Physics of W and Z Bosons at Hadron Colliders

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Goal: Learn the needed theoretical tools to perform precision measurements in hadron collisions.

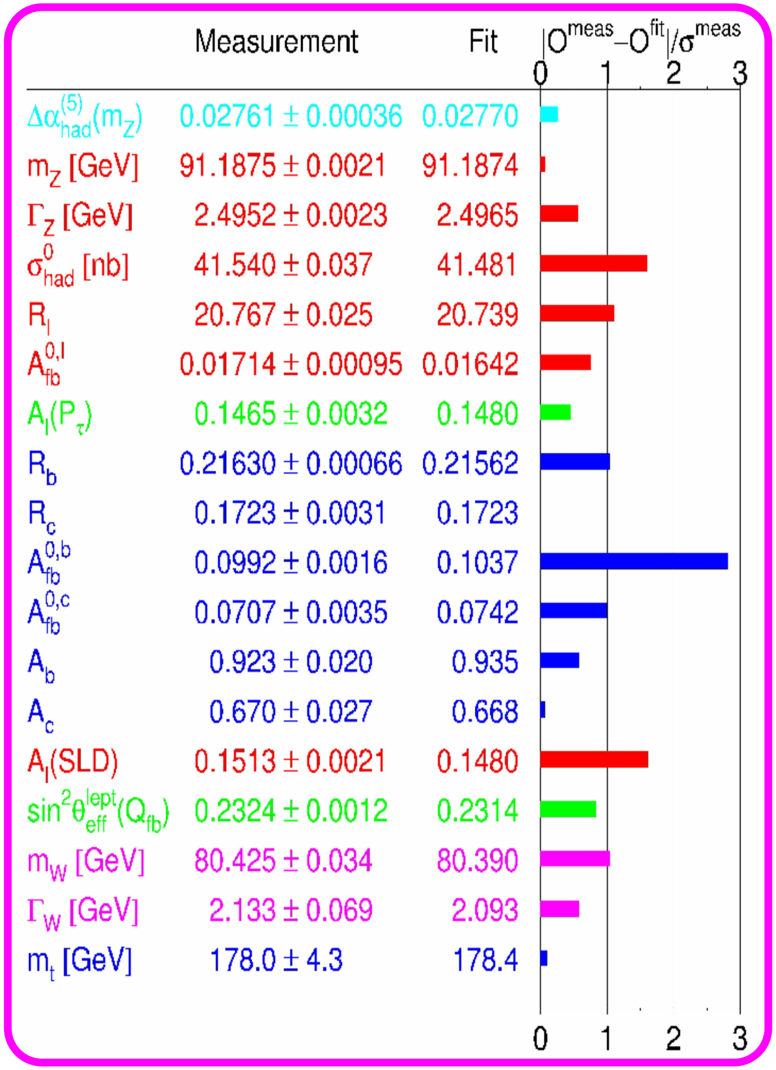
C T E Q

Era of Precision Measurements @ e^-e^+

By comparing measurements and theoretical prediction of electroweak precision observables

- the electroweak sector of Standard Model (SM) is probed at the quantum-loop level
- the consistency of SM is checked by comparing direct measurements with indirect determinations of input parameters, e.g. m_t and M_W.
- the parameters of models beyond the SM can be constrained.

Global SM fit to all electroweak data: from LEPEWWG (Winter 2004)



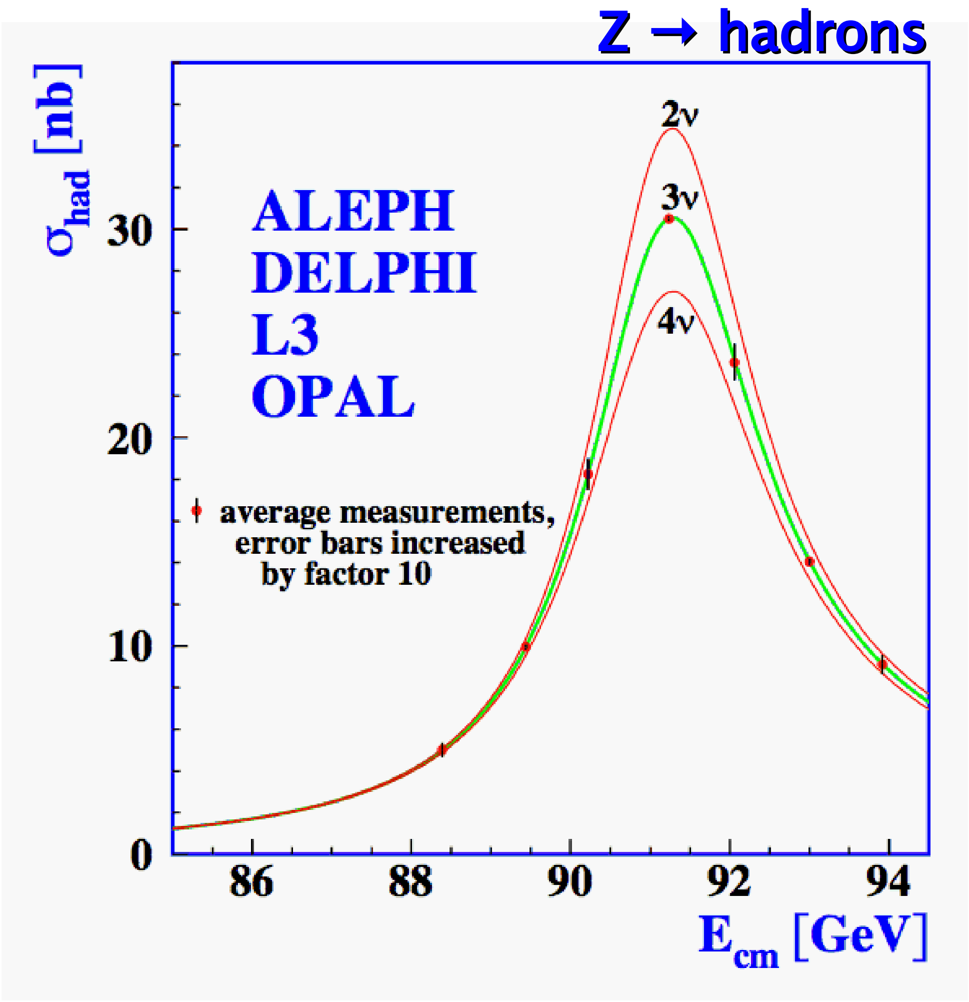
CTEQ

Precision Data

- Most precise measurement of W boson mass was done at Tevatron.
- Most precise measurement of Top quark mass was done at Tevatron.

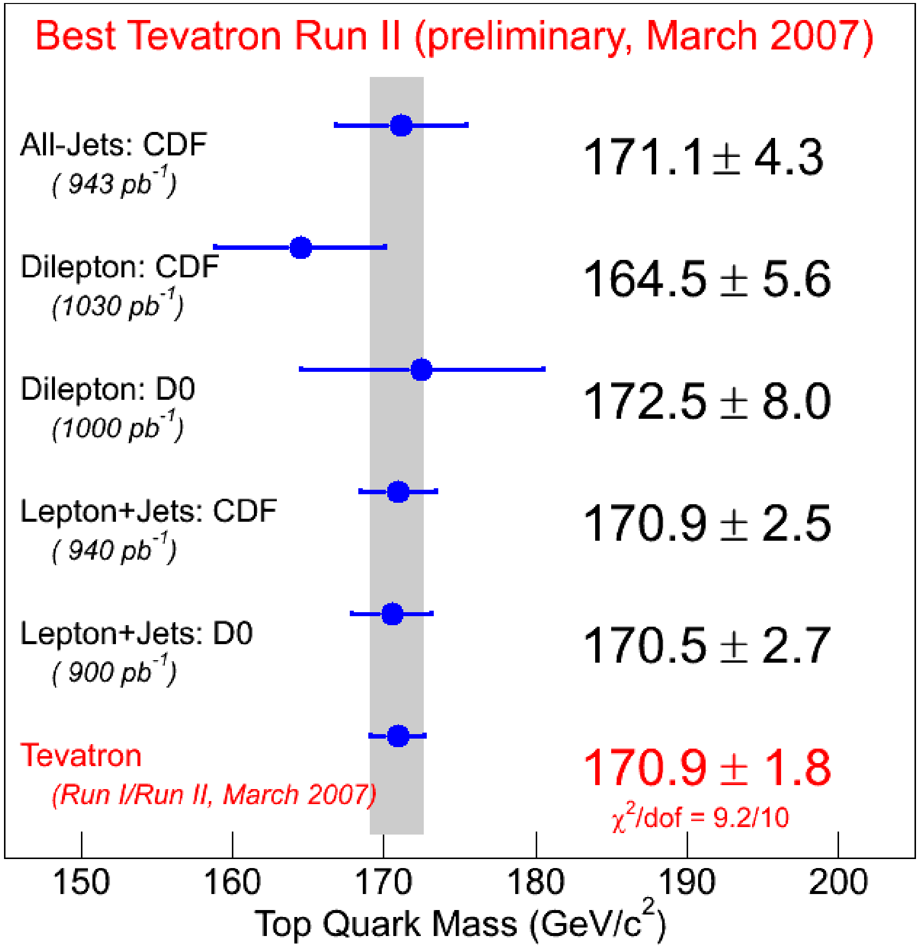
Phase II: Precision Measurements

Z boson



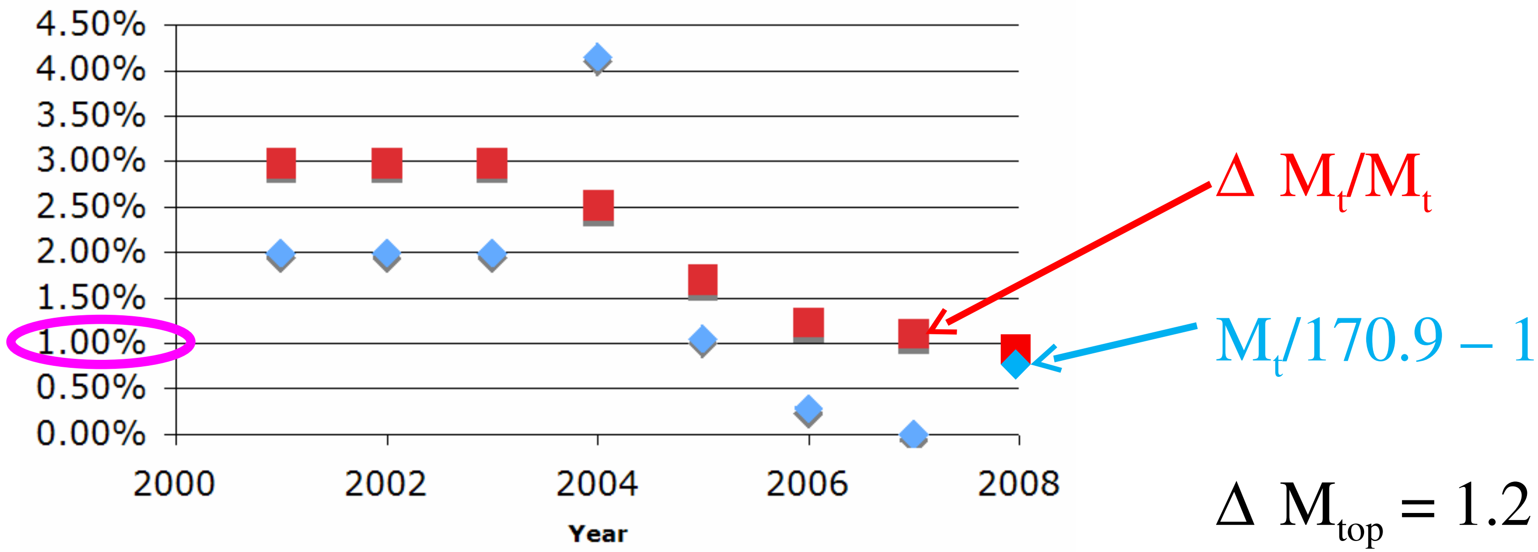
LEP experiments, CERN

top quark

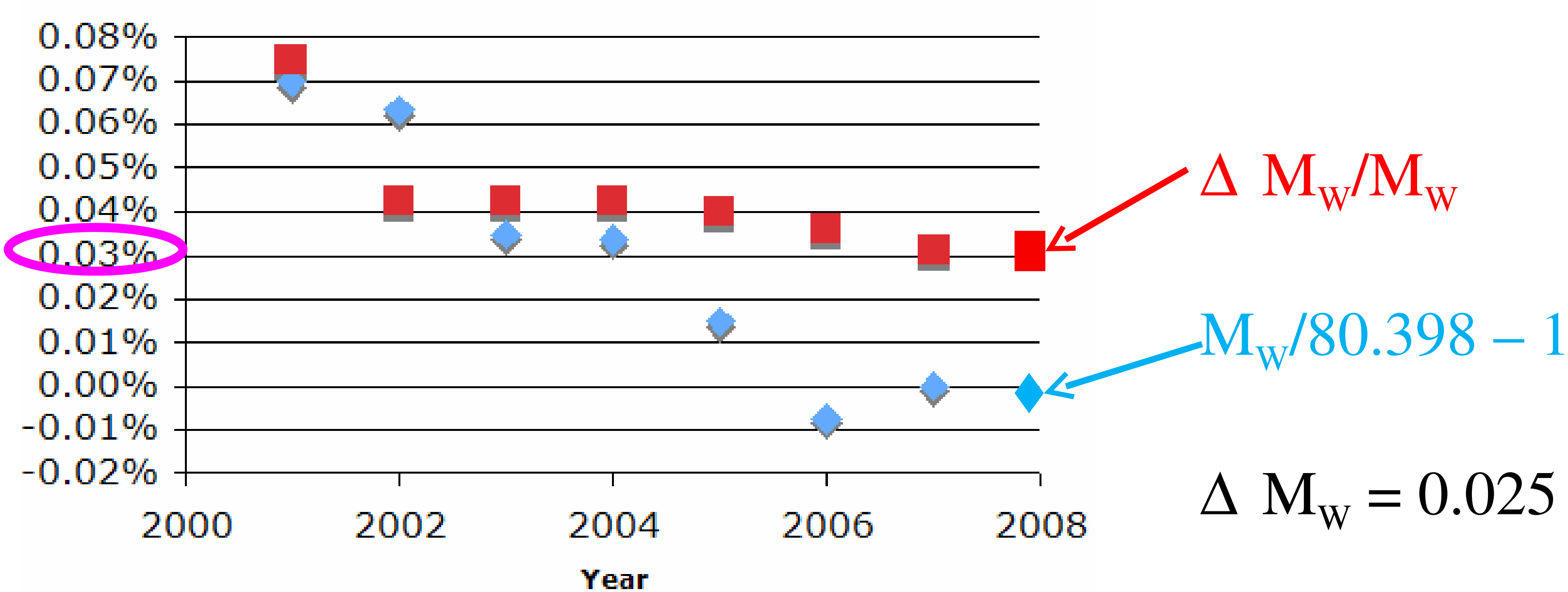


Tevatron experiments, Fermilab

M_t and M_W Uncertainties Steadily Decreasing



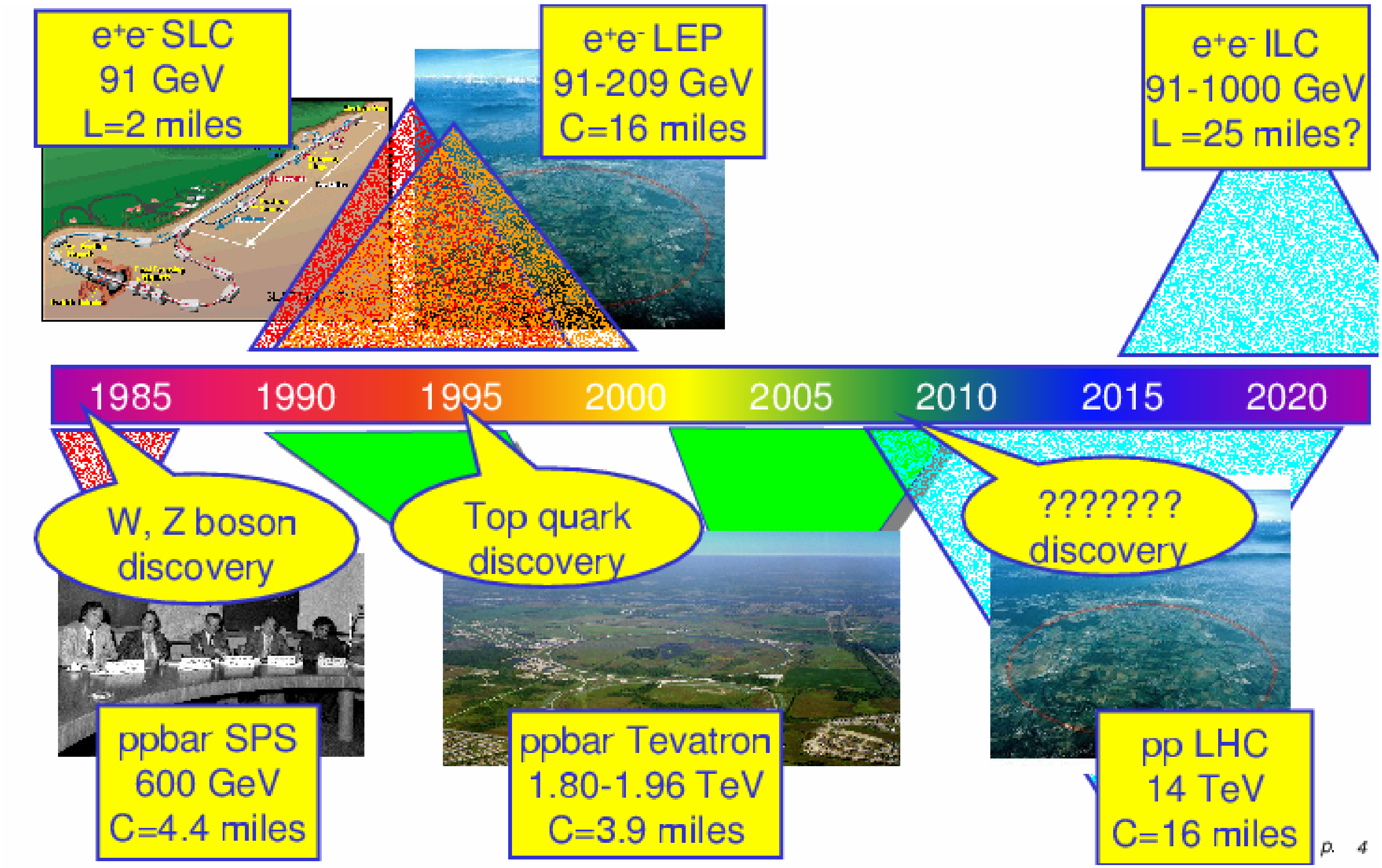
Δ M_{top} = 1.2 GeV



Δ M_W = 0.025 GeV

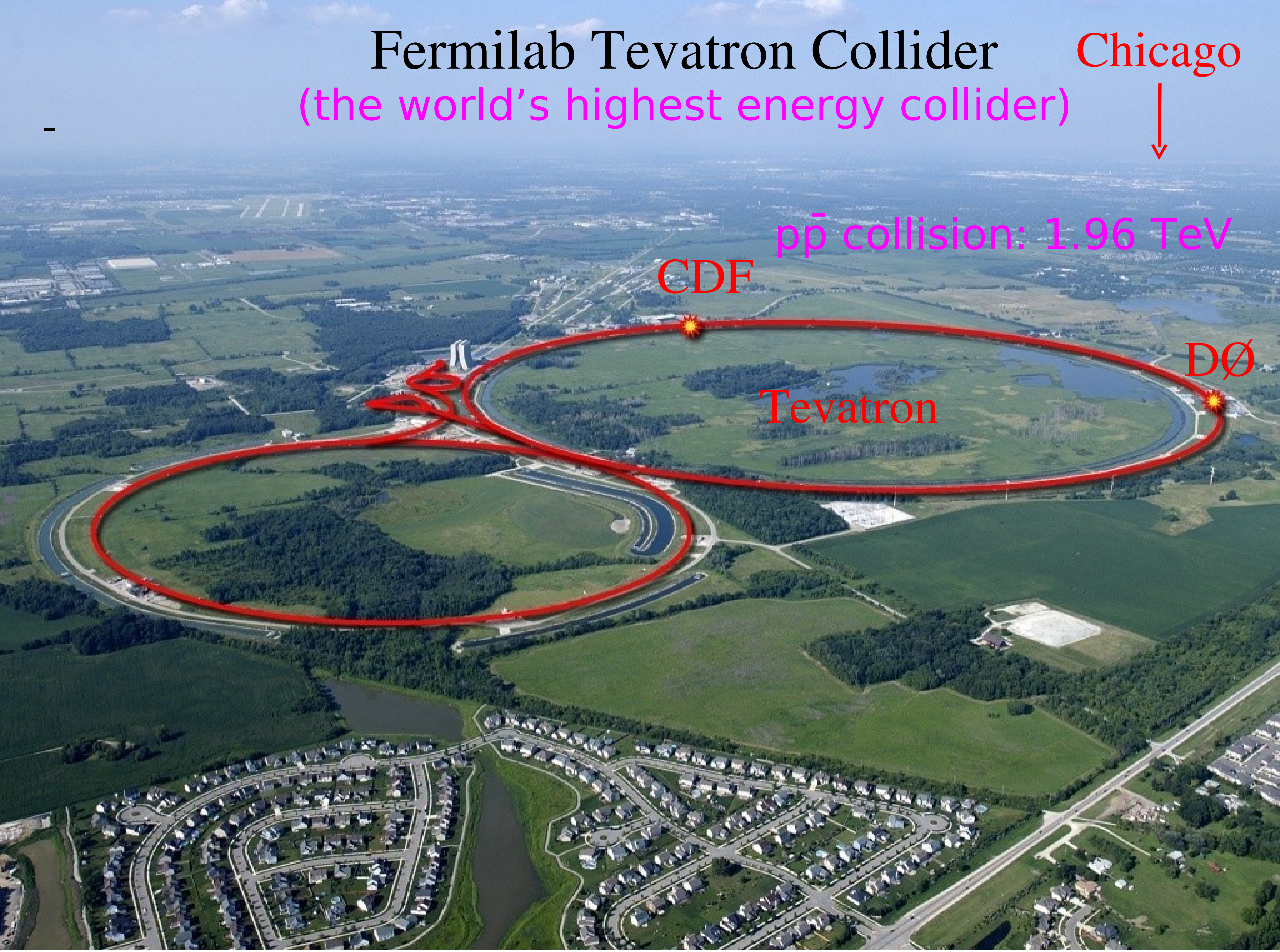
CTEQ

Era of Precision Measurements @ Hadron colliders

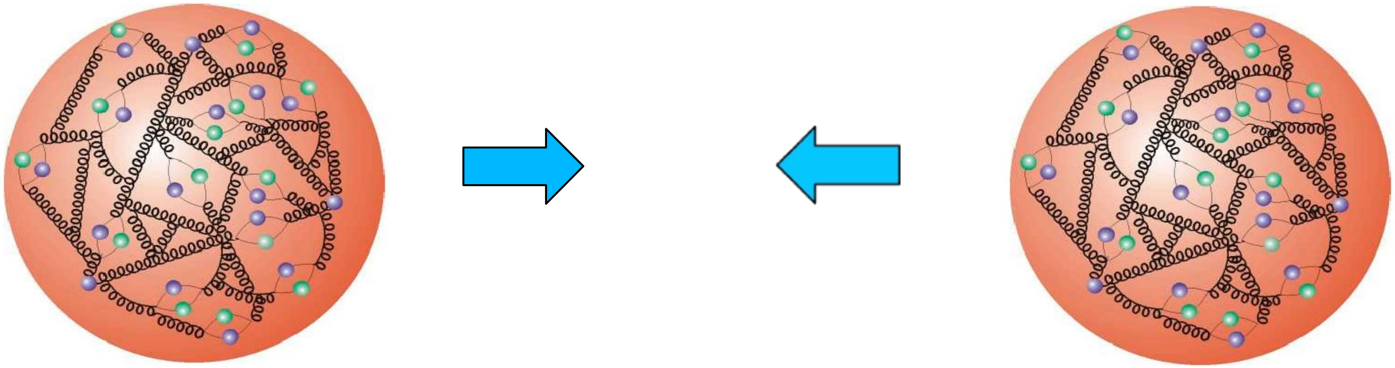


Fermilab Tevatron Collider

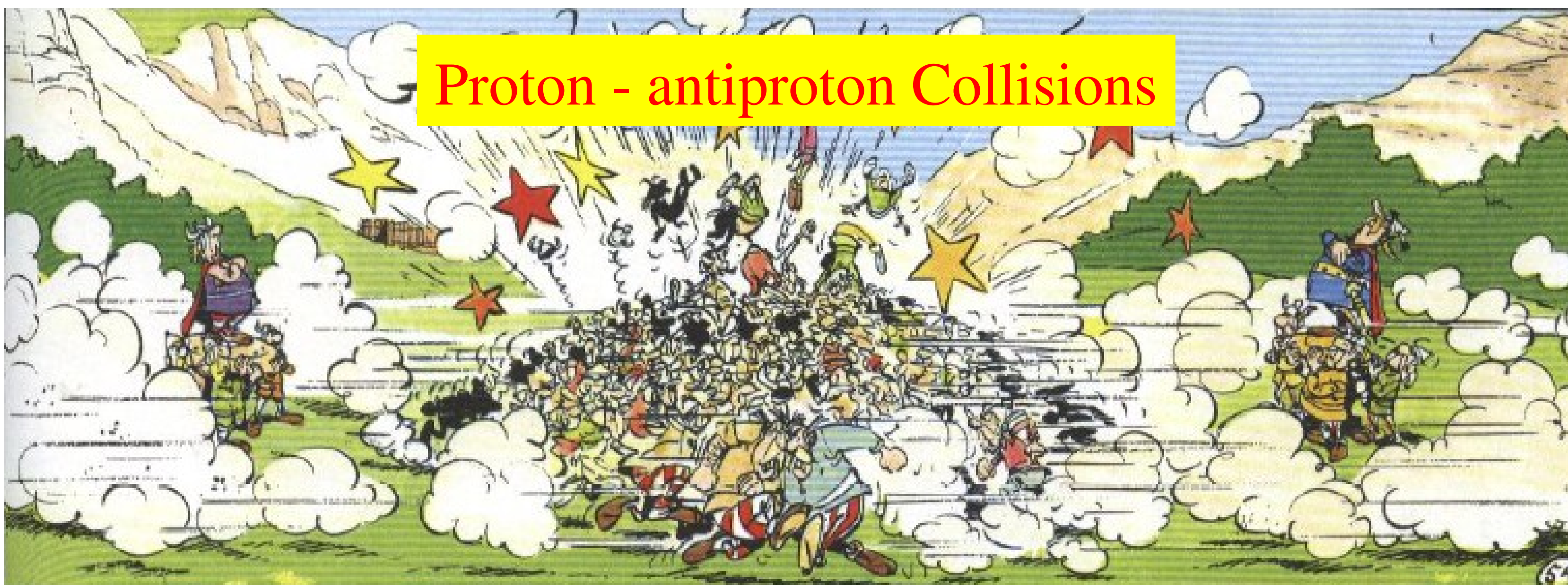
(the world's highest energy collider)



