.5 and 1.6 TeV respectively, with two beam polarizations (90% e^- and 65% e^+); the thick solid lines present the combined bounds at 90% C.L.

Does the detection of a light scalar imply Nothing Interesting in the TeV region ?

No !

- A CP-odd scalar does not couple to $W^{\pm}$ and Z

  $\Rightarrow$ $W_L W_L \to W_L W_L$ becomes strong in TeV region

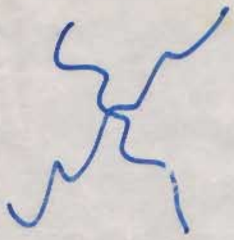
- A non-SM CP-even scalar spoils the cancellation of bad high energy behavior

  $\Rightarrow$ $\mathcal{M} \sim \frac{s}{v^2}$

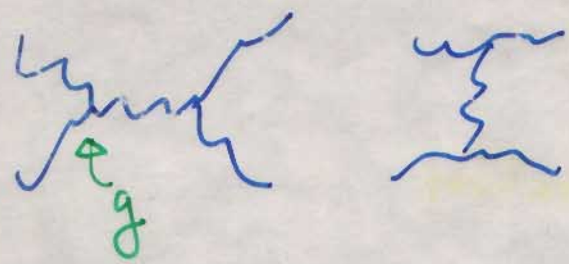
  $\Rightarrow$ $W_L W_L \to W_L W_L$ becomes strong in TeV region

SM

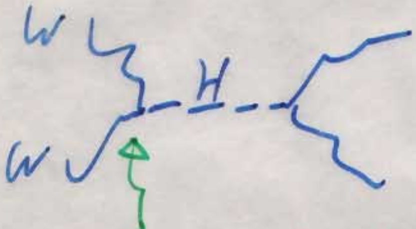
$W_L W_L \to W_L W_L$



$\sim \frac{s^2}{v^4}$



$\Big\} \sim \frac{s}{v^2}$



$\sim \frac{s}{v^2}$

$gM_W = \frac{1}{2} g^2 v$ $(v = 246\ \mathrm{GeV})$

$\Big\} \sim \mathcal{O}(1)$

If the coupling of  is non-SM, then the cancellation is not exact.

$\Rightarrow$ Strong interaction in TeV.

Even if a light (below TeV) resonance
is found at LHC / LC
(Linear Collider e^-e^+, e^-e^-, $e^-\gamma$, $\gamma\gamma$)
it remains
important to test the predictions
of the $W_L W_L$ interactions
in the TeV region.

* Is it (the light resonance) related to
the electroweak symmetry breaking sector ?
Does it play the role to unitarize the
$W_L W_L$ amplitudes in the TeV region ?