Dark-state polaritons for quantum memory in a five-level $M$-type atomic ensemble

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Dark-state polaritons in a five-level $M$-type atomic ensemble are proposed in the electromagnetically induced transparency. Under conditions of the low excitation limit and adiabatic following, the quantum field can be mapped onto the atomic polarization states, forming the shape preserving quantum states of polaritons. Two channels for robust quantum memory are provided, which enables the potential applications in quantum-information processing.

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I. INTRODUCTION

With the development of quantum information [1], quantum memory has become essential in quantum-information processing, which involves quantum states transferring among the quantum network. It is known that photons are the ideal carriers of quantum information and ensemble of atoms can be acted as the long lived storage and processing units. In the atomic ensemble of the light resonantly interacting with the atoms, various schemes have been proposed to implement quantum state transfer and storage [2]. Recently, the three-level Λ-type atomic ensemble has been extensively studied for practical quantum memory [3] in the electromagnetically induced transparency (EIT) [4]. Physically the photon storage in an EIT system grounds on the formation of shape preserving dark-state polariton (DSP), which is a mixture of the quantum field and collective polarization of the atomic system under the adiabatic conditions. The recent success in experiments [5] has demonstrated the photon storage in EIT media with the group velocity reduction via adiabatic following, and thus motivated further theoretical approach on the long-distance quantum communication [6]. In order to avoid the spatial-motion induced decoherence, the quasispin wave collective excitations of Λ-type atoms in crystal has been proposed to implement robust quantum memories [7].

Besides three-level Λ-type atomic ensemble, the study on the quantum-information processing has been extended to the multilevel atomic ensemble [8–12]. In the four-level inverted-Y scheme, DSPs are generalized and can act as a quantum state copier or divider [8]. In the double-Λ atomic ensemble [9], DSPs are also obtained to act as a controllable beam splitter. Recently, the five-level $M$-type atomic ensemble has attracted interest [11,12]. It has been experimentally demonstrated that the group velocity can be reduced in the $M$-type configuration of cesium atomic vapor [12]. Similar to three-level Λ-type atoms where coherent population trapping (CPT) (or dark state) [13] and EIT are based on the two-photon resonance, for five-level $M$-type atoms, CPT and EIT are also reported in condition of the collaboration of two sets of two-photon resonances [11]. As EIT is the foundation of the shape preserving DSP, it is natural that DSP should also exist in this five-level $M$-scheme. The goal of this work is to propose the DSP and discuss the quantum state transfer in the five-level $M$-type atomic ensemble.

In the following, we propose a quantum information storage protocol based on collective atomic excitations and dark states with an ensemble of $N$ five-level $M$-type atoms. A quantum field is propagating in the five-level atomic media under conditions of EIT. In the low excitation limit and adiabatic following [14], a shape preserving dark-state polariton can be obtained, with a group velocity that can be manipulated. In such system, most atoms stay in the ground state and the four quasispin collective excitations behave as four types of bosons, thus a four-mode exciton system is formed. We prove that in the large $N$ limit with low excitations, this system possesses a hidden dynamic symmetry described by the semidirect sum $SU(4)\otimes h$ of quasispin $SU(4)$ and the exciton Heisenberg algebra $h$ [15]. Based on the above consideration we construct the general dark states of the four-mode exciton-photon system, which can serve as a robust quantum memory. The advantage of this system is that it can preserve quantum states of light in two different collective atomic polarization states of the same atomic medium, which provides channelization of robust quantum memory. It should have potential applications in quantum information processing.

II. DARK-STATE POLARITONS

$M$ scheme is shown in Fig. 1 [11], where five atomic levels are labeled as the ground state $|b\rangle$, the two excited

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states $|c\rangle$ and $|d\rangle$, and the two metastable states $|a\rangle$ and $|e\rangle$. The quantum field described by Eq. (2) is resonantly interacting with the transition $|b\rangle \leftrightarrow |c\rangle$ with the coupling coefficient $g$, and three classical control fields resonantly with the transitions $|e\rangle \leftrightarrow |d\rangle$, $|a\rangle \leftrightarrow |d\rangle$, and $|a\rangle \leftrightarrow |c\rangle$ with Rabi frequencies $\Omega_1(t)$, $\Omega_2(t)$, and $\Omega_3(t)$, respectively. Here the two-photon resonances condition is automatically satisfied [11], so the CPT and EIT are obtained, which is the foundation of the following discussion. Under the dipole and rotating wave approximations, the interaction Hamiltonian of $N$ identical five-level $M$-type atoms read (let $\hbar = 1$)