Proton-Antiproton Collisions

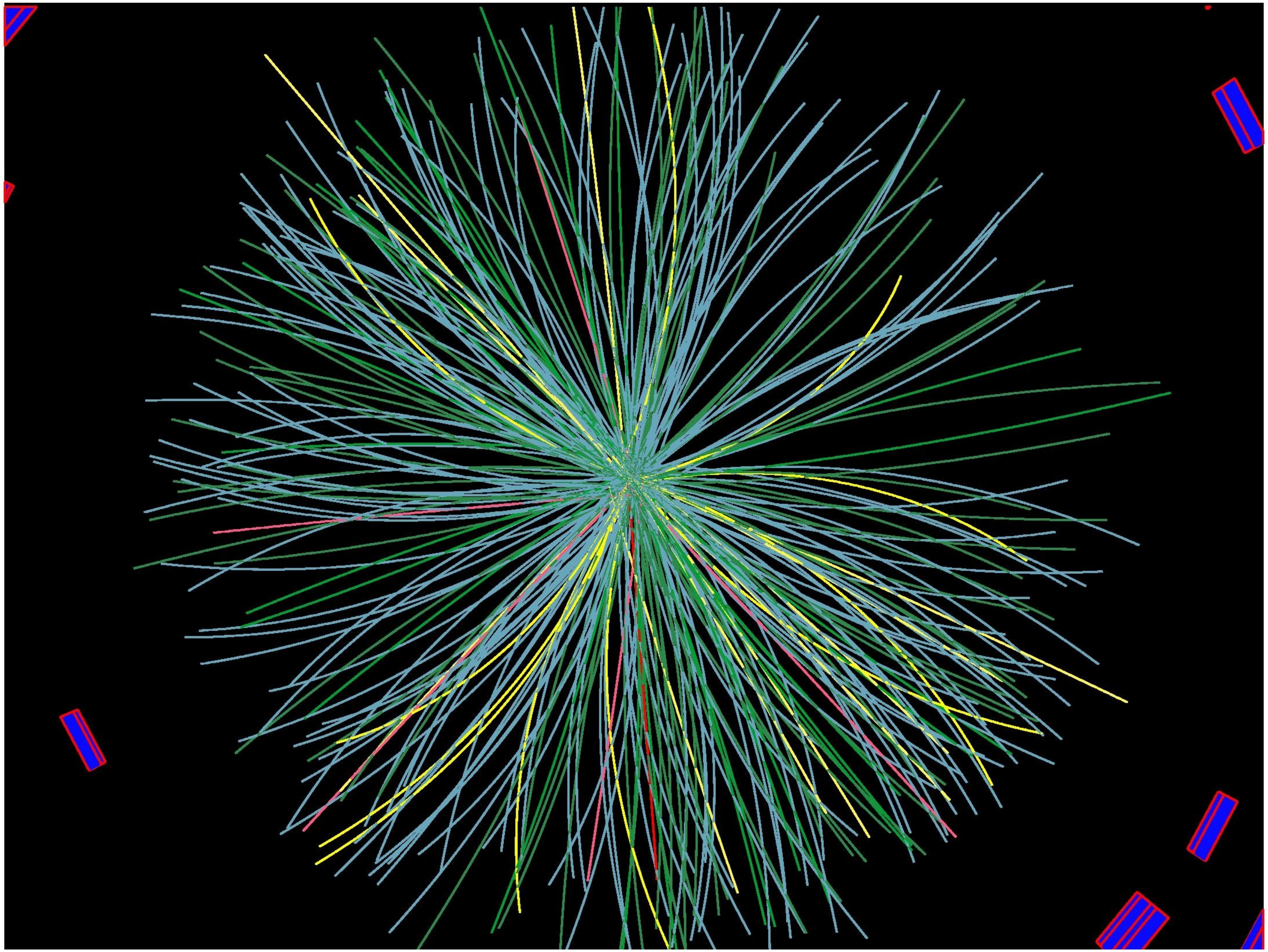


Proton: Bag of quarks and gluons (partons)

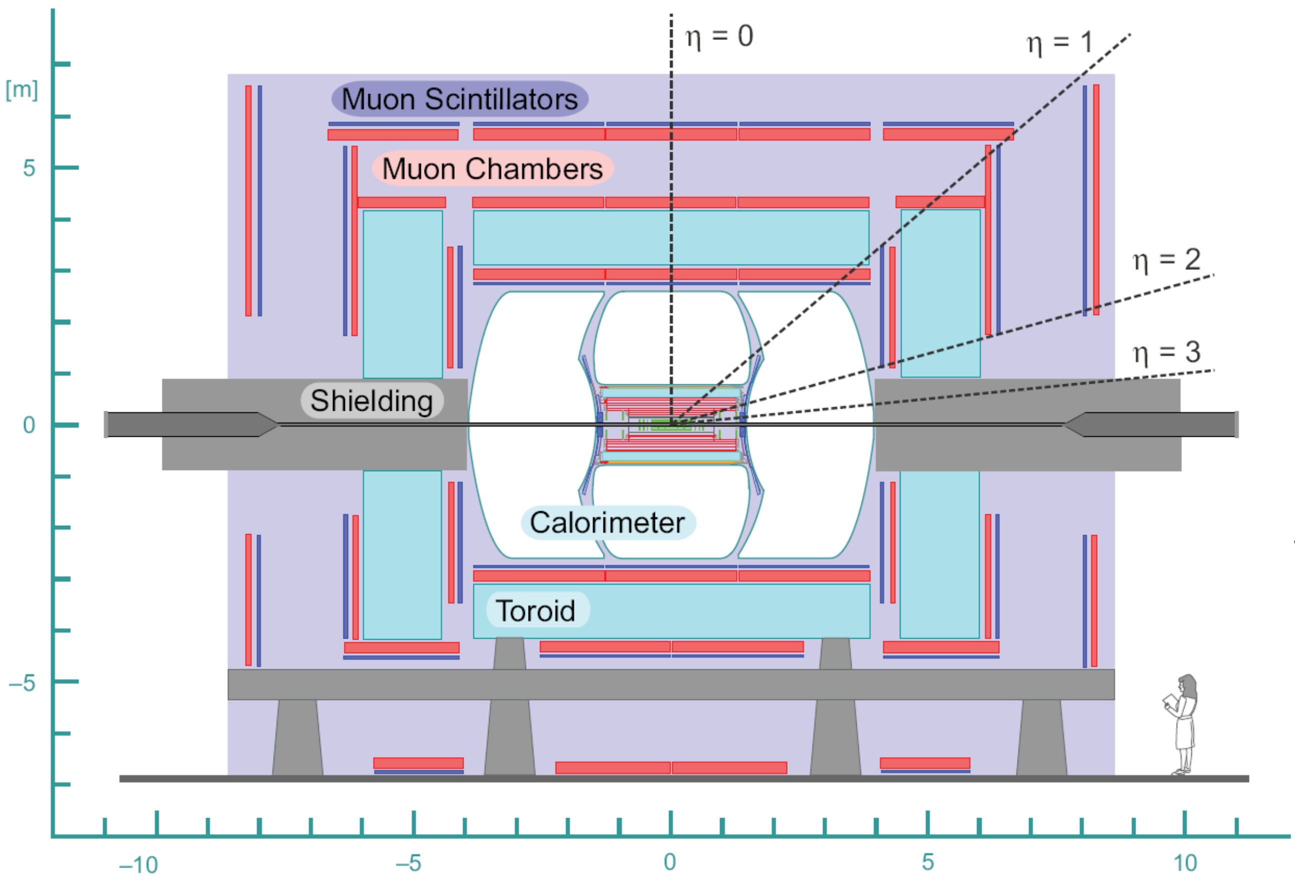


Proton - antiproton Collisions

A Real Collision



DØ Detector



30’ × 30’ × 50’
5,000 tons

550 collaborators
~ 10 tons/person

Electroweak Physics at DØ

EW physics: an excellent laboratory for precision studies! Observed discrepancies with the predictions would indicate new physics!

W/Z boson →

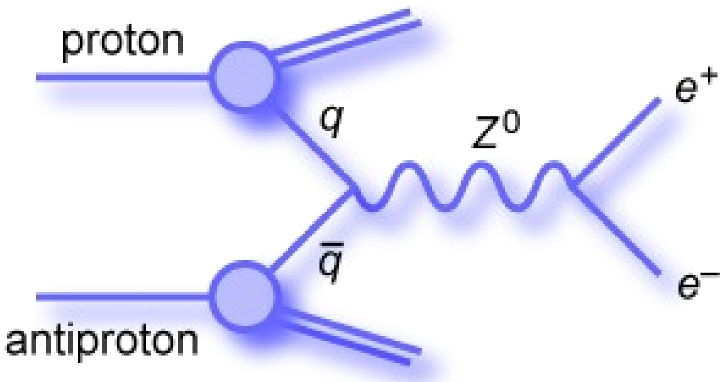
- Inclusive cross sections
- Differential cross sections $d\sigma/dm$, $d\sigma/dp_T$, $d\sigma/dy$ — PRL 100, 102002 (2008)
- W charge asymmetry — PRL 101, 211801 (2008)
- Z Forward-backward Asymmetry — PRL 101, 191801 (2008)
- W boson mass — To be submitted to PRL
- W boson width

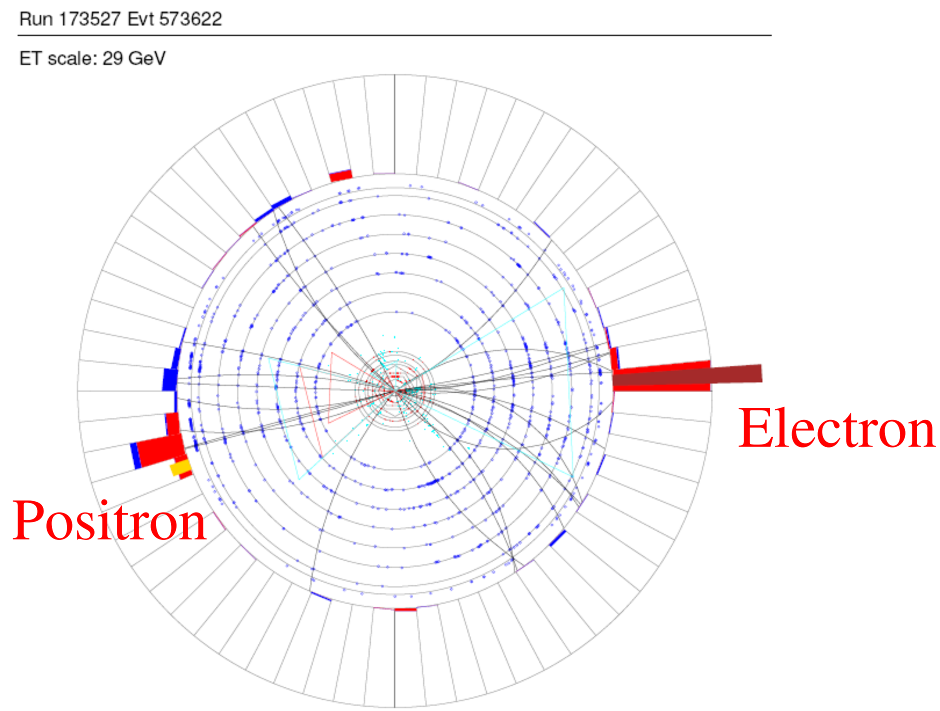
di-boson →

- $W\gamma$, $Z\gamma$, WW, WZ, ZZ

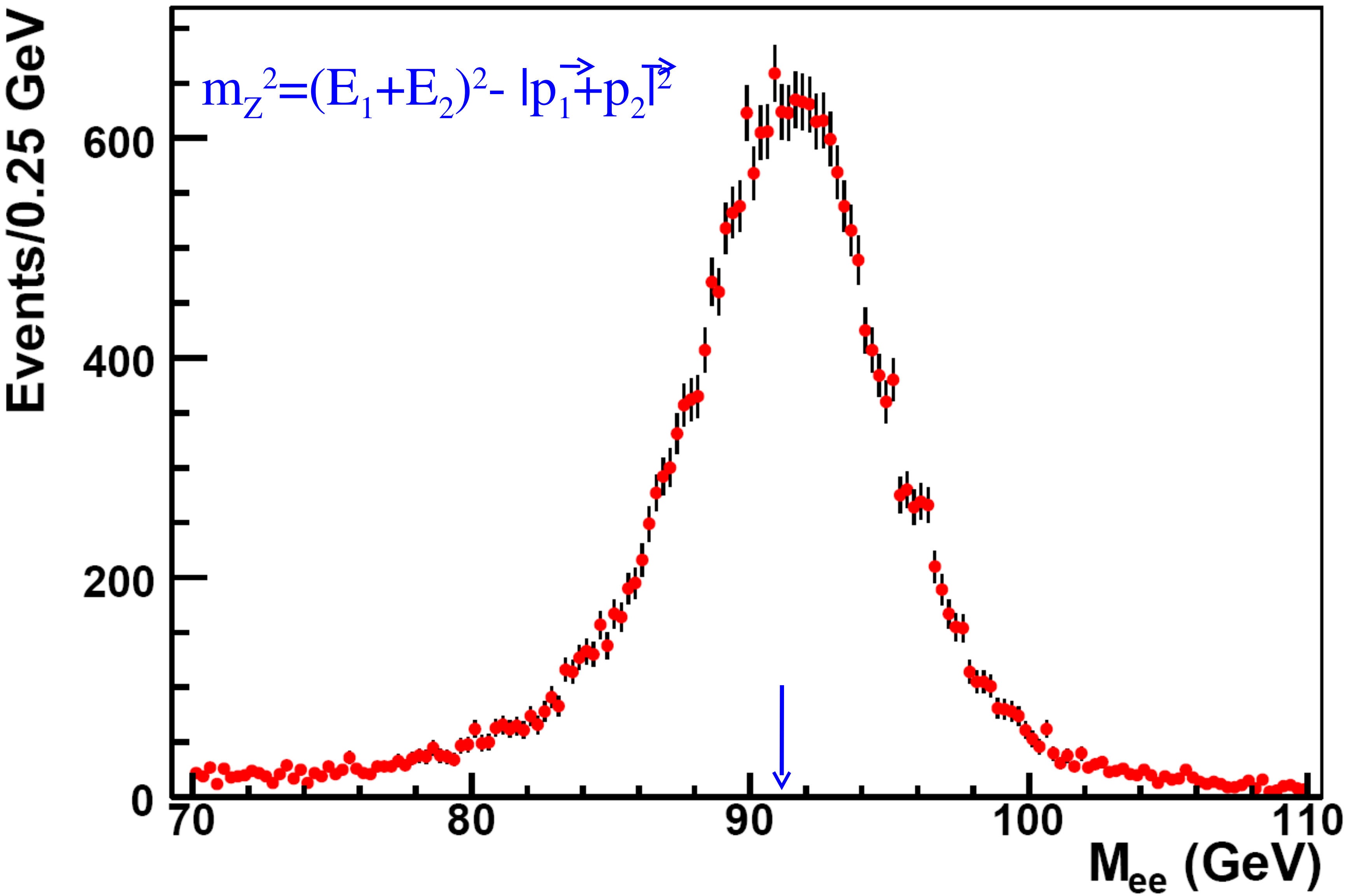
Will only talk about W/Z decays to electron(s)

$Z^0 \rightarrow e^+e^-$





Invariant Mass for $Z^0 \rightarrow e^+e^-$



W-boson physics

❶ W-boson production and decay at hadron collider

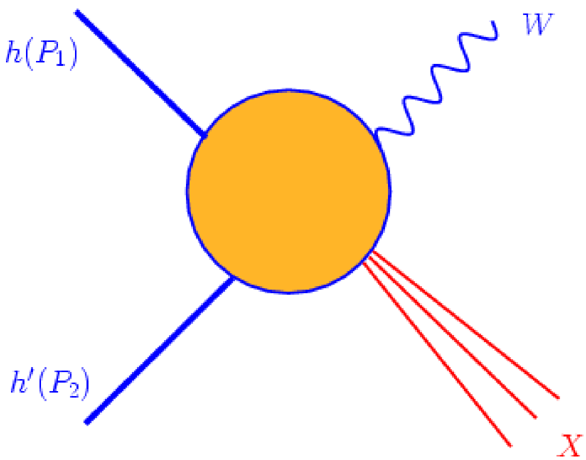
❷ How to measure W-boson mass and width?

❸ High order radiative corrections:

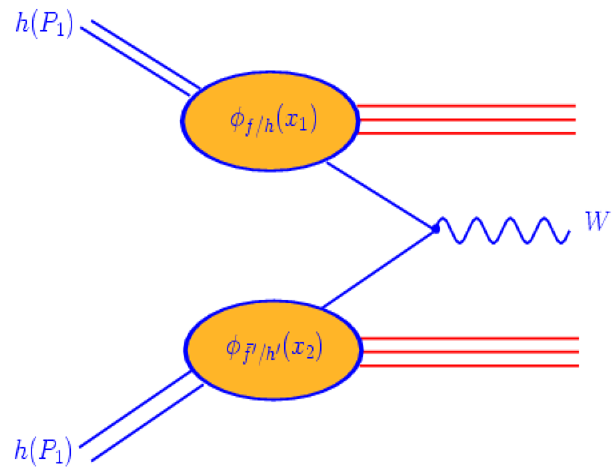
- ☞ QCD (NLO, NNLO, Resummation)
- ☞ EW (QED-like, NLO)

❹ ResBos and ResBos-A

W-boson production at hadron colliders



parton model $\Longrightarrow$

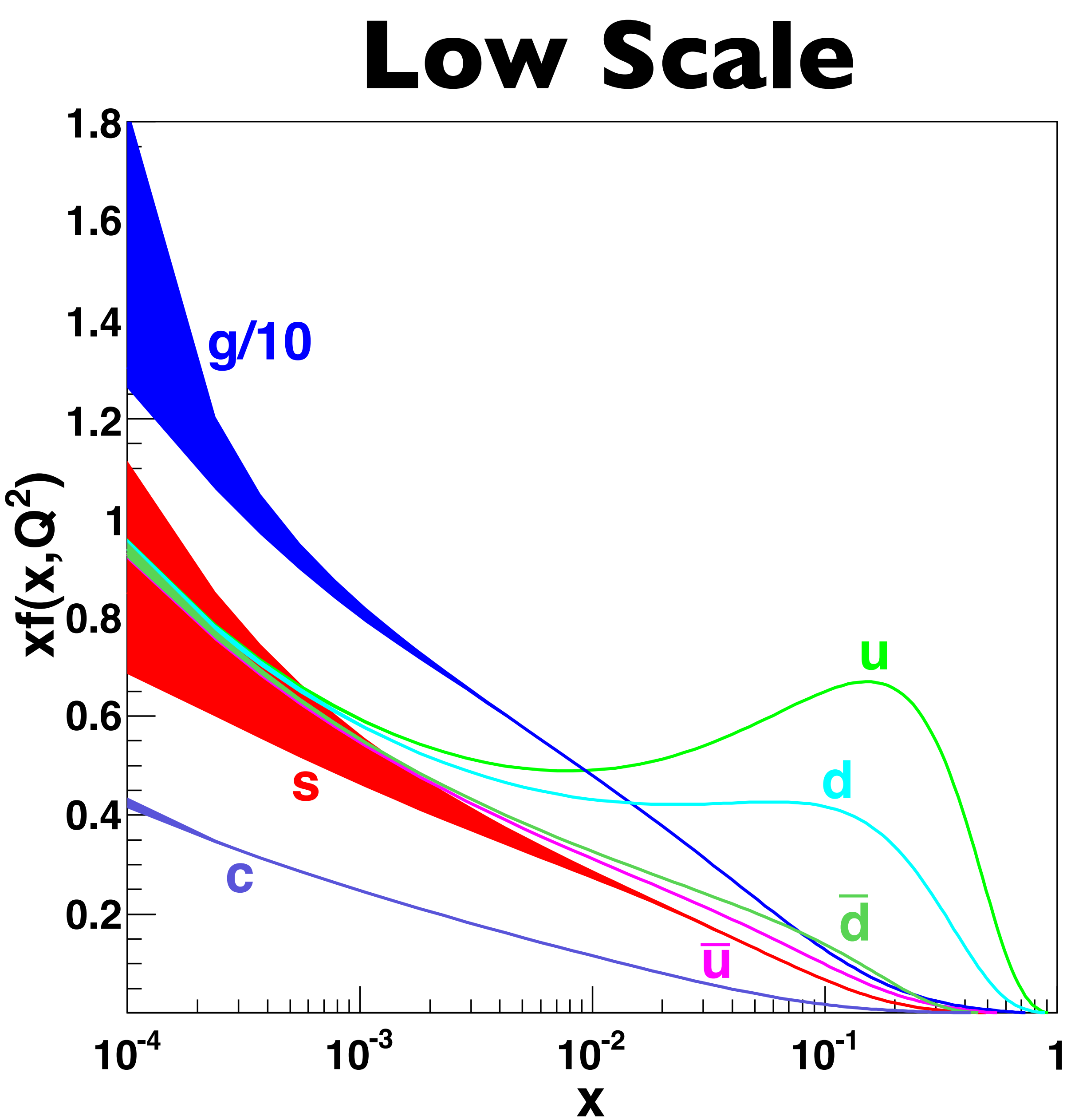


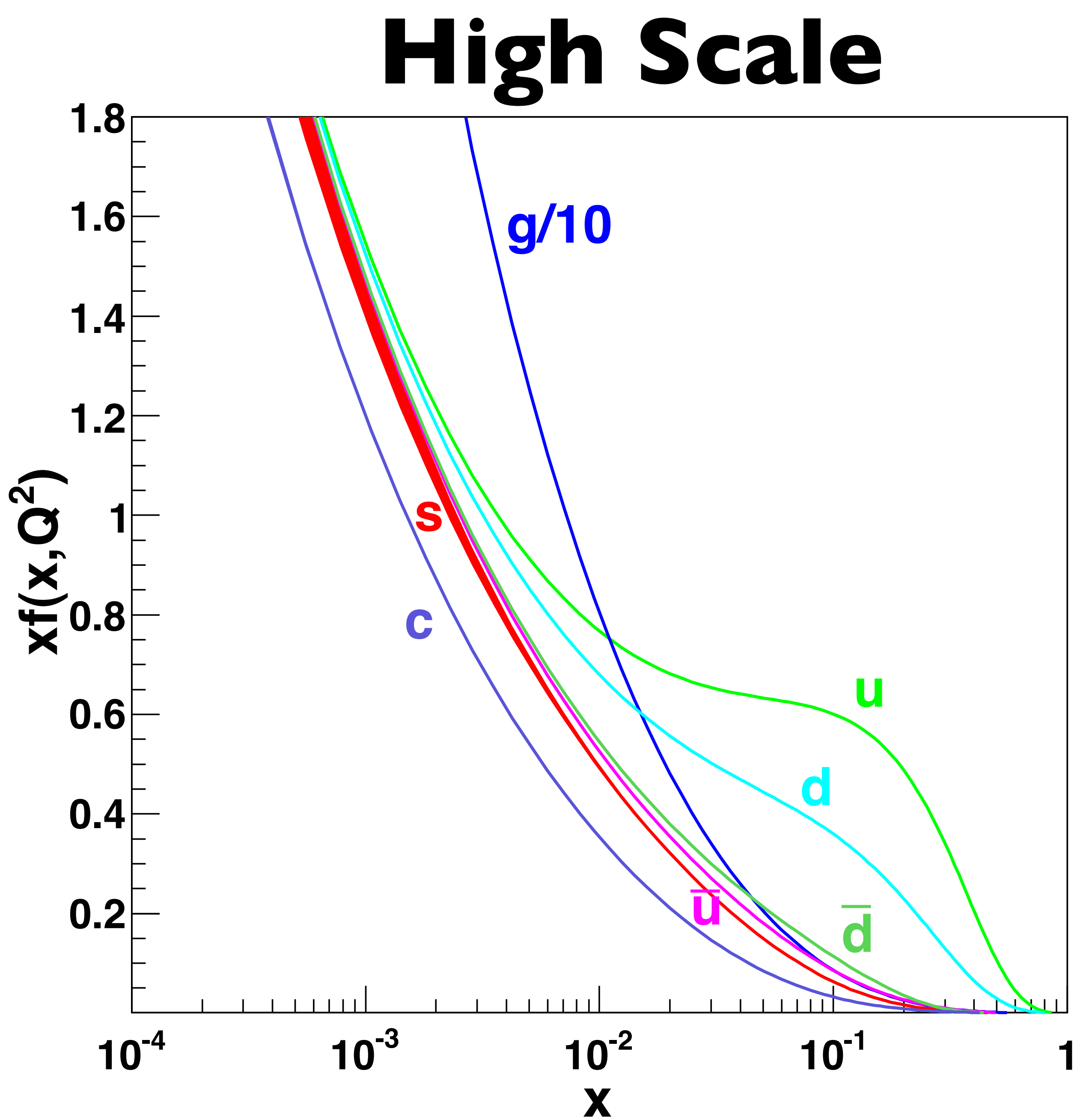
$$\sigma_{hh' \to W+X} = \sum_{f,f'} \int_0^1 dx_1 dx_2 \left\{ \phi_{f/h}(x_1)\, \hat{\sigma}_{ff'}\, \phi_{\bar{f}'/h'}(x_2) + (x_1 \leftrightarrow x_2) \right\}$$

