PHYSICAL REVIEW A **81**, 054302 (2010)

Noise effect on fidelity of two-qubit teleportation

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(Received 25 December 2009; revised manuscript received 19 February 2010; published 18 May 2010)

We investigate the effect of noise on a class of four-qubit entangled channels for two-qubit teleportation from Alice to Bob. These entangled channels include both parallel Bell pairs and inseparable channels with genuine multipartite entanglement. For the situation where only Bob's share of the entangled channel is subject to decoherence, we show by deriving a general expression for the teleported state that teleportation using noisy inseparable channels is equivalent to teleportation using noisy Bell pairs. When Alice's qubits are also subject to noise, we find that the inseparable channels never give a higher teleportation fidelity than Bell pairs, even in the presence of collective noise. Our results can shed some light on practical two-qubit teleportation.

DOI: 10.1103/PhysRevA.81.054302 PACS number(s): 03.67.Hk, 03.65.Yz, 05.40.Ca

Quantum teleportation, a protocol that transports an unknown quantum state from Alice to Bob without transferring the physical carrier of the state, is one of the most unusual communication tasks enabled by quantum theory [1]. The quantum resource that makes teleportation possible is the shared entangled state. A maximally entangled state enables perfect teleportation not only of a qubit, but also of the state of a composite system. However, unavoidable interaction with the environment results in decreasing or even sudden loss of entanglement [2]. A nonmaximally entangled quantum resource affects the quality of teleportation [1,3–6]. Optimal protocols for teleporting a single particle have been studied for both pure [4] and mixed [5] nonmaximally entangled channels. The dynamics of teleportation fidelity in the presence of some specific types of noise has also been investigated for single-qubit teleportation [6]. Interestingly, the fidelity of single-qubit teleportation is not a monotone of the entanglement [7] and can even be increased by local noise for some mixed states [8].

For teleportation of a composite system, there are generally two methods that achieve unit fidelity in the noiseless case. One method is to teleport it "piece by piece" using parallel entangled channels containing only pairwise entanglement [9,10], which has been experimentally realized [11]. One can also teleport the system as a whole using an inseparable entangled channel containing genuine multipartite entanglement [12–14], where multiqubit operations are required. For two-qubit teleportation, an explicit protocol $\mathcal{E}_0$ has been proposed [13] and extended to the case of noisy entangled channels [14]. In the regime of $\mathcal{E}_0$, the enhancement of fidelity by local noise has also been observed [15].

In this Brief Report, we aim at solving the following problem: To teleport a two-qubit state, which of the entangled channels, Bell pairs or inseparable channels, are less affected by local noise? For the case where only Bob's partition of the channel is subject to noise, we derive a general expression for the teleported state and find that teleportation using the noisy inseparable channels is equivalent to teleportation using the noisy Bell pairs in the regime of $\mathcal{E}_0$. When Alice's qubits are also subject to noise, we investigate the effect of some specific noises and find that a tensor product of Bell pairs is less affected by local noises, even when collective noises are considered. The interesting effect of enhancing the teleportation fidelity by local noises is also discussed.

As a preparation, we first give a general description of a standard teleportation protocol $\mathcal{R}_0$, which can be reduced to specific teleportation protocols such as $\mathcal{E}_0$. Suppose Alice wants to transport to Bob an unknown state

$$|\psi\rangle_c = \sum_{i=0}^{d-1} \alpha_{\tilde{i}} |\tilde{i}'\rangle, \tag{1}$$

where $|\tilde{i}'\rangle = S|\tilde{i}\rangle$, S is a given $d \times d$ unitary operator, and the $|\tilde{i}\rangle$'s are orthogonal state bases of a d-level particle or a system of n particles with $d = d_1 d_2 \cdots d_n$ (where d_j is the dimension of particle j). She prepares a bipartite maximally entangled state

