Spooky action at a distance!

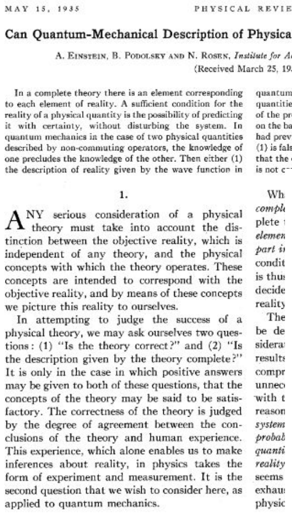
ref

"Can Quantum Mechanical Description of Physical Reality Be Considered Complete?"

A. Einstein B. Podolski N. Rosen

Physical reality must be local! - Podolsky

EPR Paradox


MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, Institute for Advanced Study, Princeton, New Jersey
(Received March 25, 1935)

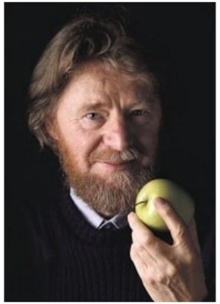
In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not c...

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

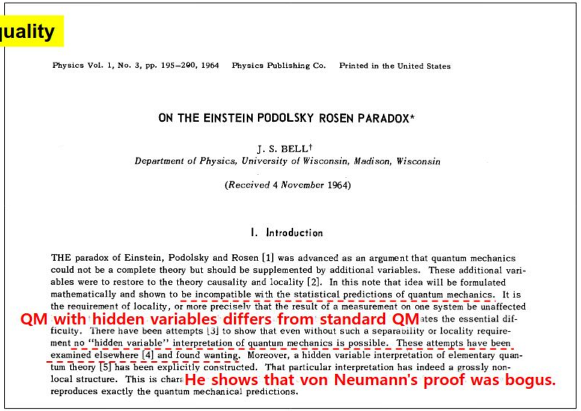
1964 QM with hidden variables differs from QM



1928~1990
John Stewart Bell

In the 1980s, he was always mentioned as a candidate for the Nobel Prize.

Bell's Inequality


Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(Received 4 November 1964)

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected ...ates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is chara... reproduces exactly the quantum mechanical predictions.

Upon observation, the cat was found to be alive.

Huh? The cat suddenly died.

Planet A

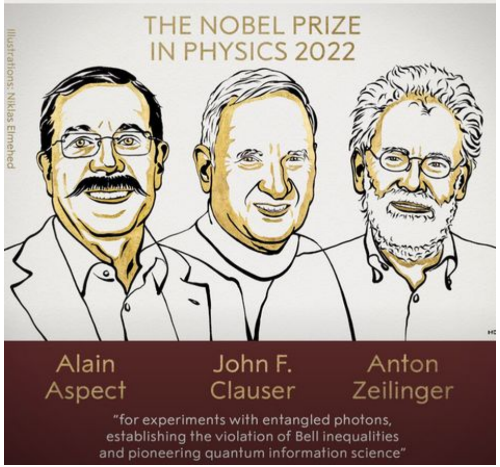
Planet B

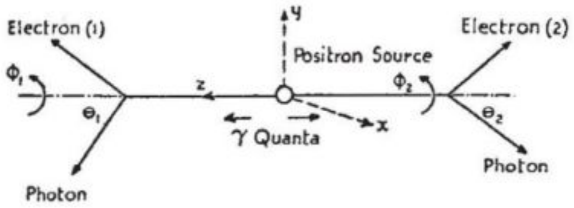
However, it still takes 1 light year for A and B to exchange answers.

Quantum mechanics is nonlocal

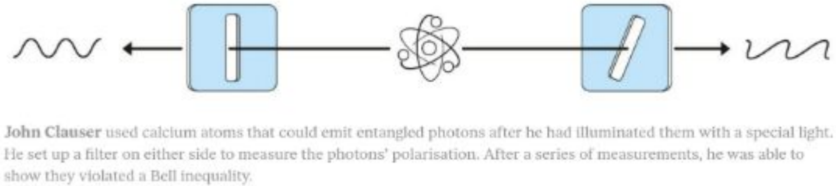
Quantum entanglement tests

- As reviewed by C. N. Yang, the first experiment on quantum entanglement is the Wu-Shaknov Experiment published in 1950 in which the angular correlation of two Compton scattered photons arising from e+e− annihilation are measured.
- The violation of Bell inequality was demonstrated in 1970s using entangled photons, confirming the non-locality of our universe.
- Alain Aspect, John Clauser and Anton Zeilinger won Nobel Prize in Physics in 2022 for demonstrating the potential to investigate and control particles (photons) that are in entangled states





Wu-Shaknov Experiment



Clauser's photon entanglement experiment

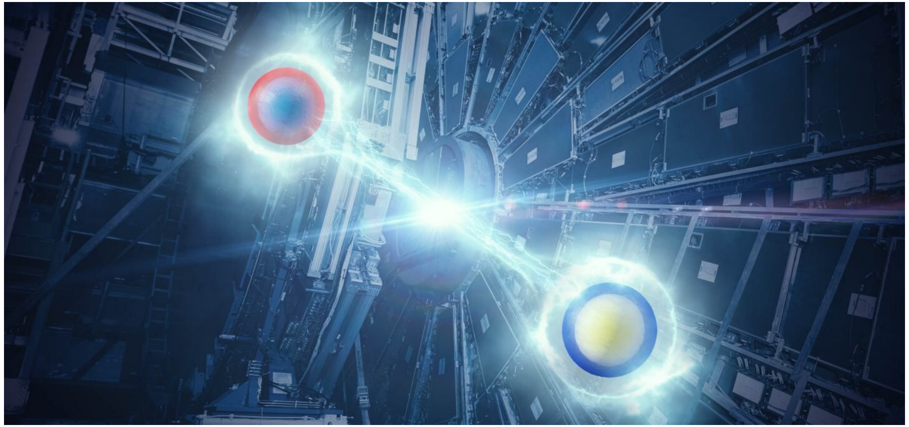
Quantum entanglement at high energy

Nature volume 633, pages 542–547 (2024)