PDFs are known from deep inelastic scattering

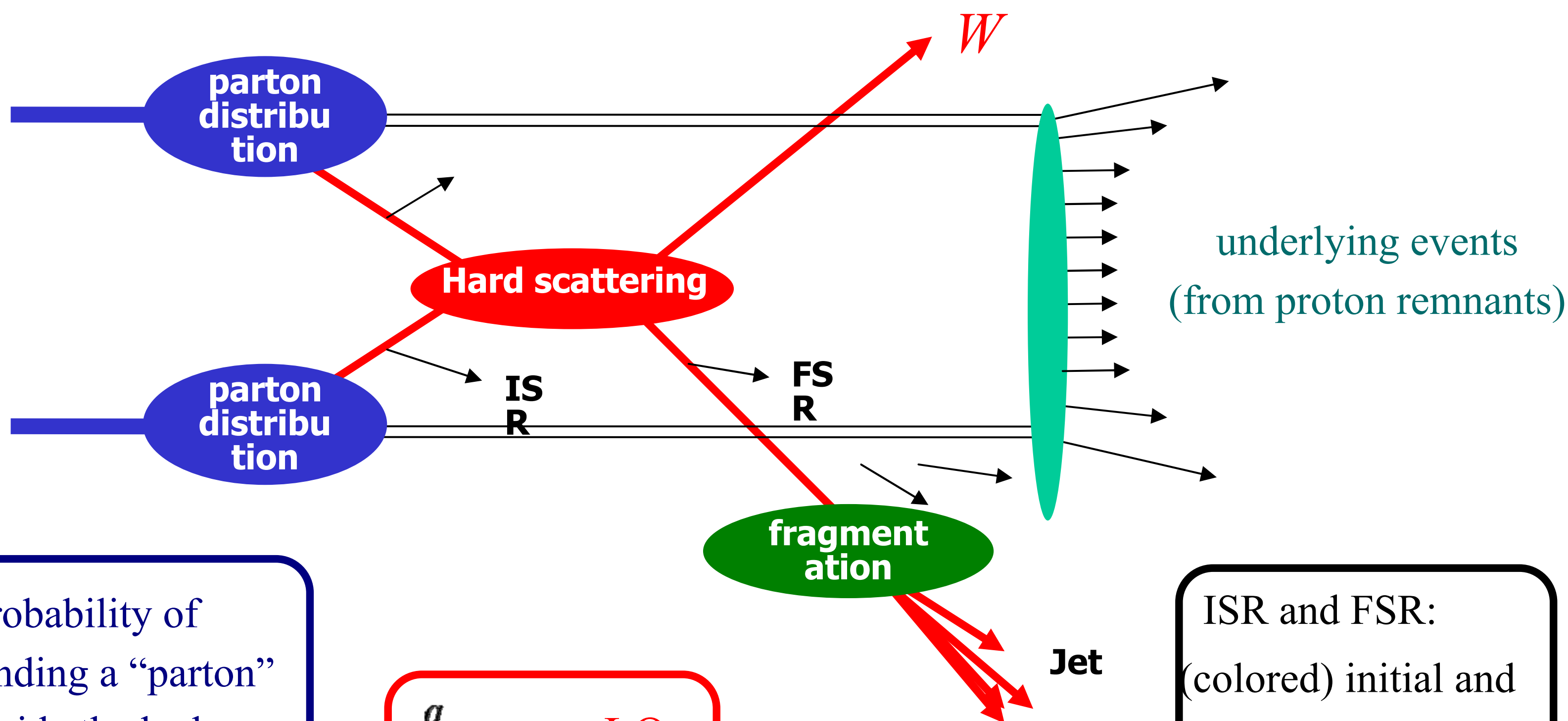
partonic "Born" cross section of $f\bar{f}' \to W^+$

CT10 PDF plots

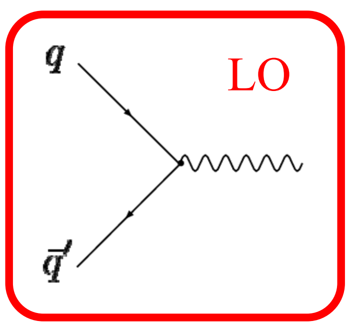




W-boson production at hadron colliders



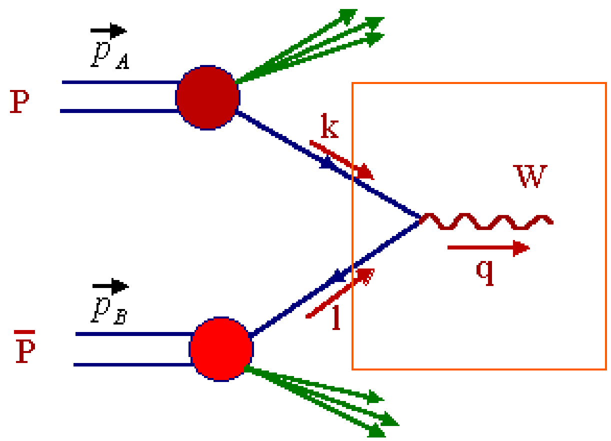
PDFs: probability of finding a "parton" inside the hadron



ISR and FSR: (colored) initial and final states can radiate gluons

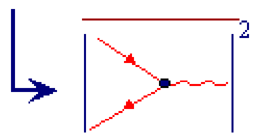
Fixed order pQCD prediction

$\alpha_S^{(0)}$



$$\sigma = \frac{1}{2S}\int \frac{d\xi_A}{\xi_A}\frac{d\xi_B}{\xi_B} f_{i/A}(\xi_A,\mu) f_{i/B}(\xi_B,\mu)\cdot d\hat{\sigma}$$

$$d\hat{\sigma} = \underbrace{\overline{|M|^2}} (2\pi)^4 \delta^{(4)}(q-k-l)\frac{d^3q}{(2\pi)^3 2q_0}$$



$$\frac{d\sigma}{dq_T^2 dy dQ^2} = \frac{1}{S}\int \frac{d\xi_A}{\xi_A}\frac{d\xi_B}{\xi_B} f_{i/A}(\xi_A,\mu) f_{i/B}(\xi_B,\mu)$$

$$\cdot\left(\frac{\pi^2}{Q^2}\right)\cdot\overline{|M|^2}\cdot\delta\left(1-\frac{x_A}{\xi_A}\right)\cdot\delta\left(1-\frac{x_B}{\xi_B}\right)$$

$$\cdot\delta(q_T^2)\cdot\delta(Q^2 - M_W^2)$$

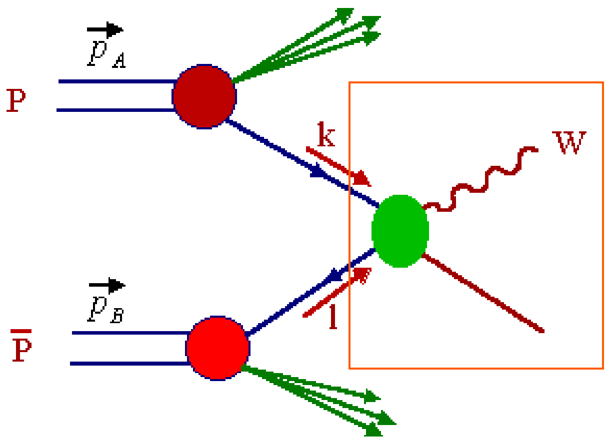
$$s = (p_A + p_B)^2$$

$$k = \xi_A p_A$$

$$l = \xi_B p_B$$

$$Q \equiv \sqrt{Q^2} = \sqrt{q^2},\ \mu = Q = M_W,\ x_A = \frac{Q}{\sqrt{S}}e^{y},\ x_B = \frac{Q}{\sqrt{S}}e^{-y}$$

$\alpha_S^{(1)}$

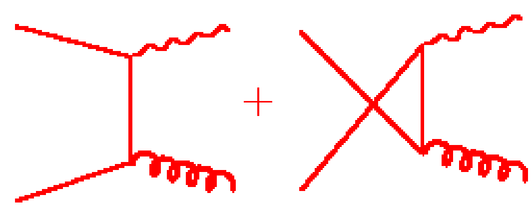


$$\frac{d\sigma}{dq_T^2 dy dQ^2} = \int \frac{d\xi_A}{\left(\xi_A S + U - Q^2\right)} \left(\frac{\hat{s} d\hat{\sigma}}{d\hat{t}}\right) \cdot f_{i/A}(\xi_A, \mu) \cdot f_{j/B}\left(\xi_B = \frac{-Q^2 - \xi_A\left(T - Q^2\right)}{\xi_A S + U - Q^2}, \mu\right) \cdot \delta\left(Q^2 - M_W^2\right)$$
$$+ \int \frac{d\xi_B}{\left(\xi_B S + T - Q^2\right)} \left(\frac{\hat{s} d\hat{\sigma}}{d\hat{t}}\right) \cdot f_{j/B}(\xi_B, \mu) \cdot f_{i/A}\left(\xi_A = \frac{-Q^2 - \xi_B\left(U - Q^2\right)}{\xi_B S + T - Q^2}, \mu\right) \cdot \delta\left(Q^2 - M_W^2\right)$$

$$T = Q^2 - \sqrt{q_T^2 + Q^2}\sqrt{S}\, e^{-y},$$
$$U = Q^2 - \sqrt{q_T^2 + Q^2}\sqrt{S}\, e^{y},$$

$$\hat{s} = \xi_A \xi_B S$$
$$\hat{t} = \xi_A\left(T - Q^2\right) + Q^2$$

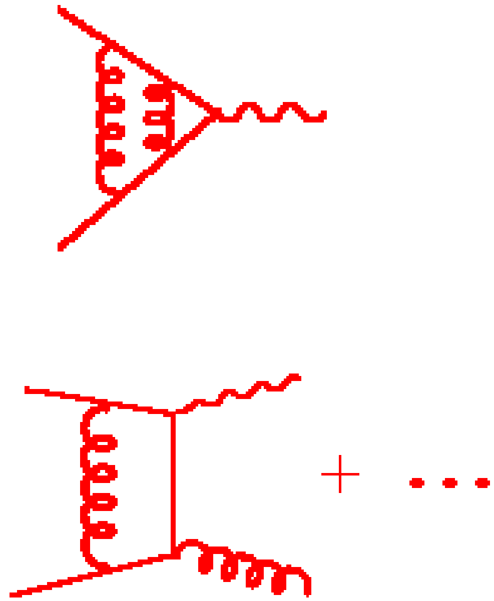
$$\frac{\hat{s}\, d\hat{\sigma}}{d\hat{t}} = \frac{1}{16\pi^2}\overline{|M|^2}$$

$M =$ 

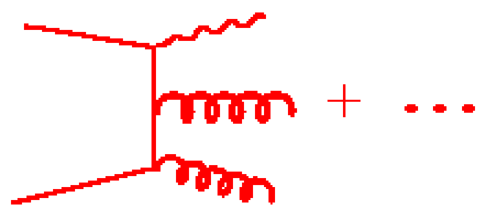
(For simplicity, only consider qq→Wg)

$\alpha_S^{(2)}$

- Virtual Corrections



- Real emission contributions



Theory Calculations

There are a variety of programs available for comparison of data to theory and/or predictions.

- Tree level (Alpgen, CompHEP, Grace, Madgraph…)

  ↓ Les Houches accord

- Parton shower Monte Carlos (Herwig, Pythia,…

  ↑ MC@NLO

- N^nLO (EKS, Jetrad, Dyrad, Wgrad, Zgrad,..Horace

  ↓ recover NLO (NNLO?) normalization

- Resummed (ResBos)

Important to know strengths/weaknesses of each.

NLO Electroweak Calculations

- $\mathcal{O}(\alpha)$ QED corrections to W/Z lepton decays
  F.A. Berends *et al.* Z. Physik **C27** (1985) 155,365
- Electroweak corrections to W production
  - ⋆ Pole approximation ($\sqrt{\hat{s}} = M_W$)
    D. Wackeroth and W. Hollik, Phys. Rev. **D55** (1997) 6788
    U. Baur, S. Keller, D. Wackeroth, Phys. Rev. **D59** (1999) 013002 WGRAD
  - ⋆ Complete $\mathcal{O}(\alpha)$ corrections
    V.A. Zykunov, Eur. P. J. **C3** (2001) 9, Phys. Atom. Nucl. **69** (2006) 1522
    S. Dittmaier and M. Krämer, Phys. Rev. **D65** (2002) 073007 DK
    U. Baur and D. Wackeroth, Phys. Rev. **D70** (2004) 073015 WGRAD2
    A. Arbuzov *et al.*, Eur. Phys. J. **C46** (2006) 407 SANC
    C.M. Carloni Calame *et al.*, JHEP **12** (2006) 016 HORACE
- Electroweak corrections to Z production
  - ⋆ $\mathcal{O}(\alpha)$ photonic corrections
    U. Baur, S. Keller, W.K. Sakumoto, Phys. Rev. **D57** (1998) 199 ZGRAD
  - ⋆ Complete $\mathcal{O}(\alpha)$ corrections
    U. Baur *et al.*, Phys. Rev. **D65** (2002) 033007 ZGRAD2

Higher Order QCD Corrections

- NLO/NNLO corrections to W/Z total production rate

  G. Altarelli, R.K. Ellis, M. Greco and G. Martinelli, Nucl. Phys. **B246** (1984) 12

  R. Hamberg, W.L. van Neerven, T. Matsuura, Nucl. Phys. **B359** (1991) 343

- NLO calculations for $W, Z + 1, 2$ jets (DYRAD, MCFM ...)

  W.T. Giele, E.W.N. Glover and D.A. Kosower, Nucl. Phys. **B403** (1993) 633

  J.M. Campbell and R.K. Ellis, Phys. Rev. **D65** (2002) 113007

- resummation of leading/next-to-leading $p_\perp^W/M_W$ logs (ResBos)

  C. Balazs and C.P. Yuan, Phys. Rev. **D56** (1997) 5558

- NLO corrections merged with HERWIG Parton Shower (MC@NLO)

  S. Frixione and B.R. Webber, JHEP **0206** (2002) 029

- Multi-parton matrix elements Monte Carlos (ALPGEN, SHERPA...) matched with vetoed Parton Showers

  M.L. Mangano *et al.* , JHEP **0307** (2003) 001

  F. Krauss *et al.*, JHEP **0507** (2005) 018

- fully differential NNLO corrections to W/Z production (FEWZ)

  C. Anastasiou *et al.* , Phys. Rev. **D69** (2004) 094008

  K. Melnikov and F. Petriello, Phys. Rev. Lett. **96** (2006) 231803, Phys. Rev. **D74** (2006) 114017

Multiple Photon Emissions

- Higher-order (real+virtual) QED corrections to W/Z production
  - → HORACE (Pavia): QED Parton Shower + NLO electroweak corrections to W/Z production (Z production available soon)

    C.M. Carloni Calame *et al.*, Phys. Rev. **D69** (2004) 037301
    C.M. Carloni Calame *et al.*, JHEP **05** (2005) 019; JHEP **12** (2006) 016
  - → WINHAC (Cracow): YFS exponentiation + electroweak corrections to W decay

    S. Jadach and W. Placzek, Eur. Phys. J. **C29** (2003) 325
- Perfect agreement between HORACE and WINHAC on multiphoton corrections to all W observables

  C.M. Carloni Calame *et al.*, Acta Phys. Pol. **B35** (2004) 1643
- Recent effort to improve the treatment of multiphoton radiation in HERWIG (with SOPHTY via YFS) and PHOTOS (via QED Parton Shower)

  K. Hamilton and P. Richardson, JHEP **0607** (2006) 010
  P. Golonka and Z. Was, Eur. Phys. J. **C45** (2006) 97

⋆ W-mass shift due to multiphoton radiation is about 10% of that caused by one photon emission ⟶ non-negligible for precision W mass measurements!

C.M. Carloni Calame *et al.*, Phys. Rev. **D69** (2004) 037301

Combine QCD and Electroweak

- First attempt: combination of soft-gluon resummation with NLO final-state QED corrections

Q.-H. Cao and C.-P. Yuan, Phys. Rev. Lett. **93** (2004) 042001
ResBos-A

- Electroweak and QCD corrections can be combined in factorized form to arrive at

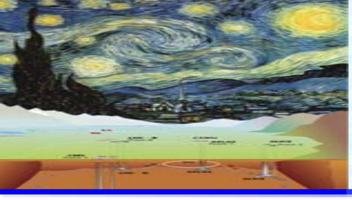
$$\left[\frac{d\sigma}{d\mathcal{O}}\right]_{\text{QCD}\otimes\text{EW}} = \left\{\frac{d\sigma}{d\mathcal{O}}\right\}_{\text{QCD}} + \left\{\left[\frac{d\sigma}{d\mathcal{O}}\right]_{\text{EW}} - \left[\frac{d\sigma}{d\mathcal{O}}\right]_{\text{LO}}\right\}_{\texttt{HERWIG}\text{ PS}}$$

- QCD $\Rightarrow$ ResBos, MC@NLO, ALPGEN (with CKKW-MLM Parton Shower matching and standard matching parameters), FEWZ, ...
- EW $\Rightarrow$ Electroweak + multiphoton corrections from HORACE convoluted with HERWIG QCD Parton Shower
  - ⋆ NLO electroweak corrections are interfaced to QCD Parton Shower evolution $\Rightarrow$ $\mathcal{O}(\alpha\alpha_s)$ corrections not reliable when hard non-collinear QCD radiation is important
  - ⋆ Beyond this approximation, a full two-loop $\mathcal{O}(\alpha\alpha_s)$ calculation is needed (unavailable yet)

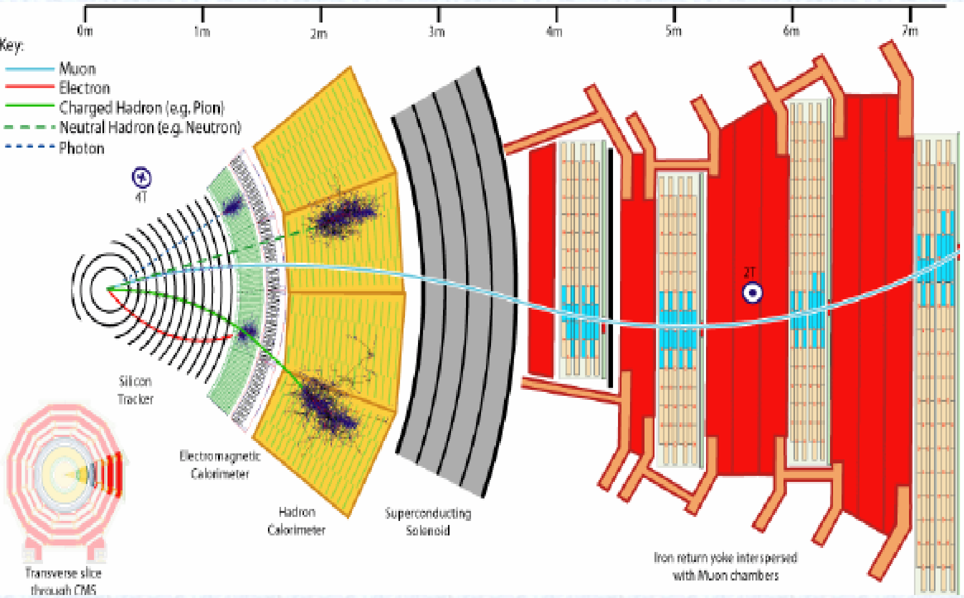
J.H. Kühn *et al.*, hep-ph/0703283
NLO/NNLO$_{\text{EW}}$ to $pp \to Wj$

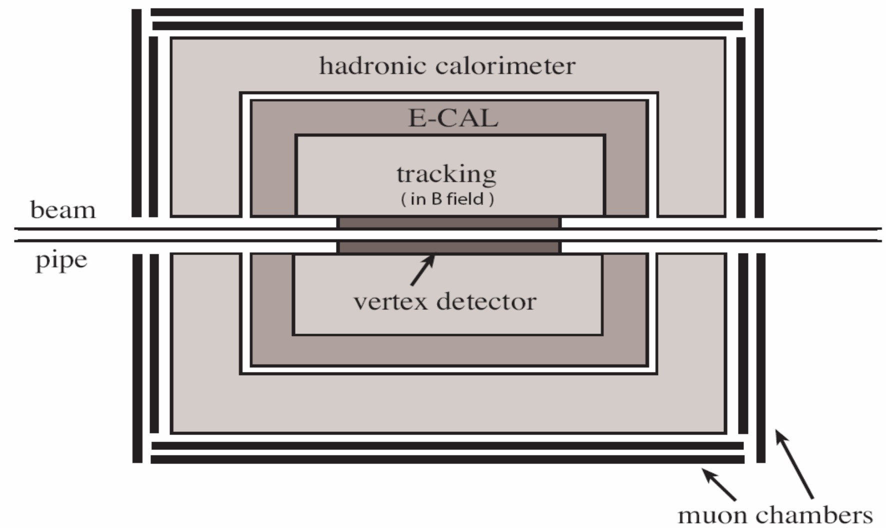
Fixed order Perturbative calculations

- Higher order in $\alpha_S^{(n)}$
  ⇒ Less sensitive to Factorization Scale μ
- High q_T and smaller y (i.e. more central)
  ⇒ PDF (parton distribution function) better known
- With larger Luminosity
  ⇒ Test QCD in one large scale problem (i.e. $q_T \sim Q$)
- Up to now, most of the Data used in Testing QCD were
  One large scale observables, e.g., Jet-P_T.
- Observables involving Multiple Scales, e.g., q_T of W-Boson with mass M_W, can only be accurately described in QCD after including effects of Resummation.



