The Magic of Units

1) $[c] = m/s$

$[\hbar] = J\cdot s = MeV\cdot s = \dfrac{[mass][length]^2}{[time]}$ $\qquad \hbar \equiv \dfrac{h}{2\pi}$

$[e] = \text{Coulomb} = \sqrt{\dfrac{[mass][length]^3}{[time]^2}}$ $\leftarrow$ $\left(\dfrac{e^2}{r} = ma\right)$

$$\boxed{\hbar c = 197.3 \text{ MeV}\cdot\text{fm}}$$

$$\boxed{\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.036}}$$

(1) Uncertainty principle : $\Delta p \Delta x \sim \hbar$, $\Delta E \Delta t \sim \hbar$ $\leftarrow$ (MeV·s)

Classical physics $\xrightarrow{\hbar}$ quantum physics

Classical physics $\xrightarrow{\hbar c}$ quantum field theory

Classical physics $\xrightarrow{c}$ relativistic kinematics

(2) The strength of E.M. interaction $\sim e^2$.

(3) The mass of electron: $m_e = 0.511 \dfrac{MeV}{c^2}$

The mass of proton: $m_p = 938.27 \dfrac{MeV}{c^2}$

$\sim 1 \dfrac{GeV}{c^2}$

(mass of neutron: $m_n = 939.57 \dfrac{MeV}{c^2}$)

2) Take $\hbar = c = 1$

$c = 3\times10^8\ m/s \to 1 \Rightarrow 1\ sec = 3\times10^8\ m$

$\hbar c = 197.3\ MeV\cdot fm \Rightarrow 1\ fm = \frac{1}{197.3\ MeV}$

$\sim \frac{1}{200\ MeV}$

$(1\ fm = 10^{-15}\ m)$

Thus, $[length] = [time] = \frac{1}{MeV}$

$GeV = 10^3\ MeV$, $\quad MeV = 10^6\ eV$

$TeV = 10^3\ GeV$, $\quad 1\ eV = 1.6\times10^{-19}\ J$

$= 10^{12}\ eV$

3) Estimate the relavent scales

(1) Strong interaction.

$1\ fm \sim \frac{1}{200\ MeV}$

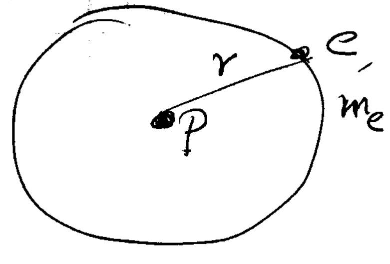
nucleus size

nucleus energy scale

$\sim (200\ MeV)$

② E.M. interaction

Bohr radius of H-atom



$$v \sim \alpha c$$

$$(m_e v)\, a_0 \sim \hbar$$

$$\Rightarrow a_0 \sim \frac{\hbar}{m_e v} = \frac{\hbar}{m_e \alpha c} = \frac{\hbar c}{m_e \alpha c^2} = \frac{\hbar c}{m_e \left(\frac{e^2}{\hbar c}\right) c^2}$$

$$\left(\hbar = c = 1,\ m_e = 0.511\ \text{MeV},\ \alpha = \frac{1}{137}\right)$$

$$\Rightarrow a_0 \sim \frac{1}{(0.511)\cdot\left(\frac{1}{137}\right)\ \text{MeV}} = \frac{197\ \text{fm}}{(0.511)\left(\frac{1}{137}\right)}$$

$$\sim 4\ \text{Å} \qquad (1\ \text{Å} \sim 10^{-10}\ \text{m})$$

atom size

Also, the energy level

$$\frac{1}{2} m_e v^2 = \frac{1}{2} m_e (\alpha c)^2 = \frac{1}{2}(0.511\ \text{MeV})\cdot\left(\frac{1}{137}\right)^2$$

$$\sim 13.6\ (\text{eV})$$

Or, $1\ \text{Å} \sim 10^{-10}\ \text{m} = 10^5\ \text{fm} = 10^5 \cdot \frac{1}{197\ \text{MeV}} = \frac{1}{1970\ (\text{eV})}$

$$\Rightarrow \text{energy level} \sim \alpha \cdot (1970\ \text{eV})$$

$$\sim \left(\frac{1}{137}\right)(1970\ \text{eV})$$

$$\sim 14.4\ \text{eV} \quad (\sim 13.6\ \text{eV})$$

4) Compton wavelength $\lambdabar = \frac{\hbar}{Mc}$

$$\lambdabar(1\,\text{GeV}) \sim \frac{\hbar c}{Mc^2} = \frac{1}{M} = \frac{197\ \text{MeV}\cdot\text{fm}}{1\ \text{GeV}} = 0.2\ \text{fm}$$

$\Rightarrow$

$$1\ \text{GeV} \xrightarrow{\text{Probing}} 10^{-16}\ \text{m}$$
$$100\ \text{GeV} \longrightarrow 10^{-18}\ \text{m}$$
$$1\ \text{TeV} \longrightarrow 10^{-19}\ \text{m}$$

$(\Delta p \Delta x \sim \hbar)$

$(E \sim pc \sim p)$

5) Cross Sections:

(Cross Section) $\sim$ (area) $\sim m^2$

Define

$$\boxed{1\ \text{barn} = 10^{-24}\,(\text{cm})^2 = 10^{-28}\,m^2 = 100\,(\text{fm})^2}$$

1

$1\ \text{mb} \sim 10^{-3}\ \text{b}$

$1\ \mu\text{b} \sim 10^{-6}\ \text{b}$

$1\ \text{nb} \sim 10^{-9}\ \text{b}$

$1\ \text{pb} \sim 10^{-12}\ \text{b}$

$1\ \text{fb} \sim 10^{-15}\ \text{b}$

(2) The cross section of a low energy photon ($E_\gamma \ll m_e c^2$) scattered by a non-relativistic (almost at rest) electron (Thomson scattering).

$$\sigma_{Tot}^{classical} = 2\pi r_e^2$$ (from classical physics: reflection + diffraction)

where the "classical radius of electron"

$$r_e = \frac{e^2}{m_e c^2}$$

Definition: $\left(\frac{e^2}{r_e} \equiv m_e c^2 \right)$

$\frac{e^2}{r_e}$: electrostatic potential energy

$m_e c^2$: rest energy / mass

$$\sim \frac{\left(\frac{1}{137}\right)}{0.511 \text{ MeV}}$$

$$\sim \frac{\frac{1}{137} \cdot 197 \text{ fm}}{0.511}$$

$$\sim (2.8 \text{ fm})$$

Actually, from quantum physics,

$$\sigma_T = \left(\frac{8}{3}\right) \pi r_e^2 = 67 \ (\text{fm})^2$$

$$= 0.67 \text{ barn}$$

③ Cross sections at high energy colliders

Rough estimate: $\sigma \sim \pi \lambda\!\!\!^{-}{}_p^2$ (for proton-proton scattering)

$$\sim \frac{\pi}{(1\,\mathrm{GeV})^2} = \frac{\pi\ (fm)^2}{10^6 \cdot \left(\frac{1}{197}\right)^2}$$

$$\sim 10^{-3} b = mb$$

It turns out that

at LHC, 14 TeV p-p collider,

$$\sigma(pp)_{total} \sim 110\ mb$$

at Tevatron, 1.96 TeV p-$\bar{p}$ collider,

$$\sigma(p\bar{p})_{tot} \sim 60\ mb.$$

Note. If we take the $\Lambda_{QCD} \sim 200$ MeV, then

$$\sigma \sim 2\pi \left(\frac{1}{\Lambda_{QCD}}\right)^2 \sim 50\ mb$$