$$\left|\Phi_S^{\tilde{0}}\right\rangle_{ab} = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |\tilde{i}\tilde{i}'\rangle_{ab} = \mathbb{I}_a \otimes S_b \left|\Phi_{\mathbb{I}}^{\tilde{0}}\right\rangle_{ab} \tag{2}$$

and sends the partition b to Bob in order to build the entangled channel. First, Alice projects her two partitions a and c on the bases

$$\left|\Phi_S^{\tilde{j}}\right\rangle_{ac} = \mathbb{I}_a \otimes U_c^{(\tilde{j})} \left|\Phi_S^{\tilde{0}}\right\rangle_{ac}, \quad j = 0,1,\ldots,d^2-1. \tag{3}$$

Here the unitary operations $U^{(\tilde{j})}$ are orthogonal in the sense that $\mathrm{Tr}(U^{(\tilde{j})\dagger} U^{(\tilde{k})}) = d\delta_{\tilde{j}\tilde{k}}$, which ensures that Alice's measurement bases are normalized and orthogonal. The result $\tilde{j}$ is then sent to Bob through a classical channel, and Bob knows that the (unnormalized) state of his partition becomes ${}_{ac}\langle\Phi_S^{\tilde{j}}|(|\psi\rangle_c \otimes |\Phi_S^{\tilde{0}}\rangle_{ab}) = (1/d)U^{(\tilde{j})\dagger}|\psi\rangle_b$, so he needs only to implement the unitary operation $U^{(\tilde{j})}$ on his partition to perfectly recover the unknown input state.

This protocol $\mathcal{R}_0$ reduces to $\mathcal{T}_0$ for single-qubit teleportation when $d = 2$, $S = U^{(\tilde{0})} = \mathbb{I} \equiv \sigma^0$, and $U^{(\tilde{1})} = \sigma^x \equiv \sigma^1$, $U^{(\tilde{2})} = i\sigma^y \equiv \sigma^2$, $U^{(\tilde{3})} = \sigma^z \equiv \sigma^3$ (where $\mathbb{I}$ is the identity operator and $\sigma^{x,y,z}$ are Pauli matrices). In contrast, $\mathcal{R}_0$ reduces

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to $\mathcal{E}_0$ when $d=4$, $\{\tilde{0},\tilde{1},\tilde{2},\tilde{3}\}=\{00,01,10,11\}$, $U^{(ij)}=\sigma^i\otimes\sigma^j$,

$$S=V(\theta,\phi)\equiv\begin{pmatrix}\cos\theta & 0 & 0 & \sin\theta\\ 0 & -\sin\phi & \cos\phi & 0\\ 0 & \cos\phi & \sin\phi & 0\\ -\sin\theta & 0 & 0 & \cos\theta\end{pmatrix}, \quad (4)$$

where $\theta,\phi\in(-\pi/2,\pi/2]$, and the entangled channel $|\Phi^{00}_{V(\theta,\phi)}\rangle_{ab}\equiv|Y^{00}(\theta,\phi)\rangle_{1234}$. It is easy to check that, for $\theta=\phi=0$ or $\theta=\phi=\pi/2$, $|Y^{00}(\theta,\phi)\rangle_{1234}$ is a tensor product of two Bell pairs, and otherwise it is an inseparable channel. When a mixed entangled channel is considered, $\mathcal{E}_0$ is revised such that the parameters θ',ϕ' in Alice's measurement basis $|\Phi^{ij}_{V(\theta',\phi')}\rangle_{ac}\equiv|Y^{ij}(\theta',\phi')\rangle_{1256}$ are chosen to maximize the teleportation fidelity.

