The Casimir-Polder force in a stationary environment out of equilibrium thermal

吴普训

宁波大学、北大高能中心

13th LHC Mini-Workshop, Zhejiang University, Hangzhou

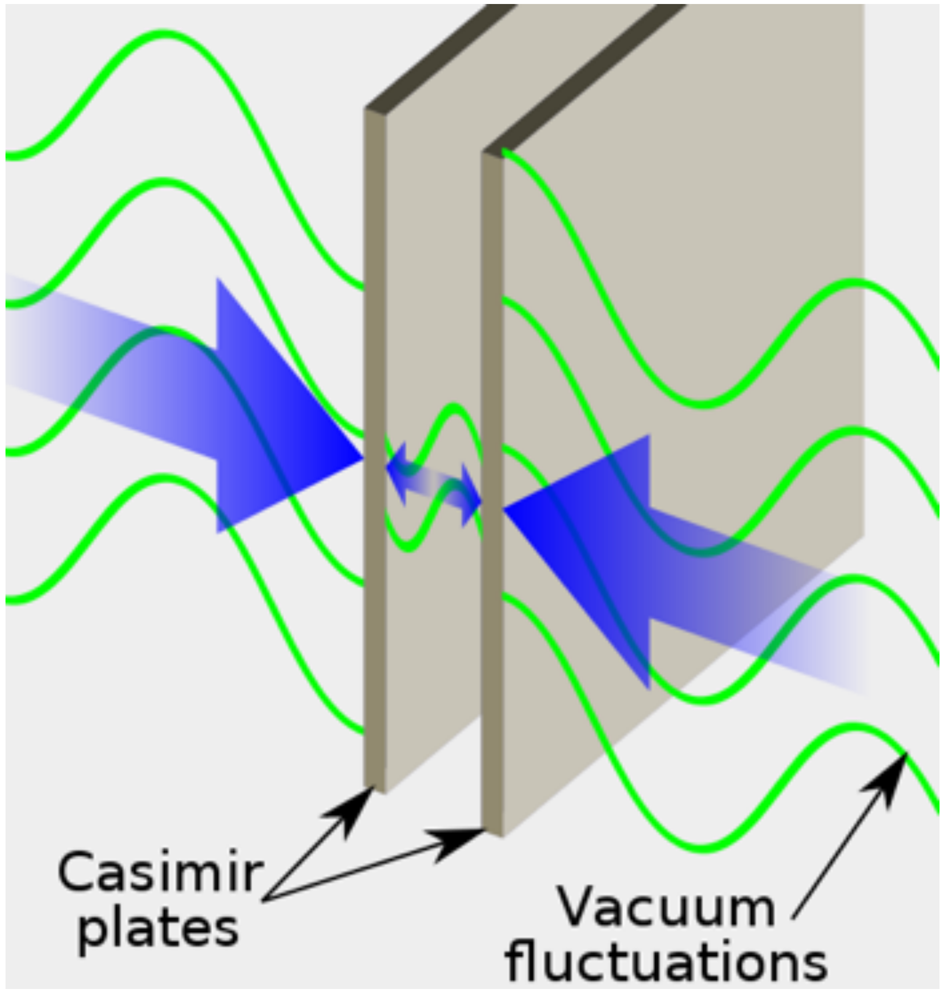
Puxun wu and Hongwei Yu: PRA **90**, 032502 (2014)
Puxun wu and Hongwei Yu: In preparation

Outline

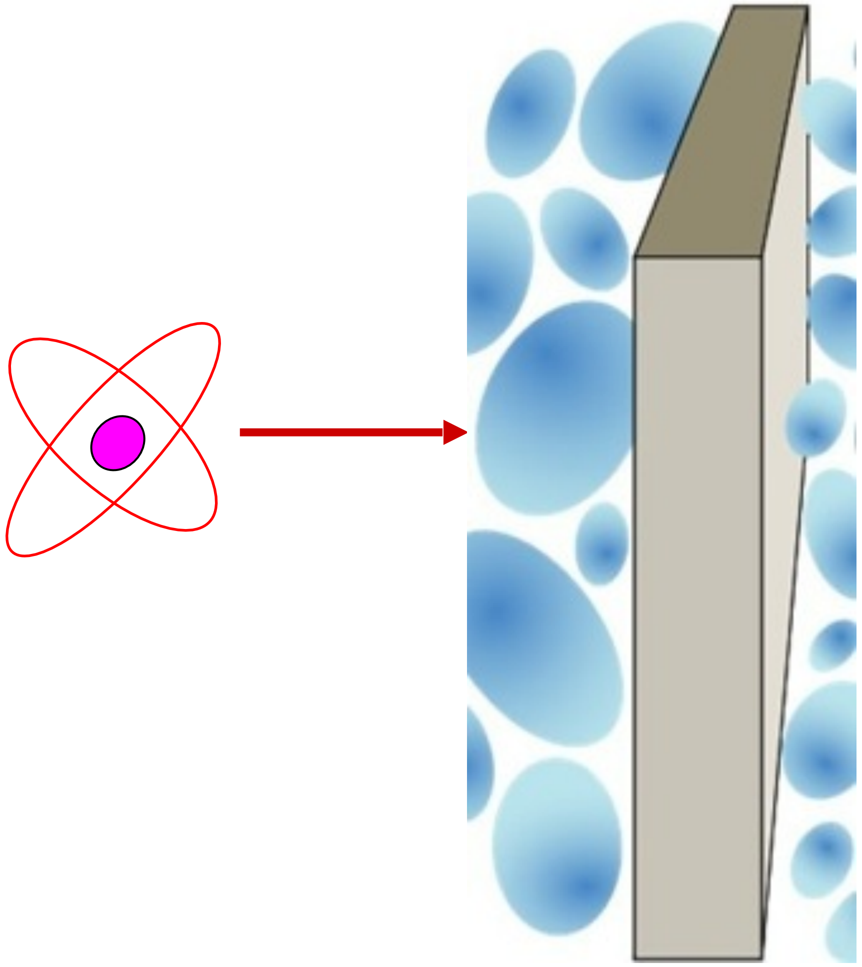
- The Casimir-Polder (CP) force: vacuum and equilibrium thermal fluctuations
- The out of thermal equilibrium effect
- The CP force and thermalization in multilayer planar dielectric system
- Conclusion