$$\hat{H}_I = g \sum_{j=1}^{\tilde{N}} \hat{a}(z_j,t) \hat{\sigma}_{ch}^j + \sum_{j=1}^{\tilde{N}} \Omega_1(t) \hat{\sigma}_{cb}^j + \sum_{j=1}^{\tilde{N}} \Omega_3(t) \hat{\sigma}_{ca}^j$$

$$+ \sum_{j=1}^{\tilde{N}} \Omega_3(t) \hat{\sigma}_{ca} + H.c.,$$

where the quasispin operators $\hat{\sigma}_{\alpha\beta} = \hat{\sigma}_{\alpha\beta} \exp[-\frac{i}{\hbar} \omega_{\alpha\beta}(z_j - ct)]$, $\hat{\sigma}_{\alpha\beta} = |\alpha\rangle_{j} \langle \beta| \ (\alpha, \beta = a, b, c, d, e)$ for $\alpha \neq \beta$, denote the flip operator of the jth atom located at position $z_j$ from the state $|\beta\rangle_j$ to $|\alpha\rangle_j$, $\omega_{\alpha\beta}$ is the frequency of the $|\alpha\rangle \leftrightarrow |\beta\rangle$ transition, and $\hat{a}(z,t) = \sum_j \hat{a}(t)e^{i\omega_j} \hat{a}(t)$ the annihilation operator of quantum field.

Now consider the propagation of quantum field in five-level atomic media under the EIT. A quantized electromagnetic field $\hat{E}(z,t)$ is written as

$$\hat{E}(z,t) = \sqrt{\frac{\hbar v}{2\epsilon_0}} \hat{a}(z,t) \exp\left(\frac{v}{c}(z-ct)\right).$$

Here $V$ is some quantization volume, which for simplicity is chosen to be equal to the interaction volume. The evolution of the Heisenberg operator $\hat{a}(z,t)$ corresponding to the quantum field in the slowly varying amplitude approximation is

$$\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial z}\right) \hat{a}(z,t) = i g N \hat{\sigma}_{bc}.$$

The evolution of atomic variables is governed by a set of Heisenberg-Langevin equations [16]

$$\frac{\partial}{\partial t} \hat{\sigma}_{\alpha\beta} = - \gamma_{\alpha\beta} \hat{\sigma}_{\alpha\beta} + i \left[\hat{H}_I, \hat{\sigma}_{\alpha\beta}\right] + F_{\alpha\beta},$$

where $\hat{\sigma}_{\alpha\beta}(z,t) = \frac{1}{N} \sum_{j=1}^{\tilde{N}} \hat{\sigma}_{\alpha\beta}^{j}(\alpha, \beta = a, b, c, d, e)$ are averaged over small but macroscopic volumes containing $N \gg 1$ particles at position $z$, $\gamma_{\alpha\beta}$ are the decay rates, and $F_{\alpha\beta}$ are deterministic Langevin noise operators [with correlation functions $\langle F_{\alpha}(t)F_{\beta}(t)\rangle = D_{\alpha\beta}\delta(t-t')$] [16], where $D_{\alpha\beta}$ are time-independent functions. These decay rates in some degree can compensate with the quantum noise.

We now assume that the Rabi frequency of the quantum field is much smaller than those corresponding to the classical control fields and that the number density of photons in the quantum field is much less than the number density of atoms. In such a case the atomic equations can be treated perturbatively in $\hat{a}(z,t)$. In zeroth order only $\hat{\sigma}_{ab} = 1$ and in first order one finds

$$\frac{\partial}{\partial t} \hat{\sigma}_{bc} = - \gamma_{bc} \hat{\sigma}_{bc} + i g \hat{a} + i \Omega_3 \hat{\sigma}_{ba} + F_{bc},$$

$$\frac{\partial}{\partial t} \hat{\sigma}_{bd} = - \gamma_{bd} \hat{\sigma}_{bd} + i \Omega_1 \hat{\sigma}_{be} + i \Omega_2 \hat{\sigma}_{ba} + F_{bd},$$

$$\frac{\partial}{\partial t} \hat{\sigma}_{ba} = i \Omega_2 \hat{\sigma}_{bd} + i \Omega_3 \hat{\sigma}_{bc} + F_{ba},$$

$$\frac{\partial}{\partial t} \hat{\sigma}_{be} = i \Omega_1 \hat{\sigma}_{bd} + F_{be}.$$
field can be manipulated by controlling three classical fields. While in the three-level Λ-type atoms, only one classical field is used to control the group velocity of the quantum field [3].