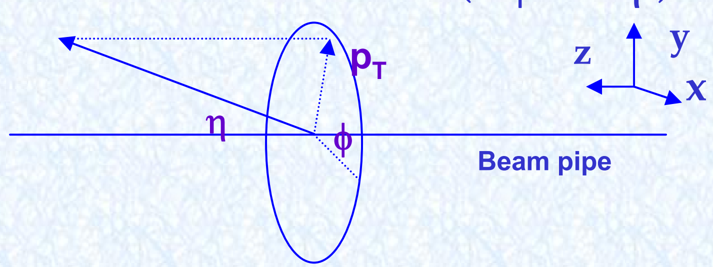
Objects at the LHC





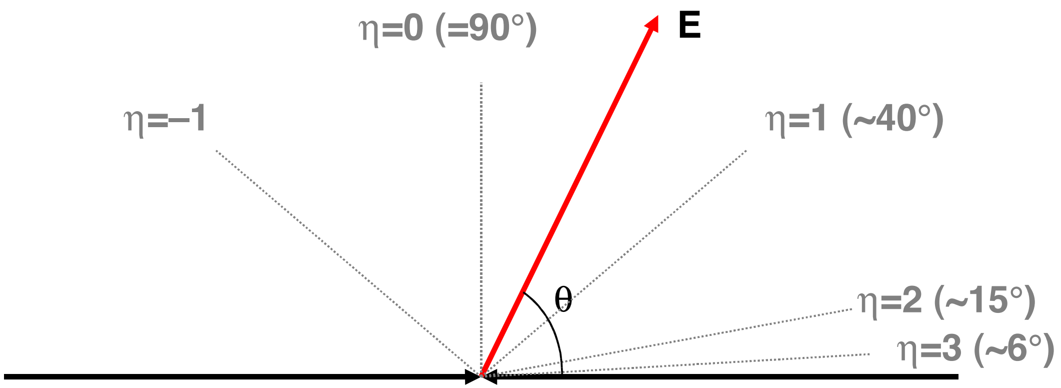
- **objects**
  - Photons: no tack, energy in ECAL, no energy in HCAL
  - Electrons: tack, energy in ECAL, no energy in HCAL
  - Muons: tack, tack in the muon chamber
  - Jets: tracks and energy in the calorimeter
  - Missing transverse enegy (MET) : inferred from the conservation of momentum in a plane perpendicular to the beam direction

- **Typical variables**
  - Transverse momentum: p_T
  - Azimuth angle: ϕ
  - Pseudorapidity: $\eta = -\ln(tg(\theta/2))$
  - Relative isolation: $\Delta R = (\Delta \phi^2 + \Delta \eta^2)^{1/2}$



Hadron Collider Variables

- The incoming parton momenta x_1 and x_2 are unknown, and usually the beam particle remnants escape down the beam pipe
  - ☞ Longitudinal motion of the centre of mass cannot be reconstructed



- Focus on transverse variables (invariant under boost along z-axis)

  Transverse Energy $E_T = E \sin\theta$ (= p_T , if mass = 0)

- and longitudinally boost-additive quantities (along z-axis)
  - ☞ Pseudorapidity $\eta = -\log(\tan\theta/2)$ (= rapidity y, if mass = 0)

    $$\eta(\text{lab}) = \eta^* + \eta(\text{cm})$$

  - ☞ Particle production typically scales per unit rapidity

(p_T : transverse momentum)

C T E Q

Theory requirements for Tevatron Run-II and LHC:

- Theory framework for Tevatron Run-I
  - $O(\alpha_S)$ (NLO-QCD) corrections
  - $O(\alpha)$ (QED) corrections

  } Adequate for comparison to Run-I data

- Run-II experimental targets:

$$\delta\sigma_{tot}/\sigma_{tot} \sim 2 - 3\%$$

$$\delta M_W \sim 30 \text{ MeV}$$

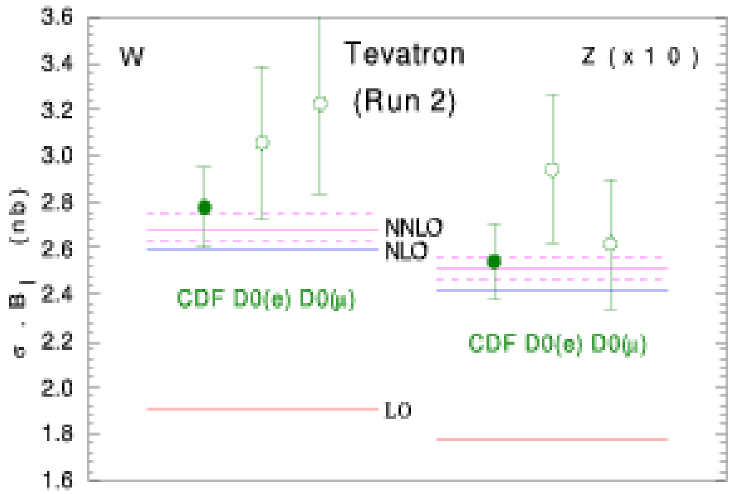
- Many factors contribute at a percent level:
  - $O(\alpha_s^2)$ (NNLO-QCD) corrections
  - $O(\alpha)$ (NLO-EW) corrections
  - uncertainties of parton distributions
  - power corrections to resummed cross sections

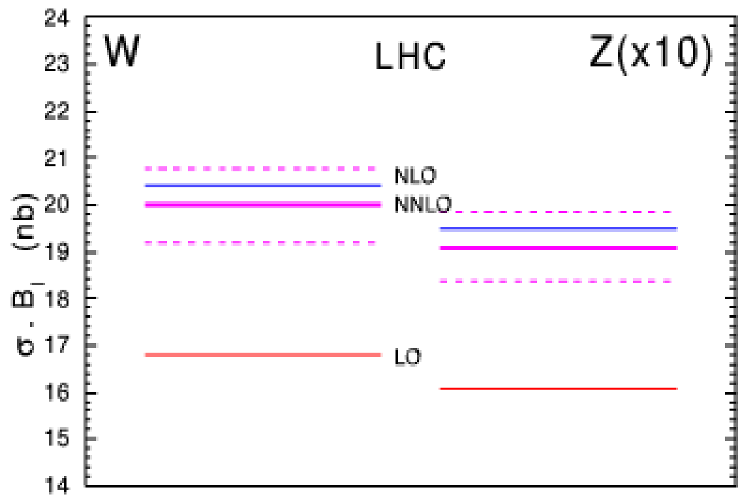
  } Task: consistent and efficient implementation of these effects

C T E Q

NNLO QCD corrections to the cross section

- NNLO hard cross section





☞ NNLO *K*-factor : 1.04 at Tevatron
0.98 at LHC

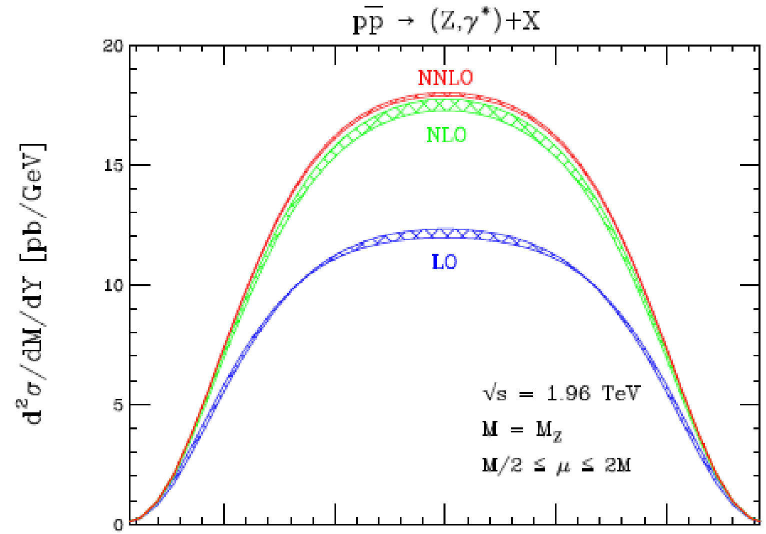
MRST'03

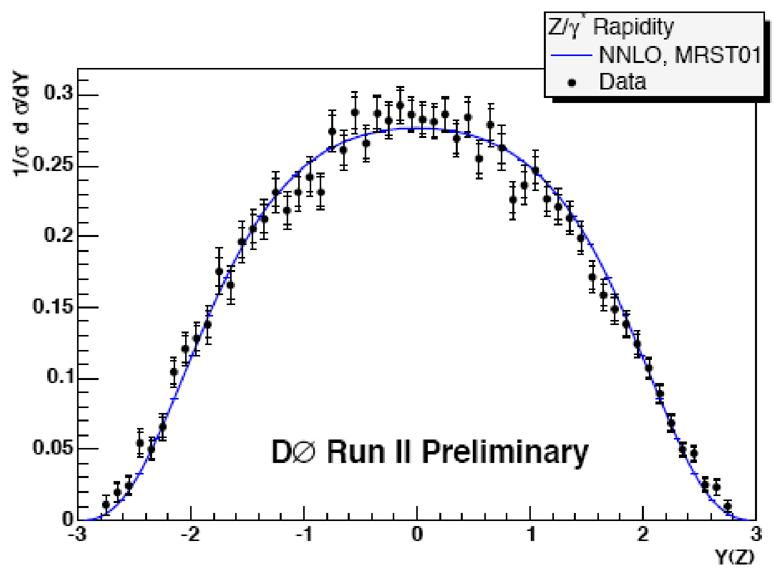
☞ scale dependence of order 1%

C T E Q

Rapidity distributions

- Little shape difference from NLO to NNLO
  - K-factor should be sufficient
- Z rapidity distributions could/will be used as input for pdf fits

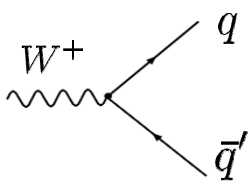




How to Measure the Mass of W Boson

W-boson Decay

- Hadronic mode

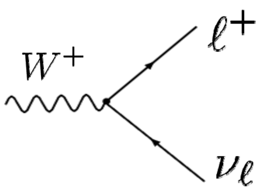


$$Br \sim \frac{6}{9}$$

$$\begin{pmatrix} u & u & u & c & c & c \\ \bar{d} & \bar{d} & \bar{d} & \bar{s} & \bar{s} & \bar{s} \end{pmatrix}$$

hard to detect,
due to huge QCD backgrounds

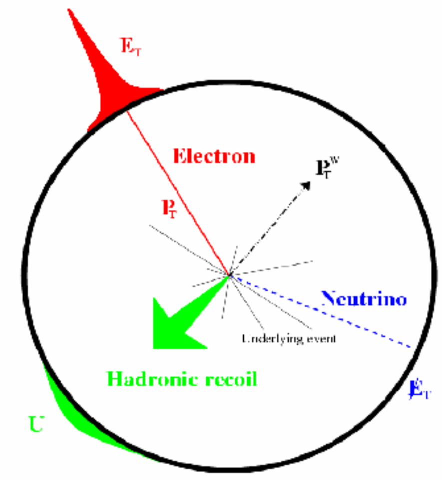
- Leptonic mode



$$Br \sim \frac{3}{9}$$

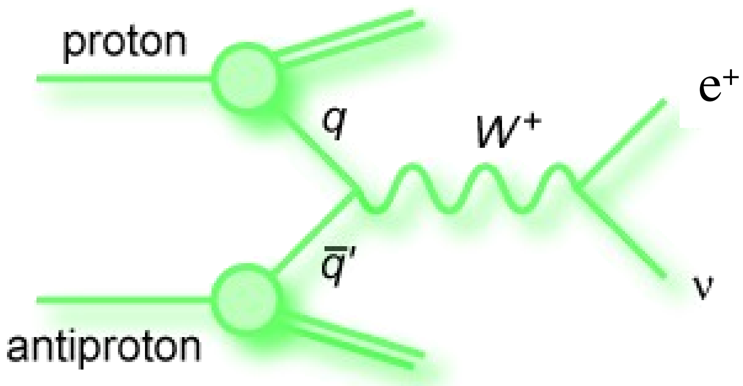
$$\begin{pmatrix} e^+ & \mu^+ & \tau^+ \\ \nu_e & \nu_\mu & \nu_\tau \end{pmatrix}$$

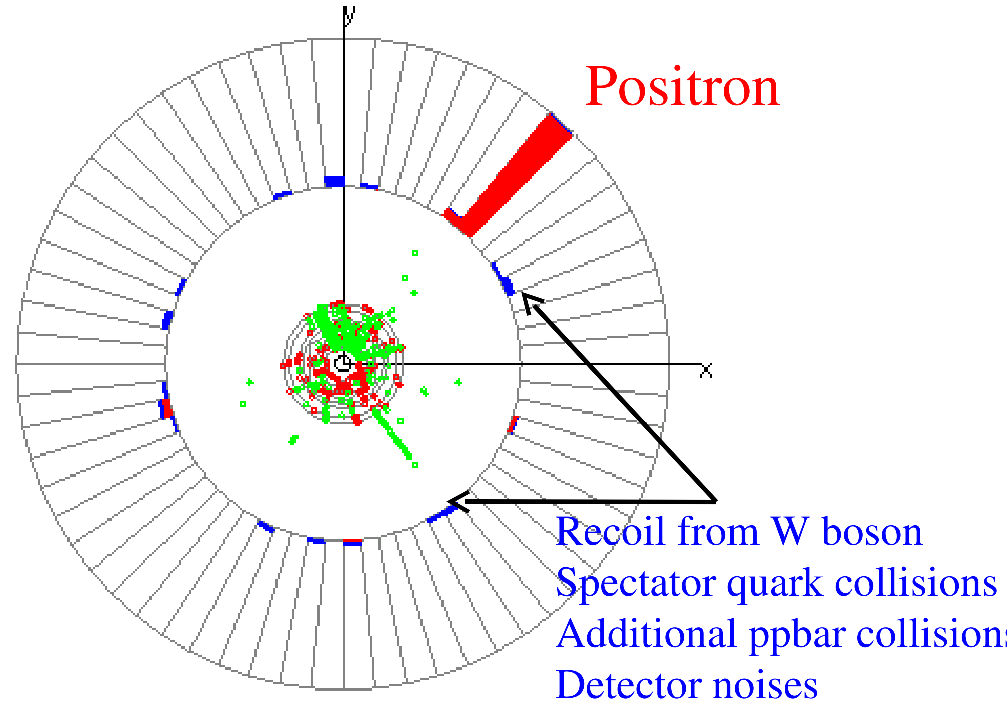
easy to identify,
but lack of p_z^ν



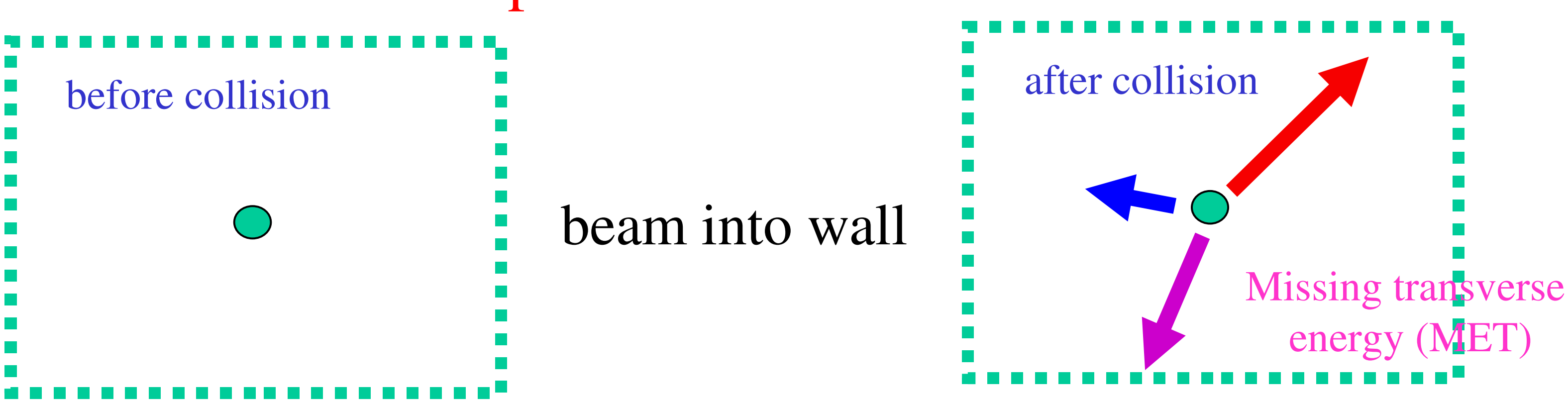
unknown p_z^ν ⇒ cannot reconstruct invariant mass of W boson

$W^+ \rightarrow e^+ \nu_e$



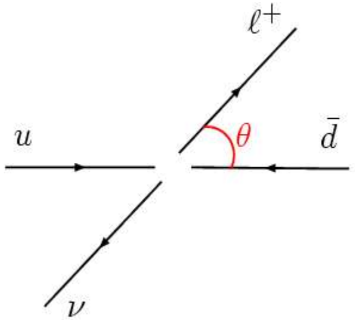


Sum of the momenta in the direction transverse to the beam must add up to zero



Transverse momentum of the charged lepton (p_T^e)

- In (ud) c.m. system,

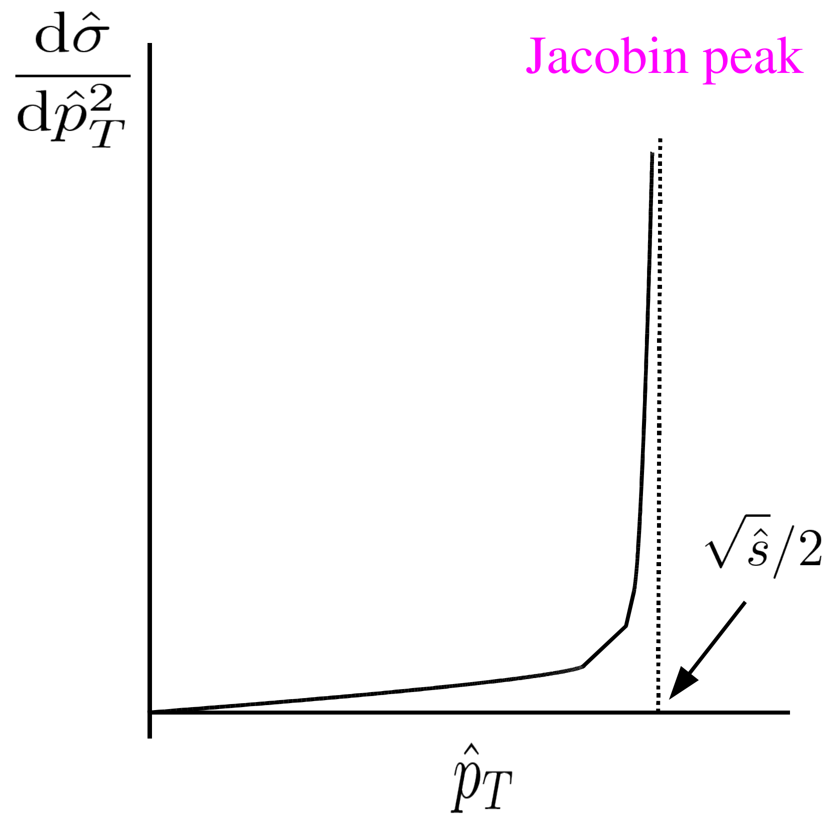


$$\hat{p}_T^2 = \frac{1}{4}\hat{s}\sin^2\theta$$

Jacobin factor

$$\frac{d\cos\theta}{d\hat{p}_T^2} = -\frac{2}{\hat{s}}\frac{1}{\sqrt{1-\frac{4\hat{p}_T^2}{\hat{s}}}}$$

$$\Longrightarrow \frac{d\hat{\sigma}}{d\hat{p}_T^2} \sim \frac{d\hat{\sigma}}{d\cos\theta} \times \frac{1}{\sqrt{1-4\hat{p}_T^2/\hat{s}}}$$



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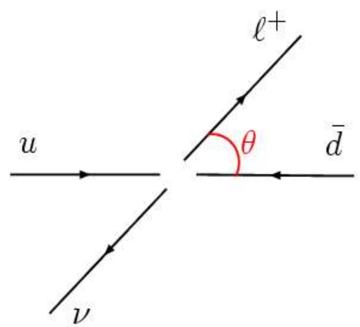

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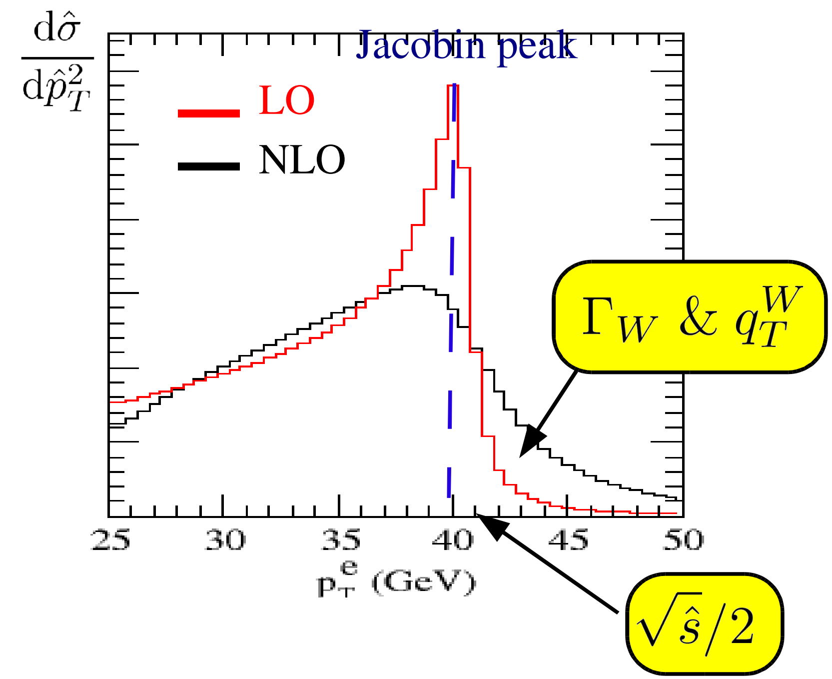


sensitive region for measuring

M_W : $p_T^e \sim 30 - 45$ GeV

Γ_W : not a good observable

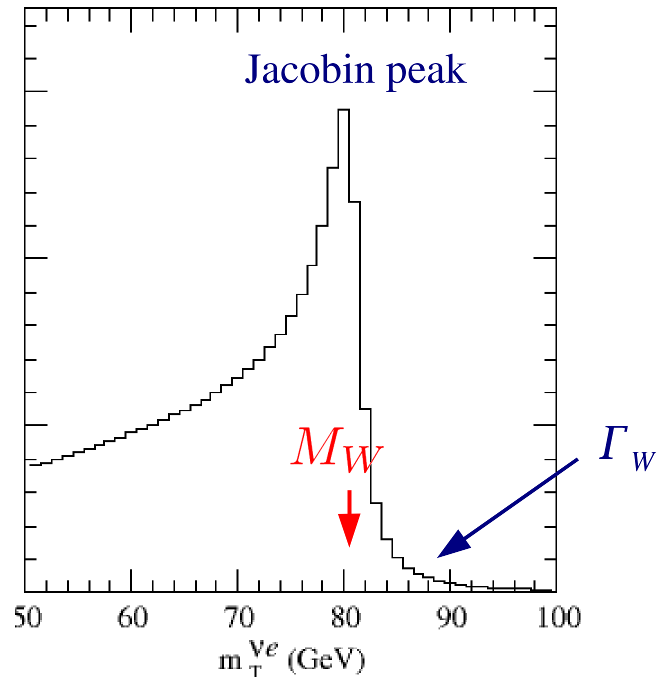
Transverse mass of the W-boson (M_T^W)

- Definition:

$$m_T^2(\ell,\nu) = 2\, p_T^\ell\, p_T^\nu (1-\cos\phi_{\ell\nu})$$

from overall p_T imbalance

$$\Longrightarrow \quad \frac{d\hat{\sigma}}{dm_T^2} \sim \frac{1}{\sqrt{1-m_T^2/\hat{s}}}$$



- unaffected by longitudinal boosts of $\ell\nu$ system
- not sensitive to q_T^W
- tail knows about Γ_W (direct measurement)