• The Casimir-Polder (CP) force: vacuum and equilibrium thermal fluctuations

1. Casimir effect



2. Casimir-Polder force



The interaction Hamiltonian

$$\hat{H}_{AF} = -\hat{\boldsymbol{d}}_A \cdot \hat{\boldsymbol{E}}(\boldsymbol{r}_A) - \hat{\boldsymbol{m}}_A \cdot \hat{\boldsymbol{B}}(\boldsymbol{r}_A) + \sum_{\alpha \in A} \frac{q_\alpha^2}{8 m_\alpha} [\hat{\bar{\boldsymbol{r}}}_\alpha \times \hat{\boldsymbol{B}}(\boldsymbol{r}_A)]^2$$

<table>
<tr><th rowspan="2">Atom</th><th colspan="2">Perfectly conducting</th></tr>
<tr><th>Retarded limit</th><th>Nonretarded limit</th></tr>
<tr><td>Electric</td><td>$-\frac{1}{z_A^4}$</td><td>$-\frac{1}{z_A^3}$</td></tr>
<tr><td>Paramagnetic</td><td>$+\frac{1}{z_A^4}$</td><td>$+\frac{1}{z_A^3}$</td></tr>
<tr><td>Diamagnetic</td><td colspan="2">$-\frac{1}{z_A^4}$</td></tr>
</table>

Thermal fluctuations:

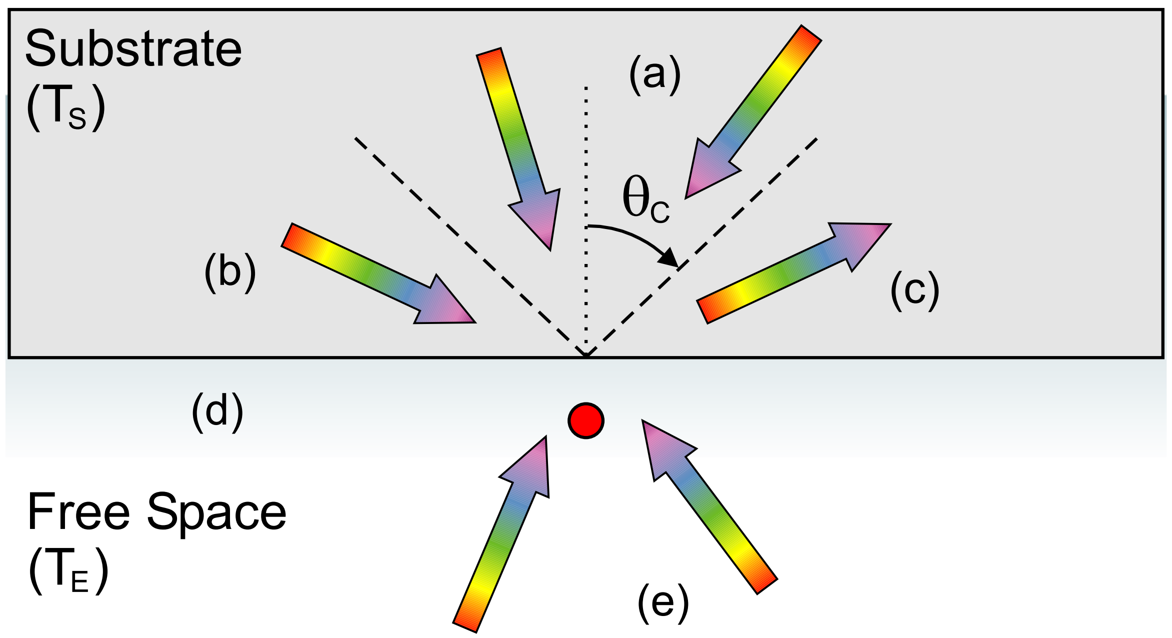
<table>
<tr><td>Electric</td><td>$-\frac{1}{z_A^3 T}$</td><td>$-\frac{1}{z_A^4 T}$</td></tr>
<tr><td>Diamagnetic</td><td colspan="2">$-\frac{1}{z_A^3 T}$</td></tr>
</table>

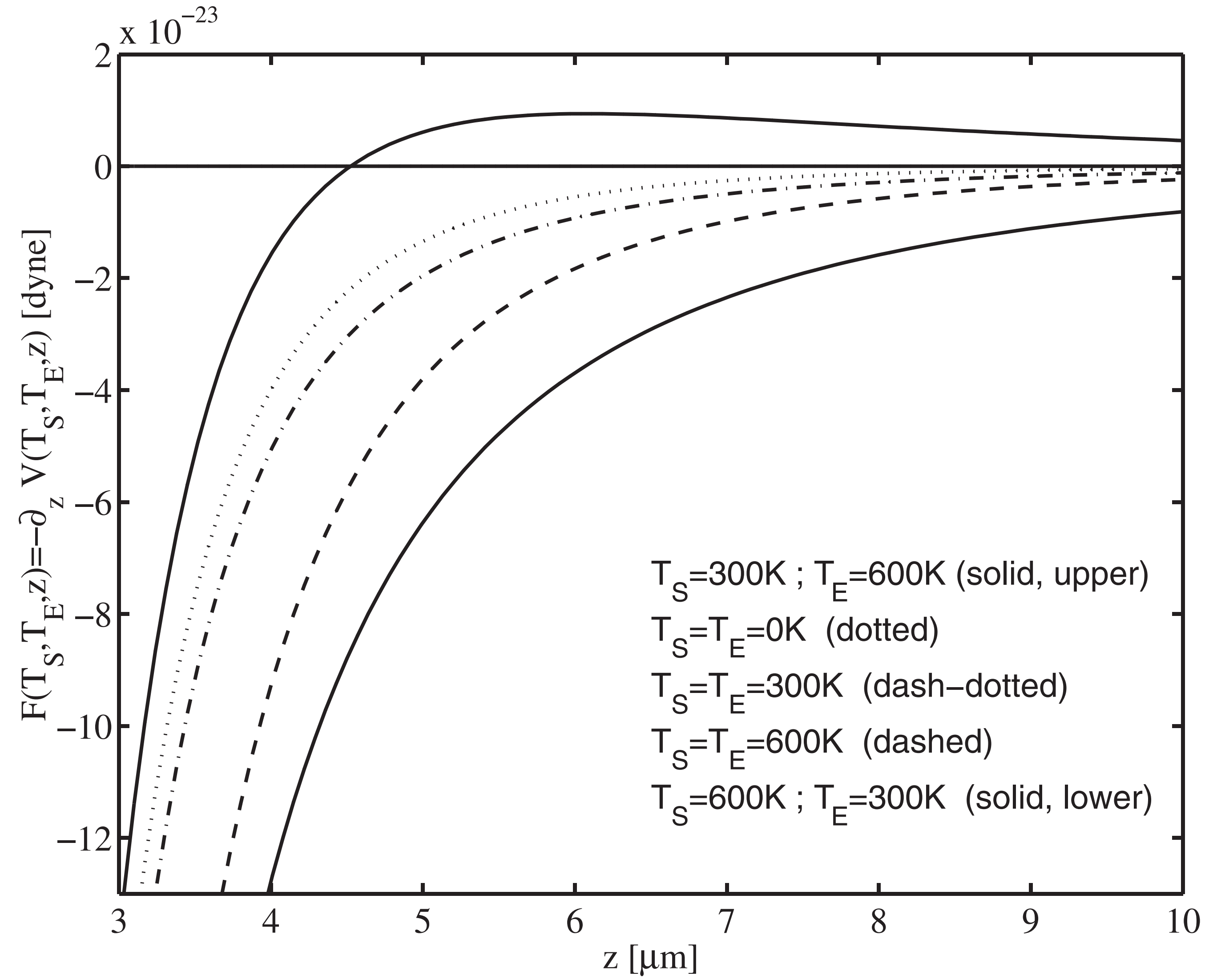
Buhmann et al. PRA **87**, 012507 (2013)

