# Shear Flows and Transitions in a ‘Tangled’ Magnetic Field

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PSFC Seminar, MIT

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# Contributions from many, yet especially:

- Samantha Chen, UC San Diego
- Rameswar Singh, UC San Diego
- Robin Heinonen, UC San Diego
- Steve Tobias, Univ. of Leeds
- Arash Ashourvan, PPPL
- Guilhem Dif-Pradalier, CEA
- Weixin Guo, HUST

# Outline

- Why? → Some thoughts — history
- Shear Flows: OV + Selected Recent Developments
  - real space: Patterns and staircases
  - k-space: Noise + Modulation
- Disordered Magnetic Fields:
  - planar tangled field: $\beta$ –plane MHD and 'viscosity' in solar tachocline
  - stochastic magnetic field: Reynolds stress decoherence and L-H Threshold with RMP

Flows and particle in $\tilde{B}$.

- Other thoughts + Look Ahead

# Part I:

# Why?  - Some Philosophy…

# Evolution of MFE Theory

Prehistory: 3D

- Beginnings: 60's ~ 1980

| | |
|---|---|
| Trieste | T3 |
| Micro-stability | Alcator A |
| Neoclassical theory | PLT |
| Disruption models | TFTR |
| Taylor Relaxation | |

- Understanding Good Confinement: 1980 ~ 2010

[Self-Organization]

| | |
|---|---|
| ExB shear, ZF's | ASDEX → H-mode (1982) |
| Transport Bifurcations | Alcator C, C-Mod → pellet, n-limit |
| Gyrokinetics, Simulation | TFTR, JET → D-T |
| AE modes | DIII-D → ETBs, ITBs |
| Intrinsic Rotation | JT-60U → ETBs, ITBs |

# Evolution of MFE Theory, cont'd

Kikuchi

- Good Confinement + Good Power Handling → ITER: 2010 – Present, and beyond

QH → EHO, turb. ped.

| | |
|---|---|
| ELMs, Peeling-Ballooning | DIII-D, AUG |
| RMP, QH-mode | Alcator C-Mod |
| Multi-scale problems | LHD |
| Core-Edge coupling, | W7X |
| Turbulence Spreading | RFX-QSH * |
| Disruptions (?) | EAST, KSTAR |
| SOL Heat Loads (?) | … |
| | … |

ELM mitigation

Cappello

N.B.:
Return to 3D !

➔ Theory must address trade-offs

➔ Challenge to understanding of confinement self-organization

# Shear Flows

Negative Triang ← TCV
weak H ELMs?
Good Conf.
DIII-D
I-mode
Alcator C-Mod

- Intensively Studied
- Not 'trendy' → c.f. contrast to Disruption, SOL heat load
- But:
  - much remains to understand
  - lots happening
- Renewed interest via:
  - LH transition – especially with RMP
  - Pedestal structure – c.f. Ashourvan, 2018
  - Density limit – c.f. Hajjar, et al '18, Hong, et al '18

# Part II:

# a) OV of Basic Shear Flow Physics

**For reviews, see:**

**- P.D. Itoh, Itoh, Hahm ’05, PPCF – ‘k-space’**
**- Gurcan, P.D.  ’15, J. Physics A – ‘patterns, real space’**
**- Hahm, P.D. ‘19, J. Korean Phys. Soc. – ‘Avalanches, spreading, and staircases’**

# Part II:

# b) Selected Recent Developments

– Staircases – ‘real space’

– c.f. Hahm, P.D. review

Dif-Pradalier N.F. ’17

– Noise + Modulation – ‘k-space’

– R. Singh, P.D. ~~submitted~~ ‘20

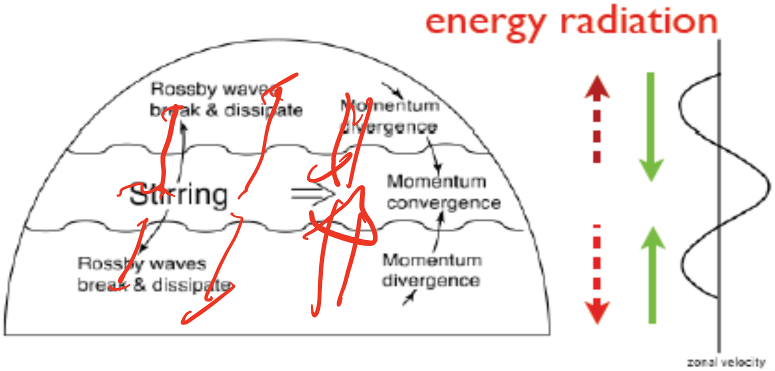
PPCF in press

→ How do Zonal Flow Form?

Simple Example: Zonally Averaged Mid-Latitude Circulation

- classic GFD example: Rossby waves + Zonal flow (c.f. Vallis '07, Held '01)
- Key Physics:

c.f. Rossby-Drift wave duality




Rossby Wave:

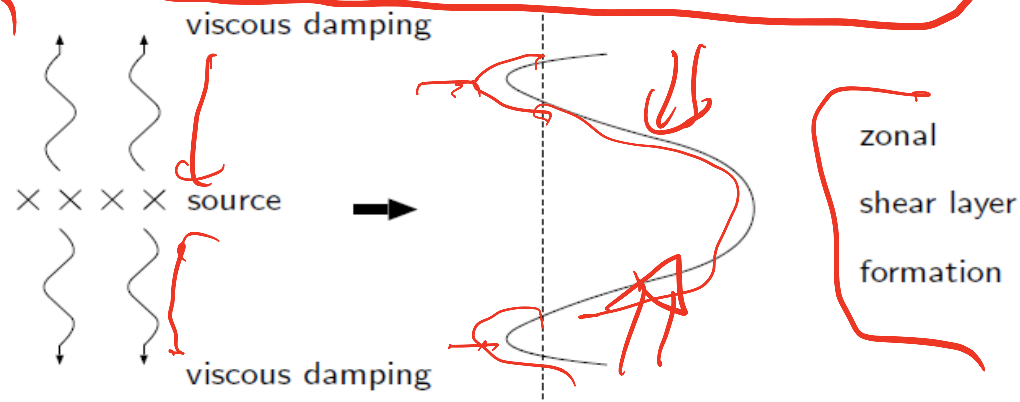
$$\omega_k = -\frac{\beta k_x}{k_\perp^2}$$

$$v_{gy} = 2\beta \frac{k_x k_y}{(k_\perp^2)^2}\,, \quad \langle \tilde{v}_y \tilde{v}_x \rangle = \sum_{\vec{k}} -k_x k_y \left|\hat{\phi}_{\vec{k}}\right|^2$$

$\therefore v_{gy} v_{phy} < 0$ → Backward wave!

$\boxed{\text{➔ Momentum convergence at stirring location}}$

- …“the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into this region.” (I. Held, ’01)
- Outgoing waves $\Rightarrow$ incoming wave momentum flux



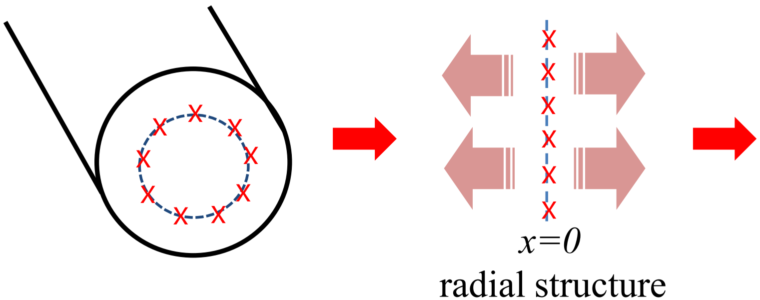

- Local Flow Direction (northern hemisphere):
  - eastward in source region
  - westward in sink region
  - set by $\beta > 0$ $\leftrightarrow$ $V_*$
  - Some similarity to spinodal decomposition phenomena
    → Both ‘negative diffusion’ phenomena
    → Cahn-Hilliard equation (c.f. Heinonen, P.D. ‘19, ‘20)

negative visc.

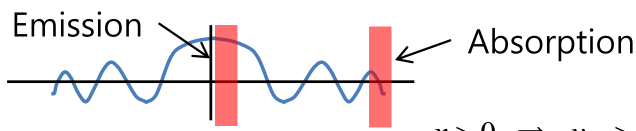
# Wave-Flows in Plasmas

## MFE perspective on Wave Transport in DW Turbulence

- localized source/instability drive intrinsic to drift wave structure




– couple to damping ↔ outgoing wave




– $v_{gr} = -2\rho_s^2 \dfrac{k_\theta k_r v_*}{(1+k_\perp^2\rho_s^2)^2}$

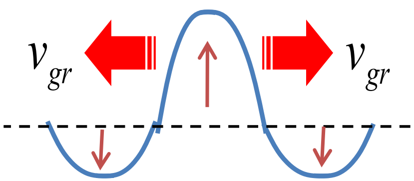
$\langle v_{rE} v_{\theta E} \rangle = -\dfrac{c^2}{B^2} |\phi_{\vec{k}}|^2 \, k_r k_\theta < 0$

$x > 0 \;\Rightarrow\; v_{gr} > 0$

$x < 0 \;\Rightarrow\; v_{gr} < 0$

$v_* < 0 \;\Rightarrow\; k_r k_\theta > 0$

- outgoing wave energy flux → incoming wave momentum flux
  → counter flow spin-up!




- zonal flow layers form at excitation regions

# Plasma Zonal Flows I

- What is a Zonal Flow? – Description?
  - $n = 0$ potential mode; $m = 0$ (ZFZF), with possible sideband (GAM)
  - toroidally, poloidally symmetric $ExB$ shear flow
- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport ($n = 0$)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

i.e. ZF's soak up turbulence energy

# Plasma Zonal Flows II

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
    → Zonal flow in magnetized plasma / QG fluid

    $$\frac{du}{dt} = 0$$

  - Kelvin's theorem is ultimate foundation
- Charge Balance → polarization charge flux → Reynolds force
  - Polarization charge → $-\rho^2\nabla^2\phi = n_{i,GC}(\phi) - n_e(\phi)$

    *polarization length scale* *ion GC* *electron density*

  - so $\Gamma_{i,GC} \neq \Gamma_e$ → $\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle \neq 0$ ↔ 'PV transport/mixing'

    *polarization flux* → What sets cross-phase?

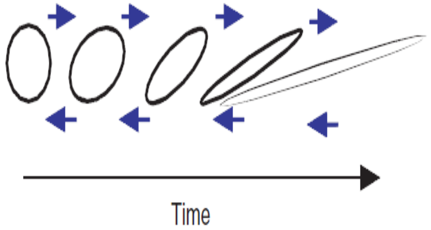
  - If 1 direction of symmetry (or near symmetry):

    $$-\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle = -\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$$ (Taylor, 1915) G. I. T.

    $-\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$ → Reynolds force → Flow

# Zonal Flows Shear Eddys I

- Shear Dispersion: (Kelvin, G.I. Taylor, Dupree'66, BDT'90)
  - radial scattering + $\langle V_E \rangle' \rightarrow$ hybrid decorrelation
  - $k_r^2 D_\perp \rightarrow (k_\theta^2 \langle V_E \rangle'^2 D_\perp / 3)^{1/3} = 1/\tau_c$
  - ➔ shearing enhances mixing!




- Other shearing effects:

  Response shift
  and dispersion

  - spatial resonance dispersion: $\omega - k_\parallel v_\parallel \Rightarrow \omega - k_\parallel v_\parallel - k_\theta \langle V_E \rangle' (r - r_0)$
  - differential response rotation → especially for kinetic curvature effects
  - Shear induced nonlinear Landau damping

TSH

- PV gradient also relevant – flow structure (Heinonen, P.D. '19 '20)

# Shearing II – Eddy Population

Langmuir V. Telanov V.E. Zakharov, Sagdeev

- Zonal Shears: Wave kinetics (Zakharov et. al., P.D. et. al. '98, et. seq.)

  Coherent interaction approach (L. Chen et. al.)

