1 Preliminaries

There are four 8-propagator families in two-loop five-light-parton scattering processes, namely, double-pentagon(dp), hexa-box(hb), penta-box(pb) and hexa-triangle(ht). Each family has five external momenta $p_1, \ldots, p_5$ satisfying the on-shell conditions $p_i^2 = 0$ and momentum conservation $\sum p_i = 0$. The Feynman diagrams and corresponding denominators are shown in the following.

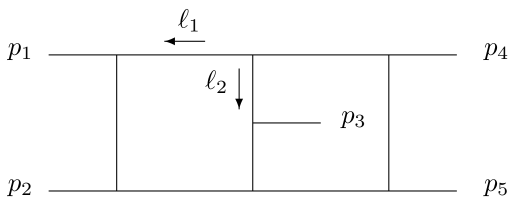

Figure 1: dp

The denominators for dp family are

$$\mathcal{D}_{\rm dp} = \{\ell_1^2,\, (\ell_1+p_1)^2,\, (\ell_1+p_1+p_2)^2,\, \ell_2^2,\, (\ell_2+p_3)^2,\, (\ell_1+\ell_2+p_1+p_2+p_3)^2,\\ (\ell_1+\ell_2-p_4)^2,\, (\ell_1+\ell_2)^2,\, (\ell_2+p_1)^2,\, (\ell_2+p_2)^2,\, (\ell_2+p_4)^2\}. \tag{1}$$

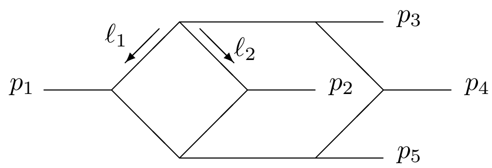

Figure 2: hb

The denominators for hb family are

$$\mathcal{D}_{\rm hb} = \{\ell_1^2,\, (\ell_1+p_1)^2,\, (\ell_2+p_2)^2, \ell_2^2,\, (\ell_1+\ell_2+p_1+p_2)^2,\, (\ell_1+\ell_2-p_3-p_4)^2,\\ (\ell_1+\ell_2-p_3)^2,\, (\ell_1+\ell_2)^2,\, (\ell_2+p_1)^2,\, (\ell_2+p_3)^2,\, (\ell_2+p_4)^2\}. \tag{2}$$

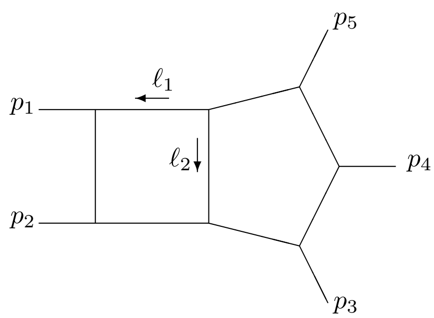

Figure 3: pb

The denominators for pb family are

$$\mathcal{D}_{\rm pb} = \{\ell_1^2,\, (\ell_1+p_1)^2,\, (\ell_1+p_1+p_2)^2,\, \ell_2^2,\, (\ell_1+\ell_2+p_1+p_2)^2,\, (\ell_1+\ell_2+p_1+p_2+p_3)^2,\\ (\ell_1+\ell_2+p_1+p_2+p_3+p_4)^2,\, (\ell_1+\ell_2)^2,\, (\ell_2+p_2)^2,\, (\ell_2+p_3)^2,\, (\ell_2+p_4)^2\}. \tag{3}$$

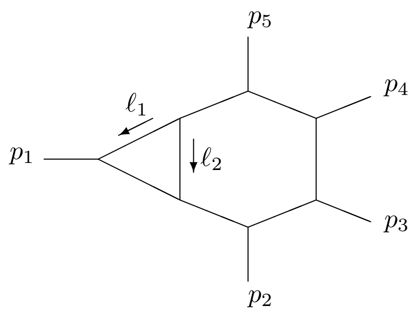

Figure 4: ht

The denominators for ht family are

$$\mathcal{D}_{\text{ht}} = \{\ell_1^2,\ (\ell_1+p_1)^2,\ \ell_2^2,\ (\ell_1+\ell_2+p_1)^2,\ (\ell_1+\ell_2+p_1+p_2)^2,\ (\ell_1+\ell_2+p_1+p_2+p_3)^2,$$
$$(\ell_1+\ell_2+p_1+p_2+p_3+p_4)^2,\ (\ell_1+\ell_2)^2,\ (\ell_2+p_2)^2,\ (\ell_2+p_3)^2,\ (\ell_2+p_4)^2\}. \tag{4}$$

For each family, we also include three irreducible scalar products(ISPs) besides inverse propagators for completeness, as can be seen in the denominator sets. As a result, any scalar integral can be identified by family ID and powers of corresponding denominators $\text{j}[\text{family}, n_1, \cdots, n_{11}]$. For example, we have

$$\text{j}[\text{dp}, 1, 1, 1, 1, 1, 1, 1, 1, -3, -1, 0] \quad \Longleftrightarrow \quad \int \prod_{i=1}^{2} \frac{\text{d}^D \ell_i}{\text{i}\pi^{D/2}} \frac{\mathcal{D}_9^3 \mathcal{D}_{10}}{\mathcal{D}_1 \cdots \mathcal{D}_8} \quad \text{with } \mathcal{D}_i \in \mathcal{D}_{\text{dp}}. \tag{5}$$

We define the sector $\text{js}[\text{family}, \theta_1, \cdots, \theta_{11}]$ for integral $\text{j}[\text{family}, n_1, \cdots, n_{11}]$ as follows:

$$\theta_n = \Theta(n_i - 1/2), \quad i = 1, \cdots, 11, \tag{6}$$

where $\Theta(x)$ is the Heaviside step function. For convenience, we also introduce the definition of sector ID S for a sector by

$$S = \sum_{i=1}^{11} \theta_i \cdot 2^{j-1}. \tag{7}$$

For example, the sector of $\text{j}[\text{dp}, 1, 1, 1, 1, 1, 1, 1, 1, -3, -1, 0]$ is $\text{js}[\text{dp}, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0]$ and the sector ID is 255.

2 The block-triangle reduction relations

In the attachment, we include block-triangle reduction relations for the four families in an implicit from. To be more precise,

`tops\family\integrals` contains all integrals in each family;

`tops\family\symrel` for dp, hb and pb, we have symmetry relations among different sectors to minimize the number of master integrals, e.g.,
$\text{j}[\text{dp}, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0] = \text{j}[\text{dp}, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0]$.

`tops\family\fitM\...` contain all rational numbers present in the reduction relations. these files have one-to-one correspondence to those in scheme;

`tops\family\scheme\...` tell us how to restore the relations, which are ordered by complexity of involved integrals, i.e. from simple sectors to complex sectors. relations among the same sector may be split into several blocks too.

To obtain readable relations, just run the package ToRel.wl. Then reduction relations will be exported to `tops\family\relations\...`. All the relations are stored by a two-dimensional vector $\{\{c_1, I_1\}, \cdots, \{c_n, I_n\}\}$, which means $\sum_{i=1}^{n} c_i I_i = 0$, where c_i are coefficients, I_i are integrals.