Wu and Yu, PRA **90**, 032502 (2014)

The out of thermal equilibrium CP force

$$\Delta d = \infty$$





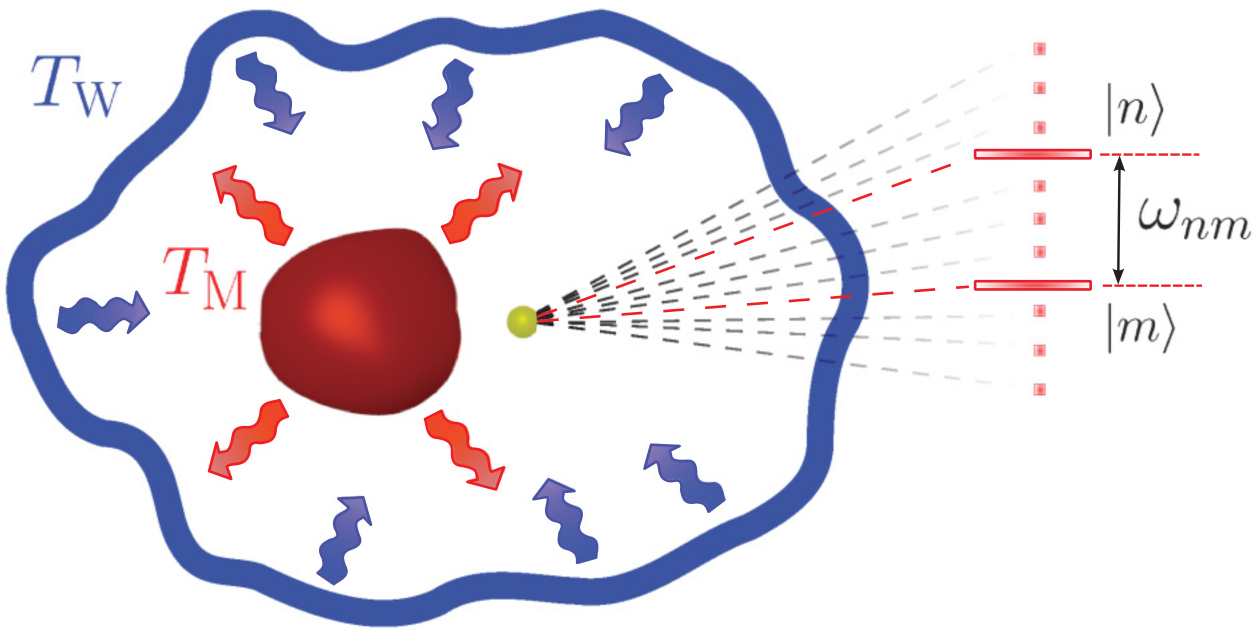
$$F^{\mathrm{neq}}(T_S, T_E, z)_{z\to\infty} = -\frac{\pi}{6}\,\frac{\alpha_0 k_B^2 (T_S^2 - T_E^2)}{z^3 c\hbar}\,\frac{\varepsilon_0 + 1}{\sqrt{\varepsilon_0 - 1}},$$