LHC experiments at CERN observe quantum entanglement at the highest energy yet

The results open up a new perspective on the complex world of quantum physics

18 SEPTEMBER, 2024



Article

Observation of quantum entanglement with top quarks at the ATLAS detector

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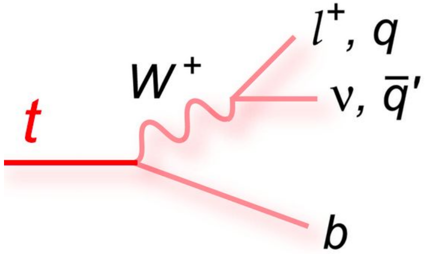
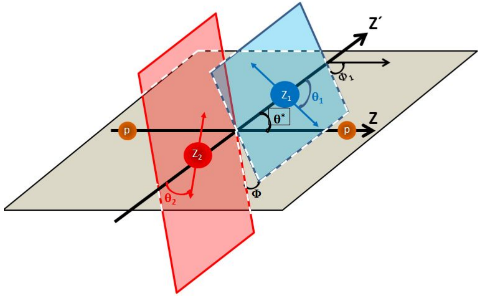
The ATLAS Collaboration*

Entanglement is a key feature of quantum mechanics[1–3], with applications in fields such as metrology, cryptography, quantum information and quantum computation[4–8]. It has been observed in a wide variety of systems and length scales, ranging from the microscopic[9–13] to the macroscopic[14–16]. However, entanglement remains largely unexplored at the highest accessible energy scales. Here we report the highest-energy observation of entanglement, in top–antitop quark events produced at the Large Hadron Collider, using a proton–proton collision dataset with a centre-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 inverse femtobarns $(\mathrm{fb})^{-1}$ recorded with the ATLAS experiment. Spin entanglement is detected from the measurement of a single observable D, inferred from the angle between the charged leptons in their parent top- and antitop-quark rest frames. The observable is measured in a narrow interval around the top–antitop quark production threshold, at which the entanglement detection is expected to be significant. It is reported in a fiducial phase space defined with stable particles to minimize the uncertainties that stem from the limitations of the Monte Carlo event generators and the parton shower model in modelling top-quark pair production. The entanglement marker is measured to be $D = -0.537 \pm 0.002\,(\mathrm{stat.}) \pm 0.019\,(\mathrm{syst.})$ for $340\ \mathrm{GeV} < m_{t\bar{t}} < 380\ \mathrm{GeV}$. The observed result is more than five standard deviations from a scenario without entanglement and hence constitutes the first observation of entanglement in a pair of quarks and the highest-energy observation of entanglement so far.

Why QE at high energy?

- **Understand quantum nature & seek for BSM effects.**
- Particle scattering/decay of unstable particles provide **a natural laboratory**
  - the momenta of observed particles are essentially commuting observables. Therefore, there is always some hidden variable theory that can explain the observed momentum data
  - However, one can focus on **spin correlation** emerges in different phase-space region
- It is plausible that **quantum mechanics undergoes modifications at some short distance scales** to achieve compatibility with gravity. Such modifications could, in principle, **be (only) detected by measuring Bell-type observables** or through quantum process tomography (ref)
- offers the potential to uncover **new insights into quantum field theory**.





https://scipost.org/10.21468/SciPostPhys.3.5.036


SciPost Phys. 3, 036 (2017)

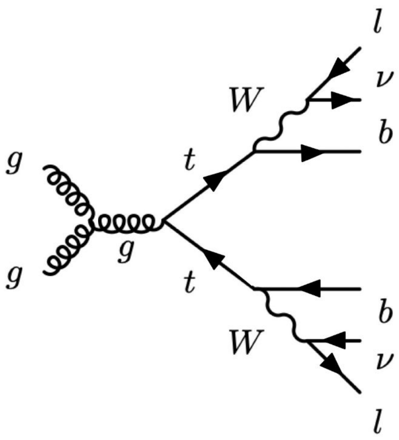
Maximal entanglement in high energy physics

Alba Cervera-Lierta[1], José I. Latorre[1,2], Juan Rojo[3] and Luca Rottoli[4]