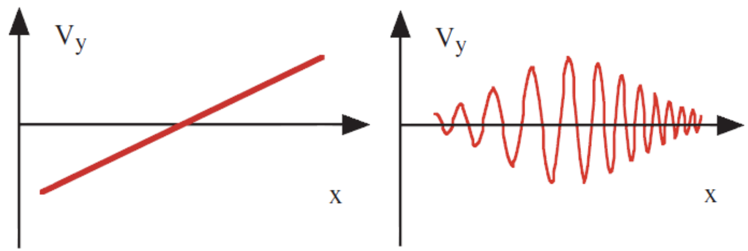
- $dk_r / dt = -\partial(\omega + k_\theta V_E)/\partial r \; ; \; V_E = \langle V_E \rangle + \tilde{V}_E$

  Mean shearing : $k_r = k_r^{(0)} - k_\theta V_E' \tau$

  Zonal Random shearing : $\langle \delta k_r^2 \rangle = D_k \tau$

  $D_k = \sum_q k_\theta^2 \left| \tilde{V}'_{E,q} \right|^2 \tau_{k,q}$

Ray chaos




- Wave ray chaos (not shear RPA) underlies $D_k \rightarrow$ induced diffusion
- Induces wave packet dispersion
- Applicable to ZFs and GAMs

- Mean Field Wave Kinetics

$N \equiv$ wave action density

$$\frac{\partial N}{\partial t} + (\vec{V}_{gr} + \vec{V}) \cdot \nabla N - \frac{\partial}{\partial r}(\omega + k_\theta V_E) \cdot \frac{\partial N}{\partial \vec{k}} = \gamma_{\vec{k}} N - C\{N\}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t}\langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N \rangle = \gamma_{\vec{k}} \langle N \rangle - \langle C\{N\} \rangle}$$

Zonal shearing

→ Evolves population in response to shearing field → statistically specified

# Shearing III

- Energetics: Books must Balance for Reynolds Stress-Driven Flows! $\left\{ \begin{array}{l} \text{shearing} \\ \text{scattering} \end{array} \right.$
- Fluctuation Energy Evolution – Z.F. shearing

$$\int d\vec{k}\,\omega\left(\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle\right) \Rightarrow \frac{\partial}{\partial t}\langle \varepsilon\rangle = -\int d\vec{k}\, V_{gr}(\vec{k}) D_{\vec{k}} \frac{\partial}{\partial k_r}\langle N\rangle \qquad V_{gr} = \frac{-2k_r k_\theta V_* \rho_s^2}{\left(1+k_\perp^2\rho_s^2\right)^2}$$

Point: For $d\langle\Omega\rangle / dk_r < 0$, Z.F. shearing depletes wave energy

- Fate of the Energy: Reynolds work on Zonal Flow

Modulational Instability

$$\partial_t \delta V_\theta + \partial\left(\delta\left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle\right) / \partial r = \gamma \delta V_\theta$$

$$\delta\left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle \sim \frac{k_r k_\theta \delta N}{(1+k_\perp^2 \rho_s^2)^2}$$

N.B.: Wave decorrelation essential:
Equivalent to PV transport

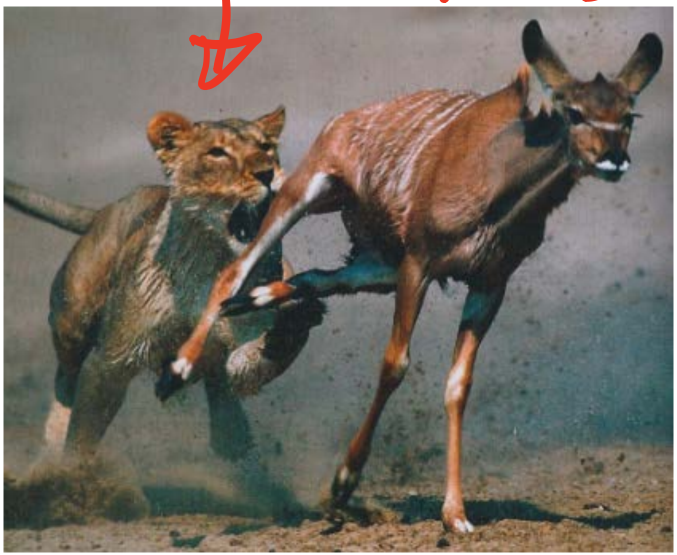
- Bottom Line:
  - Z.F. growth due to shearing of waves
  - "Reynolds work" and "flow shearing" as relabeling → books balance
  - Z.F. damping, evolution of profile → staircase

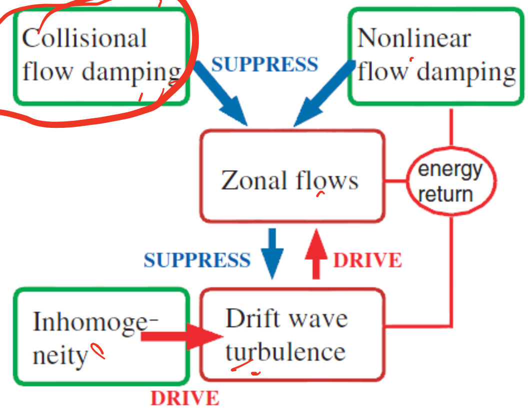
# Feedback Loops

- Closing the loop of shearing and Reynolds work
- Spectral ‘Predator-Prey’ Model, P.D. et al ‘94 et. seq

Math Biology

H. Levine






Prey → Drift waves, $\langle N \rangle$

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k \langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

→ Self-regulating system → “ecology”

→ Mixing and mixing scale regulated

and infinite extensions...

See especially: K. Miki, P.D. et al 2012-2016

Big → heat load

Goldston, et al. works

B. LaBombard

$w \sim \frac{1}{B_\theta} \sim \frac{1}{I_p}$


$w \sim v_D \tau_\parallel$

$v_D \tau_\parallel \sim \frac{\rho_\theta}{R}$

$\boxed{w \sim \frac{\rho_c}{R} q}$

Heat Load

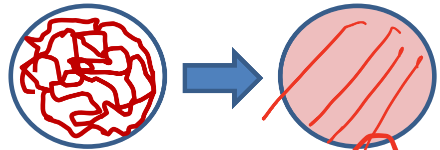
# Spatial Structure:

# Inhomogeneous Mixing and Staircases

# Dynamics in Real Space

- Conventional Wisdom → $\boxed{\text{Homogenization ?!}}$
  - Prandtl, Batchelor, Rhines:
  - PV homogenized:
    Shear + Diffusion

(2D fluid)



$\nabla q \to 0$

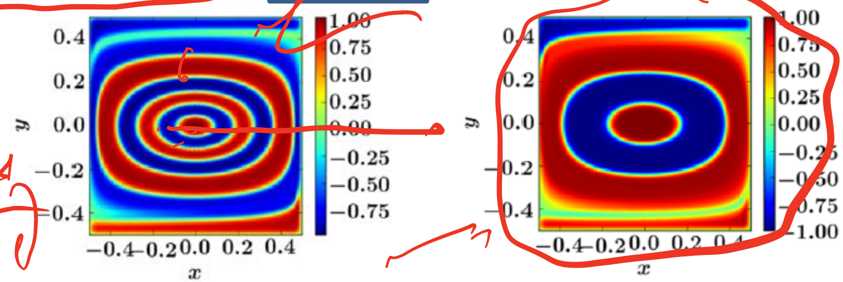
  - Mechanism: - Shear dispersion $\tau \sim \tau_{rot}\,(Re)^{1/3}$

    - Forward Enstrophy Cascade, 'PV Mixing'

  - Introduce Bi-stable Mixing → $\boxed{\text{Layers}}$

Layering

Jelly roll

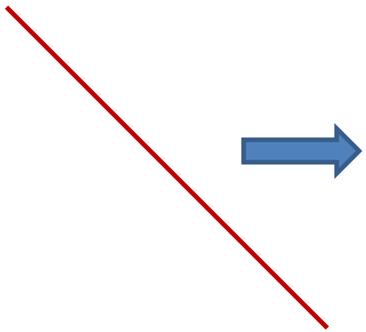



  - Cahn-Hilliard + Eddy Flow ←→ bistability

    → target pattern

(Fan, P.D., Chacon, PRE Rap. Com. '17)

# Fate of Gradient?



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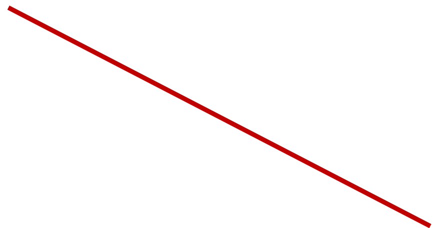

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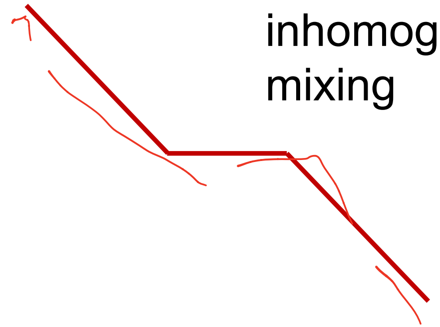
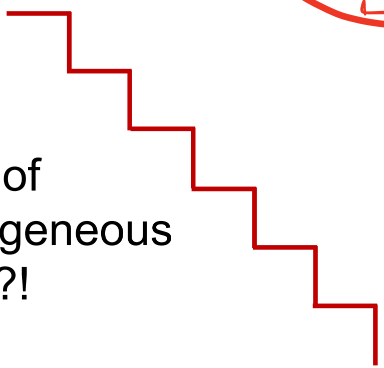


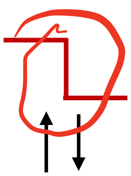
localized
inhomogeneous
mixing

OR - 'staircase'



pattern of
inhomogeneous
mixing ?!

- layers, steps, corrugations
- shear layers ←→ relation to corrugations?



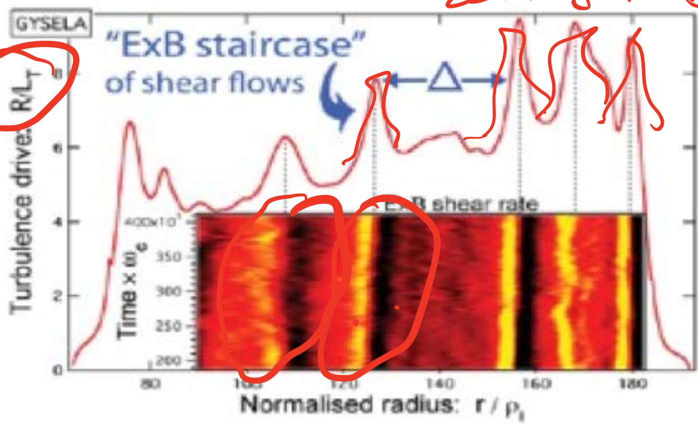
?

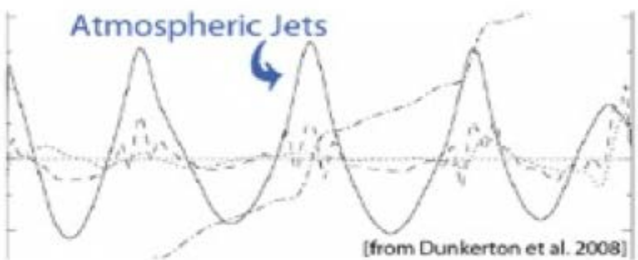
# Spatial Structure: ExB staircase formation

- ExB flows often observed to self-organize structured pattern in magnetized plasmas
- \`ExB staircase' is observed to form

(G. Dif-Pradalier, P.D. et al. Phys. Rev. E. '10)

corrugations




also: GK5D, Kyoto-Dalian-SWIP group, gKPSP, ... several GF codes

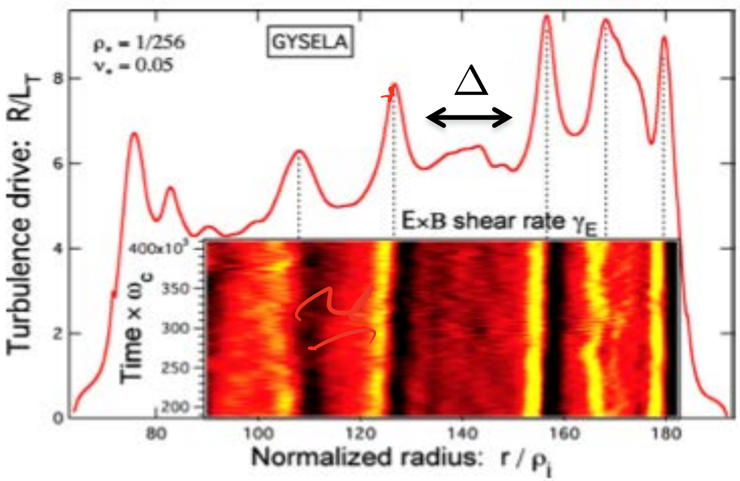
- flux driven, full f simulation
- Quasi-regular pattern of shear layers and profile corrugations (steps)
- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

  → ExB staircases

- so-named after the analogy to PV staircases and atmospheric jets
- Step spacing → avalanche distribution outer-scale
- scale selection problem

# ExB Staircase, cont'd

- Important feature: co-existence of shear flows and avalanches/spreading




- Seem mutually exclusive ?