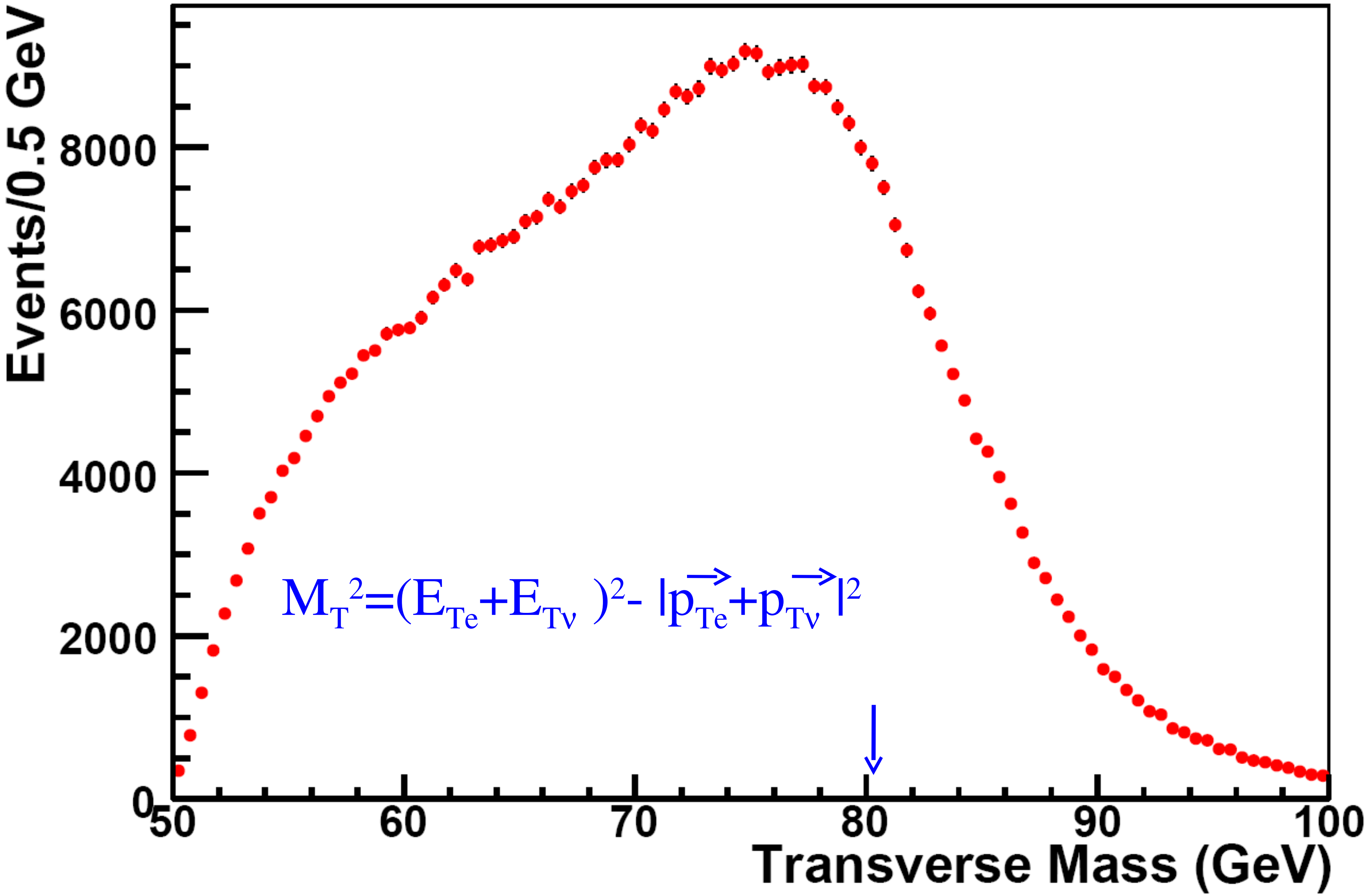
:

sensitive region for measuring

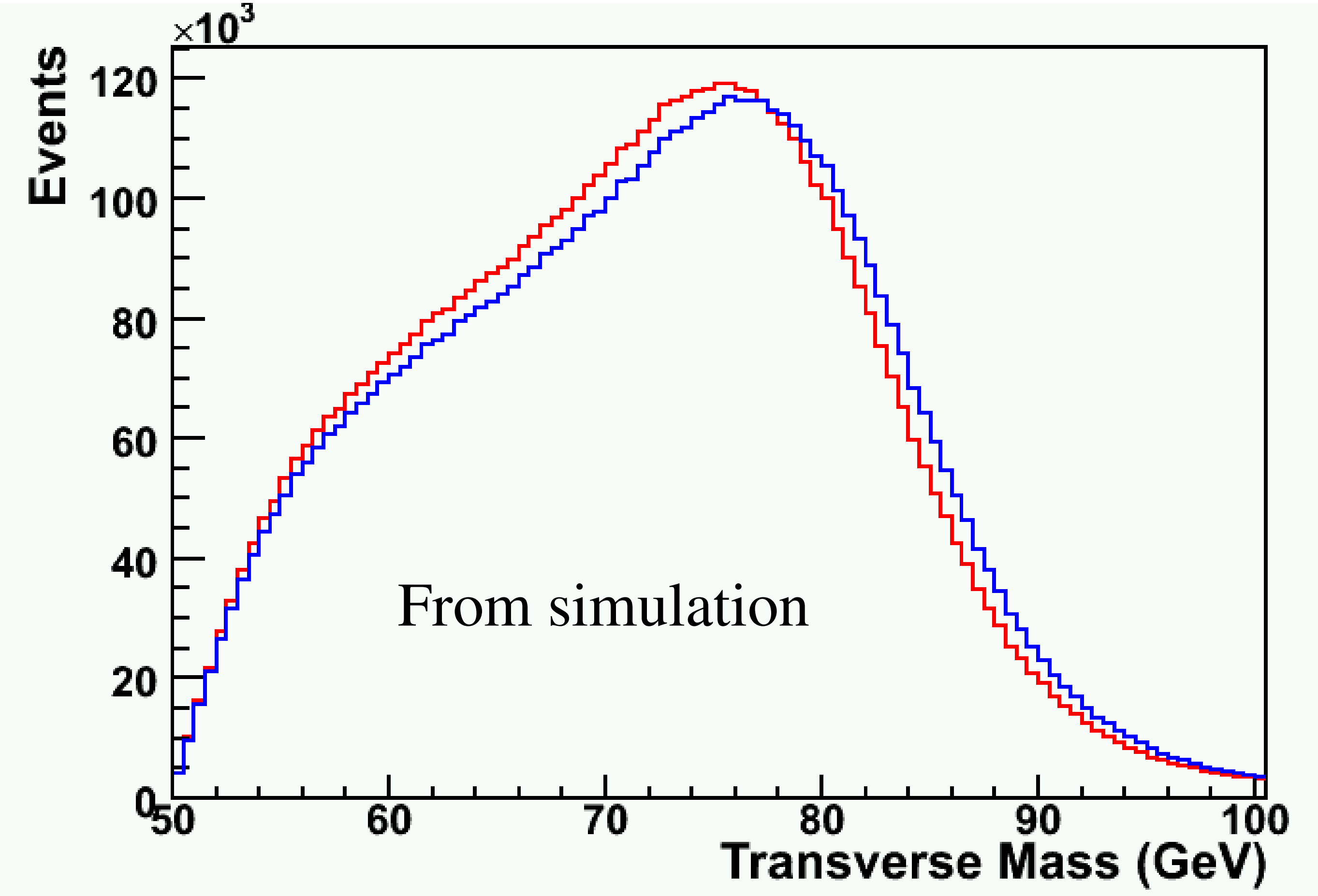
M_W : $M_T \sim 60-100\text{GeV}$

Γ_W $M_T > 100$ GeV

Transverse Mass for $W \rightarrow e\nu$



How to measure W Mass? (I)

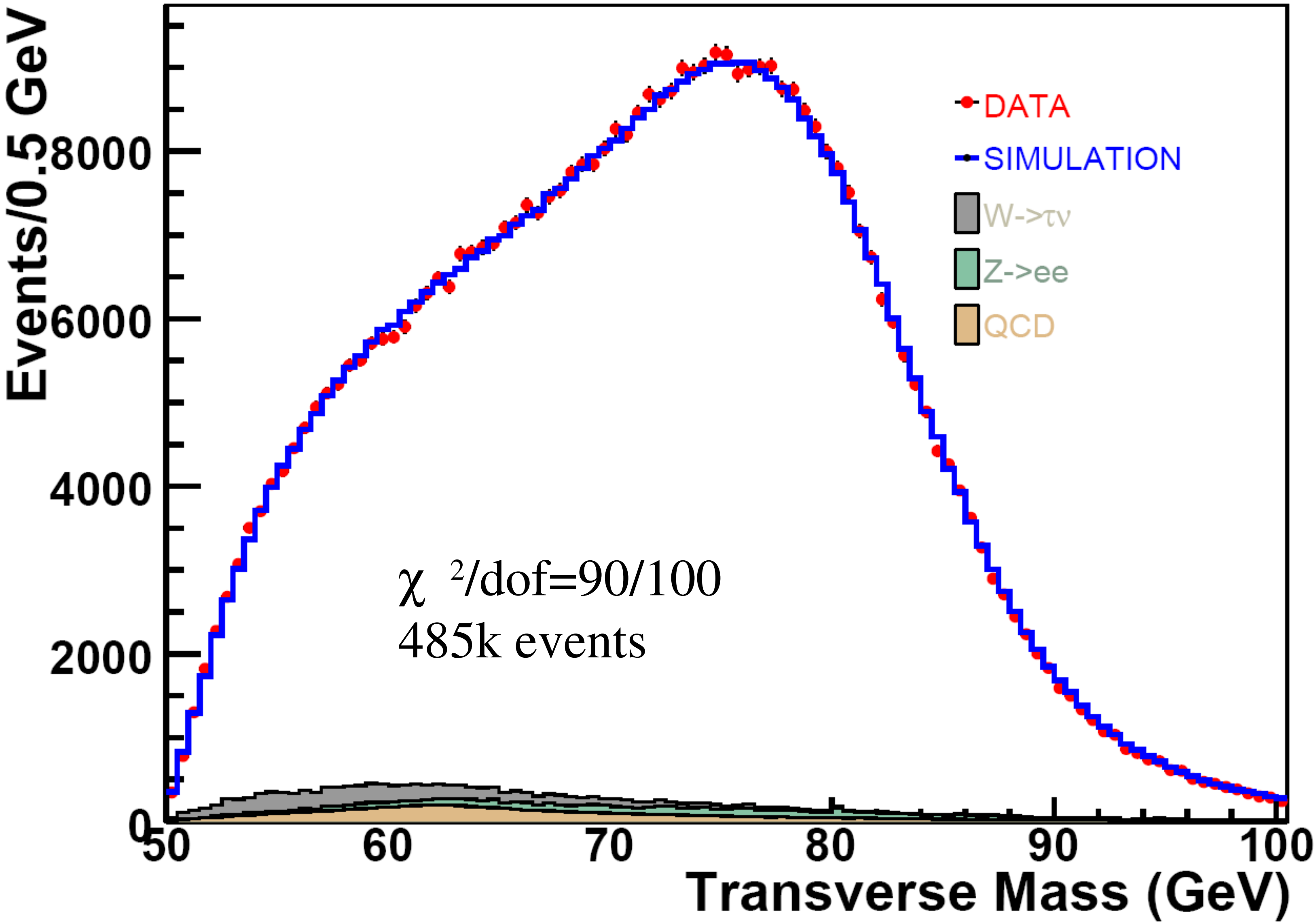


M_W = 80 GeV

M_W = 81 GeV

Similar situations for the electron p_T and MET spectra

Transverse Mass for W Boson



Transverse mass of the W-boson (M_T^W)

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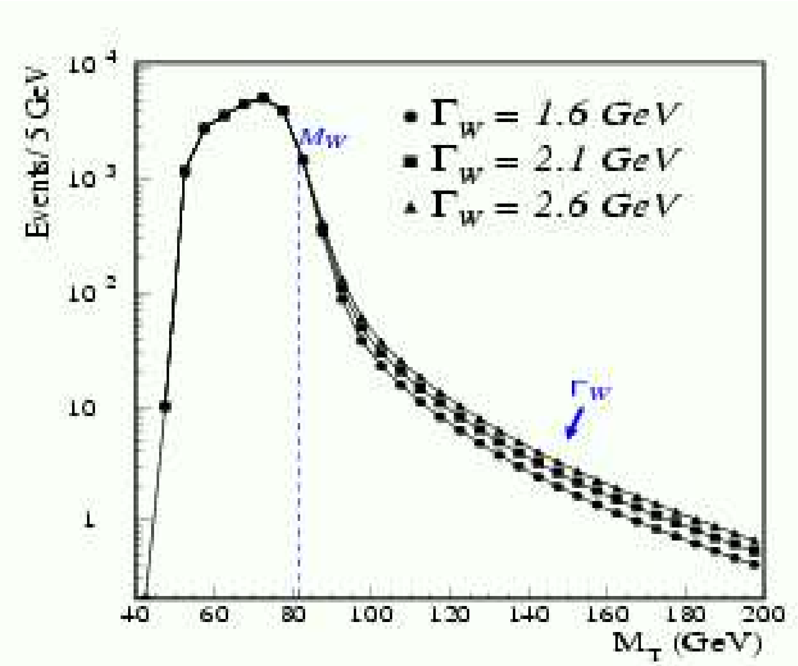
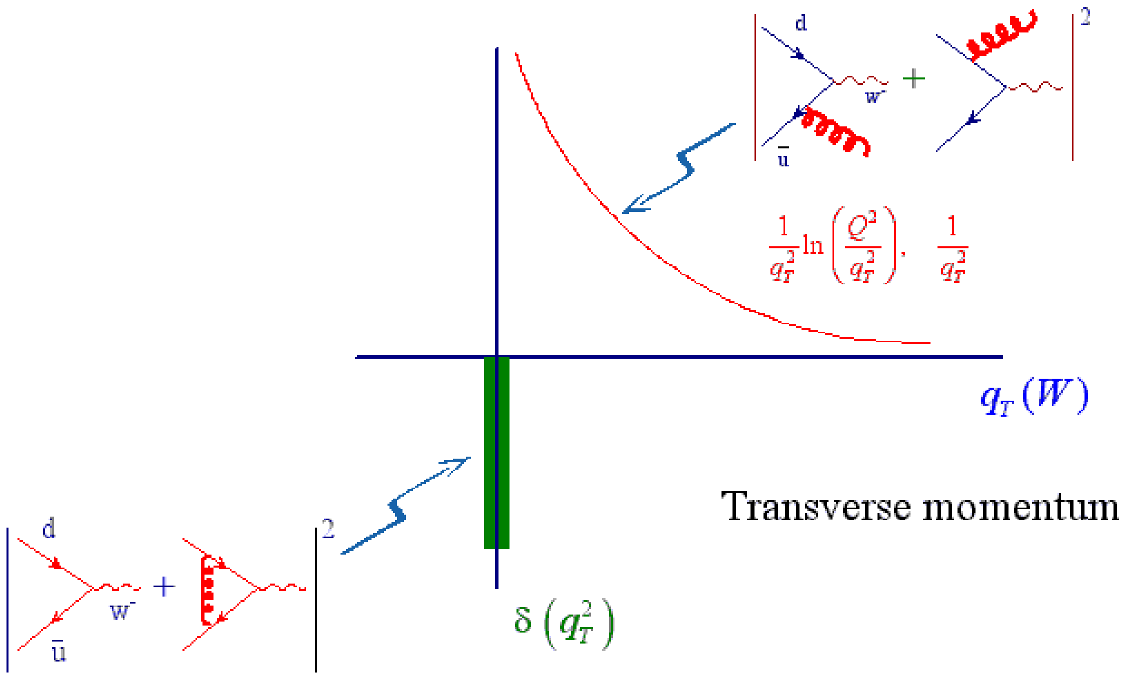


- ☞ unaffected by longitudinal boosts of $\ell\nu$ system
- ☞ not sensitive to q_T^W
- ☞ tail knows about Γ_W (direct measurement)

sensitive region:
M_W : $M_T \sim 60 - 100$ GeV
Γ_W : $M_T > 100$ GeV

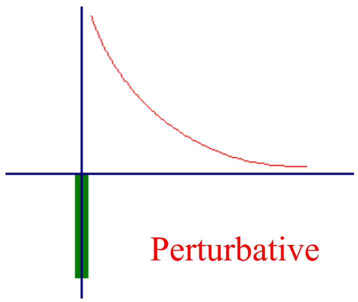
Shortcoming of fixed order calculation

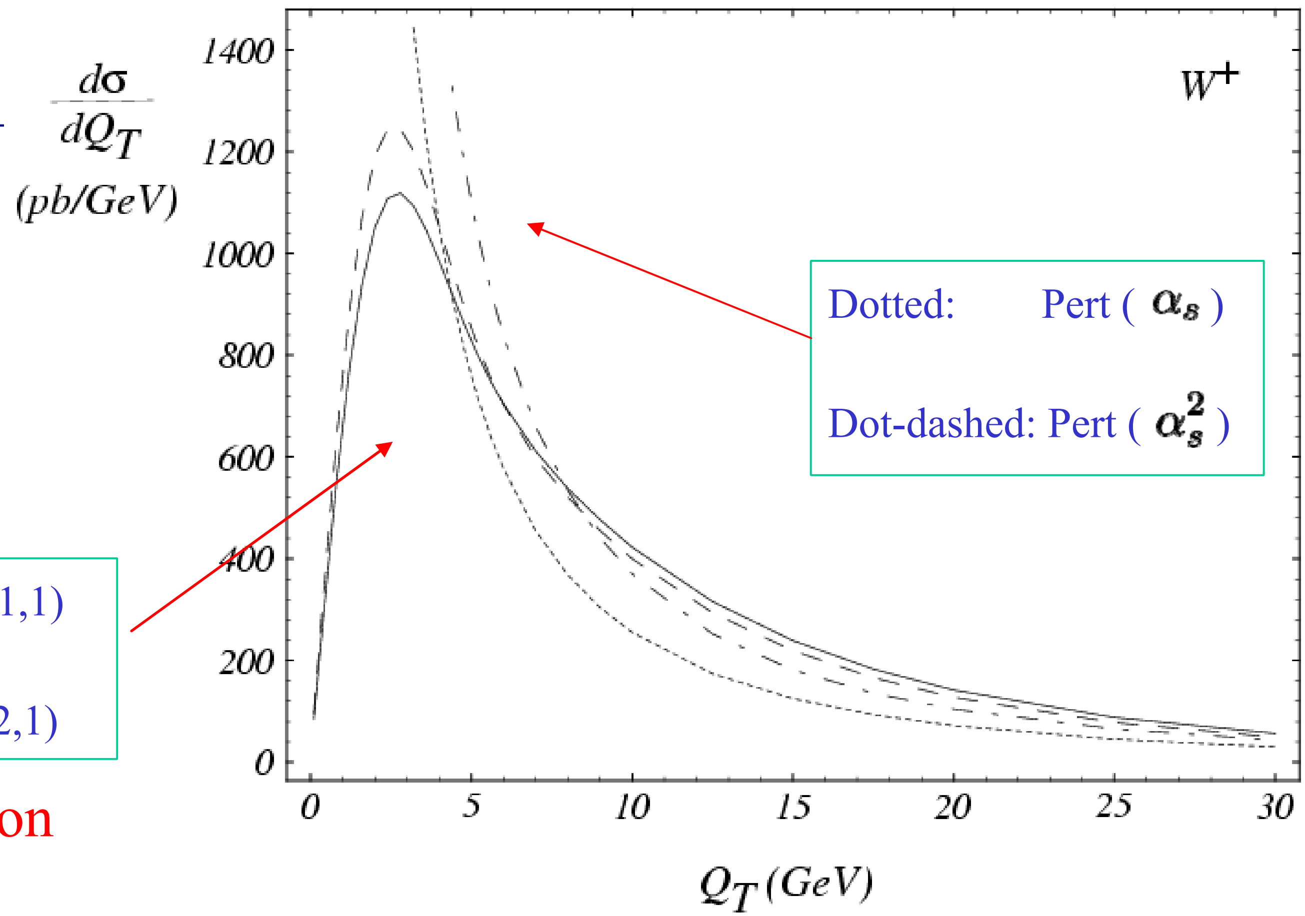
- Cannot describe data with small q_T of W-boson.
- Cannot precisely determine m_W at hadron colliders without knowing the transverse momentum of W-boson. Most events fall in the small q_T region.



(at NLO)

QCD Resummation is needed





Dashed: CSS (1,1,1)

Solid: CSS (2,2,1)

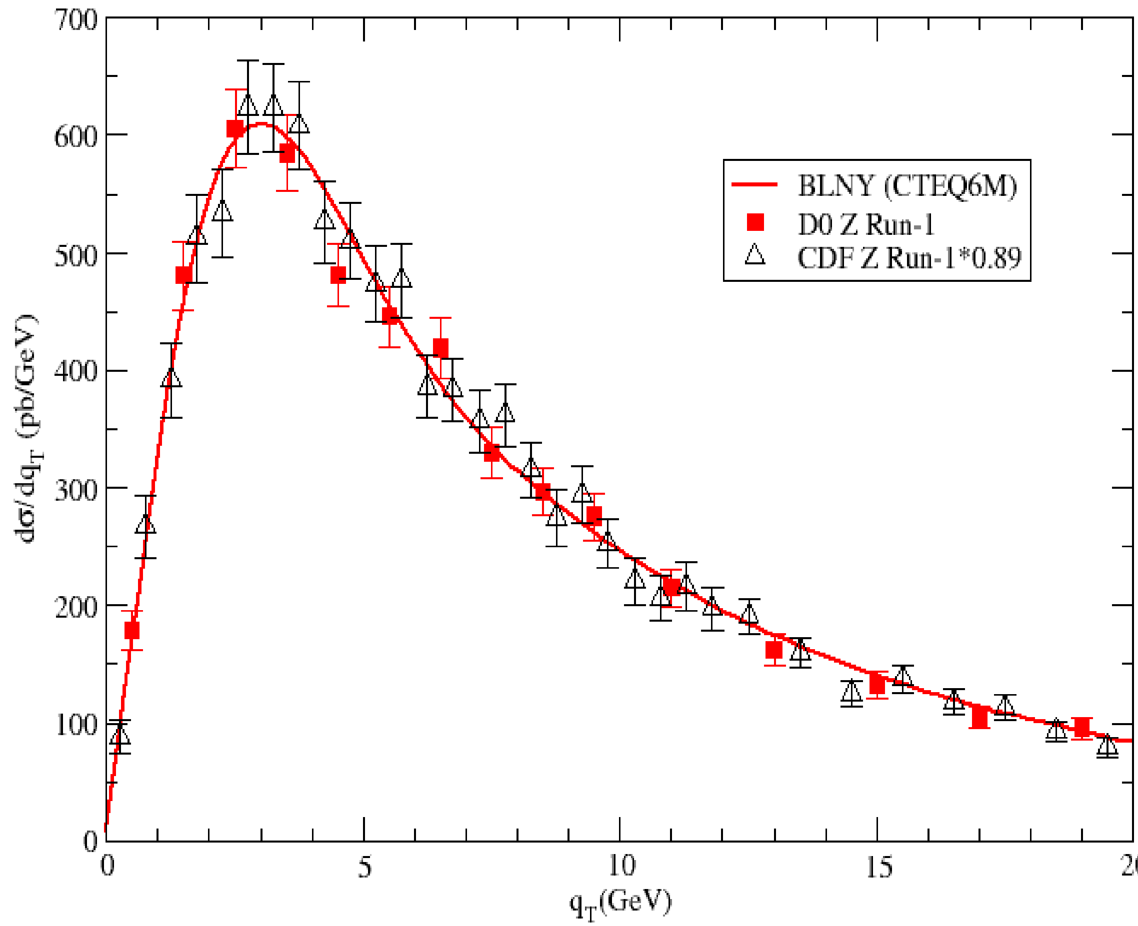
Resummation

Resummation calculations agree with data very well

Predicted by ResBos:

A program that
includes the effect of
multiple soft gluon emission
on the production of
W and Z bosons
in hadron collisions.