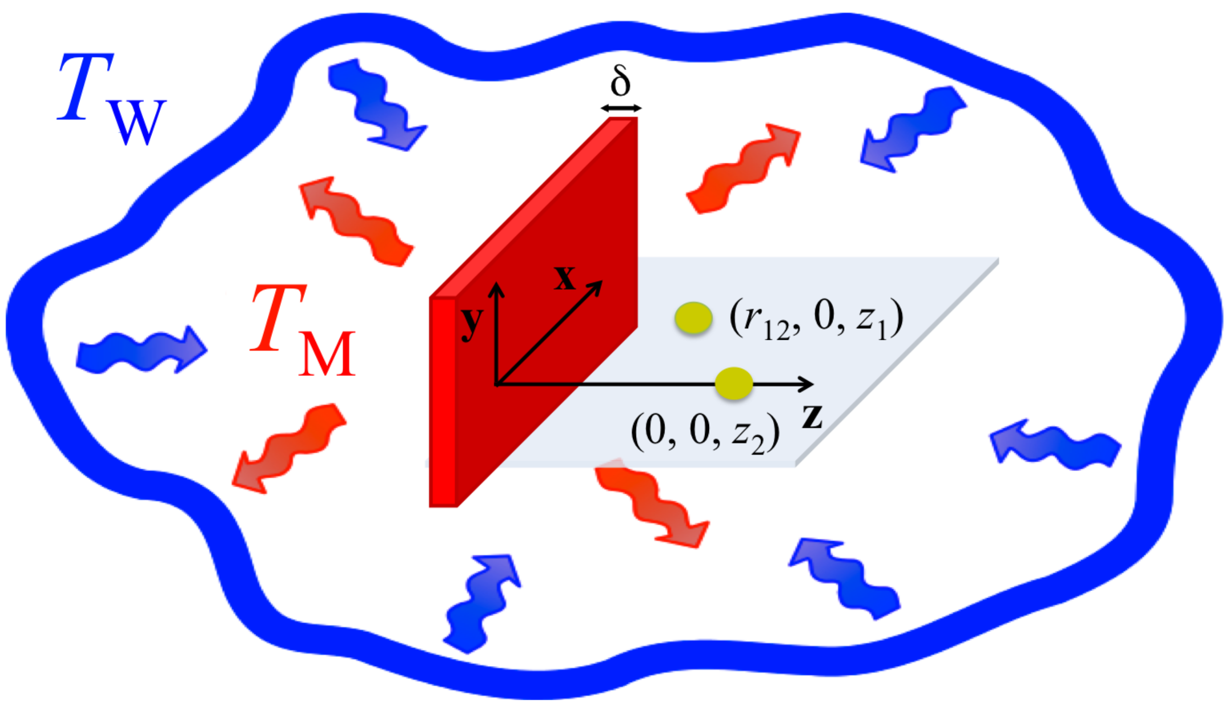
static approximation
real dielectric

Antezza et al., PRL **95,** 113202 (2005)

Zhou and Yu, PRA **90**, 032501 (2014)