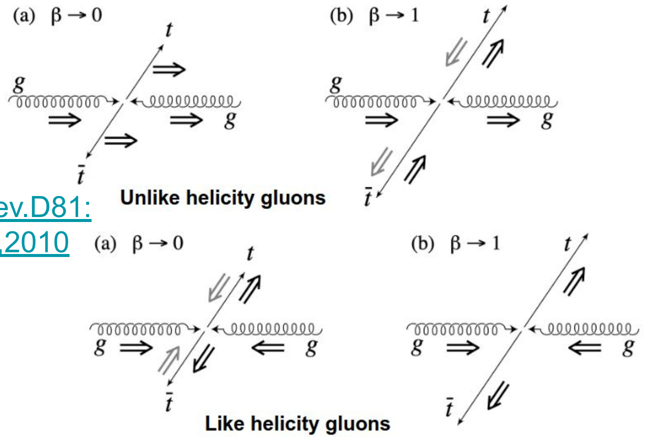
Top quark

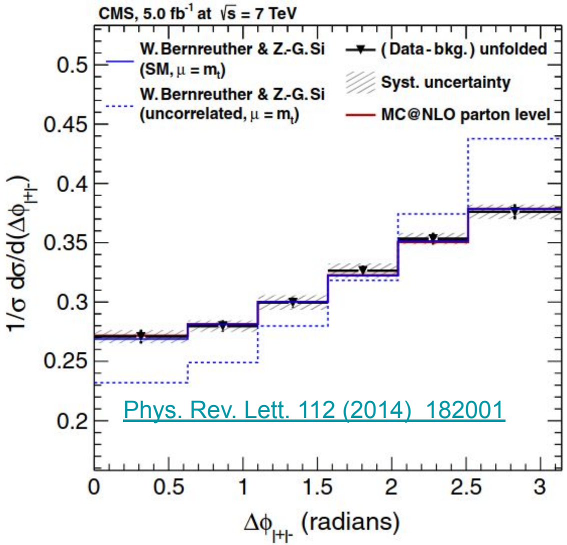
- **The most massive fundamental particle : $m_t \approx 173$ GeV**
- **Large width : $\Gamma_t \sim 1$ GeV**
  - **Short lifetime : $\tau = 1/\Gamma_t \sim 10^{-25}$ s**
    - ✓ **decay before hadronisation : $\sim 10^{-23}$ s**
      - In case of top pair production, $t\bar{t}$ spins can be measured from decay products
      - The effect due to spin correlation has already been measured in several experiments.

BR(t→Wb)~100% + weak interaction is maximally parity-violating
→ correlations are observable!



Phys.Rev.D81: 074024,2010



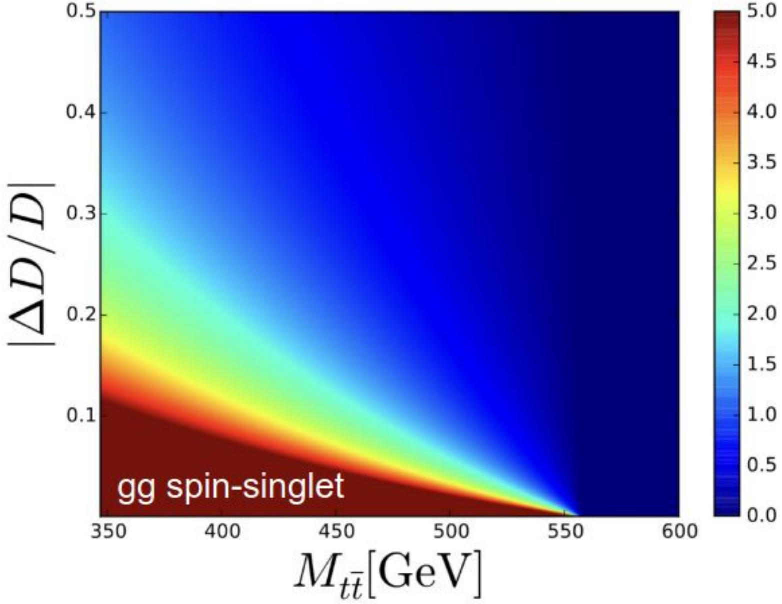


Phys. Rev. Lett. 112 (2014) 182001

(Quantum) Top quark beyond spin correlations

Eur. Phys. J. Plus (2021) 136 (March 2020) → first analysis of top quark pair production from the quantum information point of view: “bipartite qubit system”

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1\Omega_2} = \frac{1}{4\pi^2}\left(1 + \alpha_1\, \cancelto{0}{\mathbf{B}_1} \cdot \hat{\ell}_1 + \alpha_2\, \cancelto{0}{\mathbf{B}_2} \cdot \hat{\ell}_2 + \alpha_1\alpha_2\, \hat{\ell}_1\, \mathbb{C}\, \hat{\ell}_2\right)$$



$$\mathrm{Tr}\,[\mathbb{C}] < -1$$ *Peres-Horodecki criterion*

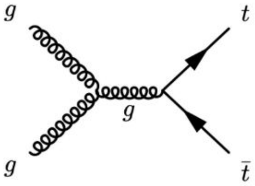
$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$ *a simple observable*

$$D = \frac{\mathrm{Tr}\,[\mathbb{C}]}{3} \Rightarrow D < -\frac{1}{3}$$ *a quantum entanglement marker!*

Density matrix, Peres-Horodecki criterion [ref]

	pure state	mixed states
wavefunction	$\lvert\psi\rangle$	$f(\lvert\psi_i\rangle)$
density matrix	$\rho = \lvert\psi\rangle\langle\psi\rvert$	$\rho = \sum_i p_i \lvert\psi_i\rangle\langle\psi_i\rvert$
trace of ρ	$Tr(\rho) = 1$	$Tr(\rho) = 1$
trace of ρ^2	$Tr(\rho^2) = 1$	$Tr(\rho^2) < 1$
entropy	$S(\rho) = 0$	$S(\rho) = -Tr(\rho \ln \rho) > 0$

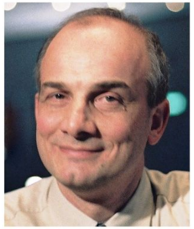
- **density matrix for 1 spin**
  - $\rho = \frac{1}{2}\left(I_2 + \sum_i B_i \sigma^i \otimes I_2\right)$
    - ✓ $B_i = \langle \sigma^i \rangle$
      - B_i 3 parameters → Polarization
- **density matrix for 2 spins**
  - $\rho = \frac{1}{4}\left(I_4 + \sum_i (B_i^+ \sigma^i \otimes I_2 + B_i^- I_2 \otimes \sigma^i)\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j)$
    - ✓ $B_i^\pm = \langle \boldsymbol{\sigma}_\pm^i \rangle$
      - 6 paramters → Polarizations
    - ✓ $C_{ij} = \langle \boldsymbol{\sigma}_+^i \boldsymbol{\sigma}_-^j \rangle$
      - 9 parameters → Correlations



- From pure state to mixed state.