  → strong ExB shear prohibits transport

  → mesoscale scattering smooths out corrugations

- Can co-exist by separating regions into:

  1. avalanches of the size $\Delta \gg \Delta_c$

  2. localized strong corrugations + jets

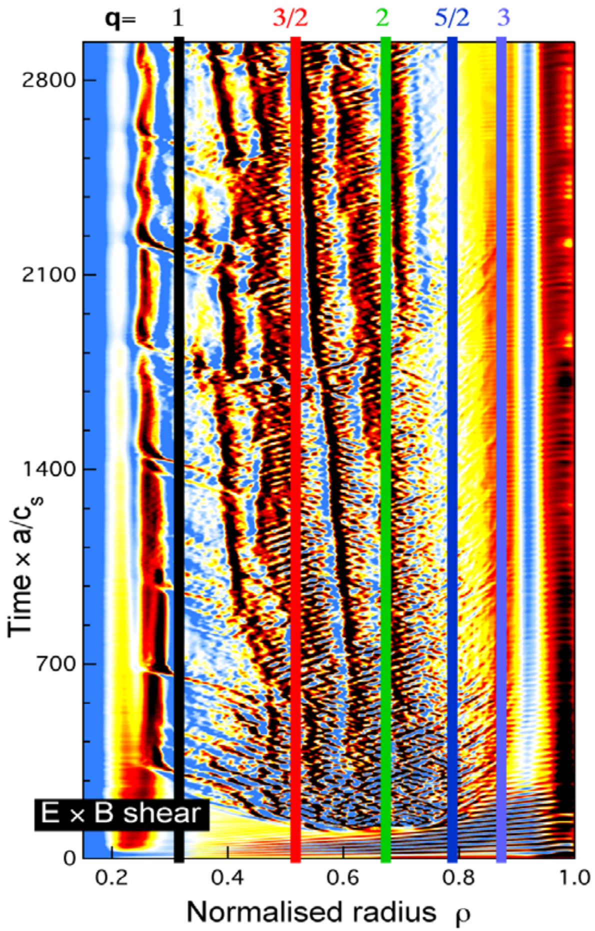
- How understand the formation of ExB staircase??
  - What is process of self-organization linking avalanche scale to ExB step scale?

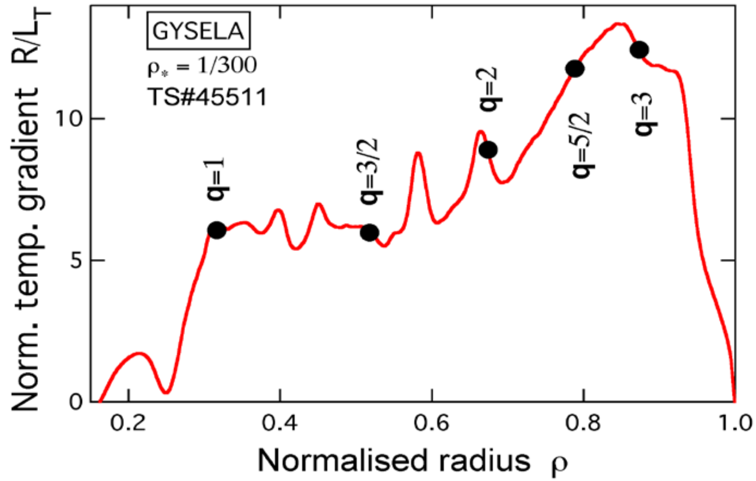
    i.e. how explain the emergence of the step scale ?

- Some similarity to phase ordering in fluids – spinodal decomposition

# Corrugation points and rational surfaces?

- No apparent relation




➔ Step location not tied to magnetic geometry structure in a simple systematic way

(GYSELA Simulation)

# Bistable Mixing – A Simple Mechanism

- Mean field model with <u>2</u> mixing scales (after Balmforth, et al. 2002)
- So, for H-W:
- Density: $\frac{\partial}{\partial t}\langle n\rangle = \frac{\partial}{\partial x}\left(D_n \frac{\partial \langle n\rangle}{\partial x}\right) + D_c \frac{\partial^2 \langle n\rangle}{\partial x^2}$
- Vorticity: $\frac{\partial}{\partial t}\langle u\rangle = \frac{\partial}{\partial x}\left[(D_n - \chi)\frac{\partial \langle n\rangle}{\partial x}\right] + \chi \frac{\partial^2 \langle u\rangle}{\partial x^2} + \mu_c \frac{\partial^2 \langle u\rangle}{\partial x^2},$

simple mixing + 2 length scale
→ staircase

$D_\varepsilon \sim \beta D_0 \mathcal{E}$

$D$

- Enstrophy (intensity): $\frac{\partial}{\partial t}\varepsilon = \frac{\partial}{\partial x}\left(D_\varepsilon \frac{\partial \varepsilon}{\partial x}\right) + \chi\left[\frac{\partial\langle n-u\rangle}{\partial x}\right]^2 - \varepsilon_c^{-1/2}\varepsilon^{3/2} + \gamma_\varepsilon \varepsilon.$ → includes crude turbulence spreading model
- $D, \chi \sim \tilde{V} l_{mix}$

$$\boxed{l_{\mathrm{mix}} = \frac{l_0}{(1 + l_0^2[\partial_x(n-u)]^2/\varepsilon)^{\kappa/2}},}$$

$l_0$ → mixing scale
$l_R$ → Rhines scale (emergent)
$\omega_{MM}$ vs $\Delta\omega$

↑
<u>two scales!</u>

- Scale cross-over → 'transport bifurcation'

$v\cdot\nabla u \sim \tilde{v}\cdot\nabla\langle u\rangle$

→ Qi de Get

→ Rhines scale

# Staircase Model – Formation and Merger (QG-HM)

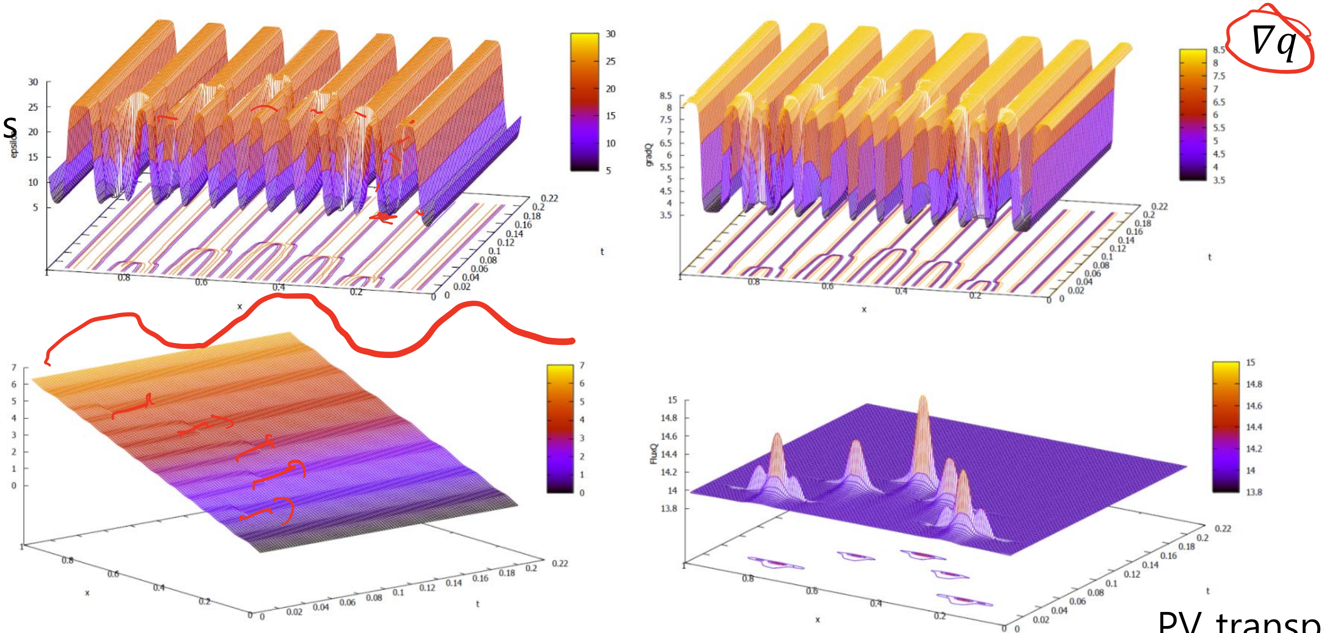
Energy

fluctuations

$q$

→

mergers



$\nabla q$

PV transport

- $\epsilon$
- $Q_y$

} top

- $Q$
- $\Gamma_q$

} bottom

- PV mixing events

Note later staircase mergers induce strong PV flux episodes!

(Malkov, P.D.; PR Fluids 2018)

# Staircase are Dynamic Patterns

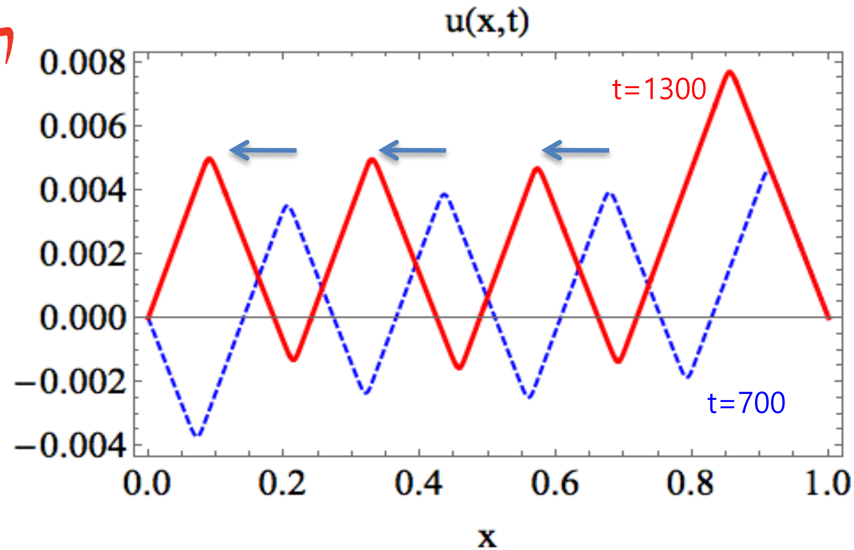
Propagating

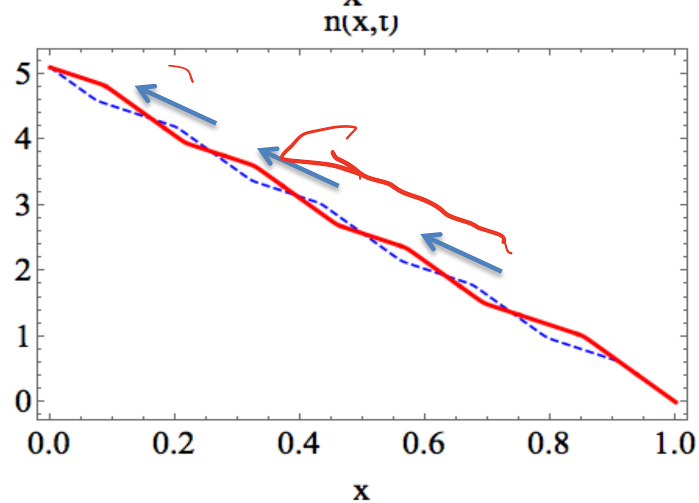
○Shear pattern detaches and delocalizes from its initial position of formation.

○Mesoscale shear lattice moves in the up-gradient direction. Shear layers condense and disappear at x=0.

○Shear lattice propagation takes place over much longer times. From t~O(10) to t~($10^4$).

○Barriers in density profile move upward in an "Escalator-like" motion.




➔ Macroscopic Profile Re-structuring

(Ashourvan, P.D. 2016)

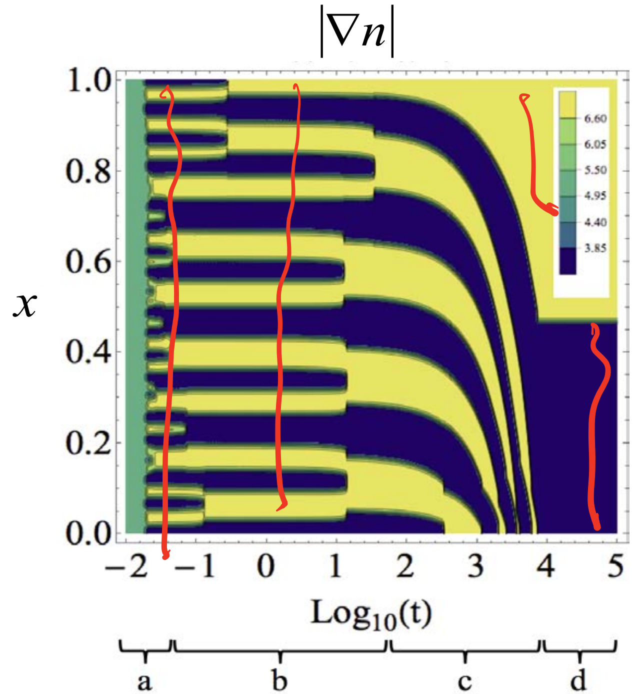
# Macro-Barriers via Condensation

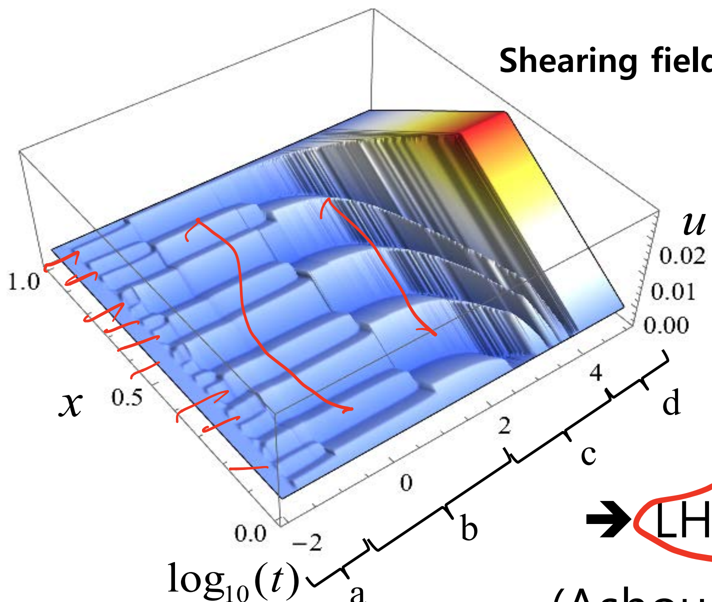
(a) Fast merger of micro-scale SC. Formation of meso-SC.

