Weak Interactions

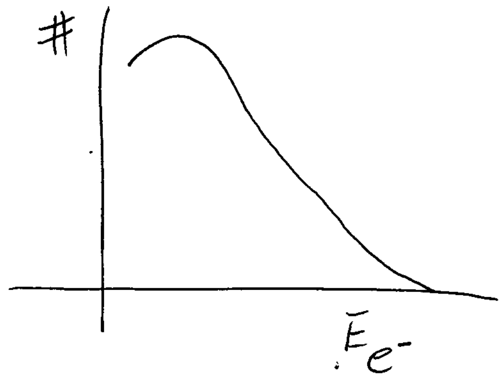
1. Nuclear β-decay

$A \to B + e^-$

${}^{40}_{19}K \quad {}^{40}_{20}Ca$

${}^{64}_{29}Cu \quad {}^{64}_{30}Zn$

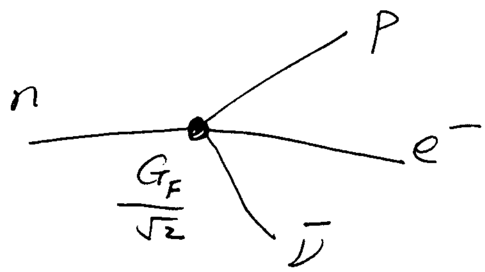
${}^{3}_{1}H \quad {}^{3}_{2}He$



Neil Bohr — ready to abandon the law of Conservation of energy

Pauli (1930) — proposed neutrino $A \to B + e^- + \bar{\nu}$
(Before the discovery of neutron, 1932 by Chadwick)

Fermi (1933) — Fermi theory
$n \to p^+ + e^- + \bar{\nu}$



$\sigma(\nu n \to p e^-) \sim G_F^2 \cdot s$
(Violate unitarity at $\sqrt{s} \gtrsim 300$ GeV)
$G_F \sim 10^{-5}\ (\text{GeV})^{-2}$

2. Charged current V − A theory

V = vector (γ_μ), A = axial vector ($\gamma_\mu \gamma_5$), $\bar{e}\gamma_\mu(1-\gamma_5)\nu$

1) Proposed by Marshak & Sudarshan
and Feynman & Gell-Mann

2) Weak hadronic current

① $\Delta S = 0$ (S = strangeness) $n \rightarrow p\, e^-\, \bar{\nu}_e$

② $\Delta S = 1$ $K^+ \rightarrow \mu^+ \nu_\mu$
$K^+ = (u\bar{s})$ ← Quark model

3) Cabibbo proposed θ_c in 1963

$$\frac{G_F}{\sqrt{2}}\left[\cos\theta_c\, J^\mu_{\Delta S=0} + \sin\theta_c\, J^\mu_{\Delta S=1}\right]\bar{e}\gamma_\mu(1-\gamma_5)\nu$$

(None)

3. Flavor changing neutral current (FCNC) & charm quark

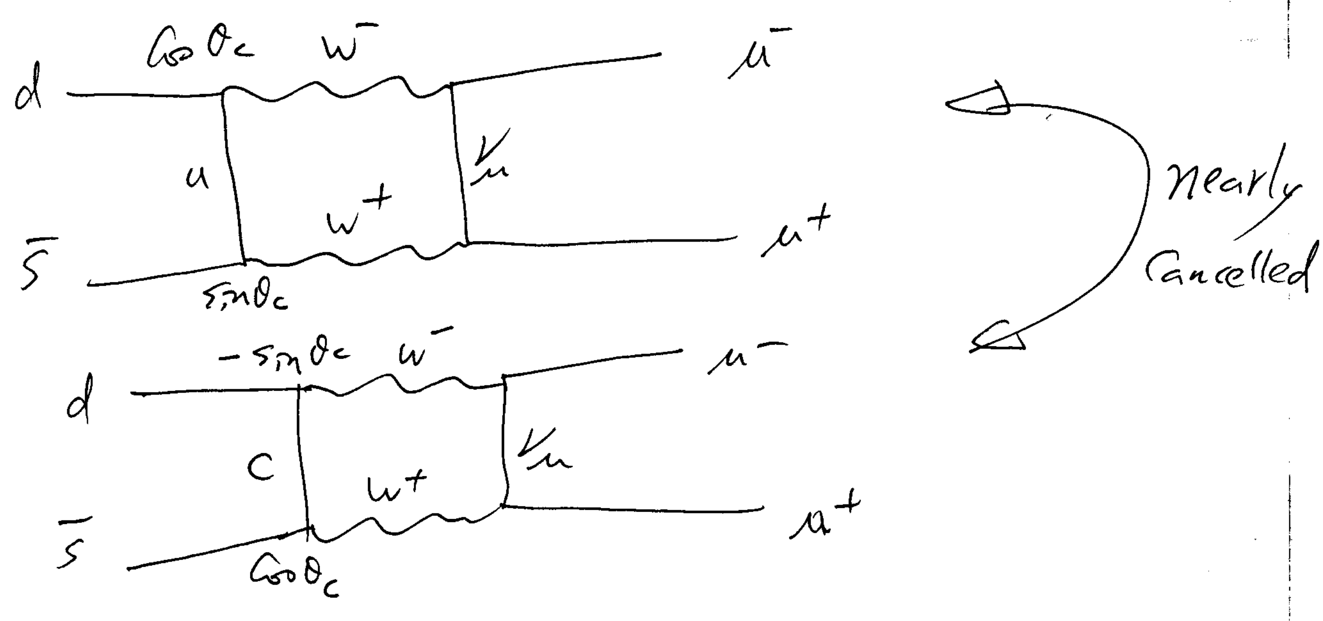
1) Until 1973, only weak charged currents were observed.
There is NO FCNC process observed.