$P\bar{P} \rightarrow Z$ @ Tevatron



ResBos

(Resummation for Bosons)

Initial state QCD soft gluon resummation
and
Final state QED corrections

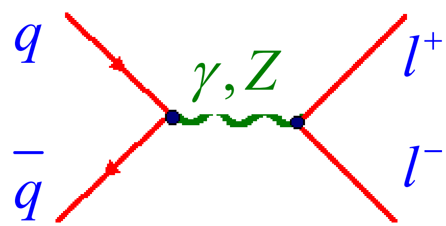
In collaboration with

Csaba Balazs, Alexander Belyaev, Ed Berger,
Qing-Hong Cao, Chuan-Ren Chen, Zhao Li,
Steve Mrenna, Pavel Nadolsky, Jian-Wei Qiu,
Carl Schmidt

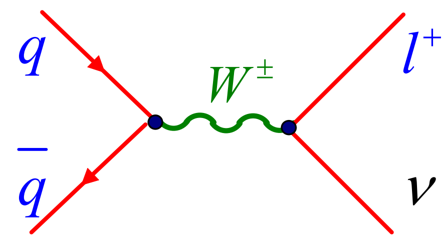
What's it for? An Example

- Transverse momentum of

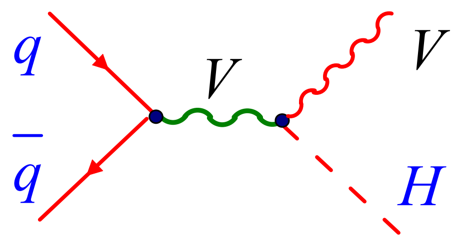
Drell-Yan



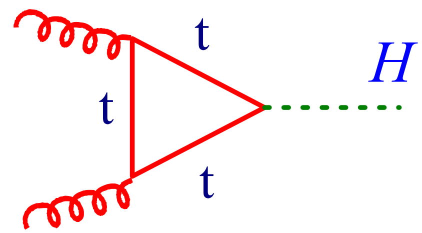
$W^\pm$



V H



$g\, g \rightarrow H$



including QCD Resummations.

- Kinematics of Leptons from the decays (Spin correlation included)

W Charge Asymmetry: A Monitor of Parton Distribution Functions

- Difference between u(x) and d(x) in proton cause $u\bar{d} \rightarrow W^{+}$ and $\bar{u}d \rightarrow W^{-}$ to be boosted in opposite directions

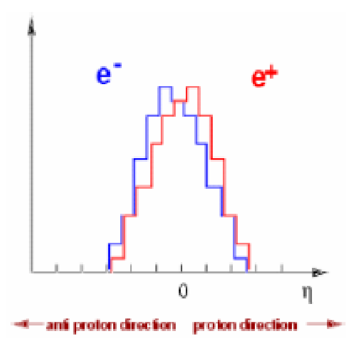
$$A(y_W) = \frac{d\sigma(W^+)/dy_W - d\sigma(W^-)/dy_W}{d\sigma(W^+)/dy_W + d\sigma(W^-)/dy_W}$$

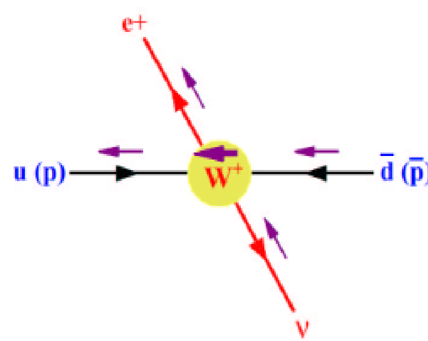
$$A(y_W) \approx \frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)}$$

Rapidity charge asymmetry is sensitive to **d(x)/u(x)** ratio at high-**x** → primary interest of PDF fitters.

- cannot reconstruct y_W directly
- measure charged lepton only

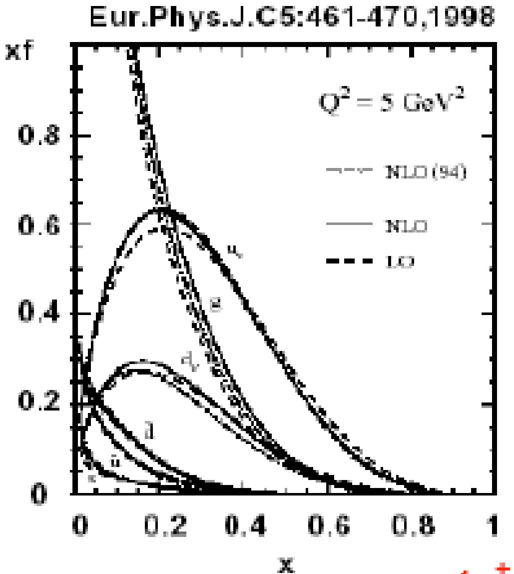
$$A(\eta_l) = \frac{d\sigma(l^+)/d\eta_l - d\sigma(l^-)/d\eta_l}{d\sigma(l^+)/d\eta_l + d\sigma(l^-)/d\eta_l}$$

$A(\eta_l) =$ 

$\otimes$ 

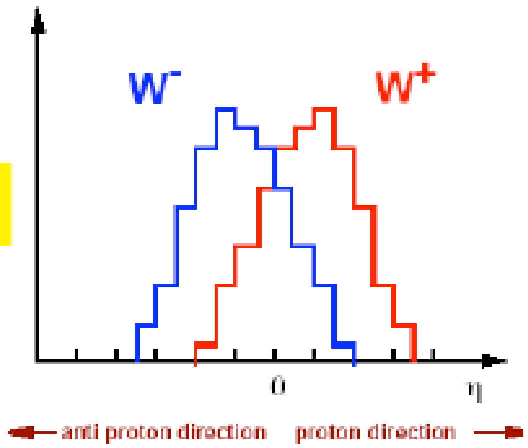
CTEQ

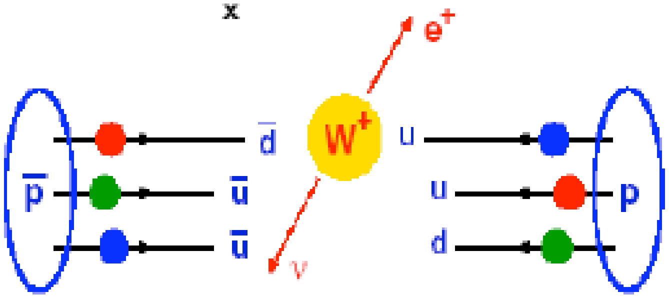
Rapidity Distribution



u higher momentum

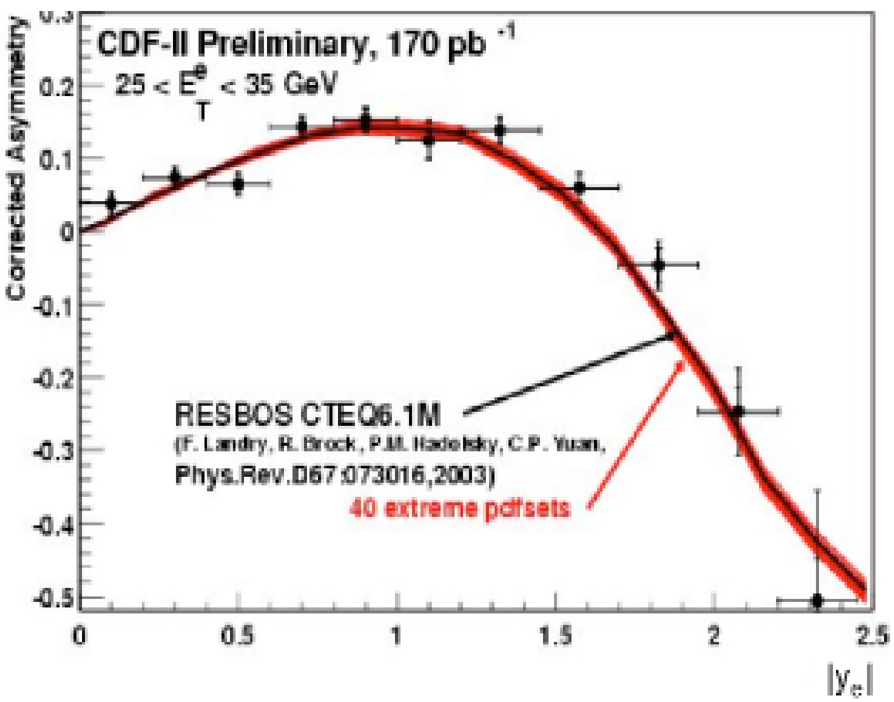
$u_V : d_V = 2 : 1$

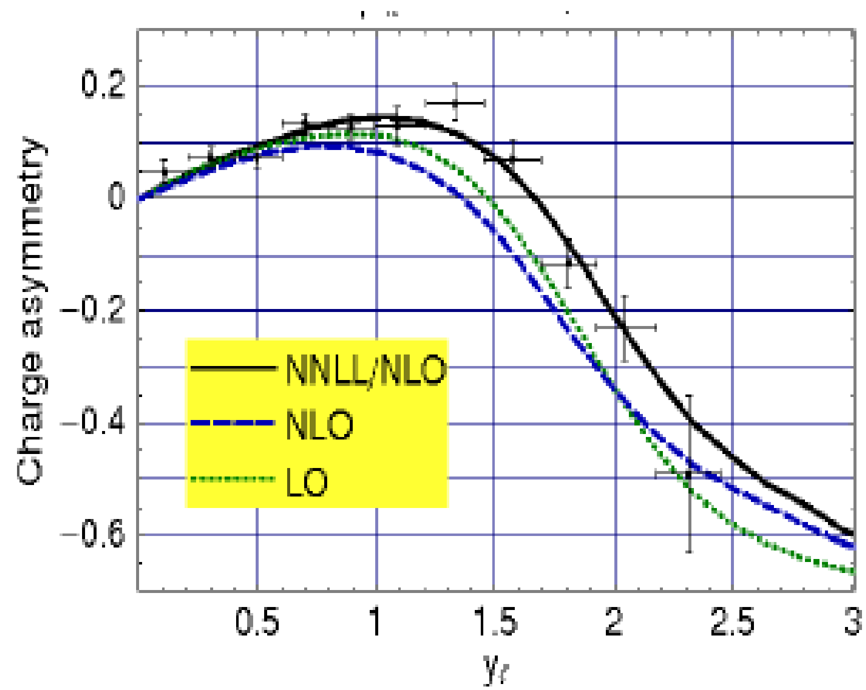




W^+ boosted in p direction
W^- boosted in $\bar{p}$ direction

ResBos is also needed for Rapidity distributions





black curve is from ResBos calculation

What's QCD Resummation?

- Perturbative expansion

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}q_T^2} \sim \alpha_S \left\{ 1 + \alpha_S + \alpha_S^2 + \cdots \right\}$$

- The singular pieces, as $\frac{1}{q_T^2}$ (1 or log's)

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}q_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_S^{(n)} \ln^{(m)}\left(\frac{Q^2}{q_T^2}\right)$$

$$\sim \frac{1}{q_T^2} \{ \quad \alpha_S \left(\underline{L+1}\right)$$

$$+\alpha_S^2 \left(\underline{L^3 + L^2} + \underline{L+1}\right)$$

$$+\alpha_S^3 \left(\underline{L^5 + L^4} + \underline{L^3 + L^2} + \underline{L+1}\right)$$

$$+ \cdots \quad \}$$

$$L \equiv \ln\left(\frac{Q^2}{q_T^2}\right)$$

Resummation is to reorganize the results in terms of the large Log's.

Resummed results:

Determined by A(1) and B(1)

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2}\Big\{ \left[\alpha_S(L+1) + \alpha_S^2\left(L^3+L^2\right) + \alpha_S^3\left(L^5+L^4\right) + \cdots \right]$$

$$+\left[\quad + \alpha_S^2(L+1) + \alpha_S^3\left(L^3+L^2\right) + \cdots \right]$$

$$+\left[\quad + \alpha_S^3(L+1) + \cdots \right]$$

$$+\cdots \Big\}$$

Determined by A(2) and B(2)

Determined by A(3) and B(3)

→ QCD Resummation

In the formalism by Collins-Soper-Sterman, in addition to these perturbative results, the effects from physics beyond the leading twist is also implemented as

[non-perturbative functions].

CSS resummation formalism

Collins-Soper-Sterman

$$\frac{d\sigma}{dq_T^2\, dy\, dQ^2} = \frac{\pi}{S}\sigma_0 \delta\left(Q^2 - M_W^2\right) \cdot$$

$$\left\{ \frac{1}{(2\pi)^2} \int d^2 b \;\; e^{i\vec{q}_T \cdot \vec{b}}\, \tilde{W}\left(b, Q, x_A, x_B\right) \cdot \text{[Non-perturbative functions]} \right.$$

$$\left. + Y\left(q_T, y, Q\right) \right\}$$

$$\tilde{W} = e^{-S(b)} \cdot C \otimes f\left(x_A\right) \cdot C \otimes f\left(x_B\right)$$

$$C \otimes f(x_A) \rightarrow \sum_j \int_{x_A}^{1} \frac{d\xi_A}{\xi_A} C_{qj}\left(\frac{x_A}{\xi_A}, b, \mu\right) \cdot f_{j/A}\left(\xi_A, \mu\right)$$

$$C \otimes f(x_B) \rightarrow \sum_k \int_{x_B}^{1} \frac{d\xi_B}{\xi_B} C_{\bar{q}k}\left(\frac{x_A}{\xi_A}, b, \mu\right) \cdot f_{k/B}\left(\xi_B, \mu\right)$$

$e^{-S(b)} \rightarrow$ Sudakov form factor

$$S(b) = \int_{\left(\frac{b_0}{b}\right)^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) A(\bar{\mu}) + B(\bar{\mu}) \right]$$

[Non-perturbative functions] are functions of (b, Q, x_A, x_B) which include QCD effects beyond Leading Twist.

- Example: for $W^{\pm}$

$$\sigma_0 = \left(\frac{4\pi^2\alpha}{3} \sum_{jj'} Q_{jj'}^{(W)} \right), \qquad Q_{jj'}^{(W)} = \frac{1}{4\sin^2\theta_W} \left(kM \right)_{jj'}^2$$

The couplings of gauge bosons to fermions are expressed in the way to include the dominant electroweak radiative corrections. The propagators of gauge bosons also contain energy-dependent width, as done in LEP precision data analysis.

Note:

$$A \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot A^{(n)}, \qquad B \equiv \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot B^{(n)},$$

$$C \equiv \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \cdot C^{(n)}$$

As $q_T \to 0$

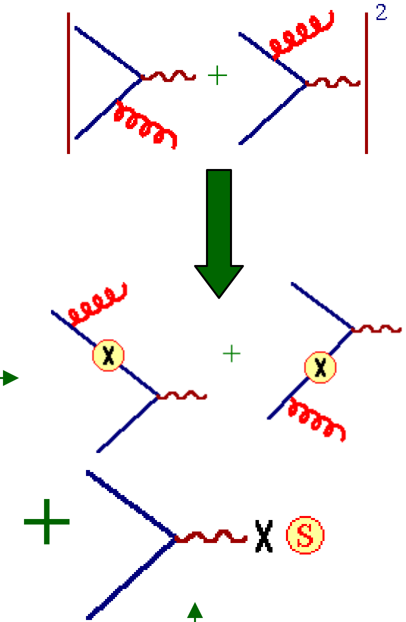
$$\left.\frac{d\sigma}{dq_T^2\,dy\,dQ^2}\right|_{q_T\to 0}$$

$$= \left(\frac{\pi}{s}\sigma_0\right)\cdot\delta\left(Q^2 - M_W^2\right)\cdot\left(\frac{1}{2\pi q_T^2}\right)\left(\frac{\alpha_s(Q)}{\pi}\right)$$

$$\cdot\Big\{ f_{q/A}(x_A,Q)\left[P_{\bar{q}\leftarrow\bar{q}}\otimes f_{\bar{q}}\right]_{x_B,Q} + \left[P_{q\leftarrow q}\otimes f_q\right]_{x_A,Q} f_{\bar{q}/B}(x_B,Q)$$

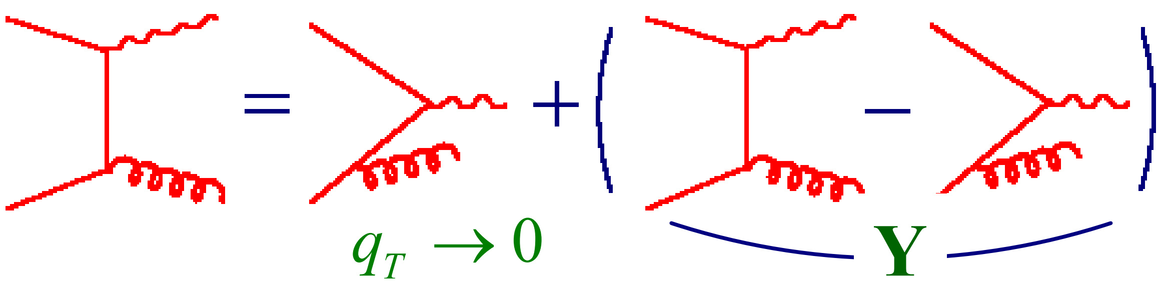
$$+ f_{q/A}(x_A,Q) f_{\bar{q}/B}(x_B,Q)\cdot\left[2\left(\frac{4}{3}\right)\ln\left(\frac{Q^2}{q_T^2}\right) + 2(-2)\right]\Big\}$$

Diagramatically,

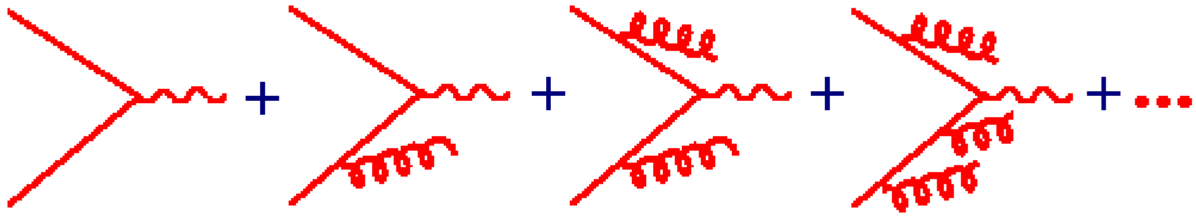


Exponentiate

To preserve transverse momentum conservation, we have to go to the impact parameter space (b-space) to perform resummation.



Diagramatically, Resummation is doing



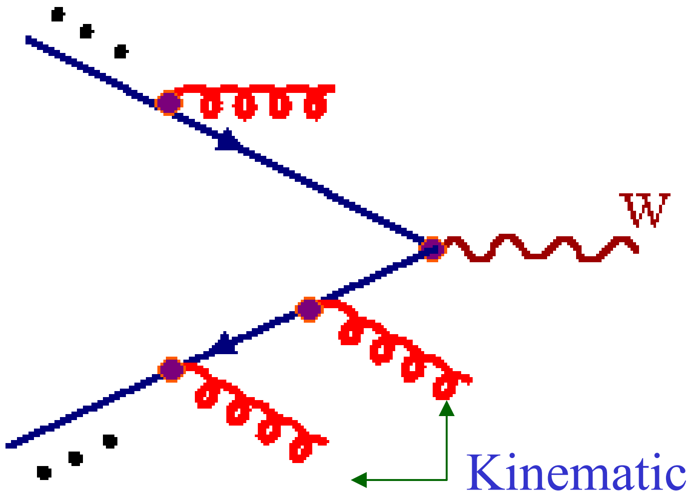
⇒ Resum large $\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)$ terms

$$\left.\frac{d\sigma}{dq_T^2 dy}\right|_{q_T \to 0} \sim \frac{1}{q_T^2}\sum_{n=1}^{\infty}\sum_{m=0}^{2n-1}\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)\cdot C_m^n$$

Monte-Carlo programs ISAJET, PYTHIA, HERWIG contain these physics.

(Note: Arbitrary cut-off scale in these programs to affect the amount of Backward radiation , i.e. Initial state radiation.)

Monte-Carlo Approach



Backward Radiation
(Initial State Radiation)

Kinematics of the radiated gluon, controlled by Sudakov form factor with some arbitrary cut-off. (In contrast to perform integration in impact parameter space, i.e., b space.)

⇒ The shape of q_T (w) is generated. But, the integrated rate remains the same as at Born level (finite virtual correction is not included).

* Recently, there are efforts to include part of higher order effect in the event generator.

Event Generators (PYTHIA, HERWIG)

Note that the integrated rate is the same as the Born level rate ($\alpha_S^{(0)}$) even though the q_T – distribution is different (i.e., not $\delta\left(q_T^2\right)$ any more).

$$\sigma = \int \mathrm{d}q_T^2 \frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \int \mathrm{d}^2 b \underbrace{\int e^{i\vec{q}_T\cdot\vec{b}}\,\mathrm{d}^2 q_T}\,\sigma_0 e^{-S(b)} \qquad \rightarrow \delta^2(b)$$

$$= \int \mathrm{d}^2 b\, \delta^2(b)\cdot\sigma_0\cdot e^{-S(b)} \qquad \rightarrow 1 \text{ at } b=0$$

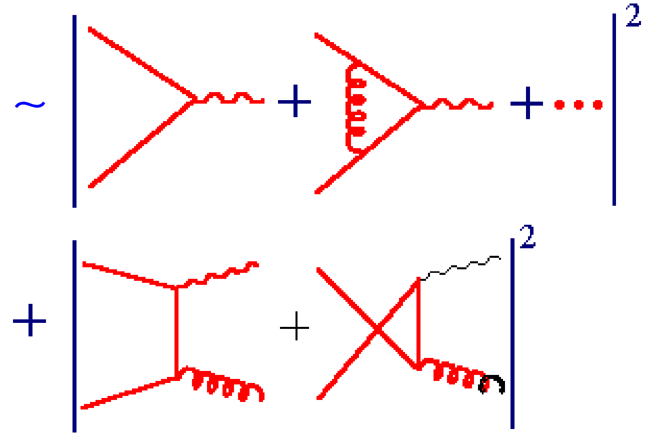
$$= \sigma_0$$

For C-Function = $\delta\left(1-\frac{x_A}{\xi_B}\right)$

To recover the "K-factor" in the NLO total rate

To include the C-Functions

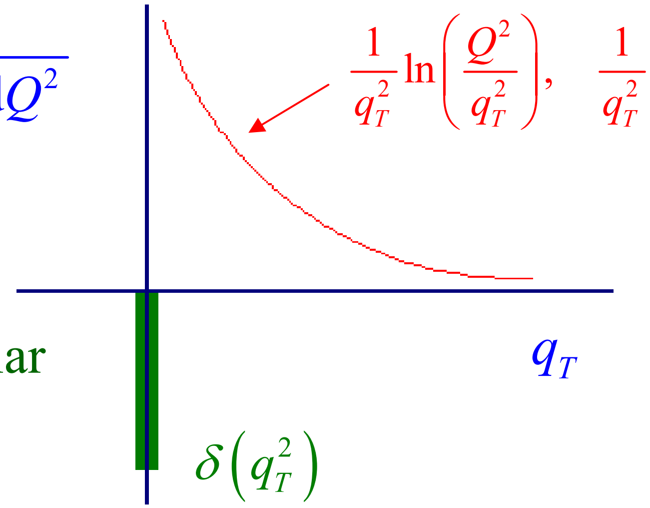
$$\frac{d\sigma}{dQ^2 dy} \sim$$

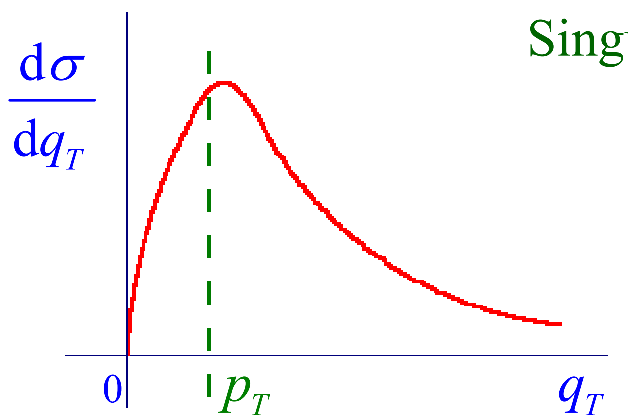


Finite

Singular

$$\frac{d\sigma}{dq_T\, dy\, dQ^2}$$





The area under the q_T – curve will reproduce the total rate at the order $\alpha_S^{(1)}$ if Y term is calculated to $\alpha_S^{(1)}$ as well.

Include NNLO in high q_T region

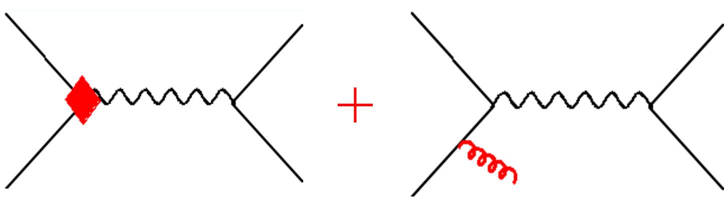
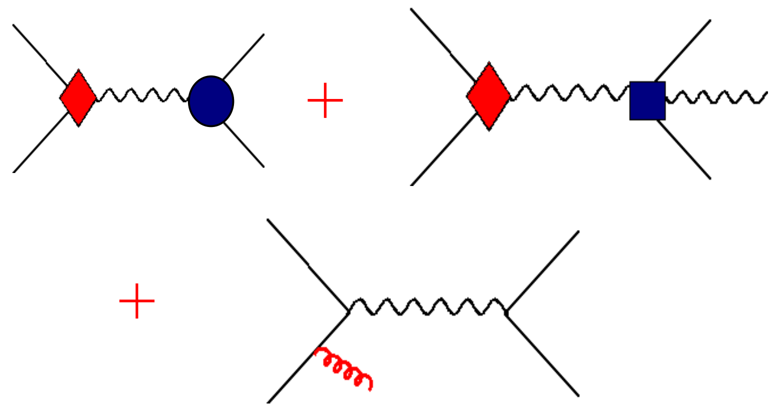
- To improve prediction in high q_T region
- To speed up the calculation, it is implemented through K-factor table which is a function of (Q, q_T, y) of the boson, not just a constant value.