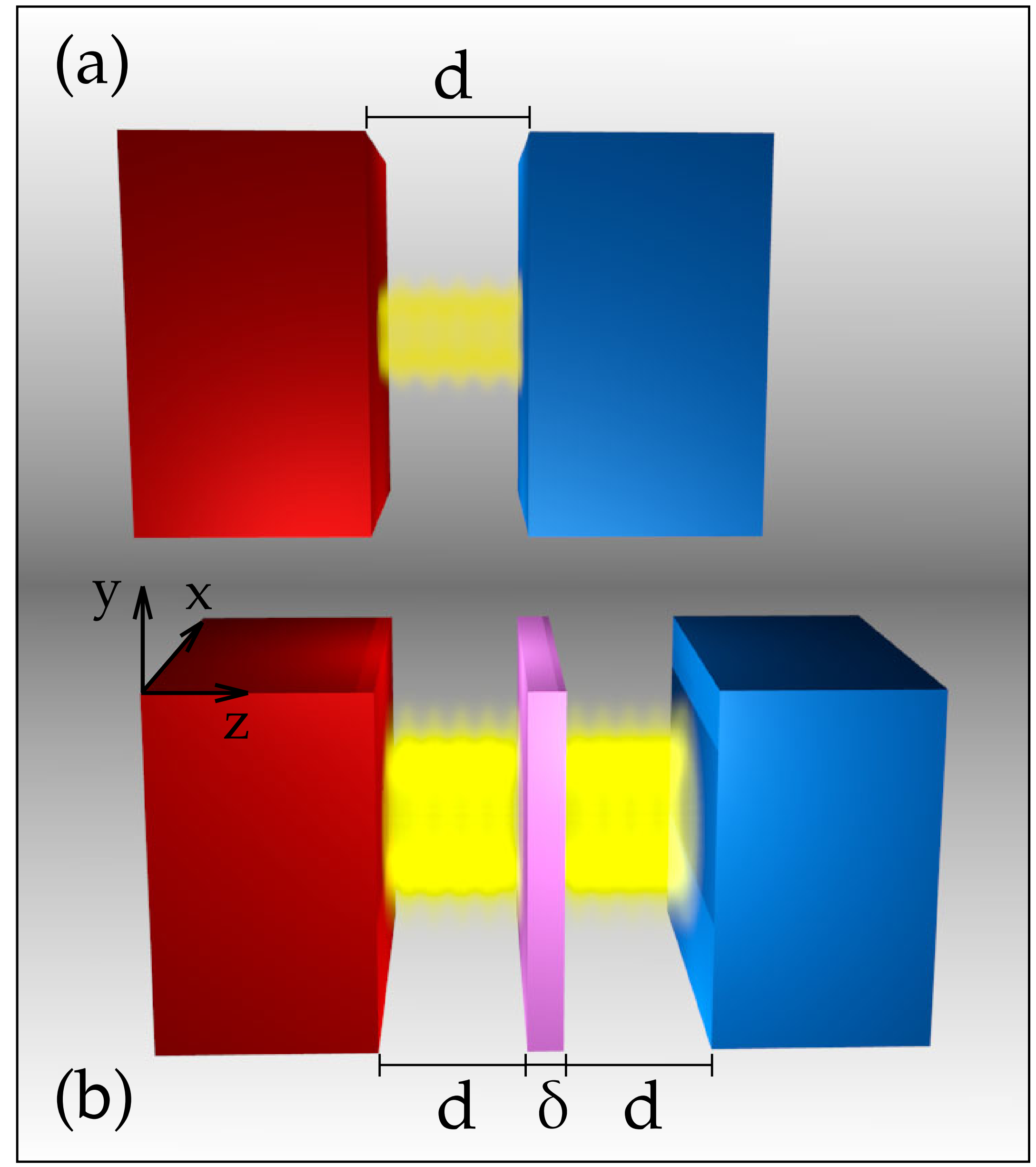
Thermalization





Entanglement

Photon Heat Tunneling



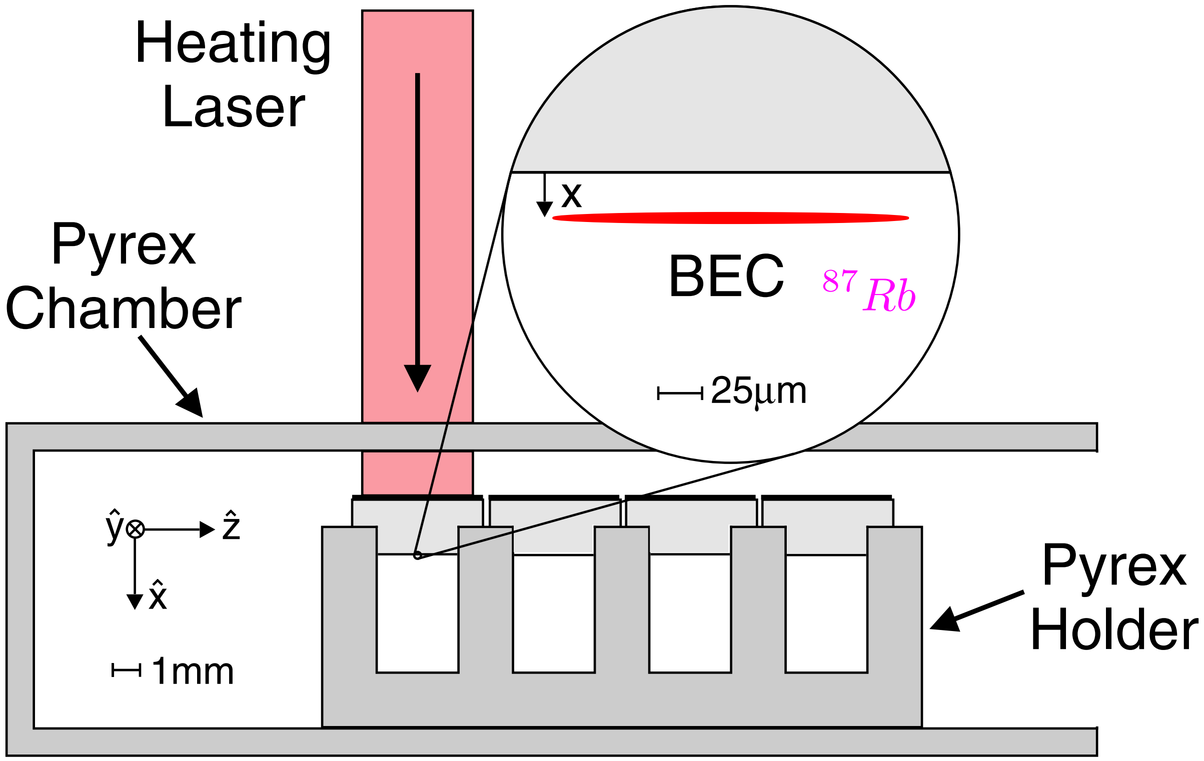
Bellomo et al., PRA 87, 012101 (2013)

Bellomo and Antezza, EPL 104, 10006 (2013)

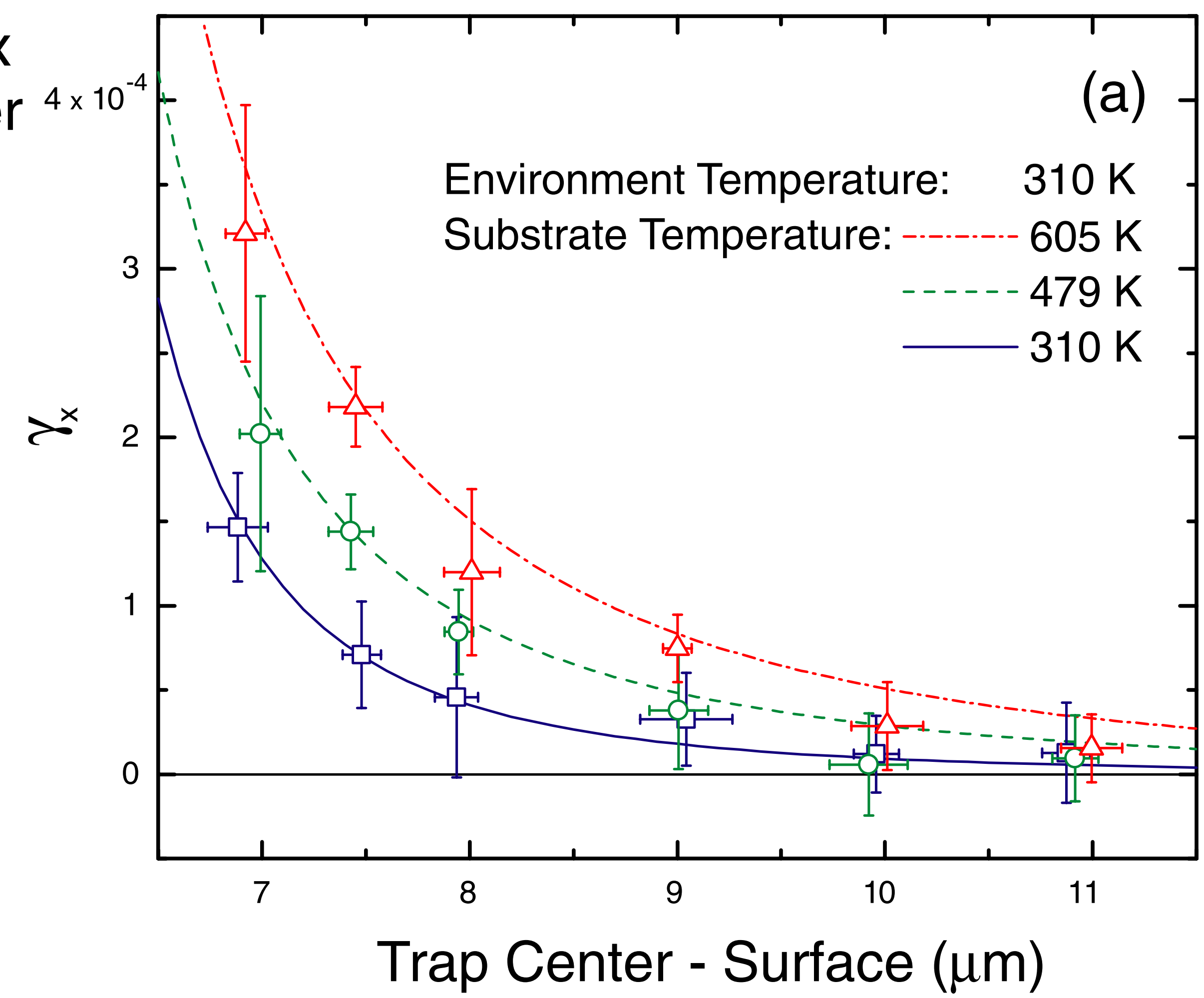
Messina et al., PRL 109, 244302 (2012)