"A quantitatively characterization of the degree of the entanglement between the subsystems of a system in a mixed state, is not unique! "

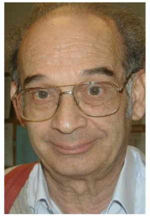
$$\rho_{AB} \overset{?}{=} \sum_{i=1}^{N} p_i \rho_A^{(i)} \otimes \rho_B^{(i)}, \quad \left(\sum_i^N p_i = 1,\, p_i > 0\right)$$



"Finally, we prove that the weak membership problem for the convex set of separable normalized bipartite density matrices is ***NP-HARD***. "

——Leonid Gurvits

- For 2×2 and 2×3 system, it is solved by Peres, and Horodeckis 1996 (Peres-Horodecki criterion, concurrence).



Asher Peres
(1934/01/30-2005/01/01)

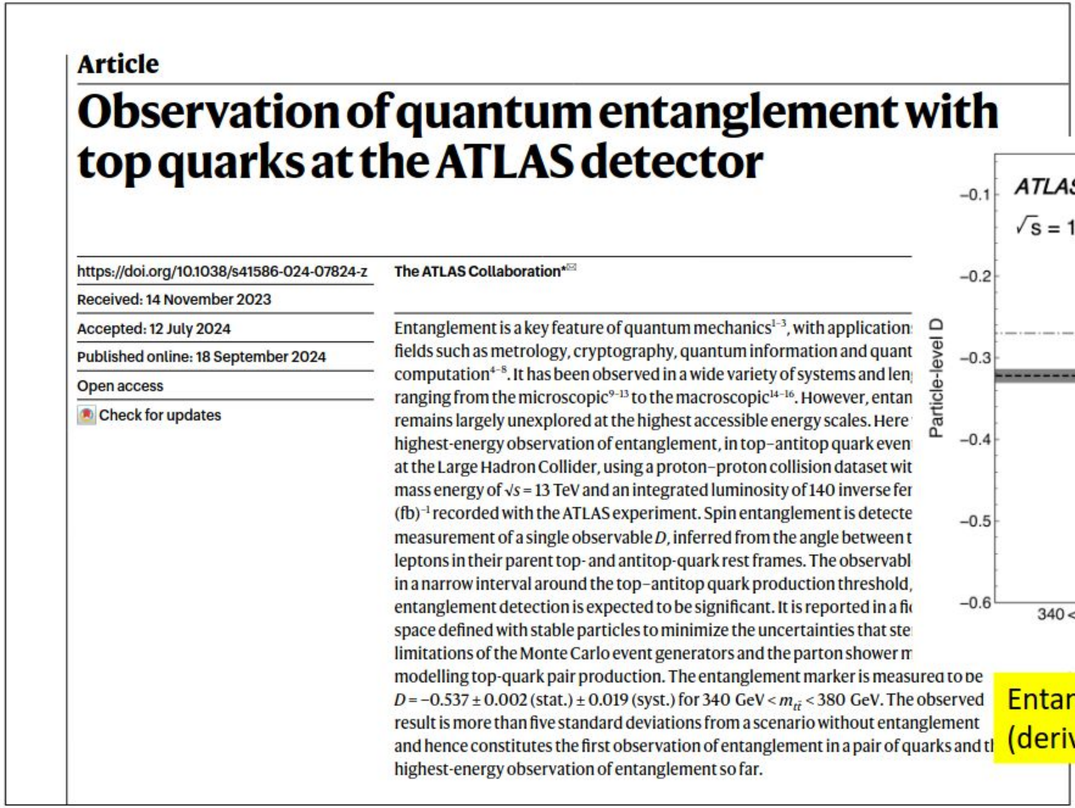
Ryszard Horodecki
(1943/09/30-)

Paweł Horodecki
(1971-)

Michał Horodecki
(1973-)

Top quark Entanglement Discovery

Nature volume 633, pages 542–547 (2024)



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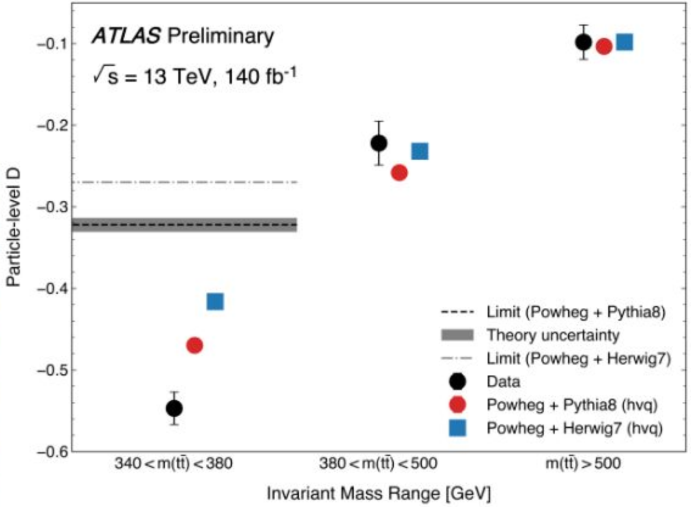
$$D = -3\langle\cos\varphi\rangle$$