By introducing the collective atomic operators

$$A_{k1} = \frac{1}{N} \sum_{j=1}^{N} e^{-ik_1j} a_{j},$$

$$T_{1}^k = \sum_{j=1}^{N} a_{j}^\dagger,$$

$$T_{2}^k = \sum_{j=1}^{N} a_{j} a_{j}^\dagger,$$

$$T_{3}^k = \sum_{j=1}^{N} a_{j}^\dagger a_{j},$$

the interaction Hamiltonian is expressed as

$$\hat{H}_I = g \sqrt{N} \sum_k \hat{a}_{k1}^\dagger + \Omega_1 T_{1}^k + \Omega_2 T_{2}^k + \Omega_3 T_{3}^k + \text{H.c.} \quad (10)$$

To properly describe the cooperative motion of the atomic ensemble stimulated by the quantum and control fields, it is necessary to introduce the following collective excitation operators:

$$A_{k1}^\dagger = \frac{1}{N} \sum_{j=1}^{N} e^{ik_1j} a_{j}^\dagger,$$

$$C_{k1} = \frac{1}{N} \sum_{j=1}^{N} e^{ik_1j} a_{j},$$

$$C_{k2} = \frac{1}{N} \sum_{j=1}^{N} e^{-ik_1j} a_{j}^\dagger.$$ In the large-N limit with the low excitation, only a few atoms occupy the four excitation states |a⟩, |c⟩, |d⟩, and |e⟩, while most of the atoms stay in the ground state |b⟩. It is not difficult to verify that the collective excitation operators $A_{k1}^\dagger A_{k1}^2, A_{k2}^\dagger A_{k2}, C_{k1}, C_{k2}, C_{k2}^\dagger$ satisfy the commutation relations [17]

$$[A_{k1}^\dagger A_{k1}] = \delta_{ij}, \quad [C_{k1}, C_{k1}^\dagger] = \delta_{ij},$$

$$[A_{k1}, C_{k1}] = 0, \quad [A_{k1}^\dagger, C_{k1}^\dagger] = 0 \quad (11)$$

Therefore, the above four mode symmetrized excitations defined by $A_{k1}^\dagger C_{k1}^\dagger$ (j = 1, 2) behave as four independent bosons in low excitation limit, forming a four-mode excitation system. This exciton system possesses a hidden dynamical symmetry described by the semidirect sum SU(4)/h of quasiparticle SU(4) and the exciton Heisenberg algebra h (with the details in the Appendix).

Based on above analysis, we can introduce a dark-state polaron operator

$$D_{k}(t) = \hat{a}_{k} \cos \theta - C_{k} \sin \theta,$$ (12)

where $C_{k}$ = (cos $\gamma$ $C_{k1}$ - sin $\gamma$ $C_{k2}$), and $C_{k}$ is a coherent mixing of two collective atomic excitations $C_{k1}$ and $C_{k2}$, and tan $\gamma$ = $\frac{\Omega}{\Omega_1}$. It can be found that in the adiabatic limit $D(z,t) = \frac{\sqrt{N}}{\Omega} D_{k}(t) e^{ikz}$ obeys the equation of motion

$$\left( \frac{\partial}{\partial t} + c \cos^2 \theta \frac{\partial}{\partial z} \right) D(z,t) = 0.$$ (13)

This equation gives a shape preserving (or quantum state preserving) solution propagating with a velocity $v = v_s(t) = c \cos^2 \theta$.

Introducing a new operator

$$T^i = \frac{\Omega}{\Omega_1} T_{1}^k + \frac{\Omega}{\Omega_2} T_{2}^k + \frac{\Omega}{\Omega_3} T_{3}^k.$$ (14)

we can rewrite the interaction Hamiltonian as

$$\hat{H}_I = g \sqrt{N} \sum_k \hat{a}_{k1}^\dagger + \Omega T^i + \text{H.c.} \quad (15)$$

It is seen that this Hamiltonian has the same form as that in the Λ-type atomic ensemble. Quantum memory is implemented by means of the quantum state mapping technique. It motivates us to define the shape preserving quantum states, i.e., dark states. As we denote by Dicke-state [18] $|\mathbf{g}_a⟩ = |b_1, d_2, \ldots, b_N⟩$ the collective ground state with all $N$ atoms staying in the single particle ground state $|b⟩$, and $|0⟩^k$ the k mode vacuum state of electromagnetic field, the ground state of coupled system is $|\mathbf{0}⟩ = |\mathbf{g}_a⟩ \otimes |0⟩^k$. The quantum memory processing via EIT technique requires that the whole system should be kept in the dark-state subspace, thus the key point is to construct the dark states of the atomic ensemble. In what follows, the details of the consideration will be presented.