→ ResBos predicts both rate and shape of distributions.

Precision measurements require accurate theoretical predictions

- ResBos-A: improved ResBos by including final state NLO QED corrections to W and Z production and decay

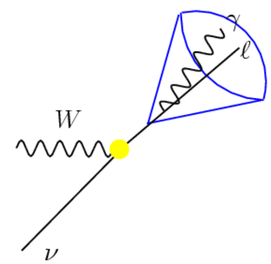
hep-ph/0401026
Qing-Hong Cao and CPY

Resum+Born	Resum+NLO
	

● and ■ denote FQED radiation corrections, which dominates the W mass shift.

Need to consider the recombination effect

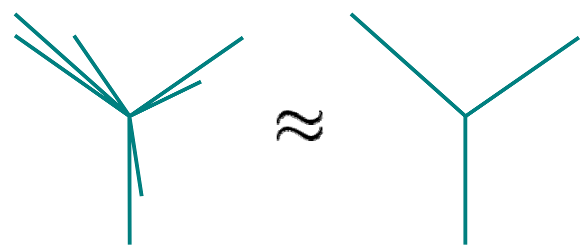
- Experimental: difficult to discriminate between electrons and photons with a small opening angle



- Theoretical: to define infra-safe quantities which are independent of long-distance physics

  Essential feature of a general IRS physical quantity:

  The observable must be such that it is insensitive to whether n or n+1 particles contributed if the n+1 particles has n-particle kinematics.



- Procedure @ Tevatron (for electron)

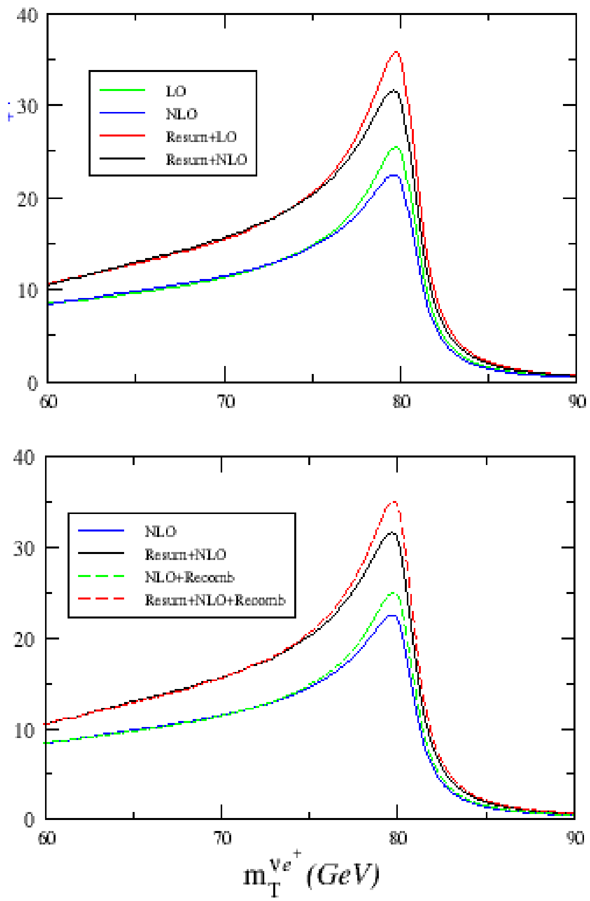
☞ $p'_e = p_e + p_\gamma$

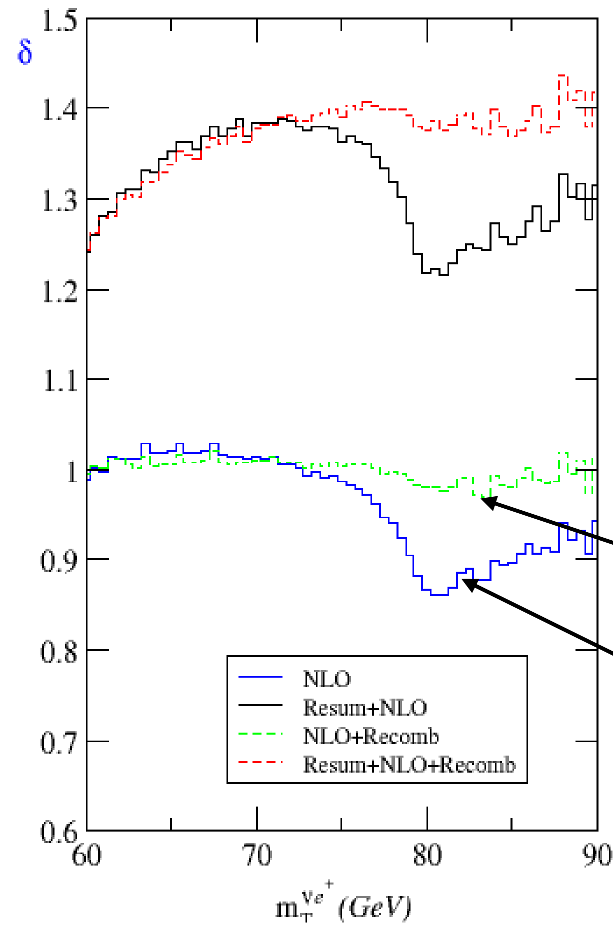
- $\Delta R(e,\gamma) < 0.2$
- $E_\gamma < 0.15 E_e$ for $0.2 < \Delta R(e,\gamma) < 0.3$

☞ rejection

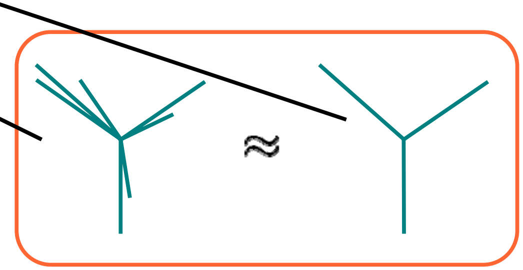
- $E_\gamma > 0.15\, E_e$ for $0.2 < \Delta R(e,\gamma) < 0.4$

Recombination Effects





Effects of EW correction decrease significantly after recombination.

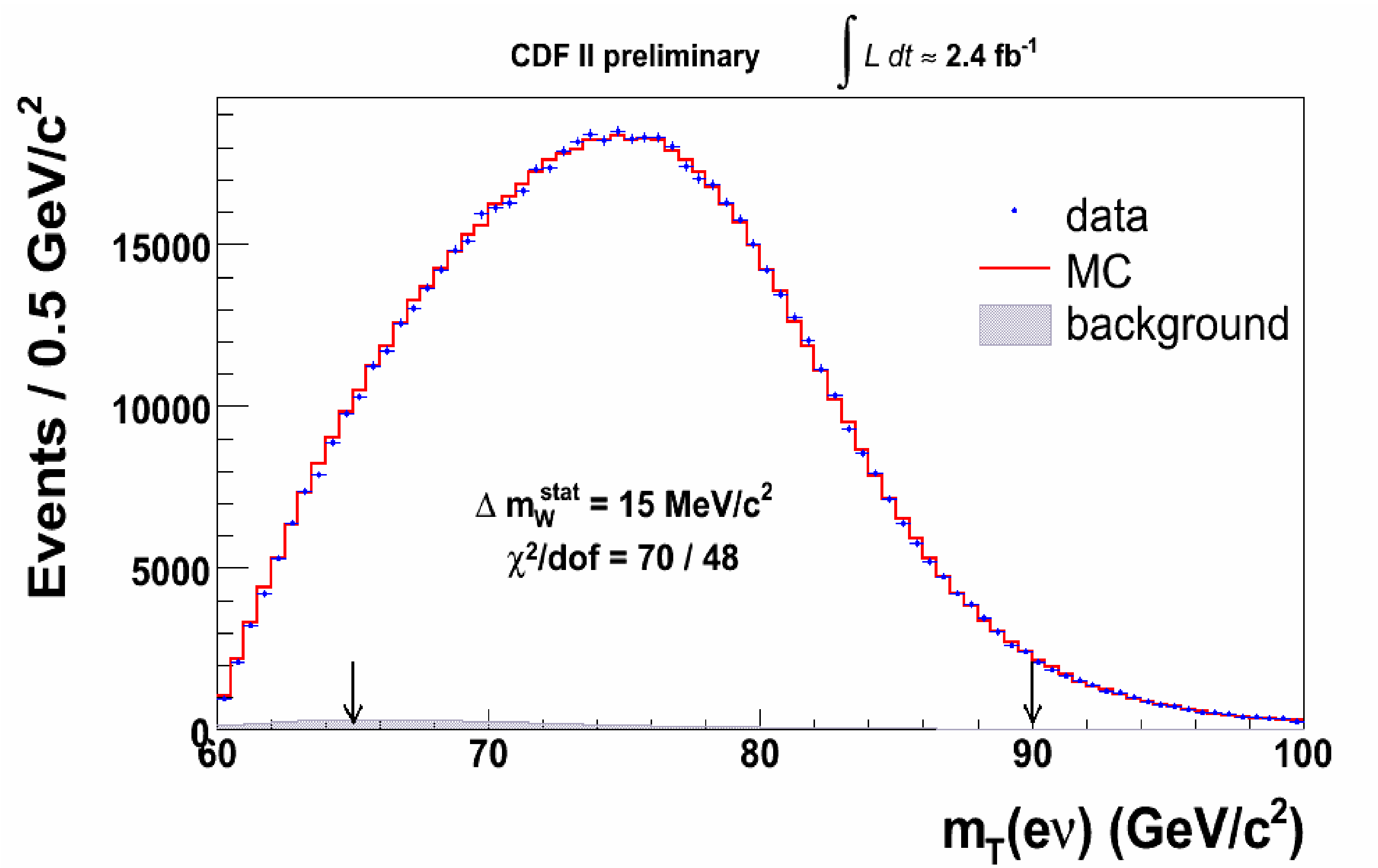


infrared-safe

CTEQ

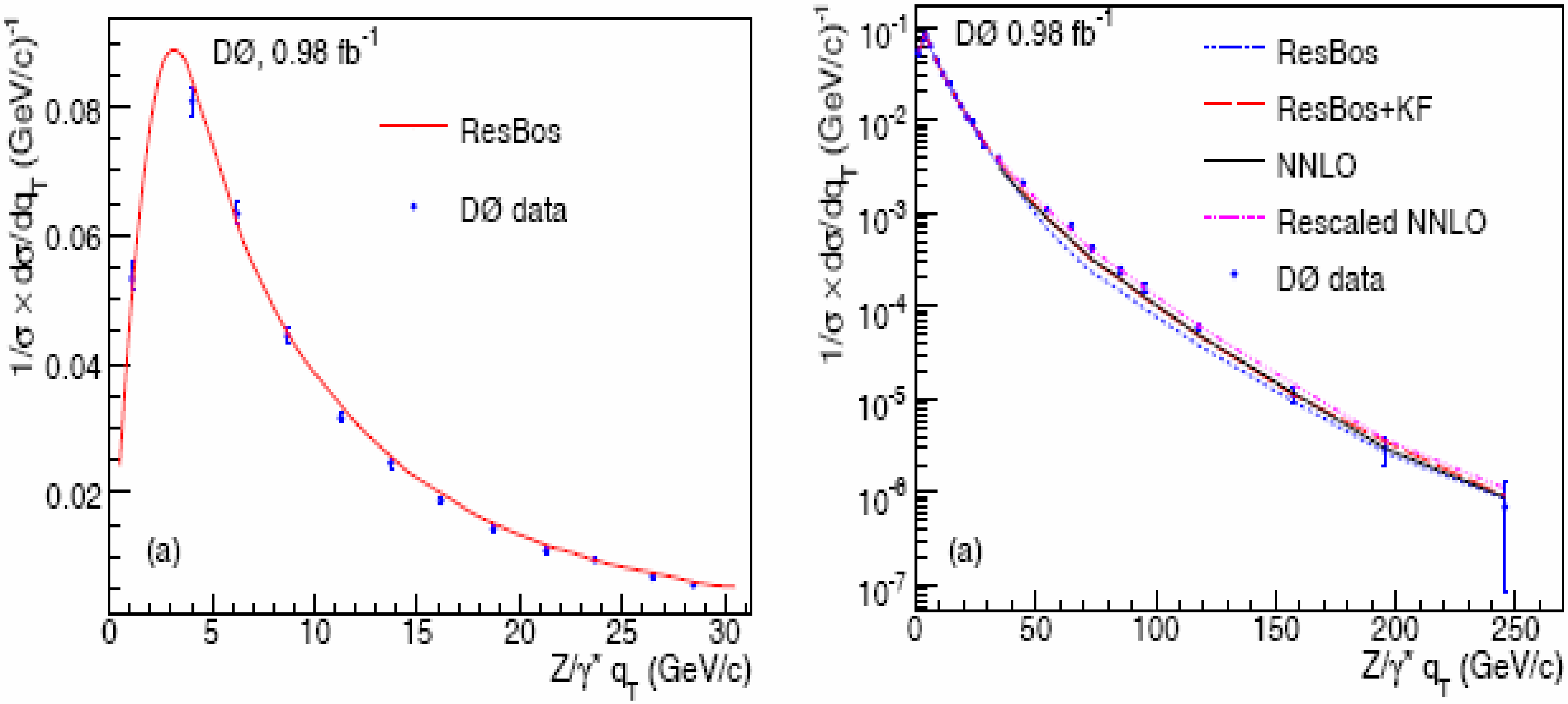
W Mass @ CDF Run-2

W→eν transverse mass distribution



Statistical error only.

W Boson q_T @ D0 Run-2

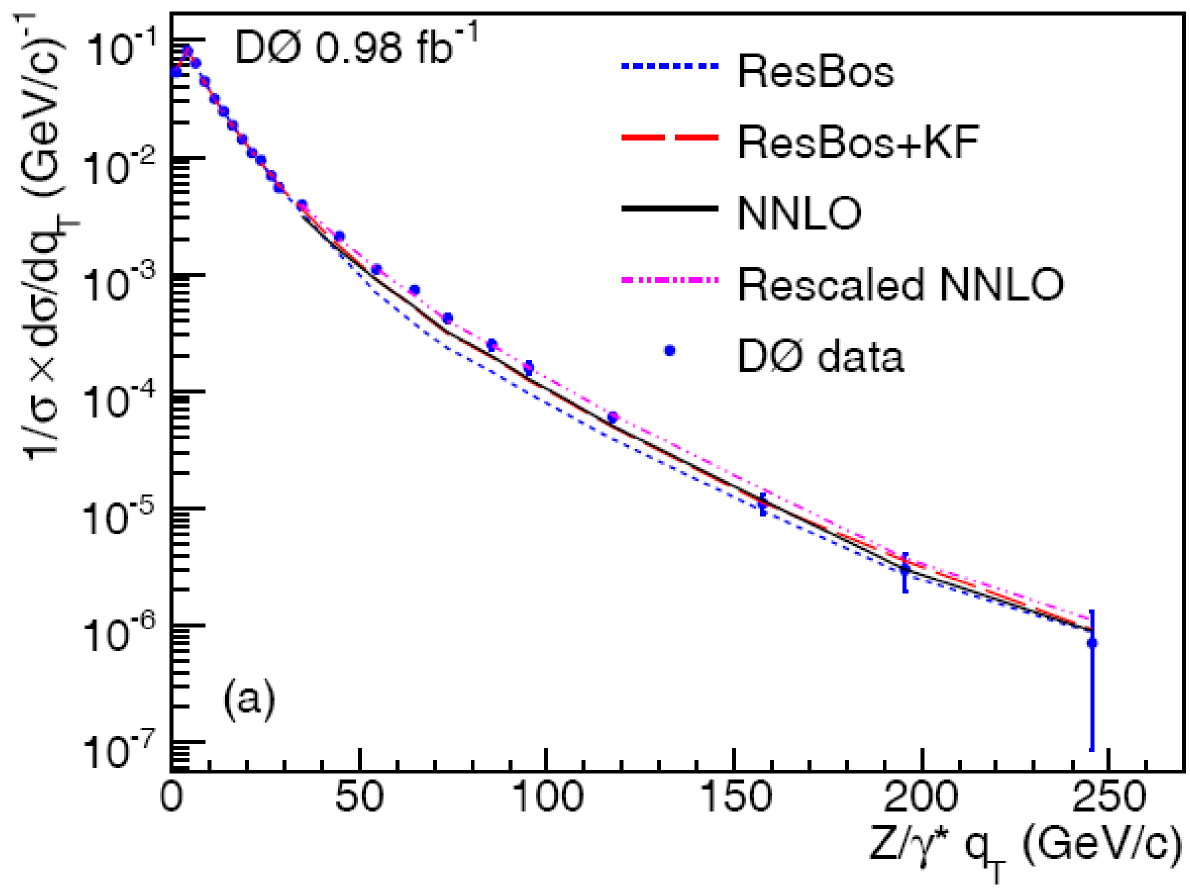


low q_T region

Figure: Phys. Rev. Lett. 100 , 102002 (2008)

CTEQ

W Boson q_T @ D0 Run-2



Need to study the difference in the intermediate q_T region.

Where is it?

- **ResBos**: http://hep.pa.msu.edu/resum/
- **Plotter**: http://hep.pa.msu.edu/wwwlegacy

ResBos-A (including final state NLO QED corrections)
http://hep.pa.msu.edu/resum/code/resbosa/
has not been updated.
Why? Because it was not used for Tevatron experiments.

The plan is to include final state QED resummation inside ResBos.

Physical processes included in ResBos

$W^{\pm}$

γ, Z

H

$\gamma\gamma, ZZ, WW$

including gauge invariant set amplitude for Drell-Yan pairs

New physics: W', Z', H^+, A^0, H^0 …

Physics processes inside ResBos

Process	$A^{(i)}$	$B^{(i)}$	$C^{(i)}$	order of Pert. part
$A+B \to W^+ \to l^+ + \nu + X$	3	2	1	NNLO
$A+B \to W^- \to l^- + \bar{\nu} + X$	3	2	1	NNLO
$A+B \to Z^0 \to l^- + l^+ + X$	3	2	1	NNLO
$A+B \to Z^0/\gamma^* \to l^+ + l^- + X$	3	2	1	NNLO
$A+B \to \gamma^* \to l^+ + l^- + X$	3	2	1	NNLO
$A+B \to gg \to H^0 \to \gamma\gamma + X$	3	2	1	NNLO
$A+B \to gg \to H^0 \to Z^0Z^0/W^+W^- \to 4l + X$	3	2	1	NNLO
$A+B \to W^{+*} \to W^+ + H^0 + X$	3	2	1	NNLO
$A+B \to W^{-*} \to W^- + H^0 + X$	3	2	1	NNLO
$A+B \to Z^{0*} \to Z^0 + H^0 + X$	3	2	1	NNLO
$A+B \to q\bar{q} \to \gamma\gamma + X$	3	2	1	NLO
$A+B \to gg \to \gamma\gamma + X$	3	2	1	NLO
$A+B \to q\bar{q} \to Z^0Z^0 + X$	3	2	1	NLO
$A+B \to W^+W^- + X$ (upcoming)	3	2	1	NLO

New Physics (upcoming)

Process	$A^{(i)}$	$B^{(i)}$	$C^{(i)}$	order of Pert. part
$A+B \to W' \to l^- + \bar{\nu} + X$	3	2	1	NNLO
$A+B \to Z' \to l^- + l^+ + X$	3	2	1	NNLO
$A+B \to bb \to A^0/H^0 + X$ (THDM)	3	2	1	NNLO
$A+B \to c\bar{s} \to H^+ + X$ (THDM)	3	2	1	NNLO

PYTHIA predicts a different shape (and rate)

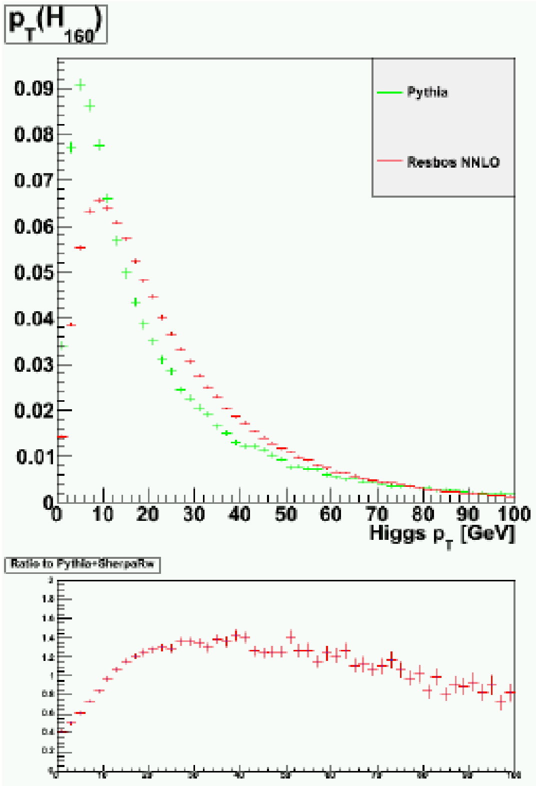
Higgs pT spectrum

All our Higgs MCs are generated with:
Pythia - using LO CTEQ6L1 PDFs

Corrections to the Higgs pT spectrum in gg→H:

In the past: reweight to Sherpa

Plan: reweight to Resbos



Limitations of ResBos

- Any perturbative calculation is performed with some approximation, hence, with limitation.
- To make the best use of a theory calculation, we need to know what it is good for and what the limitations are.

It does not give any information about the hadronic activities of the event.

⇒ It could be used to reweight the distributions generated by (PYTHIA) event generator, by comparing the boson (and it decay products) distributions to ResBos predictions.

This has been done for W-mass analysis by CDF and D0)

Potential of ResBos yet to be explored

- E.g., in the measurement of forward-backward asymmetry in Drell-Yan pairs.

ResBos can be used for Matrix Element Method by including resummed k_T-dependent parton distribution functions together with higher order matrix element contributions.

For example: The coefficients in front of the complete set of angular functions are given by ResBos

$$\mathcal{L}_0 = 1 + \cos^2\theta,\ \mathcal{A}_0 = \frac{1}{2}(1 - 3\cos^2\theta),\ \mathcal{A}_1 = \sin 2\theta\cos\phi,\ \mathcal{A}_2 = \frac{1}{2}\sin^2\theta\cos 2\phi,$$
$$\mathcal{A}_3 = 2\cos\theta,\ \mathcal{A}_4 = \sin\theta\cos\phi.$$

Angular function in Drell-Yan process

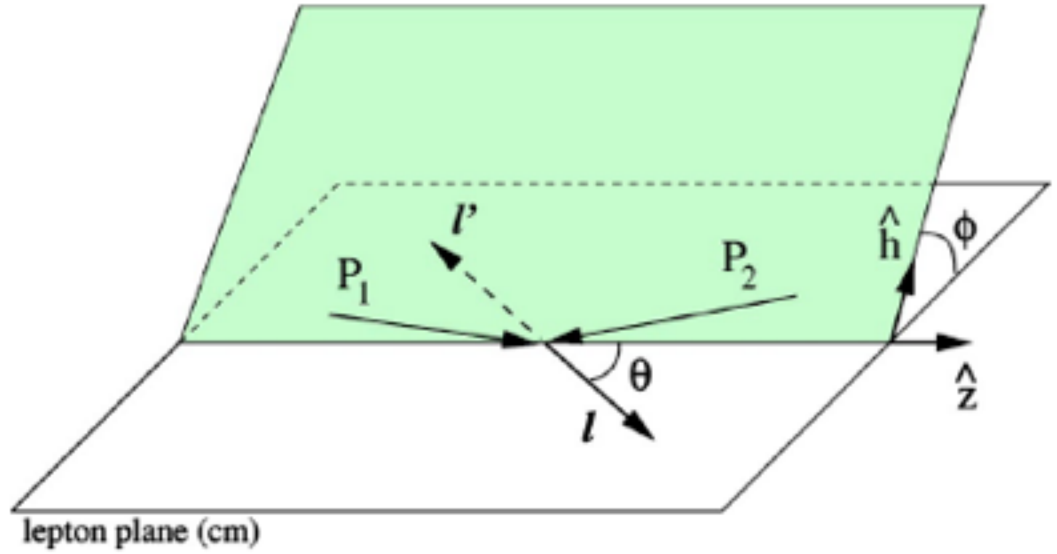
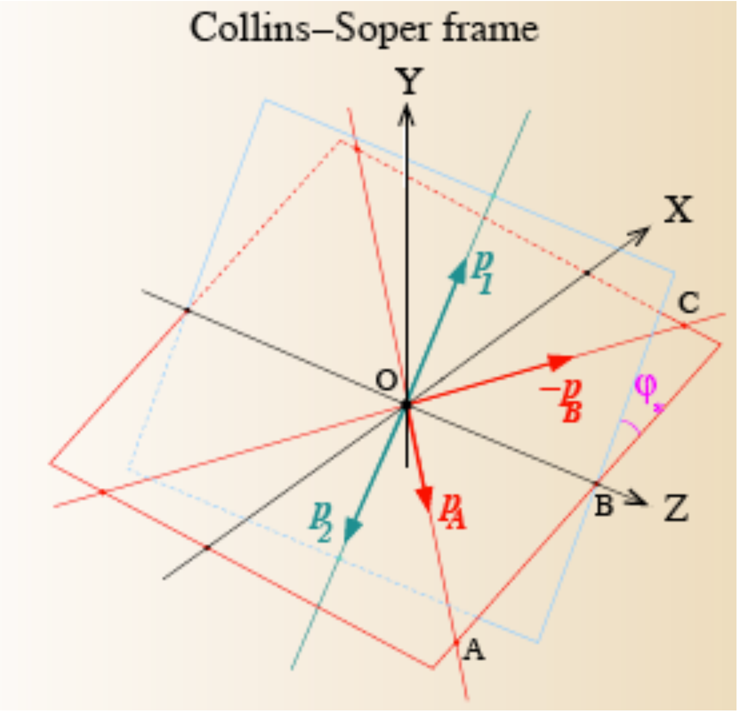


FIG. 1. Kinematics of the Drell-Yan process in the lepton center of mass frame.