(b) Meso-SC coalesce to barriers

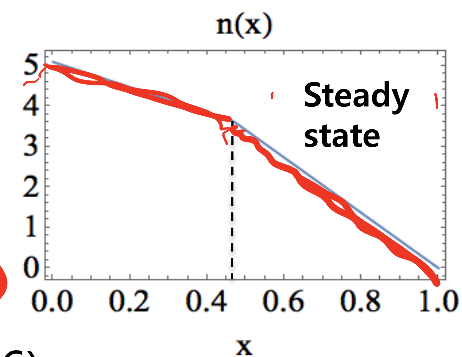
(c) Barriers propagate along gradient, condense at boundaries

(d) Macro-scale stationary profile



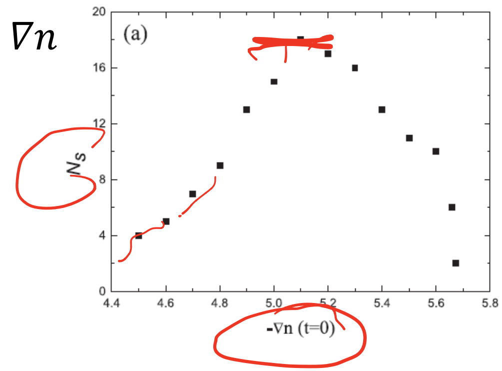

→ LH transition?




(Ashourvan, P.D. 2016)

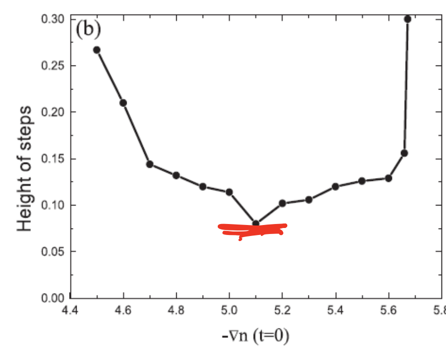
# FAQ re: Staircase Structure?

- Number of steps? - domain L
- Scan # steps vs $\nabla n$ at t=0 (n.b. mean gradient)
  - a maximum # steps (and minimal step size) vs $\nabla n$
  - rise: increase in free energy as $\nabla n \uparrow$
  - drop: diffusive dissipation limits $N_s$



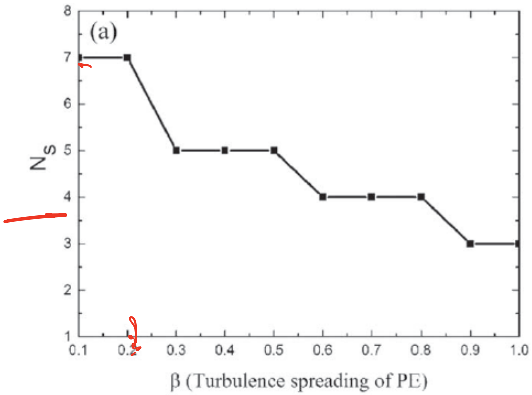

- Height of steps?
  - minimal height at maximal #

➔ system has a $\nabla n$ ‘sweet spot’ for many, small steps and zonal layers



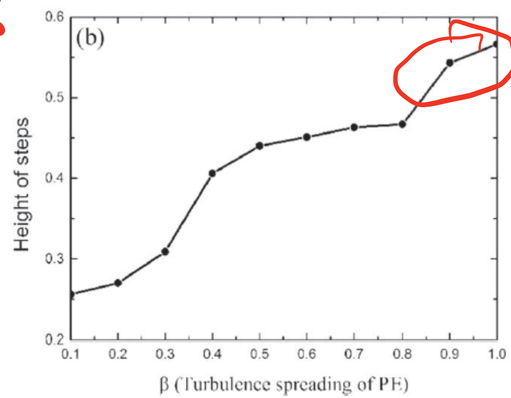
# 'Non-locality'? - Potential Enstrophy Spreading Effects?

- Scan $N_s$ vs Weighting parameters $\beta$, for potential Enstrophy Mixing




$\beta \equiv$ coefficient in $D_\epsilon$

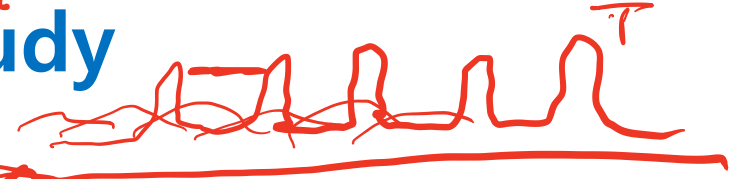
- Scan Height vs $\beta$




→ turbulence spreading tends to wash out small corrugations, limits step #

→ corrugations need not be regular size

# Status: Ongoing Study



- Explore Mechanisms
  - Bistable Mixing
  - Jams – recent hints: M. Choi, AAPPS-DPP ‘20
  - Pinch → study of layering in, say, ITG + Impurities ?!
- Layered state performance?
- Boundary effects → staircase structure?

Modula. + Bistability

staircase

Pinch + shear

# Noise + Modulations

# Noise?

- RH '98, et. seq → ZF screening and scale ($\rho_b$)

  → "residual"

- Brief mention: (N.B. rarely utilized)

$$\partial_t |\phi_q|^2 = 2\tau_c |S_q|^2 / |\epsilon_{neo}(q)|^2$$ ← screened noise

$$\boxed{S_q \leftrightarrow \tilde{V} \cdot \nabla \tilde{g}_i - \tilde{V} \cdot \nabla \tilde{g}_e \approx \tilde{V} \cdot \nabla \nabla_\perp^2 \tilde{\phi}}$$ - polarization flux

- ZF's excited by random walk, in polarization beat noise field
- Overlooked $\epsilon_{IM,NL} < 0$ → negative viscosity etc.
- Can't really formulate F-D thm ←→ screening unstable

# Noise, cont'd

- Sociological Observation: Nearly all theoretical works subdivide into
  - Screening, residual
  - Modulation, negative viscosity
- Interaction?
  - What of density, etc. corrugations?
  - What of $\langle n\phi \rangle_z$ - staircase ?!

  and

  - Noise effects on feedback processes

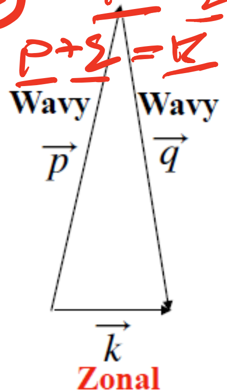
  Macroscopics → LH transition

# Zonal Intensity and Density Corrugation - Evolution

$$\left(\frac{\partial}{\partial t}+2\mu k_x^2\right)\left\langle\left|\phi_k\right|^2\right\rangle+2\eta_{1k}^{zonal}\left\langle\left|\phi_k\right|^2\right\rangle+\Re\left[2\eta_{2k}^{zonal}\left\langle n_k\phi_k^{\star}\right\rangle\right]=F_{\phi k}^{zonal}$$

$$\left(\frac{\partial}{\partial t}+2D_n k^2\right)\left\langle\left|n_k\right|^2\right\rangle+2\zeta_{1k}\left\langle\left|n_k\right|^2\right\rangle+\Re\left[2\zeta_{2k}\left\langle n_k^{\star}\phi_k\right\rangle\right]=F_{nk}$$

$p+q=k$




- $\eta_{1k}^{zonal} \propto k_x^2$ and -ve for $\dfrac{\partial I_q}{\partial q_x} < 0 \rightarrow$ transfer to large scales by NEGATIVE VISCOSITY

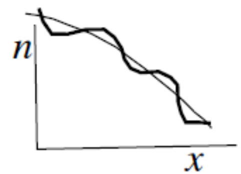
- Modulational instability when $-\eta_{1k} > \mu k_x^2$ defines a critical spectral slope

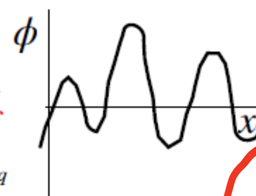
- Zonal growth is maximum when $\alpha_q \rightarrow \infty$ $\Longrightarrow$Non-adiabatic fluctuations inhibit transfer to large scales

- $\eta_{2k}^{zonal,(r)} > 0$ ALWAYS for $\dfrac{\partial I_q}{\partial q_x} < 0 \Longrightarrow$ Forward transfer when $\Re\left\langle n_k\phi_k^{\star}\right\rangle < 0$, backward transfer when $\Re\left\langle n_k\phi_k^{\star}\right\rangle > 0$

- **Noise** = Reynolds stress squared times triad interaction time. ALWAYS +ve and of envelop scale ! $F_{\phi k}^{zonal} = 4\sum_q \Pi_q^2 \Theta_{k,-q,q}^{(r)}$ ; $\Pi_q = q_y q_x I_q$

- Noise/Modulation = $q_x^2 I_q / k_x^2 I_k$ = *Turbulent KE/Zonal KE*




- Density corrugation modulational damping $\zeta_{1k}$, cross-coefficient $\zeta_{2k}$ and advection noise $F_{nk}$ ALL +ve and scale as $1/\alpha_q^2$.

  ➡Density cascade forward in $k_x$

  ➡Corrugations become weaker as the response become more adiabatic.

- Corrugation is determined by noise vs diffusion balance.

- Important for staircase

Forward cascade in k-space is supporting the idea of (inhomogeneous) mixing in real space.

# Spectral evolution of zonal cross-correlation

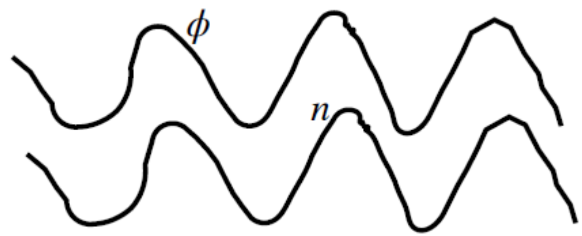
From zonal vorticity and zonal density equation one can obtain

$$\frac{\partial}{\partial t}\left\langle \bar{n}\nabla_x^2\bar{\phi}\right\rangle - (\mu + D_n)\left\langle \nabla_x^2\bar{n}\,\nabla_x^2\bar{\phi}\right\rangle = \left\langle \Gamma_{nx}\nabla_x^3\bar{\phi}\right\rangle + \left\langle \nabla_x\Pi_{xy}\nabla_x\bar{n}\right\rangle$$

- $\Longrightarrow$Zonal correlations are determined by correlation of fluxes and zonal profile

- Significant for layering or staircase structure - potential and density are aligned in staircase!

Q: When do zonal density and zonal potential align?




From spectral closure

$$\Re\left\langle n_k\phi_k^\star\right\rangle = \frac{2\eta_{2k}^{(r)}\left\langle \left|n_k\right|^2\right\rangle + 2\zeta_{2k}^{(r)}\left\langle \left|\phi_k\right|^2\right\rangle}{-(\mu + D_n)k_x^2 - 2\xi_{1k}^{(r)}} = \begin{cases} +ve & when\ -(\mu + D_n)k_x^2 - 2\xi_{1k}^{(r)} > 0 \\ -ve & when\ -(\mu + D_n)k_x^2 - 2\xi_{1k}^{(r)} < 0 \end{cases}$$

Where $\xi_{1k}^{(r)} = \eta_{1k} + \zeta_{1k}$= non-lin zonal damping rate + non-lin corrugation damping rate

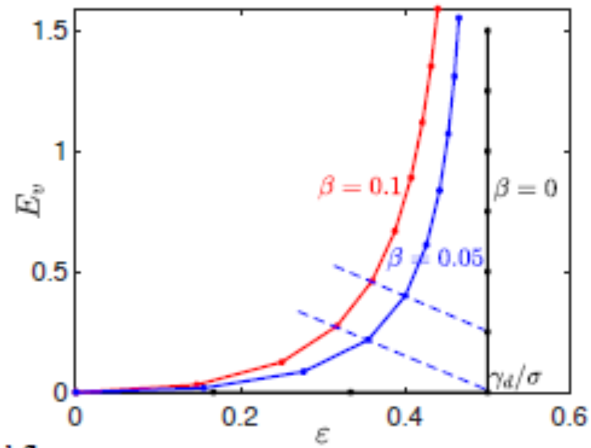
- $\Longrightarrow$Zonal density and potential are correlated (anti-correlated) when the modulational growth of zonal flow is more (less) than modulational damping of corrugations.