Here consider the following case: Alice prepares a four-qubit entangled state $\varrho_{1234}(\theta,\phi)=|Y^{00}(\theta,\phi)\rangle_{1234}\langle Y^{00}(\theta,\phi)|$ and sends qubits 3 and 4 to Bob through a noisy channel $\varepsilon_{34}(\cdot)=\sum_i E^{(i)}(\cdot)E^{(i)\dagger}$, where the $E^{(i)}$'s are the operator elements satisfying $\sum_i E^{(i)\dagger}E^{(i)}=\mathbb{I}$. We prove the following statement: Teleportation of an unknown input state $|\psi\rangle_{56}$ through the noisy entangled state $\Pi_{1234}(\theta,\phi)=\mathbb{I}_{12}\otimes\varepsilon_{34}(\varrho_{1234}(\theta,\phi))$ using $\mathcal{E}_0$ is equivalent to transferring it through the noisy channel

$$\rho^{\text{tele}}_{34}=\frac{1}{16}\sum_{ijk}U^{(jk)}E^{(i)}TU^{(jk)\dagger}|\psi\rangle_{34}\langle\psi|U^{(jk)}T^\dagger E^{(i)\dagger}U^{(jk)\dagger}, \quad (5)$$

where T takes the form $T=W(\alpha,\beta)=\cos\alpha(|00\rangle\langle 00|+|11\rangle\langle 11|)+\cos\beta(|01\rangle\langle 01|+|10\rangle\langle 10|)+\sin\alpha(|00\rangle\langle 11|-|11\rangle\langle 00|)+\sin\beta(|10\rangle\langle 01|-|01\rangle\langle 10|)$, and α and β are chosen to maximize $\sum_i|\text{Tr}[E^{(i)}W(\alpha,\beta)]|^2$.

Notice that Eq. (5) is independent of the parameters θ and ϕ in the initial entanglement resource. Recalling that these are the only elements that determine whether mutipartite entanglement exists in the entanglement resource, we can safely say that in $\mathcal{E}_0$ the inseparable channels and Bell pairs are equally affected in the case where Bob's two qubits are exposed to noise. This result can be generalized for any protocol in the regime of $\mathcal{R}_0$. Now we begin to prove the statement.

Proof. The total state of the qubits to be teleported and the entangled channel is $\Xi=|\psi\rangle_{56}\langle\psi|\otimes\Pi_{1234}(\theta,\phi)$. According to $\mathcal{E}_0$, the state of Bob's qubits 3 and 4, after the teleportation process, takes the form $\rho^{\text{tele}}_{34}=\sum_{jk}U^{(jk)}_{1256}\langle Y^{jk}(\theta',\phi')|\Xi|Y^{jk}(\theta',\phi')\rangle_{1256}U^{(jk)\dagger}=\sum_{ijk}|\varphi^{ijk}\rangle_{34}\langle\varphi^{ijk}|$, where $|\varphi^{ijk}\rangle_{34}=\frac{1}{4}U^{(jk)}E^{(i)}W(\Delta\theta,\Delta\phi)U^{(jk)\dagger}|\psi\rangle_{34}$ and $\Delta\theta=\theta-\theta',\Delta\phi=\phi-\phi'$. Here we have used the equations $\hat{O}|k\rangle=\sum_l\hat{O}_{lk}|l\rangle$ and $V(\theta,\phi)V^\dagger(\theta',\phi')=W(\Delta\theta,\Delta\phi)$. Now our task is to determine θ' and ϕ'. As shown in Ref. [14], the average teleportation fidelity associated with mixed entangled channel Π is $f=(4\mathcal{G}+1)/5$, where $\mathcal{G}=\max_{\theta',\phi'}\langle Y^{00}(\theta',\phi')|\Pi|Y^{00}(\theta',\phi')\rangle$ is the generalized singlet fraction (GSF). It is straightforward to check that the GSF of the entangled channel $\Pi_{1234}(\theta,\phi)$ is

$$\mathcal{G}_{34}=\max_{\Delta\theta,\Delta\phi}\frac{1}{16}\sum_i|\text{Tr}[E^{(i)}W(\Delta\theta,\Delta\phi)]|^2. \quad (6)$$

Clearly, we should set $\Delta\theta=\alpha$ and $\Delta\phi=\beta$, completing the proof.