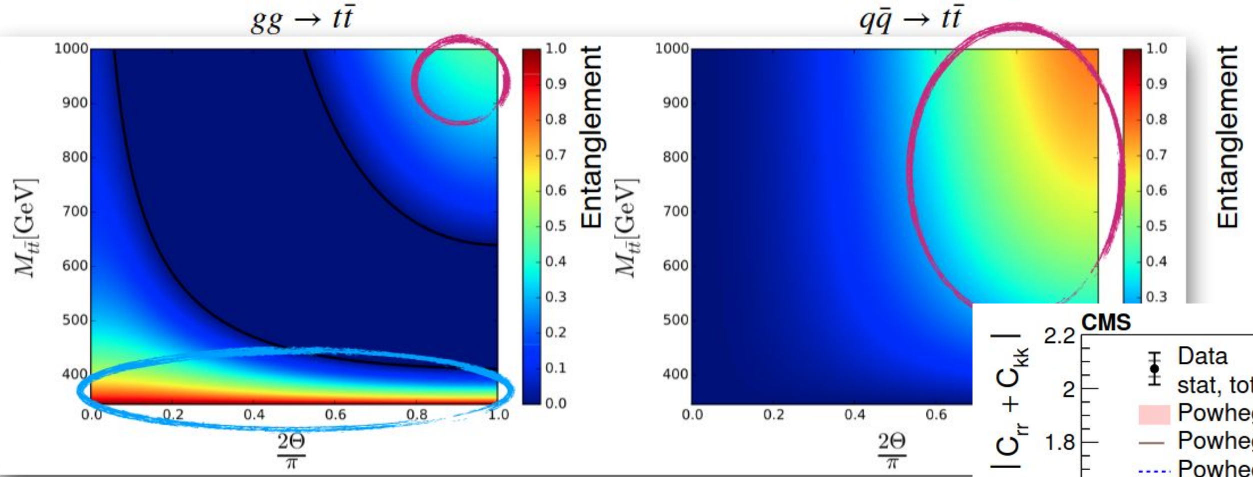
Entangled state : ***D* < −1/3**
(derived from the Peres–Horodecki criterion)

Top quark Entanglement Discovery

- SM predicts entangled states:
  - at the production threshold region in gg fusion production
  - at the boosted region for central production of the $t\bar{t}$ system

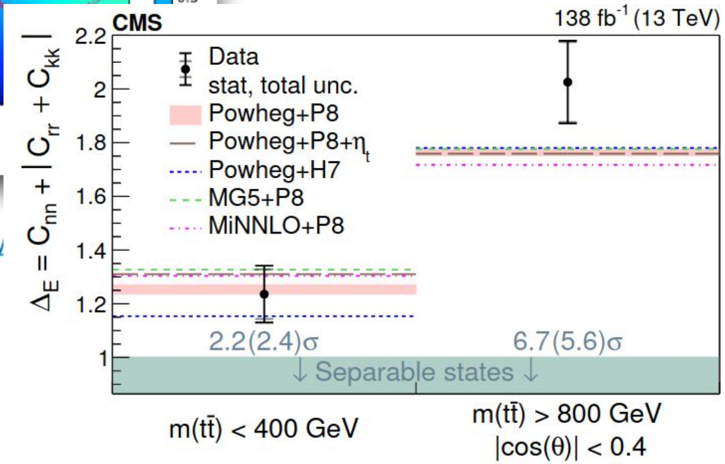
high relative velocity
of top quarks
→ space-like
separated events

top2024



low relative velocity of top quarks
→ time-like separated events

Eur. Phy



Rep. Prog. Phys. 87 (2024) 117801
Phys. Rev. D 110 (2024) 112016

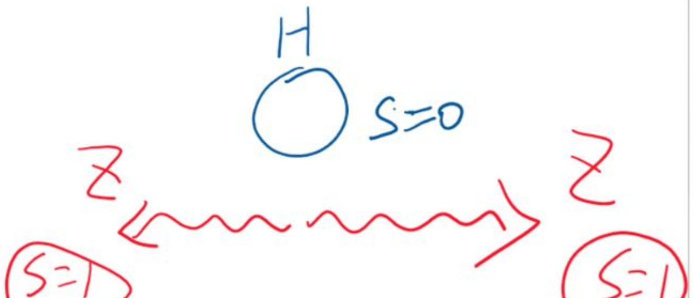
QE between Triplets: H→ VV

- The polarization density matrix(PDM) can be reconstructed from the angular distributions of the decay products:

$$\rho = |\Psi_{ZZ}\rangle\langle\Psi_{ZZ}| = |\Phi\rangle\langle\Phi|$$

$$|\Phi\rangle = \sum c_{ij}|ij\rangle \to \sum \mathcal{M}(\lambda_1, \lambda_2)|\lambda_1, \lambda_2\rangle$$

Ψ_Z has three polarization states: +1, 0, -1



$$|\psi_{ZZ}\rangle = \frac{1}{\sqrt{2+\beta^2}}(|+-\rangle - \beta|00\rangle + |-+\rangle),$$

For two-triplet system, we can expand the density matrix as

$$\rho = \frac{1}{9}[\mathbb{1}\otimes\mathbb{1}] + \sum_{a=1}^{8} f_a [T^a \otimes \mathbb{1}] + \sum_{a=1}^{8} g_a [\mathbb{1}\otimes T^a] + \sum_{a,b=1}^{8} h_{ab} [T^a \otimes T^b]$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+ \, d\Omega_-} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left[\underbrace{\rho_{V_1V_2}}_{\text{Production}} \underbrace{(\Gamma_1 \otimes \Gamma_2)}_{\text{Decay}}\right]$$

All coefficients → Quantum Tomography

- No direct spin measurements: inferred by angular distributions.
- Both the state before decay & the final state decay products inherit the SAME quantum information.