# Summary of zonal flow and corrugations interaction

| (a) Zonal flow - Vorticity equation - Polarization charge flux | | |
|---|---|---|
| **Process** | **Impact** | **Key physics** |
| Polarization noise | Seeds zonal flow | Polarization flux correlation, +ve definite |
| Zonal flow response (comparable to noise ) | Drives zonal shear using DW energy | Non-local inverse transfer in $k_x$ , -ve viscosity |
| Zonal shear straining of small scale | Regulates waves via straining | Stochastic refraction straining waves, induced diffusion to high $k_x$ |
| **(b) Density corrugations - Density equation - Particle flux** | | |
| Density advection beat noise | Seeds density corrugation | Advection beats due to non-adiabatic electrons |
| Density corrugations response | Damps and regulates density corrugations | Non-local forward transfer in $k_x$ +ve diffusivity, turbulent mixing weak for $\alpha >> 1$ |
| Zonal shear straining of small scale | Regulates waves via straining | Stochastic refraction straining waves, induced diffusion to high $k_x$ |
| **(c) Zonal cross-correlation - Vorticity and density transport processes** | | |
| ZCC response | Sets corrugation - shear layer correlation; | Growth of zonal intensity must exceed the modulational damping of corrugation |

# Feedback loop with zonal noise

Feedback + Noise – revisit Predator-Prey




Turbulence energy $\varepsilon$ evolves as

Induced diffusion/shearing

$$\frac{\partial \varepsilon}{\partial t} = \gamma\varepsilon - \sigma E_v \varepsilon - \eta\varepsilon^2$$

Nonlinear damping

Zonal flow energy $E_v$ evolves as

Modulational growth

$$\frac{\partial E_v}{\partial t} = \sigma\varepsilon E_v - \gamma_d E_v + \beta\varepsilon^2$$

**Without noise:**

- Threshold in growth rate $\gamma > \eta\gamma_d/\sigma$ for appearance of stable zonal flows.
- Turbulence energy increases as $\gamma/\eta$ below the threshold, until at $\gamma_d/\sigma$ at threshold
- Beyond the threshold, turbulence energy remains locked at $\gamma_d/\sigma$ while the zonal flow energy continues to grow as $\sigma^{-1}\eta\left(\gamma/\eta - \gamma_d/\sigma\right)$.

**With noise:**

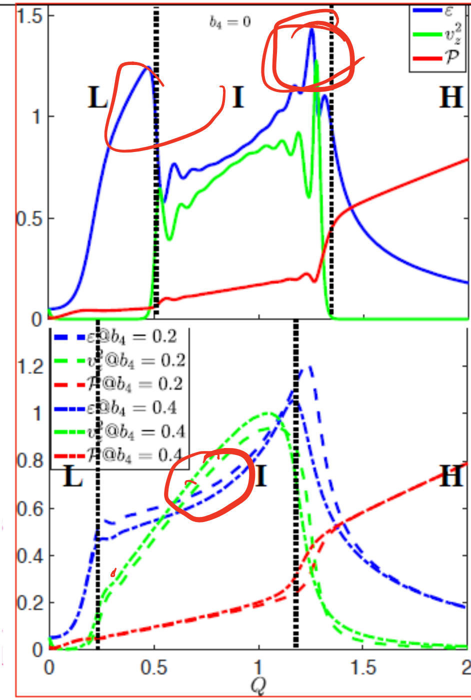
- Both zonal flow and turbulence co-exist at any growth rate – No threshold in growth rate for zonal flow excitation
- Turbulence energy never hits the modulational instability, absent noise!

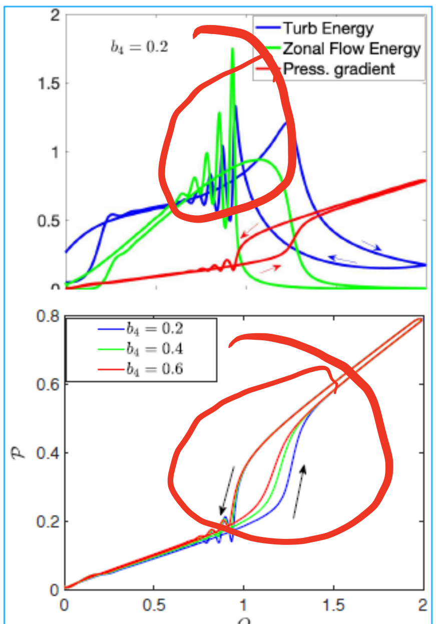
# L-H Transition

With Noise $\boxed{\text{KD 03 + Noise}}$

- Significant zonal flow appear below the modulational instability threshold. No ZF threshold in Q. Zonal flows exist at all Q.

- Turbulence level is reduced, no overshoot, zonal flow enhanced. No discernable trigger.




- The I-phase in the back transition is more oscillatory than that in the forward transition.

- Hysteresis with noise is robust w.r.t variations in initial condition

- The area enclosed by hysteresis curve decreases with noise

Noise _ S.C. Models

# Status: Ongoing Study

corrugation

- Bi-directional transfer(in HW): KE → large scale
- Int. Energy → small scale
- $\langle n\phi \rangle_Z$ → phasing of shear layers, corrugations

challenge !

→ sign? - growth shears vs corrugation damping

- Beat noise + modulations comparable
- Classic question: "If zonal flows are the trigger, then what triggers the trigger?"

  Answer: No <u>discernable</u> triggering Critical Intensity?

- Overshoot in L-H models eliminated

Physics of
L→H
transition

# Flows with Disordered Magnetic Fields

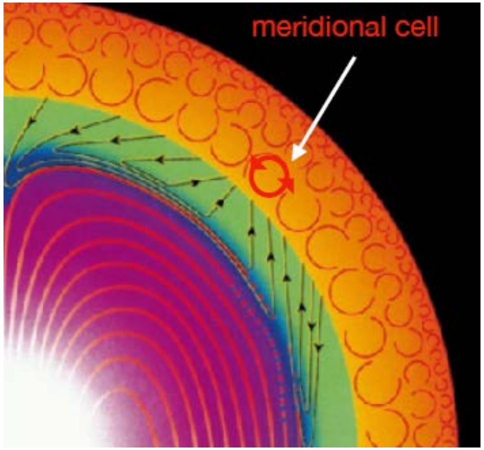
**a) planar tangled field: $\beta -$plane MHD and 'viscosity' in solar tachocline**
**C.-C. Chen, PD: ApJ'20, APS-DPP'20**

**b) stochastic magnetic field: Reynolds stress decoherence and LH Threshold with RMP**
**Chen, P.D., Singh, Tobias: APS-DPP'20, submitted to PoP**
**Others in prep.**

# What is the Tachocline?

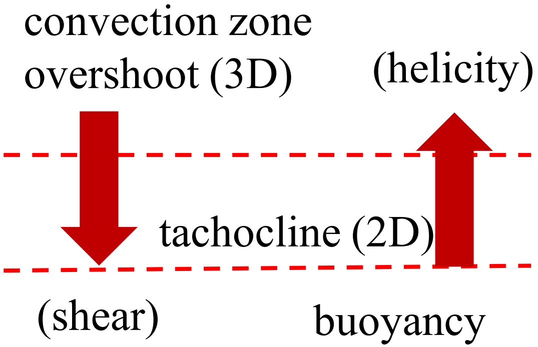
- Thin, stably stratified layer at the base of convection zone
- inferred by helioseismological inversions
- hydrostatic, $\beta \gg 1$ ~ weak $B_T$
- turbulent
- why should I care?

Interface Dynamo
(Parker 1993)



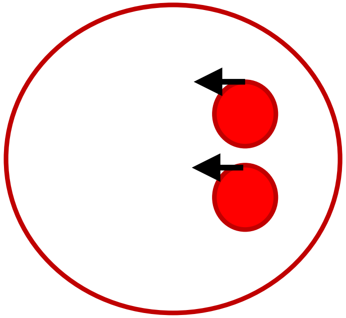

- solar dynamo!
- many problems in conventional wisdom of mean field dynamo theory ←→ multi-scale physics
- but: - shear is good!

  - stable stratification enables shear

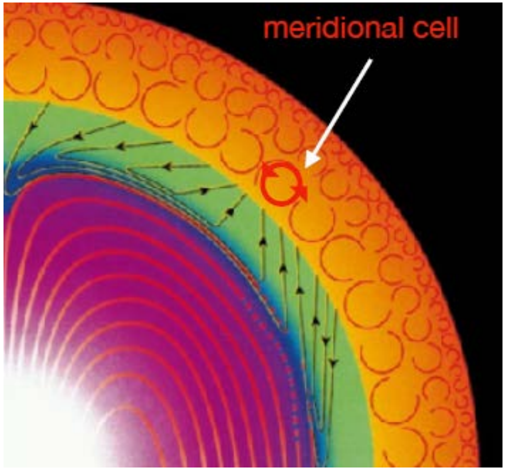


# How is the tachocline formed?

- meridional cell "burrowing" vs ?



meridional circulation ←→ $\nabla P \times \nabla \rho$ (Ertel's thm)




➔ "burrowing"

- ? Contains it ?

Momentum transport and 'viscosity' of great interest!

  – Spiegel and Zahn (1992):

  → Latitudinal viscous diffusion (2D ?)

  – Gough and McIntyre (1998):

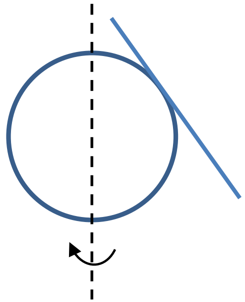
  → note PV, not momentum, mixed in 2D → negative viscosity

or

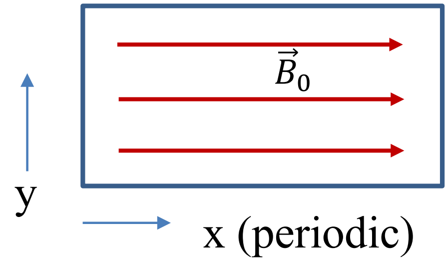
  → fossil field in radiation zone (?!)

# Model: $\beta$ –plane MHD (Tobias, P.D., Hughes ApJ Lett '07)

- Shell ➔ tangent plane



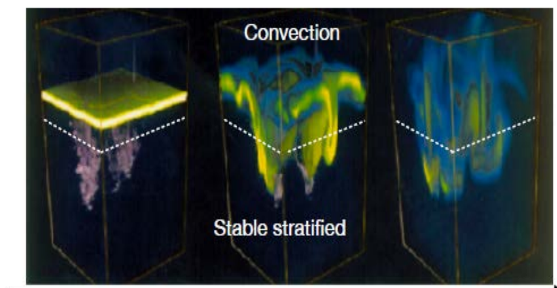
in plane




$$\beta = \frac{2\Omega}{R} cos\theta$$

($\theta$ from equator)

- $\phi, \; A$
  - Vorticity: $\left(\partial_t + \vec{V}_\perp \cdot \nabla_\perp\right)\omega \; - \beta\partial_x\phi = \frac{\vec{B}\cdot\nabla}{\rho}J + \nu\nabla^2\omega + \tilde{f}$
  - $B \rightarrow 0$ ➔ Charney (HM)
  - $\tilde{f}$ ➔ overshoot 'pumping'
  - Induction: $\left(\partial_t + \; \vec{V}_\perp \cdot \nabla_\perp\right)A = B_0\partial_x\phi + \nu\nabla^2 A$




(Tobias, et. al.)

- ala' Drift-Alfven: $\omega^2 \; - \omega\omega_R - k_x^2 V_A^2 = 0$ (R. Hide)

# Field Structure?

- Weak $\vec{B}_0$ + high $Re, Rm$

  ➔ $\langle \tilde{B}^2 \rangle \sim B_0^2 Rm$ from conservation of A (to $\eta$) in 2D

  (Zeldovich)

  $\boxed{\langle \tilde{B}^2 \rangle \gg \langle B \rangle^2}$

- disordered or 'tangled' magnetic field 'stochastic'? ←→ pumped by random overshoot. Stochastic character ←→ forcing

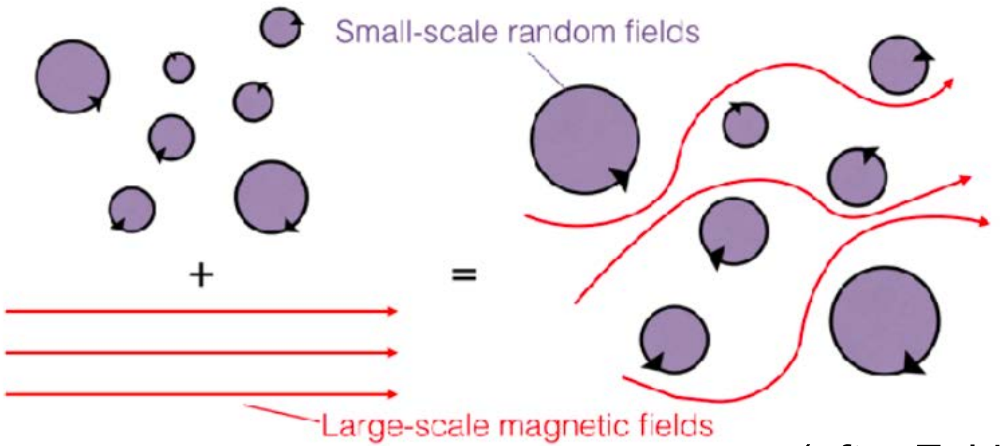
- 2 Kubo # :

$$Ku_f \ \sim \ \tilde{V}\tau_{ac} \,/\, \Delta \ \leq \ 1$$

$Ku_{mg} \ \sim \ l_{ac}\delta B \,/\, B_0\Delta \,, \quad l_{ac} \ \rightarrow \ 0$ allows $Ku < 1$ even for $\delta B / B_0$ large

