Investigation on Proof of Factorization Theorem at Operator Level

Gao-Liang Zhou

Center for High Energy Physics, PKU

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Outline

- Factorization Theorem
- Procedure of Conventional Proof
- Proof of QCD Factorization at Operator Level
- Summary

Factorization Theorem

$$\mathrm{d}\sigma = H\prod_i(\bigotimes J_i)\bigotimes S\mathrm{d}V$$

- H: Hard part which can be calculated perturbatively
- J_i: Collinear part which is independent of explicit process
- S: Soft part which can be cancelled or absorbed into jet functions

Physical Picture

There is at most one physical parton which participate in the hard collision at leading order. (Scalar polarized collinear gluons?)

Coherence effects between different jets can be absorbed into universal(process independent) functions. (Soft gluons?)

Infrared Power counting

(G. Sterman, Phys. Rev. D17, 2773(1978))
For each Feynman diagram that contributes to the hard collision process at leading order:

- There is at most one physical parton that connect to the hard subgraph
- There can be arbitrary number of scalar polarized collinear gluons connecting to the hard subgraph in covariant gauge
- Except for the hard subgraph, correlations between different jets can be caused only by soft gluons
- Soft gluons do not connect to the hard subgraph

Soft Gluons and Grammer-Yennie Approximation

$$(p+q)^2 \to p^2 + 2p^+ q^- \quad A^\mu \to A^- \delta^{\mu -}$$

$$A^\mu(x+x_n) \to n \cdot A(\bar{n} \cdot (x+x_n), n \cdot x, \vec{x_\perp})\bar{n}^\mu$$

where,

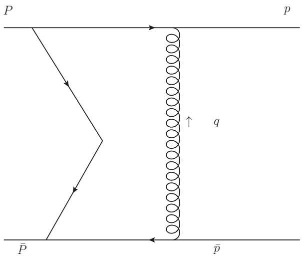
$$\bar{n}^\mu = \frac{1}{\sqrt{2}}(1, -\vec{n})$$

Glauber Gluons

For $(p^+, p^-, p_\perp) \sim Q(1, {\Lambda_{QCD}}^2 Q)$, $q^\mu \sim \Lambda_{QCD}$, we have:

$$2p^+ q^- \sim q^2 - 2\vec{p_\perp} \cdot \vec{q_\perp}$$

Glauber Gluons and Pinch Singularities



There are pinch singular points produced by the two propagators:

$$(p-q)^2 + i\epsilon \simeq p^2 - 2p\cdot q - \vec{q_\perp}^{\,2} + i\epsilon$$

$$(P-p+q)^2 + i\epsilon \simeq (P-p)^2 - 2(P-p)\cdot q - \vec{q_\perp}^{\,2} + i\epsilon$$

Cancelation of Effects of Final State Interactions in Drell-Yan Process

Drell-Yan process:

$$p_1 + p_2 \to l^+ l^- + X \tag{1}$$

We have:

$$\frac{\mathrm{d}\sigma}{dq^2 dy} = \frac{4\pi\alpha^2 Q_q^2}{3s} \int \frac{\mathrm{d}^2 \vec{q}_\perp}{(2\pi)^4} H^{\mu\nu} \frac{1}{q^2} \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) \tag{2}$$

where q denotes the total momentum of the lepton pair, $y = \frac{1}{2} \ln q^+/q^-$, Q_q is the electric charge of the active quark. $Q = \sqrt{q^2} \gg \Lambda_{QCD}$ is the hard energy scale of the process.

Cancelation of Effects of Final State Interactions in Drell-Yan Process

$$
\begin{aligned}
H^{\mu\nu}(q) \quad = \quad & \sum_{P,p,k} \Phi^*(P+k, P-k)\Phi(P+p, P-p) \\
& \int \mathrm{d}^4 x e^{-iq\cdot x} \langle P+k, \quad P-k| \\
& \Omega^\dagger(-\infty) e^{iH_{QCD}x^0} J^\mu(\vec{x}) e^{-iH_{QCD}x^0} \Omega(\infty) \\
& \Omega^\dagger(\infty) J^\nu(\vec{0}) \Omega(-\infty) |P+p, \quad P-p\rangle
\end{aligned}
$$

where Φ is the wave function of initial packet, $J^\mu(\vec{x})$ is the electromagnetic currant in Schrödinger picture.

$$
\Omega(t) \quad = \quad e^{iH_{QCD}t} e^{-iH_{free}t}
$$

H_{QCD} is the Hamiltonian of QCD, H_{free} is the free Hamiltonian of hadrons.