2) charm quark was
— first proposed by Bjorken & Glashow
(for no "obvious" reason, — no data crisis)

3) In 1970, Glashow, Iliopoulos, Maiani (GIM) proposed c-quark to explain why the

$K_L^0 \to \mu^+\mu^-$ (FCNC process) $\Delta S = 1$

decay rate is extremely small.



4) Gaillard & B.W. Lee predicted the c-quark mass to be about 1.5 ~ 2.0 GeV before the discovery of J/ψ $(c\bar{c})$, from GIM process.
$(m_{J/\psi} \sim 3$ GeV$)$.
1974

5) In 1968, Weinberg's electroweak theory, for leptons only, already includes W^+, W^- and Z^0 bosons.

By 1974, the elementary particles are

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} u \\ d \end{pmatrix}_{red} \begin{pmatrix} u \\ d \end{pmatrix}_{blue} \begin{pmatrix} u \\ d \end{pmatrix}_{green}$$

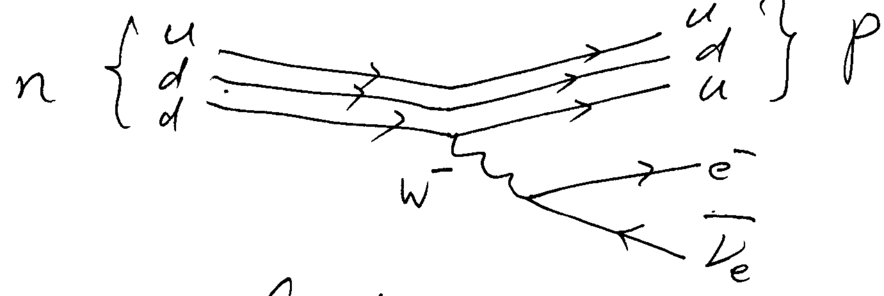
$$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix}_{red} \begin{pmatrix} c \\ s \end{pmatrix}_{blue} \begin{pmatrix} c \\ s \end{pmatrix}_{green}$$

(+) gauge bosons: γ, W^+, W^-, Z^0

and g (gluon)

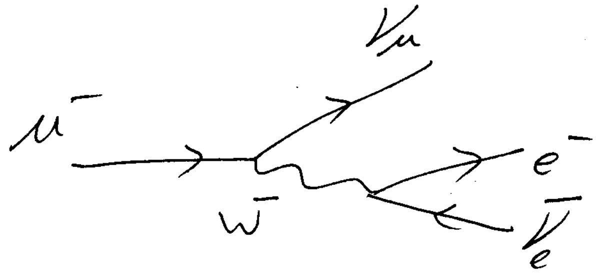
4. Standard Model's picture

1) β-decay ($n \to p + e^- + \bar{\nu}_e$)