('delta correlated')

# Field Structure, cont'd



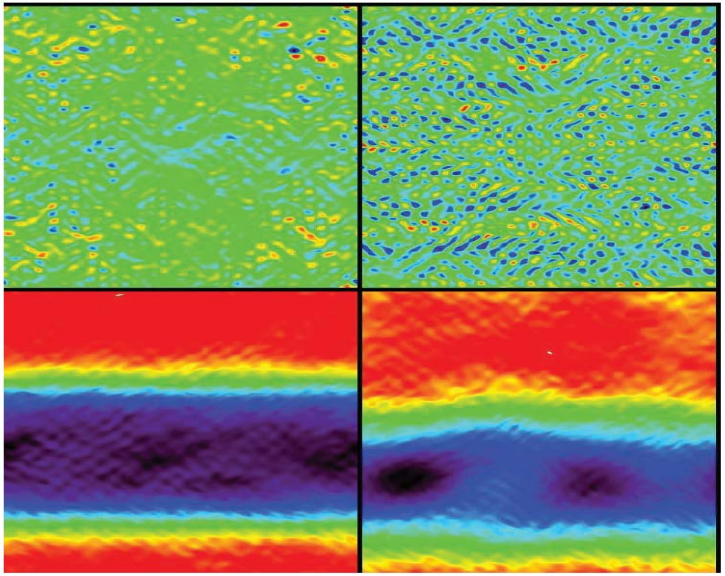

(after Zeldovich '83)

- System may be thought of as:
  - 'soup' of magnetic cells
  - threaded by 'sinews' of open lines ←→ percolation? – length of line
  - embedded in fluid, ~ frozen in ($Rm \gg 1$)

→ points toward effective medium approach

# Momentum Transport / Z.F. Production?

- Numerics: forcing via celluar array



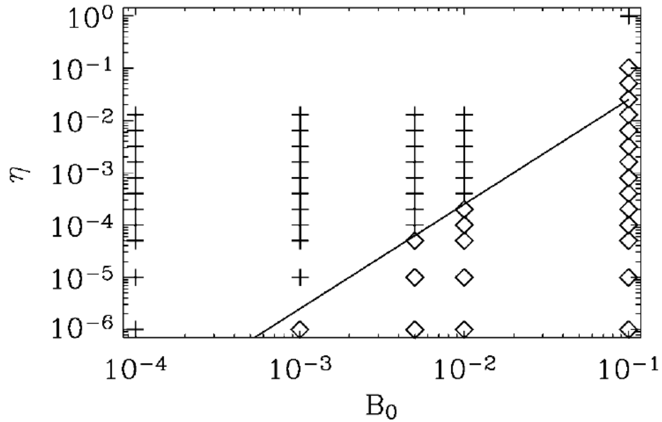
Weak $B_0$

ZF
$B_0 = 0$

  - predictably, Z.F.'s absent $B_0$
  - weak $B_0$ eliminates Z.F.'s !

# Z.F. Production, cont'd

$B_0$ and $\eta$ characterize Momentum Transport




- Systematics:

  + → Z.F.'s form

  ◇ → No Z.F.

- $B_0^2/\eta$ emerges as control parameter for character of momentum transport

- Echoes Zeldovich $\langle \tilde{B}^2 \rangle \sim Rm \langle B \rangle^2$ and,

  Reynolds-Maxwell: $\langle \tilde{V}\tilde{V} \rangle \rightarrow \langle \tilde{V}\tilde{V} \rangle - \langle \tilde{B}\tilde{B} \rangle$

➔ Tangled field retards momentum transport...

# Z.F. Production, cont'd

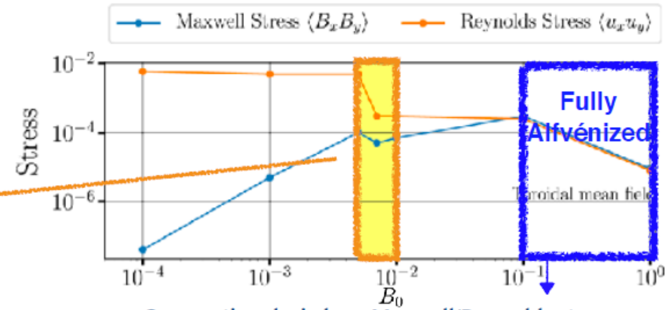
- Is it so simple? (Chen, P.D. ApJ 2020)

- Conventional wisdom: Reynolds vs Maxwell, and Alfvenization
  - Rossby, etc energy converted to Alfven wave
  - Reynolds-Maxwell equipartition

  ➔ $\Pi \rightarrow 0$

- Reality

The Reynolds stress is suppressed when mean field is weak, before the mean field is strong enough to fully Alfvénize the system.




Conventional wisdom: Maxwell/Reynolds stress balance when the system is Alfvénized.
(Chen & Diamond, ApJ 892 24, (2020))

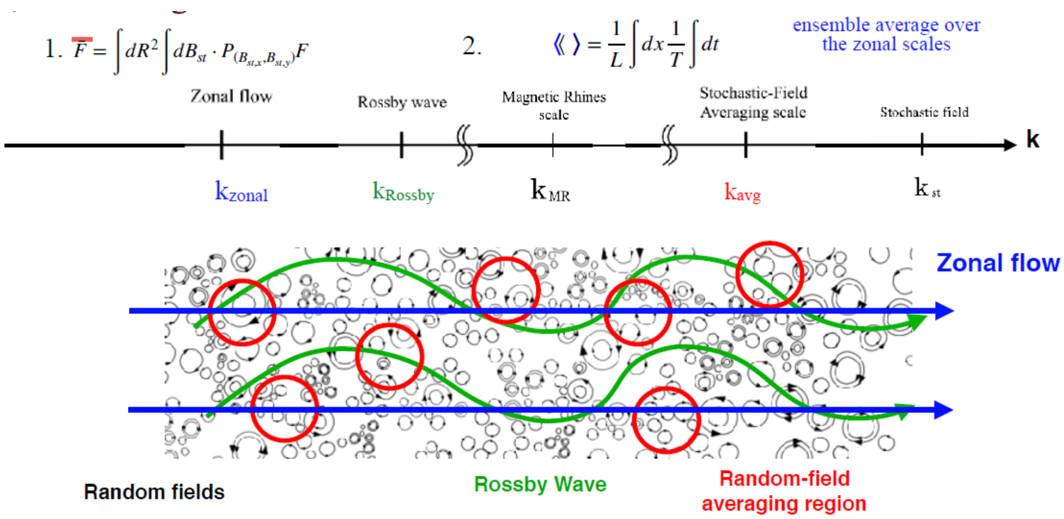
- Reynolds stress quenched by $\langle \tilde{B}^2 \rangle$ prior Alfvenization!

# Begs two related questions (Chen, P.D. '20)

- How understand the dynamics in disordered magnetic field?
  - examine PV transport <u>in prescribed disordered</u> field (replace: $\beta$ −plane MHD → $\beta$ −plane + $\tilde{B}$)
    → mean field theory
  - calculate PV flux $\langle \tilde{V}\tilde{\omega} \rangle$ or Reynolds force $\langle \tilde{V}_y \tilde{V}_x \rangle'$ in tangled field

# Effective Medium Theory - Outline

- a Multi-scale problem: (principal effect via $\langle J \times B \rangle$)
- Two-scale averaging: - stochastic field scale




- $l_{ac} \to 0$ ←→ $k_{st}$ large
- $k_{MR}$ : $k^2 \langle \tilde{V}_A^2 \rangle \sim \omega_R^2$

# Reynolds Stress Decoherence

- Recall: $\Gamma_{PV} \equiv \langle \tilde{V}_y \tilde{\omega} \rangle = \langle \tilde{V}_y \tilde{V}_x \rangle'$

**Multi-scale Dephasing:**

Mean PV Flux ($\Gamma$) and PV diffusivity ($D_{PV}$).

PV Diffusivity

Mean field

$B_0^2 < \overline{B_{st}^2}$

small-scale random

$$\overline{\Gamma} = -\boxed{\sum_k |\tilde{u}_{y,k}|^2 \frac{\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \frac{\overline{B_{st,y}^2} k^2}{\mu_0 \rho \eta k^2}}{\left(\omega - (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\omega}{\omega^2 + \eta^2 k^4}\right)^2 + \left(\nu k^2 + (\frac{B_0^2 k_x^2}{\mu_0 \rho})\frac{\eta k^2}{\omega^2 + \eta^2 k^4} + \frac{\overline{B_{st,y}^2} k^2}{\mu_0 \rho \eta k^2}\right)^2}}\left(\frac{\partial}{\partial y}\overline{\zeta} + \beta\right)$$

➤ **The large-** and **small-scale magnetic fields** have a **synergistic effect on the cross-phase** in the Reynolds stress.

**Dispersion relation of the Rossby-Alfvén wave with stochastic fields:**

(mean square) (square mean)

**Rossby frequency** $\omega_R \equiv -\beta k_x / k^2$

$$\left(\omega - \omega_R + \boxed{\frac{i\overline{B_{st,y}^2} k^2}{\mu_0 \rho \eta k^2}} + i\nu k^2\right)\left(\omega + i\eta k^2\right) = \frac{B_{0,x}^2 k_x^2}{\mu_0 \rho}$$

$$\boxed{\frac{\text{sping constant}}{\text{dissipation}} = \frac{\overline{B_{st}^2} k^2 / \mu_0 \rho}{\eta k^2}}$$

Dissipative response to Random magnetic fields

AW of the large-scale

➤ **Drag+dissipation effect**
→ this implies that the tangled fields and fluids define a **resisto-elastic medium.**

# Reynolds Stress Decoherence, cont'd

- The Point:
  - $\langle \tilde{B}^2 \rangle$ degrades Reynolds stress coherency, along with $k_{\parallel} V_{A_0}$
  - $\langle \tilde{B}^2 \rangle \gg B_0^2$
- $\langle \tilde{B}^2 \rangle$ coupling (after visco-elastic)

  → <u>'resisto-elastic medium'</u> replaces notion of ordered magnetization

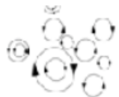
  → physics: Radiative coupling into tangled network → decorrelation
- Mean Flow?

$$\partial_t \langle U_x \rangle = \langle \bar{\Gamma} \rangle - \frac{1}{\eta\rho} \langle \tilde{B}_{st}^2 \rangle \langle U_x \rangle + \nu \nabla^2 \langle U_x \rangle$$

(previous) PV flux magnetic drag

# More Thoughts on Effective Medium

- $\overline{B_{st}^2}$ - **Resisto-elastic Medium:**



Alfvénic loops + elastic wave
= resisto-elastic medium

$$\omega^2 + i(\alpha + \eta k^2)\omega - \boxed{\left(\frac{\overline{B_{st,y}^2}k^2}{\mu_0\rho} + \frac{B_0^2 k_x^2}{\mu_0\rho}\right)} = 0,$$

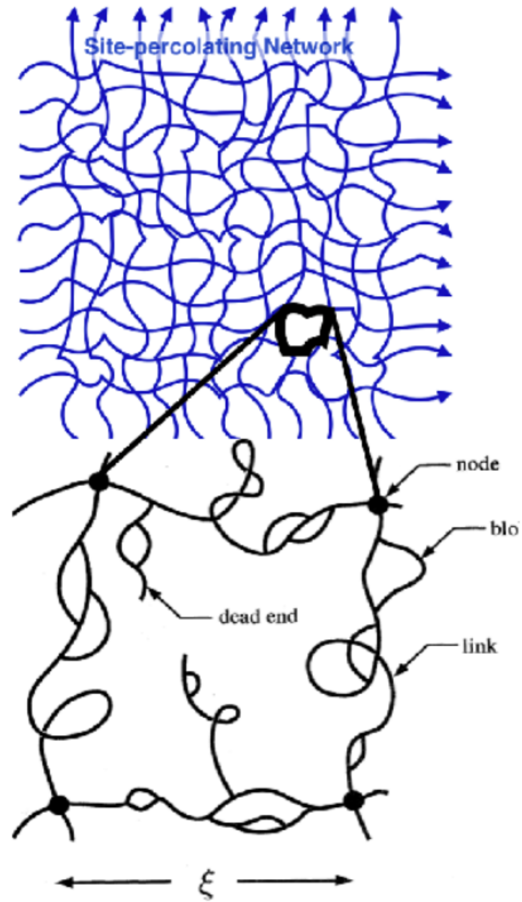
Small-scale field spring constant

Large-scale field spring constant

- Fluids couple to network elastic modes. Large elasticity degrades coherence

- This network can be **fractal (multi-scale)** and **intermittent**
(→ packing fractional factor: $\overline{B_{st}^2} \rightarrow p\overline{B_{st}^2}$ )
→ "fractons" (Alexander & Orbach 1982).

- **Similar physics— polymeric liquids.** (Oldroyd B)
We can calculate the effective spring constant, effective Young's Modulus of elasticity.
→ Elastic Energy Equation




Schematic of the nodes-links-blobs model (Nakayama & Yakubo 1994).