Until now, we have studied the dynamics of teleportation fidelity when Bob's qubits 3 and 4 of the entangled channel $\varrho_{1234}(\theta,\phi)$ are subject to noise. What would happen if Alice's qubits were also exposed to noise? After some calculations, we arrive at the GSF for this situation:

$$\mathcal{G}_{1234}=\frac{1}{16}\max_{\theta',\phi'}\sum_{ij}\left|\text{Tr}\left[E^{(i)}_{34}V(\theta,\phi)E^{(j)\text{T}}_{12}V^\dagger(\theta',\phi')\right]\right|^2, \quad (7)$$

where the $E^{(i)}_{12}$'s and $E^{(i)}_{34}$'s are the operator elements of the decoherence channels on Alice's and Bob's qubits, respectively. Here the average fidelity *does* depend on θ and ϕ. In the following, we will investigate the effect of some specific types of noise acting on the entangled channels $\{|Y^{00}(\theta,\phi)\rangle_{1234}\}$, in order to find out which of the entangled channels are the least affected by the noise.

Before starting the calculation, we briefly discuss the classical limit of the average teleportation fidelity. As proved in Ref. [16] recently, state estimation is equivalent to $1\to\infty$ quantum cloning. From Ref. [17], the optimal fidelity of $N\to M$ quantum cloning of a d-level qudit state is $f_{N\to M}(d)=N/M+(M-N)(N+1)/M(N+d)$. By setting $N=1,M\to\infty$, and $d=4$, we arrive at the optimal estimated fidelity of a two-qubit state, $f_{\text{est}}=\frac{2}{5}$, so the teleportation is nontrivial only when its fidelity surpasses f_{est}; otherwise just the estimate-and-rebuild process will do the job. Correspondingly, the classical limit of the GSF is $\mathcal{G}_{\text{cla}}=\frac{1}{4}$.

When qubits are located separately, they interact with their own environment independently, causing single-qubit decoherence. The operator elements of the usual single-qubit decoherence channels are well defined [18]. Therefore, we are ready to calculate the dynamics of the teleportation fidelity from Eq. (7) with $\{E^{(l)}_a\}=\{E^{(i)}_1\otimes E^{(j)}_2\}$ and $\{E^{(l)}_b\}=\{E^{(i)}_3\otimes E^{(j)}_4\}$. When the two local qubits 1 and 2 (or 3 and 4) are placed closed to each other, they interact with the common reservoir, causing collective decoherence. In the Markov limit, such a decoherence process can be described by the following master equation:

$$\begin{aligned}\frac{\partial\rho}{\partial t}=&\frac{\gamma_{12}}{2}(2L_{12}\rho L^\dagger_{12}-L^\dagger_{12}L_{12}\rho-\rho L^\dagger_{12}L_{12})\\&+\frac{\gamma_{34}}{2}(2L_{34}\rho L^\dagger_{34}-L^\dagger_{34}L_{34}\rho-\rho L^\dagger_{34}L_{34}),\end{aligned} \quad (8)$$

where γ_{ij} is the coupling strength between qubits i and j and the reservoir, $L_{ij}=L_i+L_j$, and L_i is the Lindblad operator acting on qubit i. After analytically solving this master equation with the initial condition $\rho(t=0)=\varrho(\theta,\phi)$, we calculate the dynamics of the GSF. In the following, we present the main results concerning the properties of average teleportation fidelities $f_{1234}(\theta,\phi)$ associated with the noisy entangled channels.

(a) Inseparable channels never give a higher average fidelity than Bell pairs with $\theta=\phi=0$. Since the multipartite entanglement contained in the inseparable channels is fragile under single-qubit decoherence, we focus on the effect of local two-qubit collective noise. It has been suggested [12] that inseparable channels for two-qubit teleportation might be more robust under a collective noise, but we will show that this is not true.