One can readily verify that

$$[D_{k}, \hat{H}_I] = 0,$$ (16)

$$[D_{k}, D_{k}^\dagger] = 1.$$ (17)

So the dark-state polaron mode operator $D_{k}$ is a quasiparticle boson operator. Based on the above commutation relations, we can obtain the dark states of the four mode exciton-phonon system. These dark states span an instantaneous quantum storage subspace and can be presented as follows:

$$|D_{a}(t)⟩_k = \frac{1}{\sqrt{n!}} D_{k}^{m} |0⟩, \quad n = 0, 1, 2, \ldots.$$ (18)

In dark states $|D_{a}(t)⟩_k$, there is no possibility to excite the atomic ensemble to the upper states |c⟩ and |d⟩, so the atoms are immune to the spontaneous emission. Using Eq. (12), we can expand $|D_{a}(t)⟩_k$ as

$$|D_{a}(t)⟩_k = \sum_{m=0}^{n} (-1)^m \sqrt{\frac{n!}{m! (m-m)!}} \sin^m \theta \cos^{n-m} \theta$$

$$\times |C_{k}^{m}⟩_k |n-m⟩^k,$$ (19)

where

$$|C_{k}^{m}⟩_a = \frac{1}{\sqrt{m!}} C_{k}^{m} |g_a⟩.$$ (20)

$$= \sum_{l=0}^{m} \sqrt{l! (m-l)!} (-1)^l \sin^l \gamma \cos^{m-l}$$

$$\times γ (N-m)_{b,c}(m-l)_{a,d} l_{c}^k,$$

with

$$|D_{a}(t)⟩_k = |(N-m)_{b,c}(m-l)_{a,d} l_{c}^k |g_a⟩.$$ (21)

This is a superposition state of atoms and fields with a probability $|\cos \theta|^2$ in state $|1⟩_k |g_a⟩$, $|\sin \theta \cos \gamma|^2$ in state $|(N-1)_{b,c} 1_{a,d} l_{c}^k |0⟩_L$, and $|\sin \theta \sin \gamma|^2$ in state $|(N-1)_{b,c} 1_{a,d} l_{c}^k |0⟩_L$. 

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III. QUANTUM INFORMATION TRANSFER

Now we have obtained the dark states of M-type atomic ensemble and electromagnetic fields. By adiabatically controlling the Rabi frequencies of several classical optical fields, we can convert the photons into collective spin polarizations of atoms, and vice versa. This process implements the quantum memory effectively and is important in quantum information processing.

The spontaneous emission can be avoided as the dark states in a five-level M-type atom do not contain the excited states [11]. The existence of collective dark states provides an excellent way to transfer the quantum state of the single mode field to collective atomic excitations. Adiabatically rotating the mixing angle $\theta$ from 0 to $\pi/2$ leads to a complete and reversible transfer of the photonic states to a collective atomic state only if the total number of excitations is less than the number of atoms [3]. When $\theta=0$, i.e., $\cos \theta \equiv 1$, which means the strong control fields ($\Omega \gg g \sqrt{N}$), the DSP becomes a purely photonic operator with the vacuum speed of light, and the dark states are the direct product of the collective ground state of atomic ensemble and the n-photon Fock state. On the contrary, as $\theta\equiv \pi/2$, i.e., $\cos \theta=0$, then the DSP becomes spin-wave-like with a zero group velocity and has two spin components, and the dark states are the direct product of the vacuum state of quantum field and the collective atomic polarization states. When $\theta:0\rightarrow \pi/2$, the following relation can be obtained for all $n \equiv N$:

$$|D_n(t)\rangle_L \otimes |g\rangle_a \rightarrow (-1)^n \frac{1}{\sqrt{n!}} (C^n)_{aa} \otimes |0\rangle_L.$$  \hspace{1cm} (22)

Therefore, if the initial quantum state of the single-mode light field is in any mixed state described by a density matrix $\rho^{L} = \sum_{n,m} \rho_{nm} |n\rangle_{LL}(m)| \otimes |g\rangle_{aa}(g)$, the transfer process generates a quantum state of collective excitations according to

$$\sum_{n,m} \rho_{nm} |n\rangle_{LL}(m)| \otimes |g\rangle_{aa}(g) \rightarrow |0\rangle_{kk} \otimes \sum_{n,m} (-1)^{n+m} \rho_{nm} \frac{1}{\sqrt{n! m! n!}} (C^n)_{aa} (C^m).$$ \hspace{1cm} (23)