Measurement of the Temperature Dependence of the Casimir-Polder Force

Obrecht et al. PRL **98**, 063201 (2007)

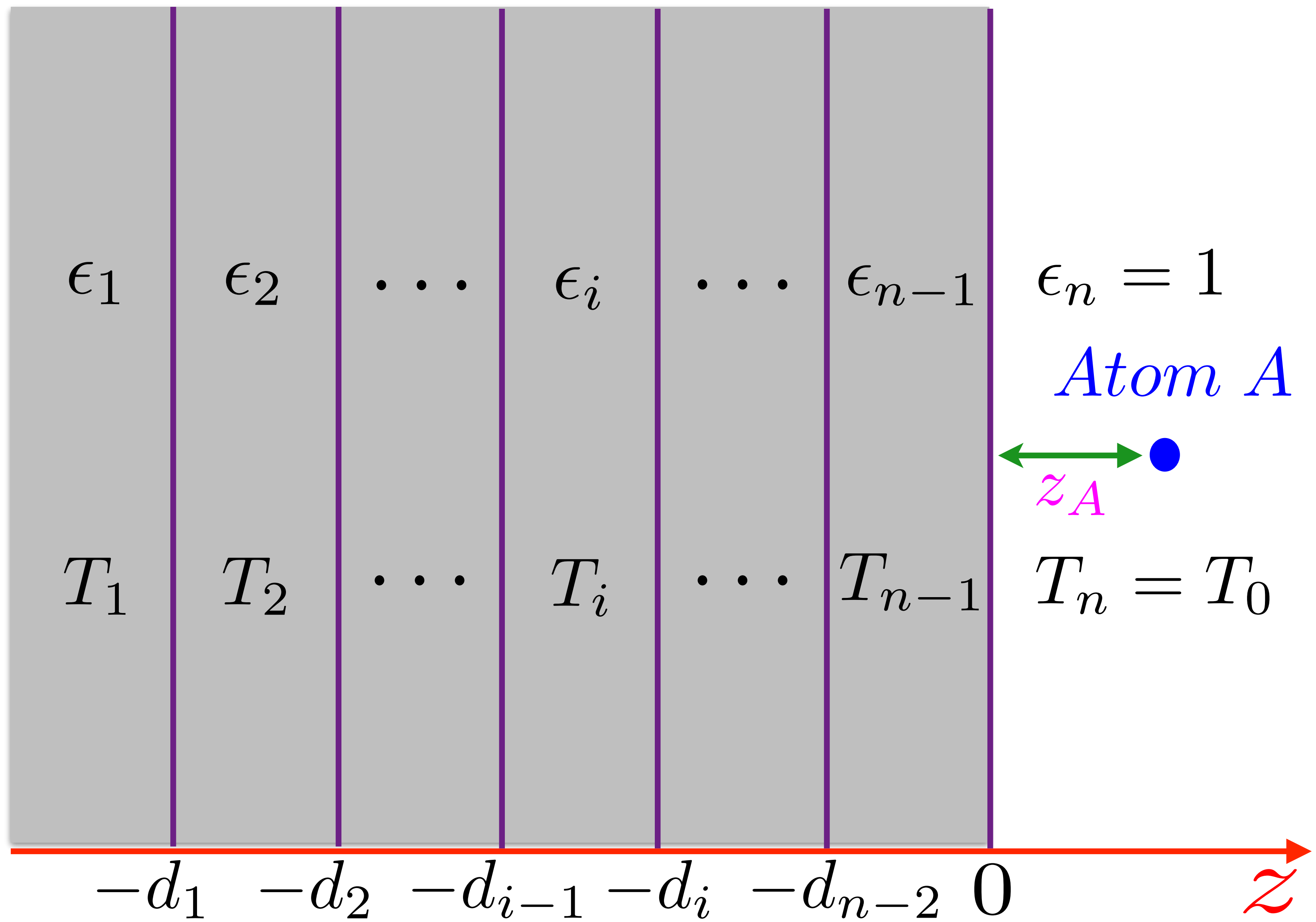


$$\gamma_x \equiv \frac{\omega_o - \omega_x}{\omega_o} \simeq \frac{1}{2m\omega_o^2}\langle \partial_x F_{\mathrm{CP}} \rangle,$$



$$2 \times 5 \times 8\ mm$$

- The CP force and thermalization in multilayer planar dielectric system



The open quantum system method

The total density matrix satisfies the von Neumann equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\mathrm{tot}}(t) = -\frac{\mathrm{i}}{\hbar}[H_I, \rho_{\mathrm{tot}}(t)]$$

The reduced density matrix obeys the master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\frac{\mathrm{i}}{\hbar}[H_{LS}, \rho(t)] + \mathcal{D}(\rho(t))$$

The Lamb-shift Hamiltonian

The dissipator term

For a two-level atom:

$$\frac{d}{dt}\rho(t) = -\mathrm{i}\left[\sum_{n=1}^{2}\omega_n|n\rangle\langle n| + S(\omega_0)|2\rangle\langle 2| + S(-\omega_0)|1\rangle\langle 1|, \rho(t)\right]$$
$$+ \Gamma(\omega_0)\left(\rho_{22}|1\rangle\langle 1| - \frac{1}{2}\{|2\rangle\langle 2|, \rho(t)\}\right) + \Gamma(-\omega_0)\left(\rho_{11}|2\rangle\langle 2| - \frac{1}{2}\{|1\rangle\langle 1|, \rho(t)\}\right)$$