mean lifetime (τ) $\simeq$ 886 sec

2) $\mu^- \to e^- \nu_\mu \bar{\nu}_e$



$$\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

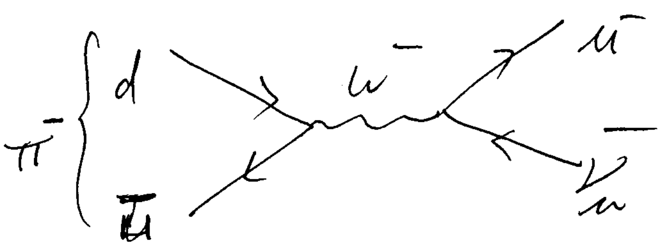
$M_\mu = 105.6$ MeV

$\tau = \frac{1}{\Gamma} = 2.2 \times 10^{-6}$ sec

$c\tau = 660$ m

$$\left(G_F = \frac{g_W^2}{4\sqrt{2} M_W^2} \right)$$

3) $\pi^- \to \mu^- \bar{\nu}_\mu$



$\tau = 2.6 \times 10^{-8}$ s

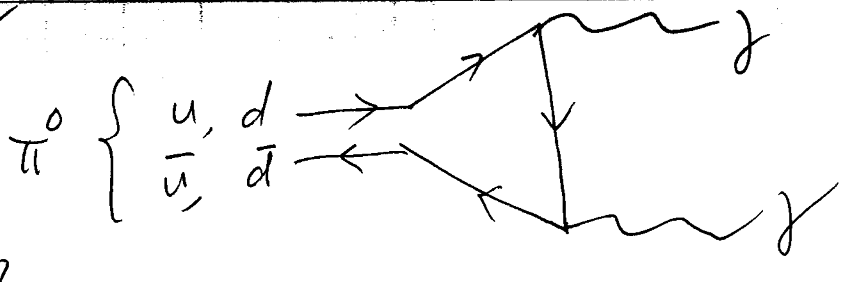
$c\tau = 7.8$ m

$m_{\pi^+} = 139.6$ MeV

$$\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) = \frac{f_\pi^2}{8\pi m_\pi^3} G_F^2 m_\mu^2 (m_\pi^2 - m_\mu^2)^2$$

$$\Rightarrow \frac{\Gamma(\pi \to e \bar{\nu}_e)}{\Gamma(\pi \to \mu \bar{\nu}_\mu)} \simeq 1.3 \times 10^{-4}$$

4) $\pi^0 \rightarrow \gamma\gamma$



$\tau = 8.4 \times 10^{-17}$ sec

$c\tau = 25$ nm

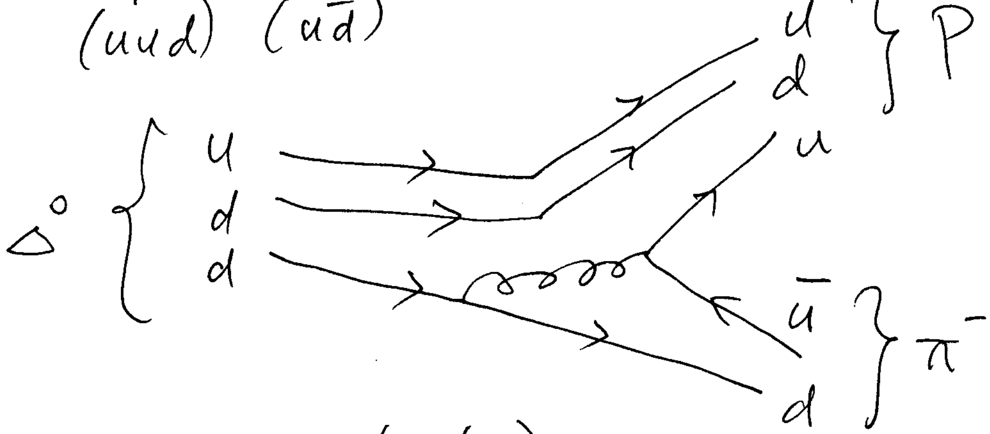
$m_\pi = 135$ MeV

$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

(It decays via E.M interaction)

5) $\Delta^0 \rightarrow p\ \pi^-$

(udd) (uud) ($u\bar{d}$)



(It decays via strong interaction)

$m_{\Delta^0} \simeq 1200$ MeV

$\Gamma_{\Delta^0} \simeq 120$ MeV

$(\Gamma \sim \alpha_s M)$

5. CKM matrix

Cabibbo-Kobayashi-Maskawa

1) Suggested by Cabibbo in 1963

$$(\bar{u}\ \bar{c})\begin{pmatrix}\cos\theta_c & \sin\theta \\ -\sin\theta_c & \cos\theta_c\end{pmatrix}\begin{pmatrix}d \\ s\end{pmatrix}$$

2) GIM in 1970 applied it to FCNC process ($K_L^0 \to \mu^+\mu^-$)

3) Kobayashi-Maskawa extended to 3-generations (before the discovery of charm). in 1973

4) The essential idea is that "mass eigenstates" are not the same as the "weak eigenstates" of quarks.

① Instead of $\begin{pmatrix}u \\ d\end{pmatrix}, \begin{pmatrix}c \\ s\end{pmatrix}, \begin{pmatrix}t \\ b\end{pmatrix}$

the weak gauge bosons couple to

$$\begin{pmatrix}u \\ d'\end{pmatrix}, \begin{pmatrix}c \\ s'\end{pmatrix}, \begin{pmatrix}t \\ b'\end{pmatrix}$$

with

$$\begin{pmatrix}d' \\ s' \\ b'\end{pmatrix} = V_{CKM}\begin{pmatrix}d \\ s \\ b\end{pmatrix}$$

(2) The CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & 0 & s_1 e^{-i\delta} \\ 0 & 1 & 0 \\ -s_1 e^{i\delta} & 0 & c_1 \end{pmatrix} \begin{pmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with $c_1 = \cos\theta_1$, $s_1 = \sin\theta_1$, etc. $\Rightarrow$ (3 angles $\theta_1, \theta_2, \theta_3$ $\oplus$ 1 CP-violating angle δ)

(3) Wolfstein's parametrization

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

with

$\lambda = \sin\theta_c \simeq 0.22$

$A = 0.8 \pm 0.075$

$\eta \simeq 0.33 \pm 0.05$

$\rho \simeq 0.20 \pm 0.09$