Cancelation of Effects of Final State Interactions in Drell-Yan Process

According to the unitarity of $\Omega(t)$, we have:

$$
\begin{aligned}
H^{\mu\nu}(q) &= \int \mathrm{d}^4 x e^{-iq\cdot x} \langle p_1 p_2 | \Omega^\dagger(-\infty) \\
&\quad e^{iH_{QCD}x^0} J^\mu(\vec{x}) e^{-iH_{QCD}x^0} J^\nu(\vec{0}) \Omega(-\infty) | p_1 p_2 \rangle
\end{aligned}
$$

We rewrite the hadronic tensor in interaction picture and have:

$$
\begin{aligned}
H^{\mu\nu}(q) &= \sum_X \int \mathrm{d}^4 x e^{-iq\cdot x} \\
&\quad \langle p_1 p_2 | \Omega_0^\dagger(-\infty) U_{QCD}^\dagger(x^0, -\infty) J_I^\mu(x) | X \rangle \\
&\quad \langle X | J_I^\nu(0) U_{QCD}(0, -\infty) \Omega_0(-\infty) | p_1 p_2 \rangle
\end{aligned}
$$

Cancelation of Effects of Final State Interactions in Drell-Yan Process

where

$$\mathcal{O}_I(x) \equiv e^{iH_0 x^0} \mathcal{O}(\vec{x}) e^{-iH_0 x^0}$$

$$\begin{aligned} U_{QCD}(t_1, t_2) &\equiv e^{iH_0 t_1} e^{-iH_{QCD}(t_1 - t_2)} e^{-iH_0 t_2} \\ &= T \exp\{-i \int_{t_2}^{t_1} \mathrm{d}t (H_I)_{QCD}(t)\} \end{aligned}$$

$$\Omega_0(t) = e^{iH_0 t} e^{-iH_{free} t}$$

H_0 is the free Hamiltonian of quarks and gluons.

We first take the substitution:

$$U_{QCD}(\infty, -\infty) \to U_{QCD}(x^0, -\infty)$$

where,

$$\begin{aligned} U_{QCD}(t_1, t_2) &\equiv e^{iH_0 t_1} e^{-iH_{QCD}(t_1 - t_2)} e^{-iH_0 t_2} \\ &= T\exp\{-i \int_{t_2}^{t_1} \mathrm{d}t (H_I)_{QCD}(t)\} \end{aligned}$$

Thus, we should make the substitution:

$$(2\pi)^4\delta^{(4)}(\sum_j l_j) \to (2\pi)^3\delta^{(3)}(\sum_j \vec{l}_j))\frac{e^{-i(\sum_j {l_j}^0 \pm i\epsilon)x^0}}{(-i)(\sum_j {l_j}^0 \pm i\epsilon)}$$

in perturbative calculations.
In the coupling between soft gluons and particles collinear-to-plus, we take the substitution:

$$\frac{\delta(\sum_j {l_j}^z)e^{-i(\sum_j {l_j}^0 \pm i\epsilon)x^0}}{(-i)(\sum_j {l_j}^0 \pm i\epsilon)} = \frac{1}{\sqrt{2}}\frac{\delta(\sum_j {l_j}^z)e^{-i(\sum_j {l_j}^0 \pm i\epsilon)x^0}}{(-i)(\sum_j {l_j}^- \pm i\epsilon)}$$

We then take the approximation:

$$e^{-i(\sum_j l_j{}^0 + i\epsilon)x^0} \simeq e^{-i(\sum_j \frac{\tilde{l}_j{}^+}{\sqrt{2}} + i\epsilon)x^0}$$

$$\begin{aligned}\delta(\sum_j l_j^3) &= \sqrt{2}\delta(\sum_j l_j^+ - \sum_j l_j^-) \\ &\simeq \sqrt{2}\delta(\sum_j \tilde{l}_j^+)\end{aligned}$$

where $\tilde{l}_j^\mu = (l_j{}^+, l_j, \vec{l}_{j\perp})$ for $l^j$is collinear-to-plus, $\tilde{l}_j = (0, l_j, \vec{l}_{j\perp})$ for l_j is soft.

One see that singular point of l_j^- in Glauber region that locate in upper or lower half plane can only be produced by the other end of l_j.

- If the other end is jet with large minus momentum, then one can deform the integral path of l_j^- to avoid the Glauber region
- If the other end is jet collinear to plus, then l_j does not affect the factorization theorem.

$$\left(\partial^\mu - n\cdot A(\bar{n}\cdot(x+x_n), n\cdot x, \vec{x_\perp})\bar{n}^\mu\right)(x_n) = Y_n\left(\partial^\mu\right) Y_n^\dagger(x_n)$$

where

$$Y_n(x_n) \equiv P\exp(ig\int_{-\infty}^{0} \mathrm{d}s n\cdot A(\bar{n}\cdot(x+x_n)+s, n\cdot x, \vec{x_\perp}))$$