In above proposed information storage protocol with M-type atomic ensemble, we can convert quantum information to stationary atomic excitation in two different channels by adiabatically changing the control fields. The stored light pulse in the two stationary spin-wave channels can be simultaneously retrieved to signal field by applying $\Omega_1$ and $\Omega_2$ simultaneously. In such a system, the quantum field is bifurcated and stored in two stationary spin waves with a weight factor governed by the two classical control fields. When $\gamma=0$, i.e., $\cos \gamma=1(\Omega_1 \ll 1)$, then $C_{kl}^{(1)} = \frac{1}{N} \sum_{j=1}^{N} e^{-i \phi_{ab}^{(1)}}$ provides the first quantum storage channel; on the other hand, when $\gamma=\pi/2$, $\cos \gamma=0(\Omega_2 \gg 1)$, then $C_{kl}^{(2)} = \frac{1}{N} \sum_{j=1}^{N} e^{-i \phi_{ab}^{(2)}}$ provides the second storage channel. The advantage of this protocol is the ability to channelize the quantum memory into two different spin waves which can then be retrieved on demand either simultaneously or with a certain time delay, which thus provides channelization of robust quantum memory. We need to note that the transfer rate is proportional to the total number of atoms $N$, which is a signature of collective coupling. This makes the proposed method potentially fast and robust.

The above discussion is based on the on-resonance EIT, but the method of atomic collective excitations can also be effective in the case of two-photon resonance EIT [19]. The experiment of the light storage under the condition of two-photon resonance EIT has been reported [20]. It has been verified that under conditions of the collaboration of two sets of two-photon resonances in a five-level M-type atomic system, the CPT and EIT exist [11], therefore DSP for quantum memory can also be implemented.

It is noted that the present treatment is valid only for the low density excitation regime where the bosonic modes of the quasispin collective excitations can be effective. It is also emphasized that though the DSPs of our protocol are generated in an ensemble of atoms with random spatial positions, the results remain valid in case of a crystal. The recent experiment of the ultraslow group velocity of light in a crystal of $Y_2S\_O_3$ proposes the possibility of implementing robust quantum memories by utilizing the solid state exciton system [21].

IV. SUMMARY

We have proposed the dark-state polaritons for quantum memory under the condition of EIT in the five-level M-type atomic ensemble. By considering the propagation of quantum field in the five-level media, we find that in low excitation limit and adiabatic following, a shape preserving dark-state polariton can be obtained, and the four quasispin collective excitations behave as four types of bosons, forming a four-mode exciton system. Then, we prove that in the large-N limit with the low excitation, this four-mode exciton system possesses a hidden dynamical symmetry described by the semidirect sum $SU(4)\otimes h$ of quasispin $SU(4)$ and the exciton Heisenberg algebra $h$. Based on these considerations, we construct the general dark states of atomic ensemble and photons, which can serve as a robust quantum memory. In the present protocol there are two different channels to implement the quantum-information storage, and thus it has potential applications in quantum-information.

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APPENDIX

In this appendix we give the details of the symmetry analysis. In order to find out the dynamical symmetry of this four-mode exciton system, we need to introduce three new collective atomic operators: $H_1 = \sum_{j=1}^{N} (\hat{\sigma}_{dd}^j - \hat{\sigma}_{cc}^j)$, $H_2 = \sum_{j=1}^{N} (\hat{\sigma}_{aa}^j - \hat{\sigma}_{dd}^j)$, $H_3 = \sum_{j=1}^{N} (\hat{\sigma}_{cc}^j - \hat{\sigma}_{aa}^j)$. It is easy to prove the commutation relations

$$ [H_j, H_k] = 0, \quad [T_j^i, T_k^j] = \delta_{jk} H_j \quad (j, k = 1, 2, 3), \quad (A1) $$

$$ [H_j, T_k^i] = 2 \delta_{jk} T_k^i - \delta_{j(k+1)} T_k^i - \delta_{j(k-1)} T_k^i, \quad (A2) $$

Thereby these operators generate a SU(4) algebra. Denoting the Lie algebra generated by the exciton operators $A_{k1}, A_{k2}, C_{k1}, C_{k2}$, $(i=1, 2)$ by $\hbar$, we find that the exciton operators and the SU(4) generators span a larger Lie algebra. By a straightforward calculation, it then follows that $[SU(4), \hbar] \subset \hbar$, which means that in the large-$N$ limit with the low excitation the operators $A_{k1}, A_{k2}, C_{k1}, C_{k2}$ $(i=1, 2), T_j^i, H_j \quad (j=1, 2, 3)$ and the identity $I$ span a semi-direct sum Lie algebra SU(4)$\oplus \hbar$. Thus the dynamical symmetry governed by $\hat{H}_j$ can be described by the semidirect sum Lie algebra SU(4)$\oplus \hbar$.