Quantum Tomography

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_+\, d\Omega_-} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left[\rho_{V_1V_2}\left(\Gamma_1 \otimes \Gamma_2\right)\right],$$

$$\Gamma(\theta,\phi) = \frac{1}{4}\begin{pmatrix} 1+\cos^2\theta - 2\eta_\ell\cos\theta & \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell \sin\theta)e^{i\phi} & (1-\cos^2\theta)e^{i2\phi} \\ \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell \sin\theta)e^{-i\phi} & 2\sin^2\theta & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell \sin\theta)e^{i\phi} \\ (1-\cos^2\theta)e^{-i2\phi} & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell \sin\theta)e^{-i\phi} & 1+\cos^2\theta - 2\eta_\ell\cos\theta \end{pmatrix};$$

$\longrightarrow$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}[1 + A^1_{LM}Y^M_L(\theta_1,\phi_1) + A^2_{LM}B_L Y^M_L(\theta_2,\phi_2)$$
$$+ C_{L_1M_1L_2M_2}B_{L_1}B_{L_2}Y^{M_1}_{L_1}(\theta_1,\phi_1)Y^{M_2}_{L_2}(\theta_2,\phi_2)],$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^M_L(\Omega_j)d\Omega_j = \frac{B_L}{4\pi}A^j_{LM}, \qquad j=1,2;$$
$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1)Y^{M_2}_{L_2}(\Omega_1)d\Omega_1 d\Omega_2 = \frac{B_{L_1}B_{L_2}}{4\pi}C_{L_1M_1L_2M_2}.$$

Integral $\longrightarrow$ events summed

Notice that the theoretical form of the density matrix imposes strong constraints on the various coefficients: this assumption can be relaxed though

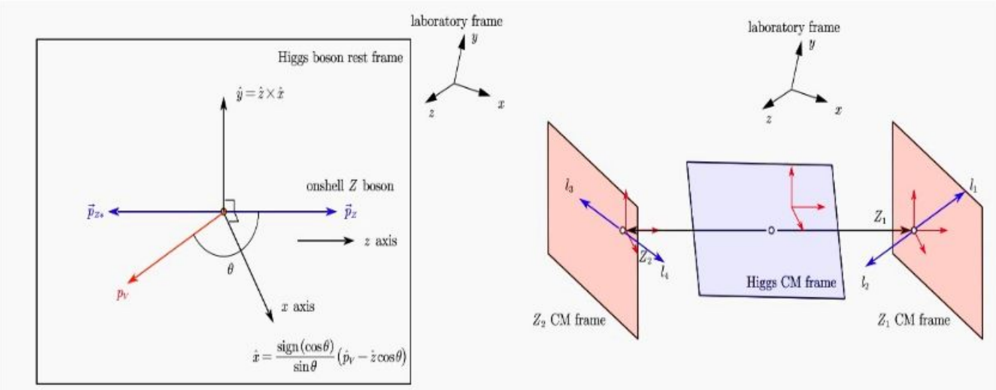
$$\rho = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&\frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right)&0&\frac{1}{3}C_{2,1,2,-1}&0&\frac{1}{3}C_{2,2,2,-2}&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&\frac{1}{3}C_{2,1,2,-1}&0&\frac{1}{3}\left(1-\sqrt{2}A^1_{2,0}\right)&0&\frac{1}{3}C_{2,1,2,-1}&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&\frac{1}{3}C_{2,2,2,-2}&0&\frac{1}{3}C_{2,1,2,-1}&0&\frac{1}{6}\left(\sqrt{2}A^1_{2,0}+2\right)&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

More details in
PRD 107 (2023) 1, 016012
JHEP 10 (2024) 211
PRD 111 (2025) 3, 036008

Quantum Tomography in Operation

In two spin-1 massive bosons' system:

- The z-axis is the direction of the on-shell Z boson's 3-momentum.
- The $\hat{x}$ axis is in the production plane: $\hat{x} = \frac{sign(cos\theta)(\hat{p}_p - cos\theta\hat{z})}{sin\theta}, \hat{p}_p = (0,0,1)$
- The $\hat{y} = \hat{z}\times\hat{x}$
- J_Z is the polarization operator.
- The eigenstates of J_Z is the basis of the spin space.



Two Lorentz Transformation:

- Higgs rest frame → determine Z axis
- Z boson rest frame(boost along Z vector) → lepton's polar angles

Obtain :(θ_1, φ_1) in Z_1 rest frame, (θ_2, φ_2) in Z_2 rest frame. The coefficients can A^l_{LM} *and* $C_{L_1M_1L_2M_2}$ can be calculated

$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM} T^L_M \otimes \mathbb{1}_3 + A^2_{LM} \mathbb{1}_3 \otimes T^L_M + C_{L_1M_1L_2M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

Quantum Tomography → Bell Inequality

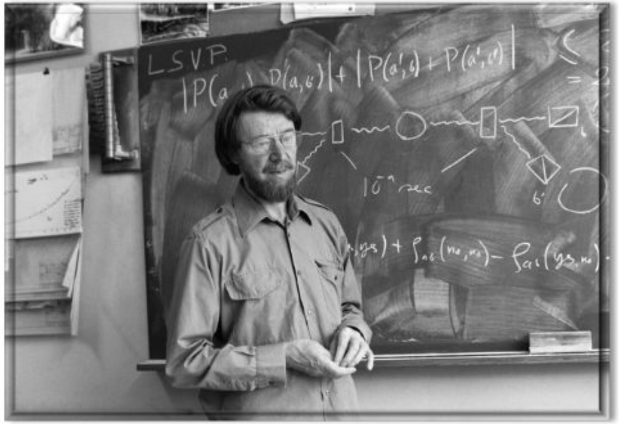
- The most original form of Bell inequalities (Clauser-Horne-Shimony-Holt Inequality):

$$P(A_1 B_1 | AB, \lambda) = P(A_1 | A, \lambda) P(B_1 | B, \lambda)$$

Classical local hidden variable theory:

$$\boldsymbol{I_3 = \langle O_{Bell} \rangle = Tr\{\rho O_{Bell}\} \leq 2}$$

ρ **: Polarization density matrix (PDM)**



- More general form (Collins-Gisin-Linden-Massar-Popescu Inequality)

$$\begin{aligned}\mathcal{I}_d = \sum_{k-0}^{[d/2]-1} \left(1 - \frac{2k}{d-1}\right)\{+[P(A_1 = B_1 + k) + P(B_1 = A_2 + k + 1) + P(A_2 = B_2 + k) \\ + P(B_2 = A_1 + k) - [P(A_1 = B_1 - k - 1) + P(B_1 = A_2 - k) \\ + P(A_2 = B_2 - k - 1) + P(B_2 = A_1 - k - 1)]\}\end{aligned}$$