# The Lesson, so far…

- Reynolds decoherence occurs via $\langle \tilde{B}^2 \rangle$ coupling, well <u>below</u> Alfvenization

  → decoheres Reynolds stress before Reynolds-Maxwell balance

- Physics:
  - tangled magnetic network
  - effective resisto-elastic medium
  - radiative decorrelation

- Tachocline?
  - both S+Z, G+M(a) wrong
  - magnetic disorder <u>impedes</u> momentum transport
  - only G+M(b) remains standing – fossil field in radiation zone?

# Reynolds Stress Decoherence and the L→H Threshold in a Stochastic Magnetic Field

# Benefit and Cost, revisited

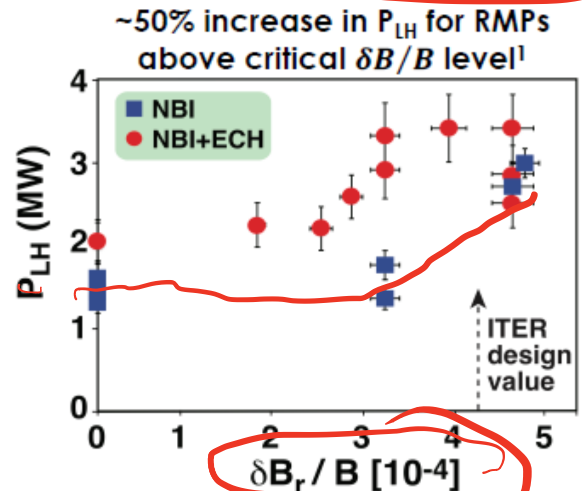
- Need make L→H Transition with RMP !
- Increase in $P_{th}$ for L→H !?
  - $(\delta B/B)_{crit}$ for

    L→H Power increase
  - Significant !
- Issues:
  - Why L→H threshold ↑ due RMP

    → decoherence of Reynolds stress
  - What physics defines $(\delta B/B)_{crit}$?

    → 'trigger' → shear flow
  - What Else?

"First ELM the largest"




(resonant vs. non-resonant)!

(Schmitz, et al 2019)

# Magnetic Field Structure, Model

RMP → stoch. layer

- Mea Culpa:
  - stochastic layer calculated
  - paradigm: 'stochastic field' as surrogate for RMP field (complex)
- Familiar story:
  - strong mean $B_0$, 3D
  - $\vec{k} \cdot \vec{B} = 0$ resonances, overlap → stochasticity / chaos
  - $Ku \approx l_{ac}\delta B_0/\Delta_\perp B_0 \leq 1$ (no 'delta correlation' assumption)
  - hereafter $b^2 \equiv (\delta B/B_0)^2$
- Model
  - 2 fluid, supported by kinetics
  - vorticity - $\omega, \phi$
  - induction - $A$
  - pressure - $P$
  - parallel velocity - $V_\parallel$

trends model insensitive, as

$\nabla \cdot J = 0$

$J = J_{pol} + J_{ps} + J_\parallel$

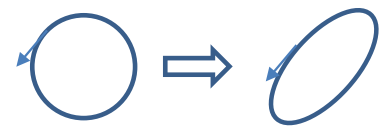
# The Plan (builds on previous)

- Understand Reynolds stress in stochastic field
  - physics argument
  - scales
  - analysis
- Implications for L$\rightarrow$H transition

# The Simple Physics (one way...)

- Shear flow generation – ‘tilting feedback’

$$\frac{dk_x}{dt} = -\partial_x(\omega + k_\theta V_E) = -k_\theta V_E' \quad \text{(small)}$$



then $\langle \tilde{V}_r \tilde{V}_\theta \rangle \sim \langle k_r k_\theta \rangle \rightarrow \; -k_\theta^2 V_E' \tau_c$

so tilt → stress

tilt induces correlation

$$\langle \tilde{V}_r \tilde{V}_\theta \rangle \approx -\sum_k \frac{c^2}{B_0^2} |\phi_k|^2 k_\theta^2 V_E' \tau_c$$

$\boxed{\text{Tilting Feedback}}$

stress → tilt

→ Modulational Instability, etc

- Stochastic field?`

# The Simple Physics, cont'd

- Recall (BBK'66) $\omega^2 - \omega_D\omega - k_\parallel^2 V_A^2 = 0$ $\qquad$ $\omega_D$ = drift wave frequency
- Consider: $k_\parallel = k_\parallel^{(0)} + \vec{b}\cdot\vec{k}_\perp$, for stochastic field
- $\omega = \omega_D + \delta\omega$

so (mean field)

- $\langle\omega\rangle \approx \omega_D + \frac{1}{2}\frac{V_A^2}{\omega_D} b^2 k_\perp^2$ → ensemble avg frequency shift due $b^2$

$\langle k_r k_\theta\rangle$

stochastic field effect on $\langle k_x k_y\rangle$

- $\langle\tilde{V}_r\tilde{V}_\theta\rangle \approx -\sum_k \frac{c^2}{B_0^2}|\phi_k|^2\left(k_\theta^2 V_E' \tau_{ck} - \frac{1}{2}\frac{k_\perp^2 V_A^2}{V_*}\frac{\partial}{\partial x}|b|^2\tau_{ck}\right)$
- → critical $\langle b^2\rangle$ to overwhelm shearing feedback
- TBC

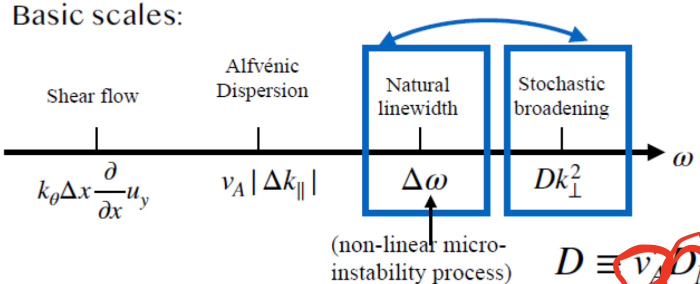
strong enough, stoch field →

Fi(l) eddy tilting feedback

# Scales

- When does stochastic dephasing become effective?

Basic scales:




Stochastic field decoherence beats self-decoherence.

$k_\perp^2 D$

$\Delta\omega_k$ vs $V_A D_M$ is $\sim$ FOM

$$D \equiv v_A D_M = v_A \underbrace{\sum_k \pi\delta(k_\parallel) b_k^2}$$

Alfvén wave propagate via stochastic fields → characteristic velocity from $\underline{\nabla} \cdot \underline{J} = 0$

Magnetic diffusivity

Auto-correlation length $l_{ac}$

Trieste '66 RSTZ

- FAQ's

  – why $V_A$? → from $\nabla \cdot J = 0$ → $\nabla_\perp \cdot J_{pol}$, so Alfvenic coupling in response

  – $B_0$ dependence? → $V_A \langle b^2 \rangle l_{ac}$ independent $B_0$!

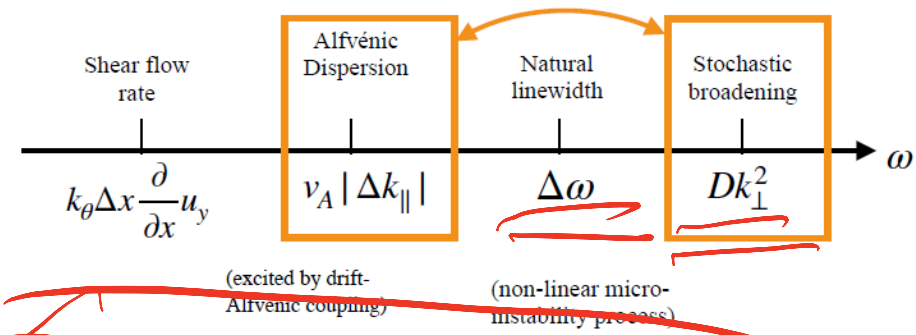
  – $V_A |\Delta k_\parallel|$ → autocorrelation rate of vorticity response → mean vorticity flux

d/

# Scales, cont'd

$C_s$

- $V_A D_M k_\perp^2$ vs $\Delta\omega$ → Dimensionless FOM for Decoherence, key parameter
- $\boxed{\alpha = (b^2/\rho_*^2 \sqrt{\beta})\, q/\epsilon}$ ~ 1 (GyroBohm)
- $b^2 > \sqrt{\beta}\, \rho_*^2\, \epsilon/\, q$ ~ $10^{-7}$, for 'typical' parameters
  - Modest field will decohere stress
  - scaling is unfavorable
- How stochastic is this?




$Ku_{mag} \simeq 1$

$$Ku_{mag} \equiv \frac{\text{stochastic field scattering length}}{\text{perpendicular magnetic fluctuation size}} \simeq 1$$

- In practice, need $\underline{Ku \sim 1}$

# Proper Analysis – Schematic

- $\nabla \cdot J = 0 \;\sim\; V_A D_M$ characterizes mixing, $D_M$ - RSTZ, R.R.

  ➔ $V_A$ is signal speed along stochastic magnetic field

- $\partial_x \langle \tilde{V}_r \tilde{V}_\theta \rangle = \langle \tilde{V}_r \nabla^2 \tilde{\phi} \rangle$ Taylor Identity

  Vorticity Perturbation

- $\nabla^2 \tilde{\phi} \;=\; (\;)\; \partial_x \langle \nabla^2 \phi \rangle \;+ (\;)\; k\, \nabla_y \tilde{P}$

  diagonal

  residual
  $\nabla P$ etc. → flow energy

- $\tilde{P}$ → Acoustic coupling → $c_s D_M$, slower

  ➔ of interest to fate of intrinsic rotation

# Outcome

$\propto \delta I$

$$\partial_x \langle \tilde{V}_x \tilde{V}_y \rangle = -D_{PV} \frac{\partial}{\partial x} \langle \nabla^2 \phi \rangle + F_{res} k \, \partial_x \langle P \rangle$$

$$D_{PV} \approx \sum_{k,\omega} \left| \tilde{V}_{r;k,\omega} \right|^2 \left[ \frac{V_A b^2 \, l_{ac} k^2}{\overline{\omega}^2 + (V_A b^2 l_{ac} k^2)^2} \right]$$

$$b^2 = \frac{\langle \tilde{B}^2 \rangle}{B_0^2}$$

$l_{ac}$ = field autocorrelation

$\Delta\omega_k$ vs stochastic broadening

$$F_{res} \sim -\sum_{k,\omega} \frac{2k_y}{\omega} D_{PV;k,\omega}$$

- Onset: $\Delta\omega_k \sim k_\perp^2 V_A D_M$

spectral linewidth

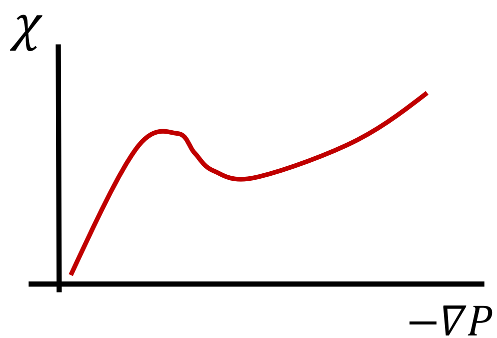
Stochastic field decorrelation must beat ambient limits on Reynolds stress phase

- In practice: $\boxed{Ku \sim 1}$ for effect, a challenge to predictions...

# To the L→H Transition…

# Theoretical Problem:
# L→H Transition in a Stochastic Magnetic Field

- What of L→H ? → Converging, though still somewhat
  (38 years +)
  controversial (c.f. $J_r$ ?
  → L. Schmitz, APS)




- Fundamentals:
  - Transport bifurcation
  - Bistability essential – S curve (c.f. A. Hubbard, et al)
  - Robust feedback channel – ExB shear flows
  - Insulation layer at the edge...

$$\chi_T = \chi_T(V'_{E\times B}/\omega)$$

$\chi_T \downarrow$ for $V'_{E\times B}/\omega$ > crit.

$$V_{E\times B} = \nabla P/n + \cdots$$

# L→H Transition, cont'd

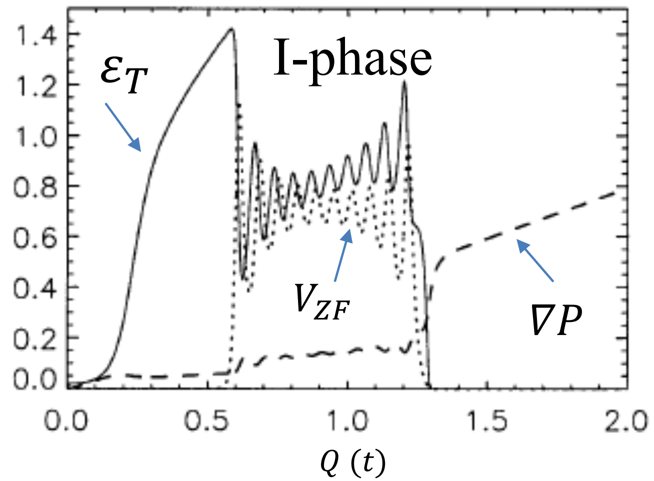
- Subtleties: $\langle J_r \rangle$
  – What is the "trigger"? → i.e.,
  – What physics allows $\nabla P$ to steepen?

candidates:
- polarization fluxes
  → Reynolds stress
- orbit loss
- NTV

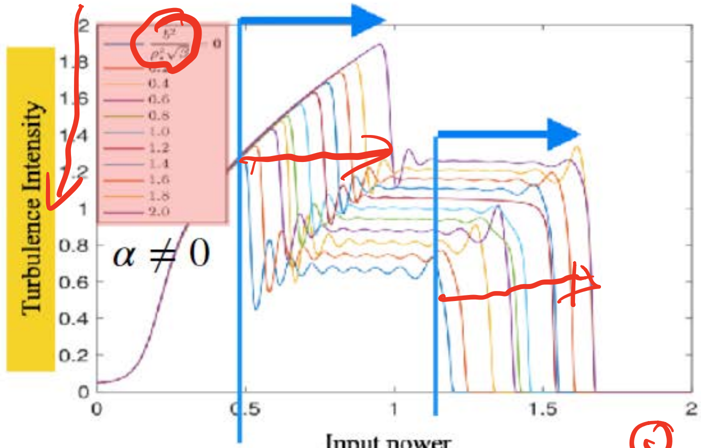
...