For the situation where collective phase-damping (CPD) channels are applied locally to Alice's qubits 1 and 2 as well as Bob's qubits 3 and 4, we solve Eq. (8) with $\gamma_{12} = \gamma_{34} = \gamma, L_i = L_i^{\mathrm{PD}} \equiv a_i^\dagger a_i$, and obtain the GSF

$$\mathcal{G}_{1234}^{\mathrm{CPD}}(\theta,t) = \frac{1}{8} \max_{\theta'} \Big[4e^{-\gamma t}\cos\Delta\theta + (1+e^{-4\gamma t})\cos^2\Delta\theta - \frac{1}{2}(1-e^{-2\gamma t})^2 \sin 2\theta \sin 2\theta' + 2\Big], \tag{9}$$

which reaches its upper bound $\mathcal{G}_{1234}^{\mathrm{CPD}}(t) = \frac{1}{8}(3 + 4e^{-\gamma t} + e^{-4\gamma t})$ for $\theta = 0,\pi/2$. Obviously, the tensor product state $|Y^{00}(0,0)\rangle_{1234}$ satisfies this condition; in other words, even when *collective* phase damping is present, total teleportation does not occur prior to a series teleportation of two qubits. For the collective amplitude-damping (CAD) channels, we solve Eq. (8) with $L_i = L_i^{\mathrm{AD}} \equiv a_i$. The expression for the GSF $\mathcal{G}_{1234}^{\mathrm{CAD}}(t)$ is quite complicated, so we present only the result in the limit $t \to \infty$:

$$\mathcal{G}_{1234}^{\mathrm{CAD}}(t \to \infty) = \frac{1}{16}[2 + (\cos\theta + \cos\phi)^2]. \tag{10}$$

The upper bound of this GSF is reached only for $\theta = \phi = 0$. The average teleportation fidelity $f_{1234}^{\mathrm{CAD}} = (4\mathcal{G}_{1234}^{\mathrm{CAD}} + 1)/5$ is depicted in Fig. 1 for various values of θ and ϕ. Clearly, the parallel entangled pairs with $\theta = \phi = 0$ (curve B) can always give higher teleportation fidelity than the inseparable channel with $\theta = 0, \phi = \pi/2$ (curve C). This means that parallel Bell pairs with $\theta = \phi = 0$ give strictly more teleportation fidelity than inseparable channels in the presence of collective amplitude damping.

Here we discuss the reason that independent channels are *not* more robust than Bell pairs when collective decoherence is present. Although the bipartite entanglements $E_{13|24}$ and $E_{14|23}$ are nonzero for inseparable entangled channels, the pairwise entanglements $E_{1|2}$ and $E_{3|4}$ are zero. The priority of multipartite entanglement under collective noises is lost when qubits 1 and 2 (or 3 and 4) are exposed to a common reservoir, because no quantum correlation exists between the two qubits 1 and 2 (or 3 and 4), just the tensor product of two singlets. Since multipartite entanglement is fragile, it is not surprising that $|Y^{00}(0,0)\rangle_{1234}$ is the most robust of the class of states $\{|Y^{00}(\theta,\phi)\rangle_{1234}\}$.

(b) Enhancement of fidelity by local noise. The average teleportation fidelity can be increased by local noise for some mixed entangled channels [8,15]. It is of interest to characterize such mixed channels and the local noises. Here, we find that local two-qubit collective amplitude damping and single-qubit amplitude damping can increase the fidelity of some mixed entangled channels.