3-dimensional form:

$$\begin{aligned}\mathcal{I}_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)].\end{aligned}$$

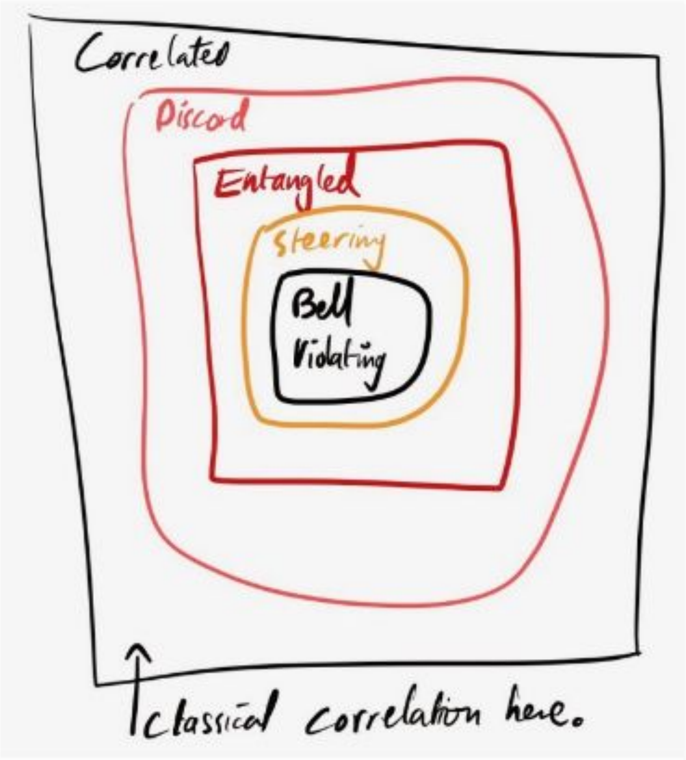
- The expectation value of the Bell operator can be written as:

$$\begin{aligned}\mathcal{B} = \Big[& \frac{2}{3\sqrt{3}} (T_1^1 \otimes T_1^1 - T_0^1 \otimes T_0^1 + T_1^1 \otimes T_{-1}^1) + \frac{1}{12} (T_2^2 \otimes T_2^2 + T_2^2 \otimes T_{-2}^2) \\ & + \frac{1}{2\sqrt{6}} (T_2^2 \otimes T_0^2 + T_0^2 \otimes T_2^2) - \frac{1}{3} (T_1^2 \otimes T_1^2 + T_1^2 \otimes T_{-1}^2) + \frac{1}{4} T_0^2 \otimes T_0^2 \Big] + \text{h.c.}.\end{aligned}$$

- Bell inequality expectation value can be calculated:

$$\mathcal{I}_3 = \frac{1}{36}\left(18 + 16\sqrt{3} - \sqrt{2}\left(9 - 8\sqrt{3}\right) A_{2,0}^1 - 8\left(3 + 2\sqrt{3}\right) C_{2,1,2,-1} + 6C_{2,2,2,-2}\right)$$

Quantum Tomography → Entanglement Criteria



SUSY2024

The theoretical form of the density matrix imposes strong constraints and leads to a entanglement criteria

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c \\ 0 & 0 & 0 & 0 & 0 & b & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & f & 0 \\ 0 & 0 & 0 & 0 & d & 0 & 0 & 0 & 0 \\ 0 & b^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & f^* & 0 & 0 & 0 & 0 & 0 \\ c^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{29}$$

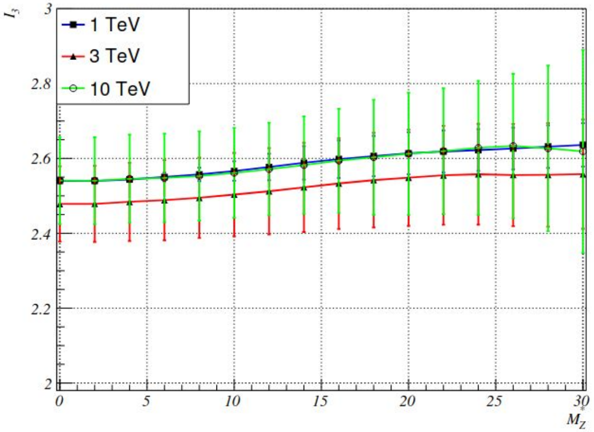
which has eigenvalues $a, d, g, \pm|b|, \pm|c|, \pm|f|$. Therefore if $b \neq 0$, $c \neq 0$ or $f \neq 0$ the density matrix is entangled. Note that the reverse is also true: if $b = c = f = 0$ the state is obviously separable, as ρ is diagonal in the separable basis. This represents a noteworthy example beyond a two-qubit system, where, thanks to an underlying symmetry, the Peres-Horodecki condition for entanglement is not just sufficient, but also necessary.

When applied this condition to our density matrix (26), it turns out that the ZZ system is entangled if and only if

$$\boxed{C_{2,1,2,-1} \neq 0 \qquad \text{or} \qquad C_{2,2,2,-2} \neq 0\,.}$$

Prospects@LHC, MuC, CEPC

- The numerical analysis shows that with a luminosity of L =300 fb−1 entanglement can be probed at >3σ level. For **L=3 ab−1 (HL-LHC) entanglement** can be probed beyond the **5σ** level, while the sensitivity to **Bell inequalities** violation is at the **4.5σ** level.
- At Muon Collider, **Quantum entanglement** can be probed up to **4σ** of significance with lower MZ2 cut or 2σ ~ 3σ with higher MZ2 cut, using either one of the correlation coefficients C2,1,2,−1 and C2,2,2,−2. The significance of the **violation of Bell inequality** can be obtained up to **2σ**.