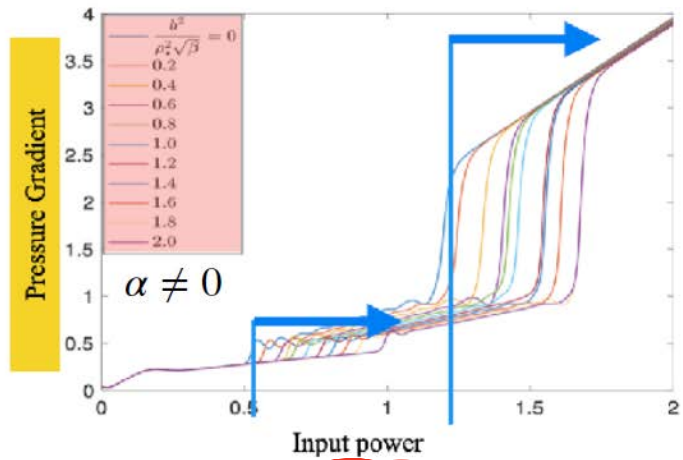
- Coupling of energy to edge zonal flow
  – Interplay of $\varepsilon_T, V_{ZF}, \nabla P$
  – $P_{Reynolds}$ crit. needed,
    measured (Tynan)
  – Crucial to note <u>$E \times B$ flow</u>
  – Zonal noise promote transition

Kim, PD, PRL'03



# Results 1, with Stochastic Reynolds Stress Decoherence

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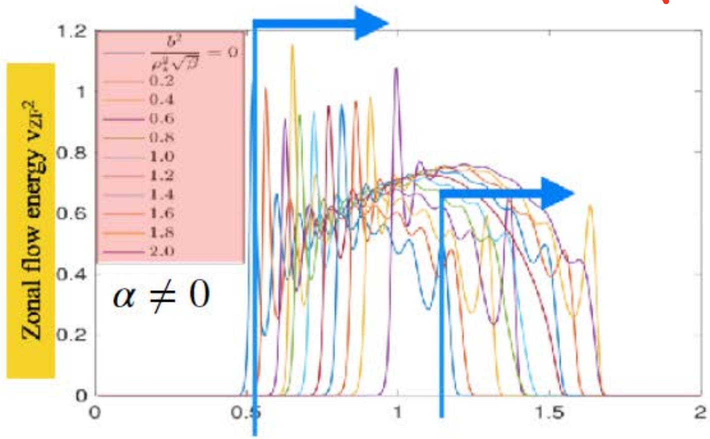



$$\alpha \equiv \frac{b^2}{\sqrt{\beta}\rho_*^2}\frac{q}{\epsilon}$$

**The threshold increase due to stochastic dephasing effect is seen in turbulence intensity, zonal flow, and pressure gradient.**

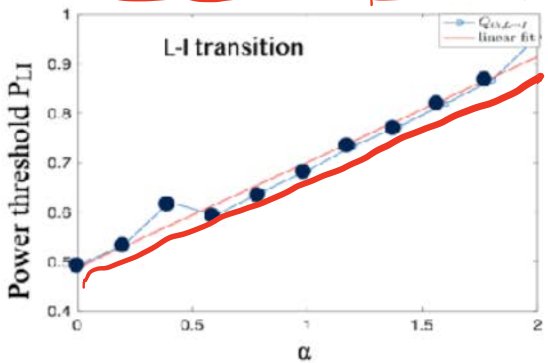
$\langle \tilde{v}_r \tilde{v}_\theta \rangle$

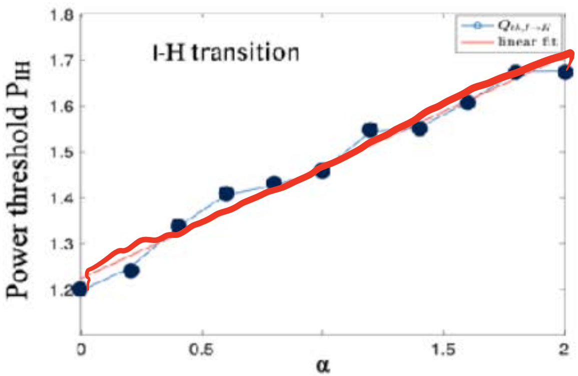
# Results II: L→H Power Increment

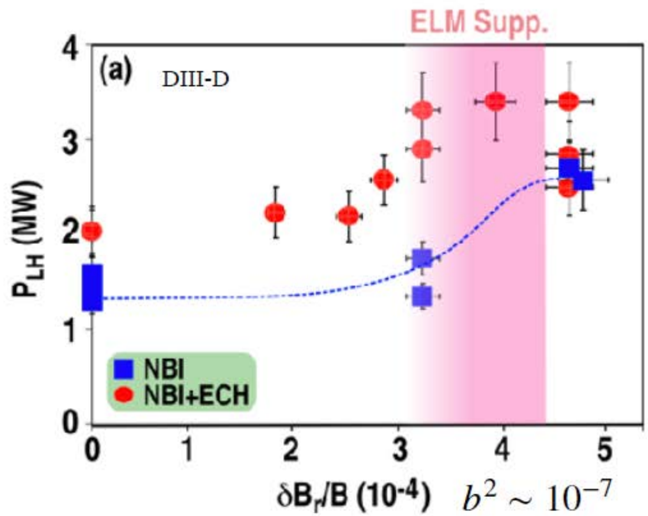
- L→H, L→I, I→H thresholds <u>all increase linearly</u> in $\alpha = \left(b^2/\rho_*^4\sqrt{\beta}\right) q/\epsilon$
- $\rho_*^{-2}$ not optimistic... (politely stated)

$\alpha \sim b \sim (\delta B/B)^2 \sim 10^{-7}$

RMP




(L. Schmitz et al, NF **59** 126010 (2019) )

# Related Work (Executive Summary)

$\nabla_\parallel J_\parallel$ $\langle \tilde{v}_r \nabla^2 \tilde{\phi} \rangle \rightarrow \partial_x \langle \tilde{v}_r \tilde{v}_\theta \rangle$

- Broad Theme: Turbulence and Transport [especially momentum, PV] in Stochastic Field
- What of intrinsic rotation? → $\boxed{\langle \tilde{V}_r \tilde{V}_\parallel \rangle}$ (local favorite)
- N.B. : 'Pedestal Torque' essential to stability in high performance discharges!
  - Parallel Flow ↔ <u>Acoustic</u> Dynamics

<u>So</u>

  - Scattering effect ~ $c_s D_M$ → modest
  - $\nu_T$ and $F_{z,res}$ persist, with modification

# Intrinsic Rotation, cont'd

But:

- Broken Symmetry required, for $\langle k_\theta k_\parallel \rangle \neq 0$
- $F_{res} \approx \; -\frac{k_z}{\omega}\, \nu_{Turb}$
- Key Question: How does stochastic field interact with symmetry breaking?

  ➔ $V_E'$ is leading candidate mechanism

→ Currently under investigation i.e. shift vs dispersion

# Direct Effects of Stochastic Field?
# → Parallel flow, pressure

$$\partial_t \langle V_\parallel \rangle + \partial_r \langle \tilde{V}_r \tilde{V}_\parallel \rangle = \boxed{-\frac{1}{P}\, \partial_r \langle b \tilde{P} \rangle}$$

$\langle b \tilde{P} \rangle$

$\langle b \tilde{P} \rangle = 0$

and:

“kinetic stress” (W.X. Ding, et al)

$$\partial_t \langle P \rangle + \partial_r \langle \tilde{V}_r \tilde{P} \rangle = -\frac{\partial}{\partial r} P_0 \, \langle b \tilde{V}_\parallel \rangle$$

- Finn, et al ’92: rate $c_s D_M / l^2$ via $\delta P \pm \delta V_\parallel$
- But... fluxes non-diffusive!

$-c_s D_M \nabla \langle P \rangle$

HW

For static stochastic field

Flow → $\vec{B} \cdot \nabla P = 0$ ⇒ $-c_s D_M \nabla \langle P \rangle$ → Residual stress

pressure → $\vec{B} \cdot \nabla V_\parallel = 0$ ⇒ $-c_s D_M \nabla \langle V_\parallel \rangle$ → Convection

# Direct Effects, Cont'd

- But: turbulence co-exists with stochastic field!
- Time scales: $k_\perp^2 D_T$ vs $k_\parallel c_s$
- Resonance: $\delta(k_\parallel) \rightarrow \quad 1/\left[k_\parallel^2 c_s^2 + (1/\tau_c)^2\right]$

turbulent scattering

shift, contrast

resonance broadening

- What balances $\tilde{b}_r\, \partial\langle P\rangle/\partial r$ ?
  - $c_s \nabla_\parallel \tilde{P}$ → weak turbulence → residual stress

    $b$ only, as previous
  - $k_\perp^2 D_r \tilde{V}_\parallel$ → strong turbulence → magnetic viscosity

    $b, v_\perp$ interplay

new effect

$$\boxed{\nu_T \approx \sum_k |b_k|^2 c_s^2 / k_\perp^2 D_T}$$

# Direct Effects, Cont'd

- Structure of flux, 'Fick's law' changes !
- Interesting new direction...

RMP

- Correlations?! (M. Cao, P.D., AAPPS-DPP 2020)
  - Are $\tilde{b}$, turbulence uncorrelated?
  - No → interaction develops $\langle b\phi \rangle$ correlation
  - ala' Kadomtsev, Pogutse, impose $\nabla \cdot J = 0$ to all orders
  - Novel small scale convective cell, $\tilde{b}$ structure develops

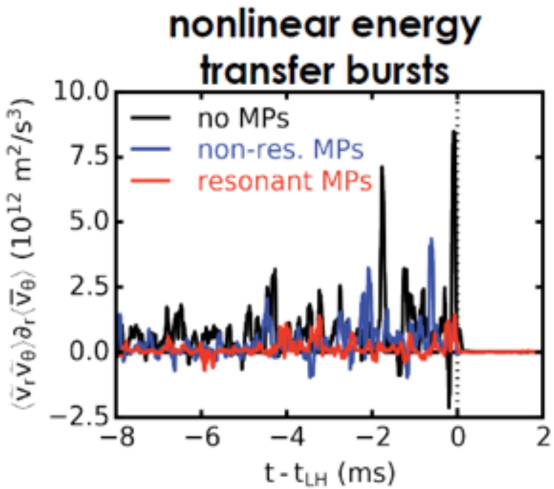
[Dynamics of Instability in stochastic field → classic question]

$\tilde{\phi}$

# Status

- Physics of Reynolds stress decoherence clarified
- Pessimistic scaling for increment in $P_{Thres}$ → linear in $\alpha = \frac{b^2}{\rho_*^2\sqrt{\beta}}\frac{q}{\epsilon}$
- degrades Reynolds coupling
- $\alpha \sim 1 \quad \leftrightarrow \quad Ku \sim 1$
- $V_A D_M$ is characteristic scattering rate
- Turbulence ←→ Stochasticity interaction enters parallel flow dynamics ($c_s D_M$)

# A Tantalizing Goodie...




(M. Kreite, G. McKee, et al.

also Z. Yan, APS'20)

- Transition → Pdf of Reynolds Power Bursts ←→ statistics!
- RMP/stochastic field alters population of large bursts, approaching transition
- Probe of power coupling statistics ?! ←→ Multiplicative Noise Process – Tilting?!

# General Conclusions – More Philosophy

- <u>40+ years</u> on from 'Rechester and Rosenbluth', dynamics in a stochastic magnetic field remains:
  - theoretically challenging
  - <u>vital</u> to MFE physics (i.e. trade-off, 3D)
- Transport in state of coexisting turbulence and stochastic magnetic fields is topic of interest. Especially, questions:
  - small scale energy tensor evolution (real space)
  - Need better understand $Ku \geq 1$ + transport
- Fractal network model promise new theoretical directions
- 1D (at least) L→H model ! Length scale of stochastic region will enter (ongoing)

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