$S(-\omega_0)$ and $S(\omega_0)$ represent the atomic eigenvalue shifts of the ground state and the excited one, respectively

$$
\begin{aligned}
S(\omega_0) &\equiv \sum_{i,j} s_{ij}(\omega_0)[\boldsymbol{d}_{21}]_i^*[\boldsymbol{d}_{21}]_j \ , \\
S(-\omega_0) &\equiv \sum_{i,j} s_{ij}(-\omega_0)[\boldsymbol{d}_{21}]_i[\boldsymbol{d}_{21}]_j^* \ ,
\end{aligned}
$$

$\Gamma(-\omega_0)$ and $\Gamma(\omega_0)$ are the transition rates associated to the down- and upward transitions, respectively

$$
\begin{aligned}
\Gamma(\omega_0) &\equiv \sum_{i,j} \gamma_{ij}(\omega_0)[\boldsymbol{d}_{21}]_i^*[\boldsymbol{d}_{21}]_j \ , \\
\Gamma(-\omega_0) &\equiv \sum_{i,j} \gamma_{ij}(-\omega_0)[\boldsymbol{d}_{21}]_i[\boldsymbol{d}_{21}]_j^*
\end{aligned}
$$

Here

$$s_{ij}(\omega) = \frac{1}{\hbar^2}\int_0^\infty d\omega' \int_0^\infty d\omega'' \left[\frac{\langle E_i(\boldsymbol{r},\omega')E_j^\dagger(\boldsymbol{r},\omega'')\rangle}{\omega-\omega'} + \frac{\langle E_i^\dagger(\boldsymbol{r},\omega')E_j(\boldsymbol{r},\omega'')\rangle}{\omega+\omega'}\right]$$

$$\gamma_{ij}(\omega) = \frac{2\pi}{\hbar^2}\int_0^\infty \mathrm{d}\omega' \begin{cases} \langle E_i(\boldsymbol{r},\omega)E_j^\dagger(\boldsymbol{r},\omega')\rangle & \omega > 0 \\ \langle E_i^\dagger(\boldsymbol{r},-\omega)E_j(\boldsymbol{r},\omega')\rangle & \omega < 0 \end{cases}$$

For a thermal state:

Vacuum fluctuations

Equilibrium thermal fluctuations

$$s_{ij}(\omega) = \frac{\mu_0}{\hbar\pi}\int_0^\infty \mathrm{d}\omega'\omega'^2 \left[\frac{1+N(\omega',\beta_0)}{\omega-\omega'} + \frac{N(\omega',\beta_0)}{\omega+\omega'}\right] \operatorname{Im} G_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega')$$

$$+\ \frac{1}{\hbar}\sum_{l=1}^{n-1}\int_0^\infty \mathrm{d}\omega' \left(\frac{1}{\omega-\omega'} + \frac{1}{\omega+\omega'}\right) \left[N(\omega',\beta_l) - N(\omega',\beta_0)\right] g^l_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega')$$

The contribution from out of thermal Equilibrium

$$N(\omega,\beta_i) = \frac{1}{e^{\beta_i\omega/c}-1} \qquad \beta_i = \frac{\hbar c}{kT_i}$$

$$\gamma_{ij}(\omega) = \frac{2\mu_0\omega^2}{\hbar}[1+N(\omega,\beta_0)]\,\mathrm{Im}\,G_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega) + \frac{2\pi}{\hbar}\sum_{l=1}^{n-1}[N(\omega,\beta_l)-N(\omega,\beta_0)]\,g^l_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega)$$

$$\gamma_{ij}(-\omega) = \frac{2\mu_0\omega^2}{\hbar}N(\omega,\beta_0)\,\mathrm{Im}\,G_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega) + \frac{2\pi}{\hbar}\sum_{l=1}^{n-1}[N(\omega,\beta_l)-N(\omega,\beta_0)]\,g^l_{ij}(\boldsymbol{r}_A,\boldsymbol{r}_A,\omega)$$

where

$$\boxed{g^l_{ij}(\boldsymbol{r},\boldsymbol{r},\omega) \equiv \frac{\mu_0\omega^4}{\pi c^2}\int \mathrm{d}^2\boldsymbol{r}'_\parallel \int_{-d_{l-1}}^{-d_l} \mathrm{d}z'\,\mathrm{Im}\,\epsilon_l\,G_{ik}(\boldsymbol{r},\boldsymbol{r}',\omega)G^*_{jk}(\boldsymbol{r},\boldsymbol{r}',\omega)}$$

For an isotropic atom, the CP force from out of thermal equilibrium is determined by

$$\mathrm{Tr}\, g^l_{ij} = g^l_{xx} + g^l_{yy} + g^l_{zz}$$

Thermalization:

after evolving for a sufficiently long period of time, the system thermalizes to a steady state with an effective temperature

$$\rho(t \to \infty) = \frac{1}{\Gamma(-\omega_0) + \Gamma(\omega_0)} \begin{pmatrix} \Gamma(\omega_0) & 0 \\ 0 & \Gamma(-\omega_0) \end{pmatrix}.$$

The transition rates can be re-expressed as

$$\boxed{\begin{pmatrix} \Gamma(\omega_0) \\ \Gamma(-\omega_0) \end{pmatrix} = \alpha(\omega_0)\Gamma_0(\omega_0) \begin{pmatrix} 1 + N_{\text{eff}}(\omega_0) \\ N_{\text{eff}}(\omega_0) \end{pmatrix}}$$

The effective number of photons becomes

$$\begin{aligned} N_{\text{eff}}(\omega_0) &= N(\omega_0, \beta_0) + \frac{6\pi^2 c}{\mu_0 \omega_0^3 \alpha(\omega_0)} \sum_{l=1}^{n-1} [N(\omega_0, \beta_l) - N(\omega_0, \beta_0)] \cdot \\ &\qquad \sum_{i,j} \frac{[\boldsymbol{d}_{21}]_i [\boldsymbol{d}_{21}]_j^*}{|\boldsymbol{d}_{21}|^2} g_{ij}^l(\boldsymbol{r}_A, \boldsymbol{r}_A, \omega_0) \\ &= N(\omega_0, \beta_0) + \frac{2\pi^2 c}{\mu_0 \omega_0^3 \alpha(\omega_0)} \sum_{l=1}^{n-1} [N(\omega_0, \beta_l) - N(\omega_0, \beta_0)] \, \mathrm{Tr}\, g_{ij}^l(\boldsymbol{r}_A, \boldsymbol{r}_A, \omega_0) \end{aligned}$$

$$\boxed{T_{\text{eff}} = \frac{\hbar \omega_0}{k} [\ln(1 + N_{\text{eff}}^{-1}(\omega_0))]^{-1}}$$