Effective Theory

$$
\begin{aligned}
\mathcal{L}_\Lambda &\equiv \sum_{n^\mu} \mathcal{L}_n^{(0)} + \mathcal{L}_s \\
&= \sum_{n^\mu} i\bar{\psi}_n^{(0)}(\not\partial - ig \not A_n^{(0)})\psi_n^{(0)} \\
&\quad + \sum_{n^\mu} \frac{1}{2g^2} tr\left\{([\partial^\mu - igA_n^{(0)\mu}, \partial^\nu - igA_n^{(0)\nu}])^2\right\} \\
&\quad + i\bar{\psi}_s(\not\partial - ig \not A_s)\psi_s \\
&\quad + \frac{1}{2g^2} tr_c\{[\partial^\mu - igA_s^\mu, \partial^\nu - igA_s^\nu]^2\}
\end{aligned}
$$

Hard Vertex

$$\Gamma_Q^\mu = \sum_{n,\bar{n}\cdot p,\ldots,m,\bar{m}\cdot p'} \Gamma_Q^\mu\big(Y_n(\widehat{\psi}^{(0)}_{n,x})_{\bar{n}\cdot p},\ldots,Y_m(\partial^{m\perp} - ig\widehat{A}^{(0)m\perp}_{m,x})_{\bar{m}\cdot p'}Y_m^\dagger\big)(x)$$

where

$$\begin{aligned}
\widehat{\psi}^{(0)}_{n,x}(x_n) &= W^{(0)\dagger}_{n,x}(x_n)\psi^{(0)}_n(x_n) \\
(\partial^\mu - ig\widehat{A}^{(0)\mu}_{n,x}) &= W^{(0)\dagger}_{n,x}(\partial^\mu - igA^{(0)\mu}_n)W^{(0)}_{n,x}
\end{aligned}$$

$$W^{(0)}_{n,x}(x_n) \equiv P\exp(ig\int_{-\infty}^{0} \mathrm{d}s\bar{n}\cdot A^{(0)}_n(n\cdot(x+x_n)+s,\bar{n}\cdot x,x_\perp))$$

At leading order in Λ_{QCD}/Q

- Γ^{μ}_{Q} is multi-linear with $\widehat{\psi}^{(0)}_{n}$ and $\widehat{A}^{(0)}_{m}$
- For each direction n^{μ}, $\widehat{\psi}^{(0)}_{n}$ and $\widehat{A}^{(0)}_{n}$ do not both appear in Γ^{μ}_{Q} simultaneously
- Γ^{μ}_{Q} can rely on soft gluons only through the Wilson lines Y_n, $Y^{\dagger}_{m}, \ldots$

We then have

$$\begin{aligned} H^{\mu\nu}(q) &= \int \mathrm{d}^4 x e^{-iq\cdot x}\langle p_1 p_2|\Omega_\Lambda^\dagger(-\infty) \\ & e^{iH_\Lambda x^0}\Gamma^\mu(\vec{x})e^{-iH_\Lambda x^0}\Gamma^\nu(\vec{0})\Omega_\Lambda(-\infty)|p_1 p_2\rangle \end{aligned}$$

where

$$\Omega_0(t) = e^{iH_\Lambda^0 t}e^{-iH_{free}t}$$

with H_Λ^0 the free part of H_Λ and H_{free} free Hamiltonian between hadrons.

$$
\begin{aligned}
S &= \langle 0|S^\dagger(x)S(0)|0\rangle \\
&\simeq \langle 0|S^\dagger(0)S(0)|0\rangle \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
H^{\mu\nu}(q) &= \sum_\Gamma \frac{1}{N_c^2}\int \frac{\mathrm{d}\bar{n}_1\cdot k_1}{\bar{n}_1\cdot k_1}\int \frac{\mathrm{d}\bar{n}_2\cdot k_2}{\bar{n}_2\cdot k_2} D(k_1,p_1)D(k_2,p_2) \\
&\quad \sigma^{\mu\nu}_{hard}(k_1,k_2,q)
\end{aligned}
$$

where

$$
\begin{aligned}
& \sigma^{\mu\nu}_{hard}(k_1, k_2, q) \\
= & \int \mathrm{d}^4 x e^{-iq\cdot x} \langle k_1 k_2 | \Omega^{\dagger}_{\Lambda}(-\infty) e^{iH_{\Lambda}x^0} \Gamma^{\mu}(\vec{x}) e^{-iH_{\Lambda}x^0} \\
& \Gamma^{\nu}(\vec{0}) \Omega_{\Lambda}(-\infty) | k_1 k_2 \rangle |_{\bar{n}_i \cdot A^{(0)}_{n_i} = \psi_s = A_s = 0}
\end{aligned}
$$

Summary

- Effects of Glauber gluons can be absorbed into Wilson lines for process in which effects of final interactions cancel out
- Effective theory in which soft gluons decouple from collinear particles can be constructed. It is equivalent to QCD in the level of cross section for processes in which pinch singular singularities caused by Glauber gluons cancel out.
- Operator method based on effective theory is important in proof of QCD factorization. It exhibits some general aspects of factorization theorem

Thank you !