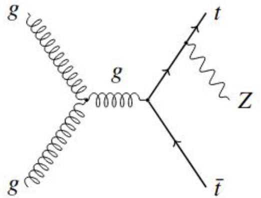
$\sqrt{s} = 1\,\mathrm{TeV}$

M_{Z_2} (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.563 ± 0.325	-0.928 ± 0.216	0.527 ± 0.164
10.000	2.596 ± 0.335	-0.943 ± 0.220	0.553 ± 0.179
20.000	2.654 ± 0.373	-0.977 ± 0.248	0.574 ± 0.192
30.000	2.663 ± 0.508	-0.979 ± 0.334	0.589 ± 0.248

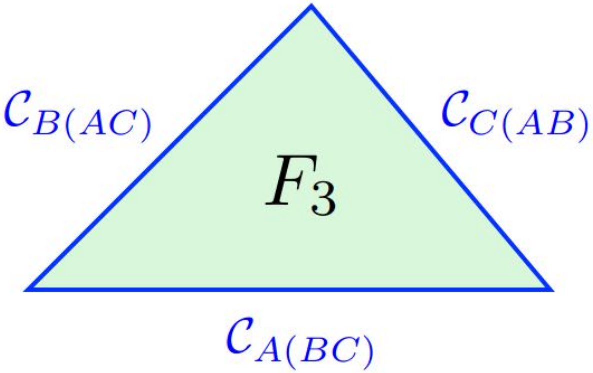
Table 2. Values of the correlation coefficients $C_{2,1,2,-1}$ and $C_{2,2,2,-2}$ as the signal for quantum entanglement and also the expectation value of the Bell operator I_3. The expected target luminosity is $30\mathrm{ab}^{-1}$ and $\sqrt{s} = 1\,\mathrm{TeV}$.

Quantum Collisions: more funs

- **Three-partite entanglement**
  - 3-body Decay: *Phys.Rev.Lett.* 132 (2024) 15, 151602; arXiv:2502.19470
  - 2 to 3 process (ttZ): arXiv:2404.03292



- **Quantum Process Tomography** (operating initial particles' flavor and spin)
  - arXiv:2412.01892



concurrence triangle

PHYSICAL REVIEW A, VOLUME 62, 062314

Three qubits can be entangled in two inequivalent ways

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(Received 26 May 2000; published 14 November 2000)

PHYSICAL REVIEW A, VOLUME 65, 052112

Four qubits can be entangled in nine different ways

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(Received 29 November 2001; published 25 April 2002)

Quantum Process Tomography: one further step

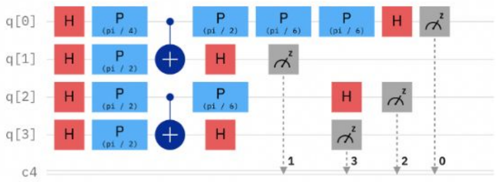
- **Spin and flavour measurements** in collider experiments as a **quantum instrument**
- **Choi matrix**, which completely determines input-output transitions, can be both **theoretically computed** and **experimentally reconstructed**
- **Polarized Beam collisions, or,**

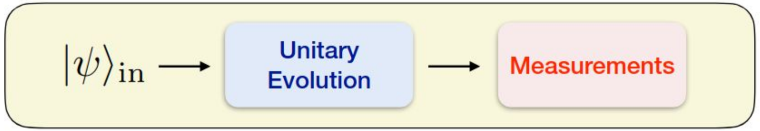
ref **lepton scattering on polarized target experiments (see next)**

Particle Collider = Quantum Computer

=

C. Altomonte, A.Barr [2312.02242]





Reconstruction of Choi matrix $\boxed{e^-e^+ \to t\bar{t}}$

- Reconstruction of the diagonal part:

$$\widetilde{\mathcal{I}}_x = \frac{1}{4}\begin{pmatrix} \mathcal{I}_x(|++\rangle\langle++|) & \mathcal{I}_x(|++\rangle\langle+-|) & \mathcal{I}_x(|++\rangle\langle-+|) & \mathcal{I}_x(|++\rangle\langle--|) \\ \mathcal{I}_x(|+-\rangle\langle++|) & \mathcal{I}_x(|+-\rangle\langle+-|) & \mathcal{I}_x(|+-\rangle\langle-+|) & \mathcal{I}_x(|+-\rangle\langle--|) \\ \mathcal{I}_x(|-+\rangle\langle++|) & \mathcal{I}_x(|-+\rangle\langle+-|) & \mathcal{I}_x(|-+\rangle\langle-+|) & \mathcal{I}_x(|-+\rangle\langle--|) \\ \mathcal{I}_x(|--\rangle\langle++|) & \mathcal{I}_x(|--\rangle\langle+-|) & \mathcal{I}_x(|--\rangle\langle-+|) & \mathcal{I}_x(|--\rangle\langle--|) \end{pmatrix}$$

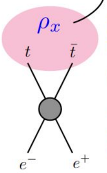
- Consider 4 purely polarised beam settings:

$$\{|i\rangle\} = \{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\} \qquad \rho_0^i = |i\rangle\langle i|$$

$$\mathcal{I}_x(|i\rangle\langle i|) = \varrho_x^i = \mathrm{Tr}\varrho_x^i \cdot \frac{\varrho_x^i}{\mathrm{Tr}\varrho_x^i}$$

measurable

$$\frac{\sigma_x[e^-e^+(\rho_0^i) \to t\bar{t}]}{\sigma[e^-e^+(\rho_0^{\text{mix}}) \to t\bar{t}]}$$



Reconstruction of spin-density matrix is possible

Quantum State Tomography

Y.Afik, J.Nova [2003.02280], Ashby-Pickering, Barr, Wierzchucka [2209.13990]