Consider a class of mixed entangled channels $\varrho_{1234}^{\mathrm{CAD}}(\theta,\phi) \equiv \varepsilon_{34}^{\mathrm{CAD}}(\varrho_{1234}(\theta,\phi))$ which are obtained from $\varrho_{1234}(\theta,\phi)$ by subjecting qubits 3 and 4 to two-qubit CAD. By solving Eq. (8) with $\gamma_{12} = 0$, $\gamma_{34} = \gamma$, and $L_i = L_i^{\mathrm{AD}}$, we analytically calculate the GSF of $\varrho_{1234}^{\mathrm{CAD}}(\theta,\phi)$, $\mathcal{G}_{34}^{\mathrm{CAD}}(t) = \frac{1}{4}(1 + e^{-\gamma t})^2$, and plot the associated fidelity $f_{34}^{\mathrm{CAD}}(t)$ as curve A in Fig. 1. Notice that $\mathcal{G}_{34}^{\mathrm{CAD}}(t \to \infty) = \frac{1}{4} = \mathcal{G}_{\mathrm{cla}}$; Alice's exposing of qubits 1 and 2 of the channel $\varrho_{1234}^{\mathrm{CAD}}(\theta,\phi)$ to CAD can increase the average teleportation fidelity after some critical time t_c, as long as $\mathcal{G}_{1234}^{\mathrm{CAD}}(t \to \infty) > \mathcal{G}_{34}^{\mathrm{CAD}}(t \to \infty)$, and, equivalently, $\cos\theta + \cos\phi > \sqrt{2}$.

Consider another class of mixed entangled channels, $\varrho_{1234}^{\mathrm{AD}}(\theta,\phi) \equiv \varepsilon_3^{\mathrm{AD}}\varepsilon_4^{\mathrm{AD}}(\varrho_{1234}(\theta,\phi))$ obtained by subjecting qubits 3 and 4 of the ideal channels $\varrho_{1234}(\theta,\phi)$ to independent amplitude damping (AD). The operator elements of channel AD are [18] $E_i^{\mathrm{AD}(0)} = |0\rangle\langle 0| + \sqrt{1-p}|1\rangle\langle 1|$, $E_i^{\mathrm{AD}(1)} = \sqrt{p}|0\rangle\langle 1|$, where p is a function of t: $p = 0$ for $t = 0$ and $p = 1$ for $t \to \infty$. From Eq. (6), the GSF of the channel $\varrho_{1234}^{\mathrm{AD}}(\theta,\phi)$ is $\mathcal{G}_{34}^{\mathrm{AD}} = \frac{1}{16}(1 + \sqrt{1-p})^4$. After Alice's qubits 1 and 2 are also exposed to AD, the associated GSF $\mathcal{G}_{1234}^{\mathrm{AP}}(\theta,\phi)$ can be directly calculated from Eq. (7). By setting $\theta = \theta'$ and $\phi = \phi'$, we obtain the

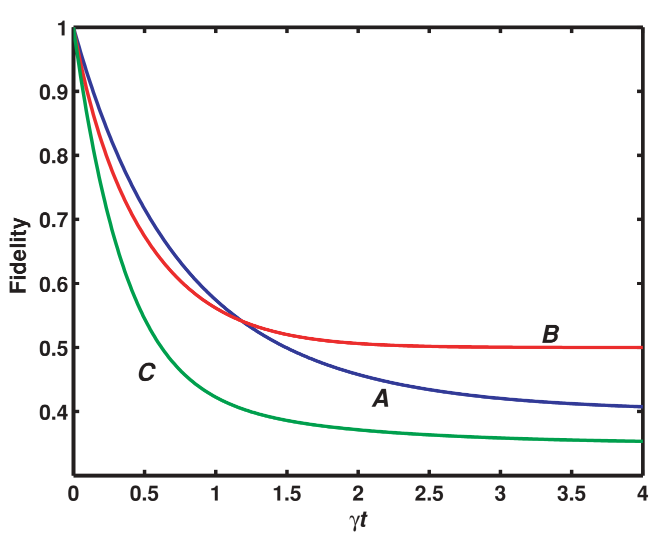

FIG. 1. (Color online) Average teleportation fidelity as a function of γt for the CAD case. Curves A, B, and C stand for the evolution of f_{34}^{CAD}, $f_{1234}^{\mathrm{CAD}}(0,0)$, and $f_{1234}^{\mathrm{CAD}}(0,\pi/2)$, respectively.