Using the Green function, we obtain

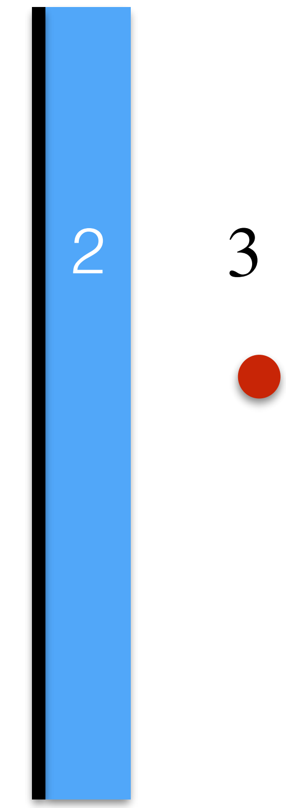
$$g^l(z,z,\omega) = \frac{\mu_0\omega^2}{8\pi^2}\int_0^\infty \frac{k\mathrm{d}k}{|b_n|^2} e^{-2\,\mathrm{Im}\,b_n z - 2\,\mathrm{Im}\,b_l \Delta d_l}\Big(\mathrm{Re}\,b_l (A_+ + \bar{A})[e^{-2\,\mathrm{Im}\,b_l d_l} - e^{-2\,\mathrm{Im}\,b_l d_{l-1}}]$$
$$-\mathrm{Re}\,b_l (A_+|r^p_{l-}|^2 + \bar{A}|r^s_{l-}|^2)[e^{2\,\mathrm{Im}\,b_l d_l} - e^{2\,\mathrm{Im}\,b_l d_{l-1}}]$$
$$-\mathrm{Im}\,b_l (A_-\,\mathrm{Re}\,r^p_{l-} + \bar{A}\,\mathrm{Re}\,r^s_{l-})[\sin(2\,\mathrm{Re}\,b_l d_l) - \sin(2\,\mathrm{Re}\,b_l d_{l-1})]$$
$$+\mathrm{Im}\,b_l (A_-\,\mathrm{Im}\,r^p_{l-} + \bar{A}\,\mathrm{Im}\,r^s_{l-})[\cos(2\,\mathrm{Re}\,b_l d_l) - \cos(2\,\mathrm{Re}\,b_l d_{l-1})]\Big)\ ,$$

$$\mathrm{Im}^2\,b_l = \frac{1}{2}\left[-\left(\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon_l - k^2\right) + \sqrt{\boxed{\frac{\omega^4}{c^4}\mathrm{Im}^2\,\epsilon_l} + \left(\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon_l - k^2\right)^2}\,\right]$$
$$\mathrm{Re}^2\,b_l = \frac{1}{2}\left[\left(\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon_l - k^2\right) + \sqrt{\boxed{\frac{\omega^4}{c^4}\mathrm{Im}^2\,\epsilon_l} + \left(\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon_l - k^2\right)^2}\,\right].$$

$$2\,\mathrm{Im}^2\,b_n = -(\tfrac{\omega^2}{c^2} - k^2) + |\tfrac{\omega^2}{c^2} - k^2| \;\Rightarrow\; k^2 > \frac{\omega^2}{c^2}$$

For the real dielectric: $\mathrm{Im}\epsilon_l = 0 \;\Rightarrow\; g^l(z,z,\omega) = 0$

A special case n=3: $r^{p}_{2-} = 1$ and $r^{s}_{2-} = -1$



$$g^2(z,z,\omega) = \frac{\mu_0\omega^2}{8\pi^2}\int_0^\infty \frac{k\mathrm{d}k}{|b_3|^2}e^{-2\,\mathrm{Im}\,b_3 z}\big[\mathrm{Re}\,b_2(A_+ + \bar{A})(1-e^{-4\,\mathrm{Im}\,b_2\Delta d_2})$$
$$+\mathrm{Im}\,b_2(A_- - \bar{A})e^{-2\,\mathrm{Im}\,b_2\Delta d_2}\sin(2\,\mathrm{Re}\,b_2\Delta d_2)\big]\,.$$

If $2\mathrm{Im}b_2\Delta d_2 > 1$, the dominated term of the transition rates will independent on Δd_2 and the result reduces to that obtained in half space dielectric case.

Assuming that the dielectric has a very small but nonzero $\mathrm{Im}\,\epsilon_2$

$$\mathrm{Im}^2\,b_2 \simeq \frac{1}{2}\left[-\left(\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon_2 - k^2\right) + \left|\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon_2 - k^2\right| + \frac{\omega^4\,\mathrm{Im}^2\,\epsilon_2}{2c^4\left|\frac{\omega^2}{c^2}\mathrm{Re}\,\epsilon_2 - k^2\right|}\right]$$

$$\Big\downarrow\; k^2 > \frac{\omega^2}{c^2}$$

$$\boxed{Min\{\mathrm{Im}\,b_2\} = \frac{\omega\,\mathrm{Im}\,\epsilon_2}{2c\sqrt{\mathrm{Re}\,\epsilon_2 - 1}}}$$

The necessary condition that the finite thick slab can be treated as a half infinite thick substrate:

$$\boxed{\frac{\mathrm{Im}\,\epsilon}{\sqrt{\mathrm{Re}\,\epsilon - 1}} \frac{\Delta d}{\lambda_0} > 1}$$

$\lambda_0 = \dfrac{c}{\omega_0}$ is the transition wavelength of the atom

结　论

- 有限厚度的实电介质板没有非平衡热效应（CP力和热化）

- 如果满足 $\frac{\mathrm{Im}\,\epsilon}{\sqrt{\mathrm{Re}\,\epsilon - 1}} \frac{\Delta d}{\lambda_0} > 1$，有限厚度的电介质可以当做半无限厚的电介质来处理

……

谢谢大家