Lam-Tung relation

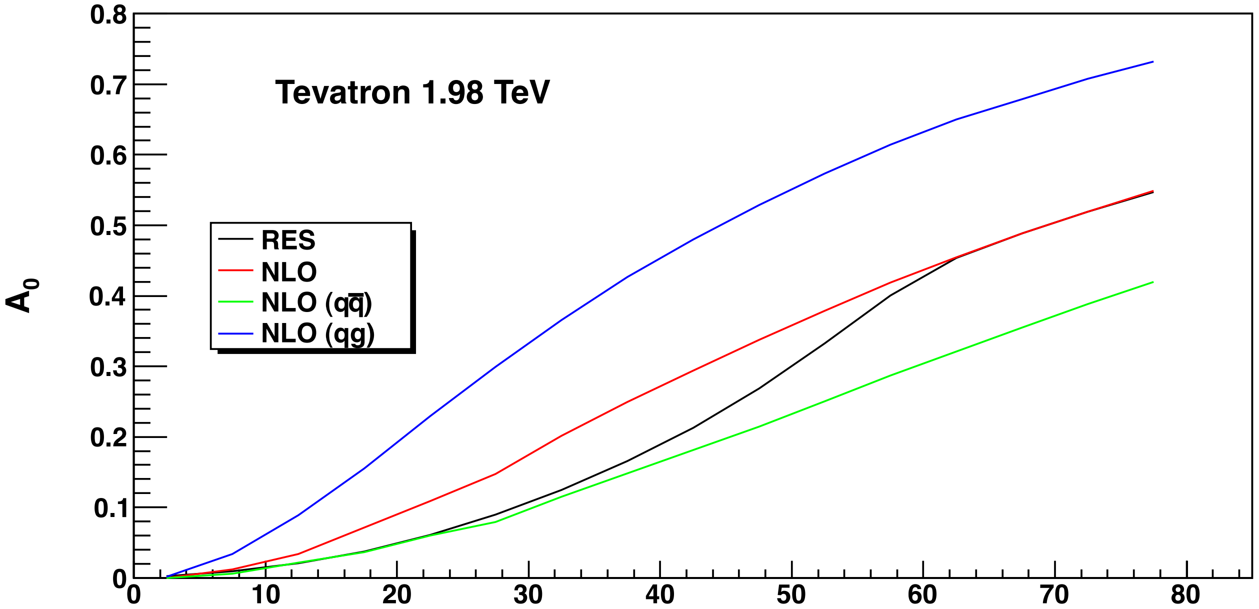
PHYSICAL REVIEW D **16,** 2219 (1977)

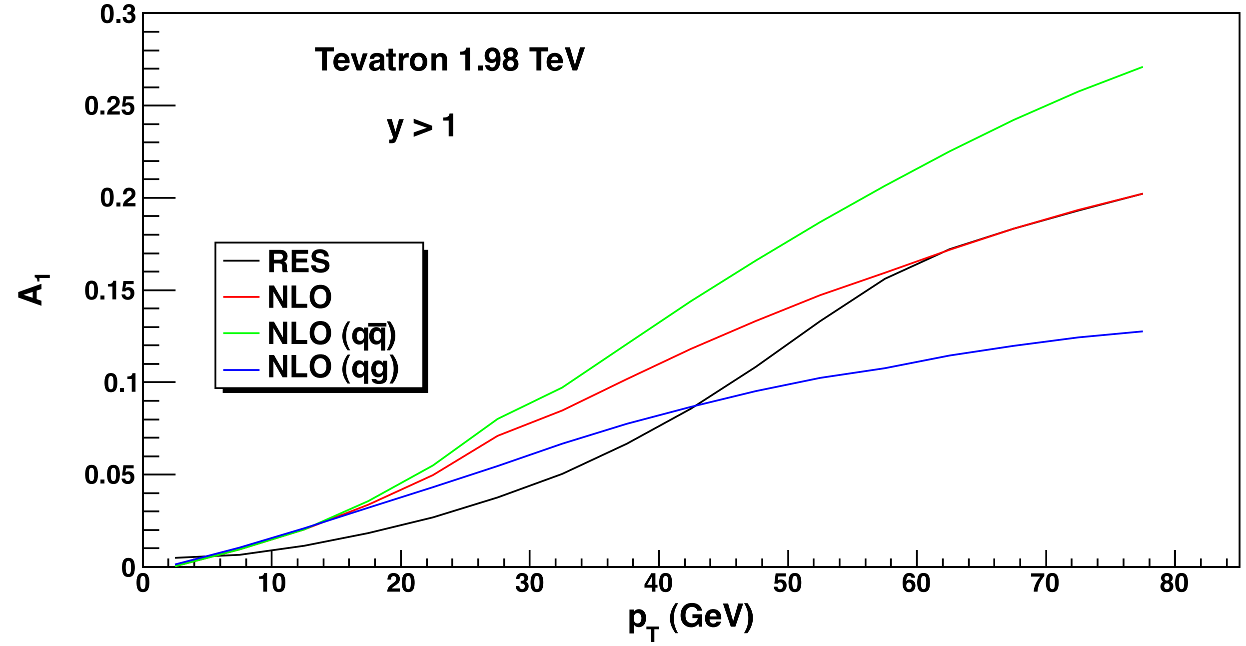
$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta + (\tfrac{1}{2} - \tfrac{3}{2}\cos^2\theta)A_0 + 2\cos\theta\sin\theta\cos\phi\, A_1 + \tfrac{1}{2}\sin^2\theta\cos 2\phi\, A_2 ,$$

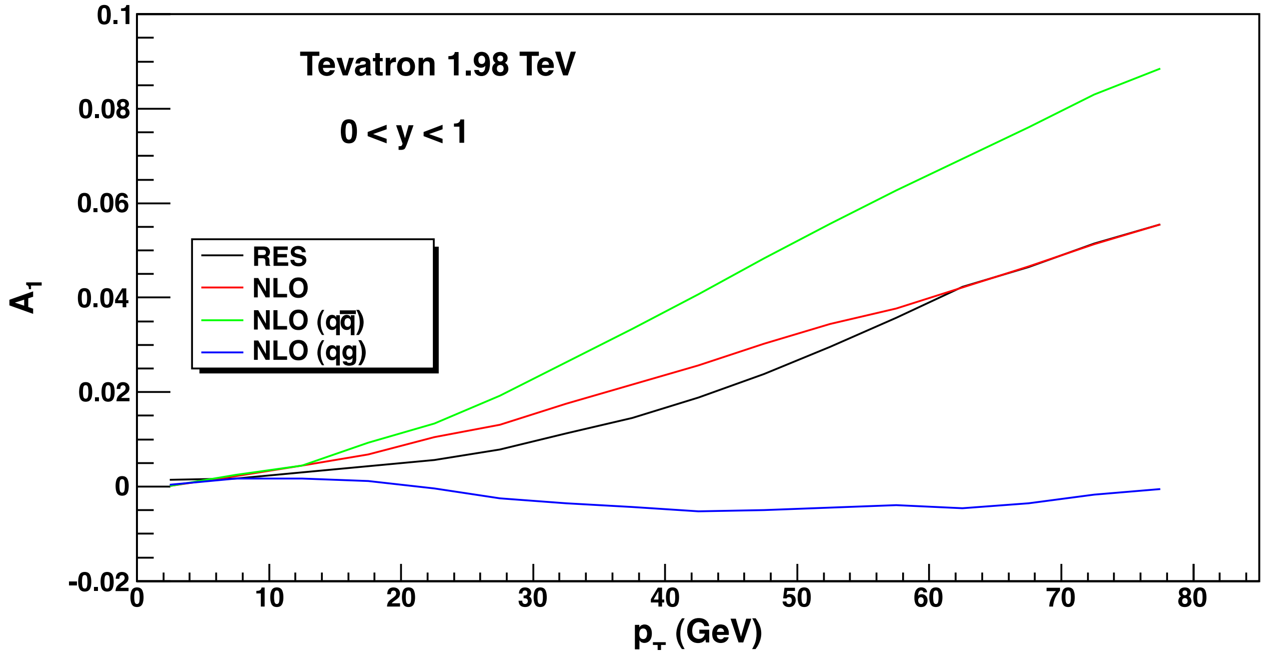
$$A_2 = A_0$$

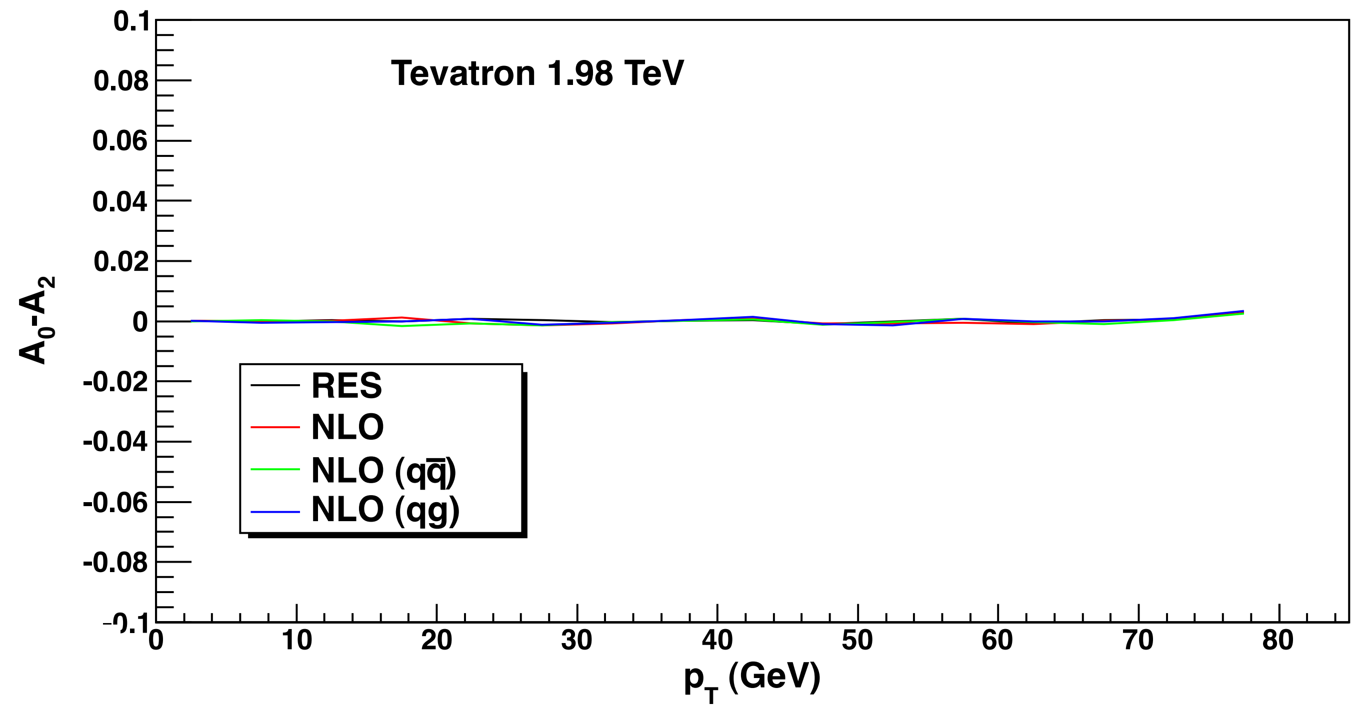
PHYSICAL REVIEW D **73,** 052001 (2006)

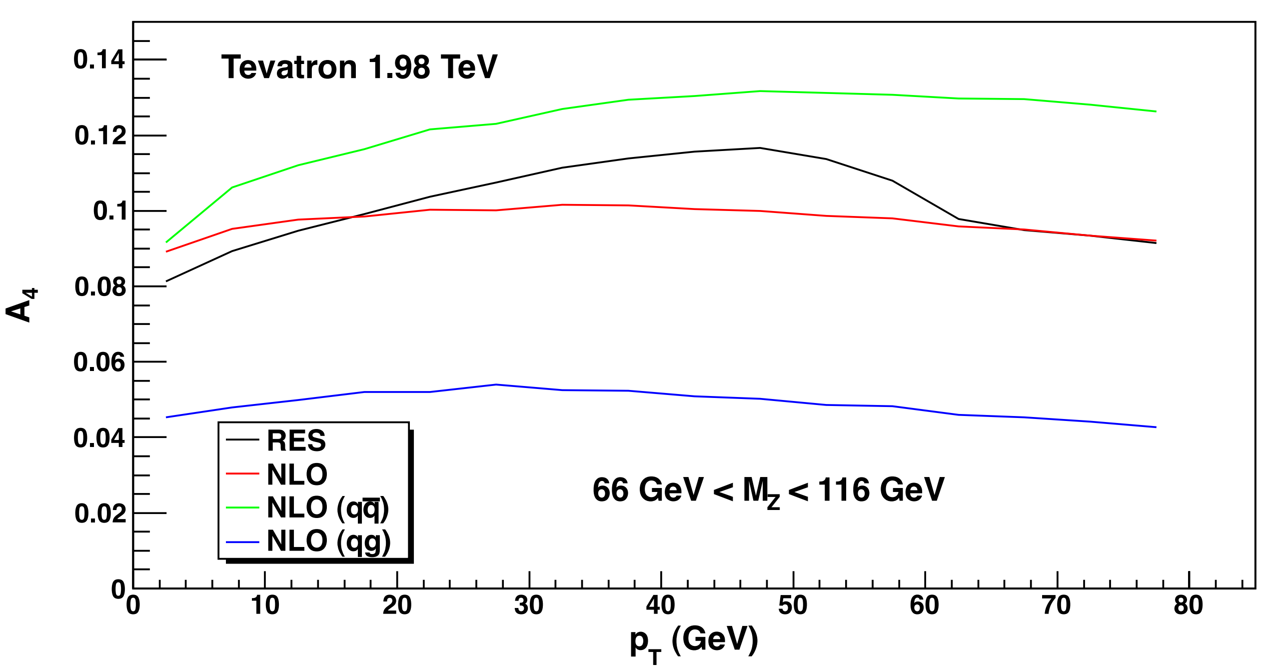
$$\frac{d\sigma}{dq_T^2\, dy\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^u}{dq_T^2\, dy}[(1+\cos^2\theta) + \tfrac{1}{2}A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi + \tfrac{1}{2}A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi], \quad (1)$$





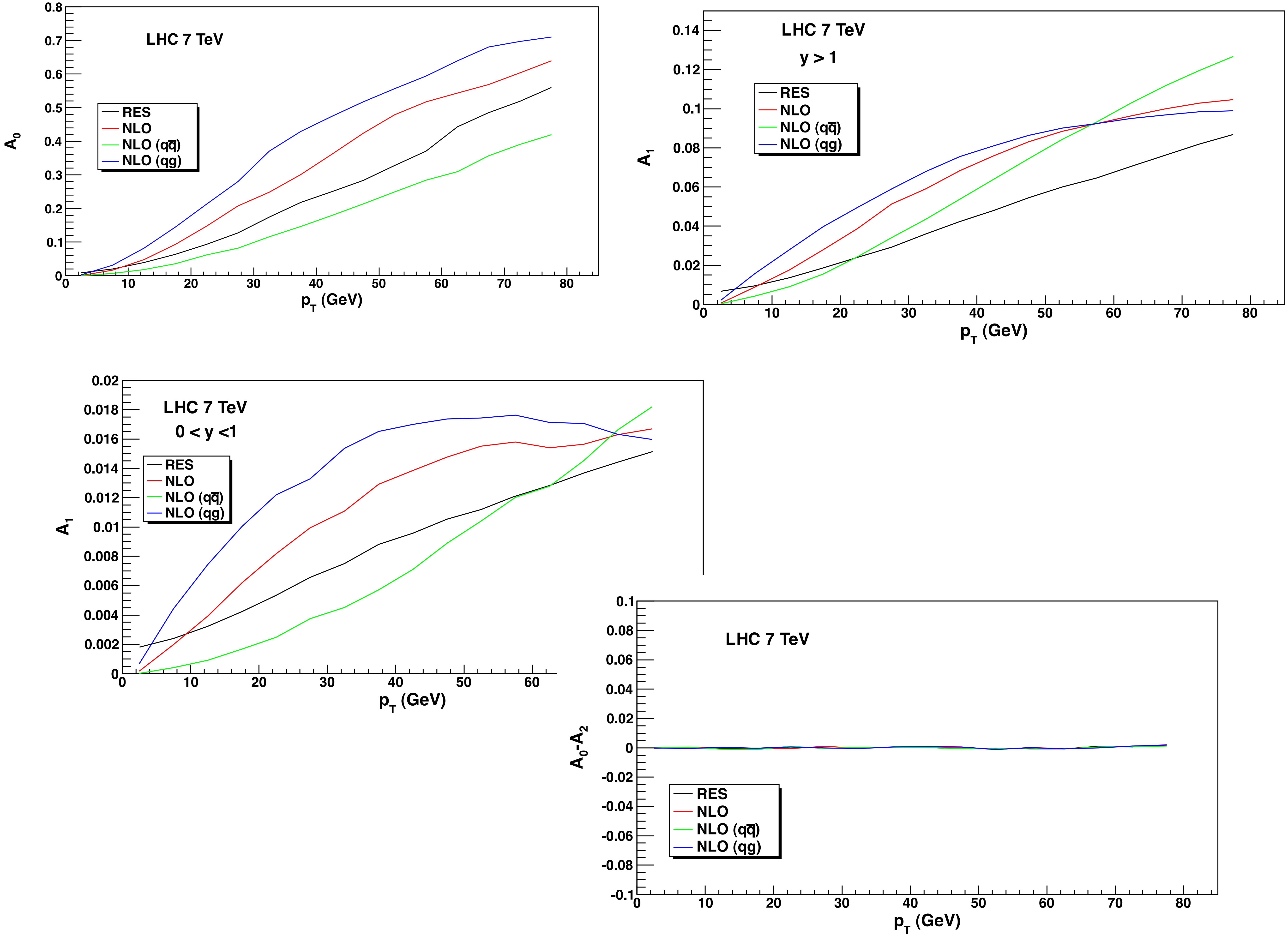






$$A_2 = A_0$$

Lam-Tung relation

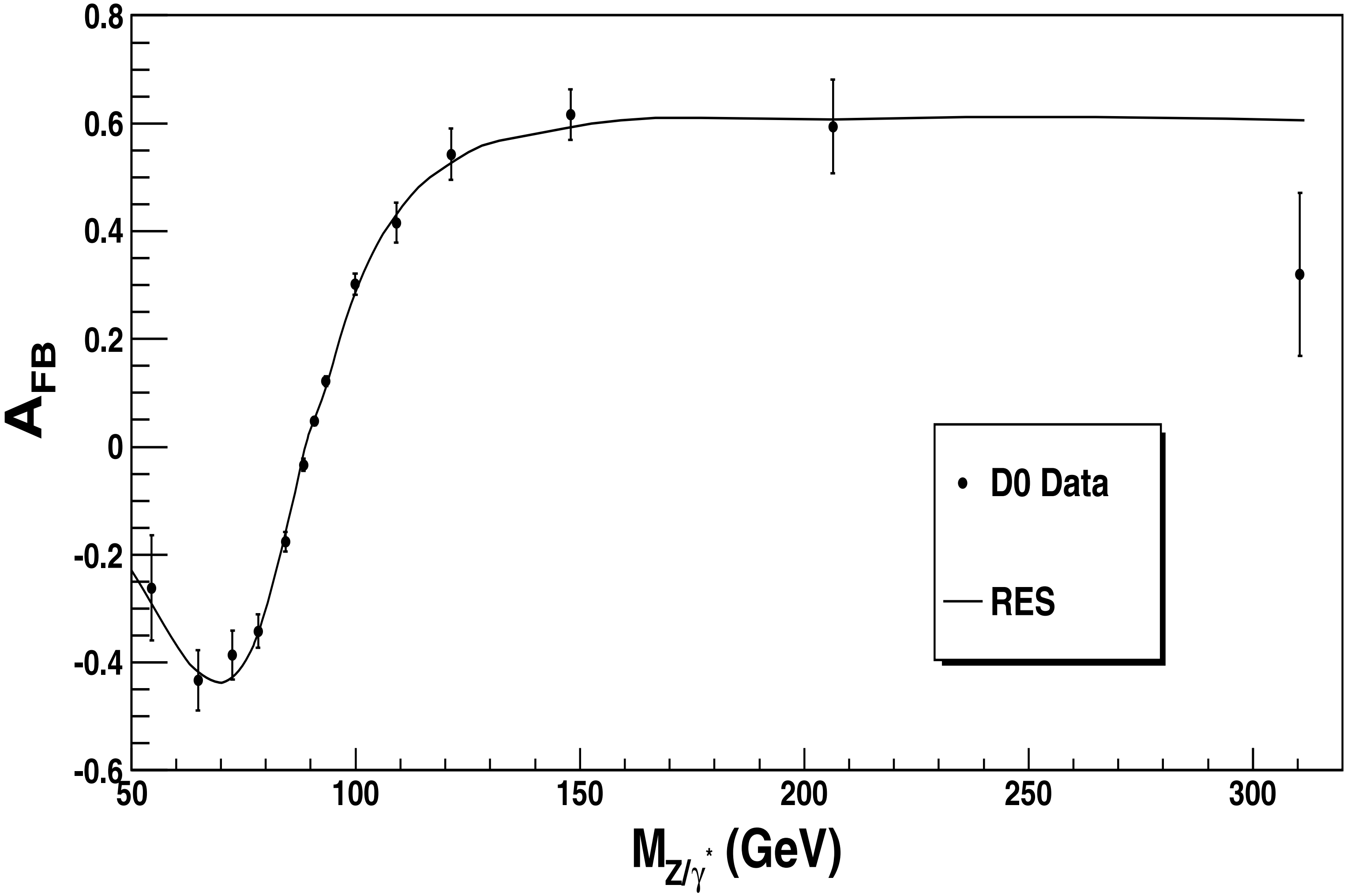


LHC 7 TeV
A0
pT (GeV)
RES
NLO
NLO (qq̄)
NLO (qg)
LHC 7 TeV
y > 1
A1
pT (GeV)
RES
NLO
NLO (qq̄)
NLO (qg)
LHC 7 TeV
0 < y <1
A1
pT (GeV)
RES
NLO
NLO (qq̄)
NLO (qg)
LHC 7 TeV
A0-A2
pT (GeV)
RES
NLO
NLO (qq̄)
NLO (qg)

Make Precision Tests possible

- Weak-mixing angle
- Z boson couplings to up- and down-type quarks.
  - This could not be done at LEP-I or SLC.
  - It is correlated to the initial state PDFs.

ResBos vs D0 Run-2 A_{FB} data



Conclusion

- ResBos is a useful tool for studying electroweak gauge bosons and Higgs bosons at the Tevatron and the LHC.
- It includes not only QCD resummation for low q_T region but also higher order effect in high q_T region, with spin correlations included via gauge invariant set of matrix elements.

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