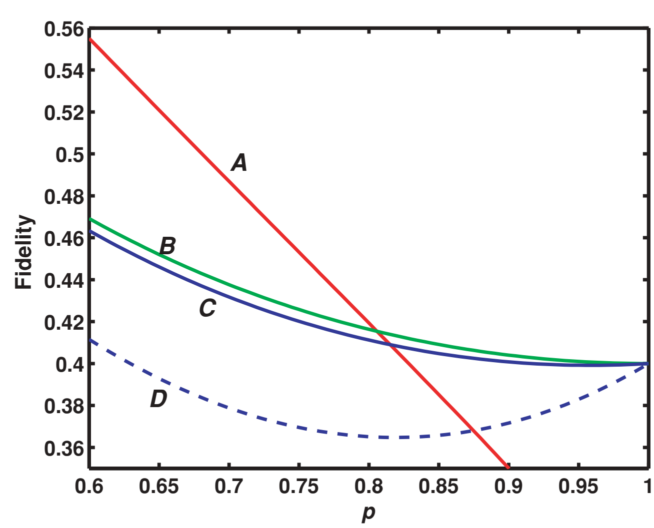

FIG. 2. (Color online) Average teleportation fidelity as a function of γt for the AD case. Curves A, B, C, and D are, respectively, the evolution of f_{34}^{AD}, and the lower bounds of $f_{1234}^{\mathrm{AD}}(0,0)$, $f_{1234}^{\mathrm{AD}}(0, \arcsin\sqrt{0.1})$, and $f_{1234}^{\mathrm{AD}}(0,\pi/2)$.

lower bound of the GSF, $\mathcal{G}_{1234}^{\rm AP}(\theta,\phi) \geqslant \frac{1}{4}[1+(1-p)^2]^2 - \frac{1}{4}p^2(2-p)^2\sin^2\theta - \frac{1}{2}p^2(1-p)\cos^2\theta\sin^2\phi$. We plot the lower bound of the associated teleportation fidelity $f_{1234}^{\rm AP}(0,\phi)$ with $\phi = 0$, $\arcsin\sqrt{0.1}$, and $\pi/2$, as well as $f_{34}^{\rm AP} = (4\mathcal{G}_{34}^{\rm AD}+1)/5$, in Fig. 2. Notice that $f_{1234}^{\rm AP}(0,\pi/2)$ (curve D) becomes larger than $f_{34}^{\rm AP}$ (curve A) after it has fallen below the classical limit $f_{\rm est} = 2/5$, so the enhancement of fidelity by noise on Alice's qubits is trivial. For $\phi = 0$ or $\arcsin\sqrt{0.1}$ (curves B and C), there exist regions of p where $f_{1234}^{\rm AP}(0,\phi) > f_{34}^{\rm AP} \geqslant f_{\rm est}$ holds. This means that, after Bob's qubits 1 and 2 have inevitably been subjected to individual amplitude damping, Alice's exposing of her two qubits to individual amplitude damping can enhance the ability of the four-qubit entangled channel to teleport the two-qubit quantum state. Recalling that *single-qubit* amplitude damping cannot increase the GSF of the class of mixed entanglement resource discussed in Ref. [15], we see that our result gives a complementary result for those states whose associated fidelity can be enhanced by local noise.

In summary, we have investigated two-qubit teleportation through a class of four-qubit noisy entangled states and found that, in the presence of noise, the tensor product of Bell pairs is more suitable as an entangled channel than inseparable entangled channels. For the situation in which Bob's partition of the entangled channel is subject to decoherence, we have shown that Bell pairs and inseparable channels are equally affected. For the case where Alice's qubits 1 and 2 are also exposed to noise, we have investigated the effect of some explicit noises by calculating the associated teleportation fidelity of the noisy entangled channels. Both single-qubit and local two-qubit collective noisy channels have been considered, and we concluded that inseparable entangled channels never give a higher teleportation fidelity than the Bell pairs.

This work was supported by the National Natural Science Foundation of China under Grants No. 10874004 and No. 10821062 and the National Key Basic Research Program No. 2006CB921601.

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