RADIATIVE COOLING AND BROADBAND PHENOMENON IN LOW-FREQUENCY WAVES

FU Zun-tao (付遵涛)\textsuperscript{1,2}, JIANG Xun (蒋)\textsuperscript{1} and LIU Shi-kuo (刘式适)\textsuperscript{1}

(1. Department Of Geophysics, Peking University, Beijing 100871 China; 2.SKLTR, Peking University, Beijing, 100871, China; 3. First National Institute of Oceanography Key Laboratory of Marine Science and Numerical Modeling, Qingdao 266000, China)

ABSTRACT: In this paper, we analyze the effects of radiative cooling on the pure baroclinic low-frequency waves under the approximation of equatorial $\beta$-plane and semi-geostrophic condition. The results show that radiative cooling does not, exclusively, provide the damping effects on the development of low-frequency waves. Under the delicate radiative-convective equilibrium, radiative effects will alter the phase speed and wave period, and bring about the broadband of phase velocity and wave period by adjusting the vertical profiles of diabatic heating. When the intensity of diabatic heating is moderate and appropriate, it is conductive to the development and sustaining of the low-frequency waves and their broadband phenomena, not the larger, the better. The radiative cooling cannot be neglected in order to reach the moderate and appropriate intensity of diabatic heating.

Key words: radiative cooling; low-frequency waves; broadband phenomena

CLC number: P433

Document code: A

1 INTRODUCTION

Since Madden and Julian (1971) discovered through spectrum analysis that there are 40-50-day low-frequency oscillations in the variations of atmospheric wind and surface pressure fields in the tropics, and confirmed in further investigation (1972) that there exist 40-50-day low-frequency oscillations over the global tropics, investigations about atmospheric low-frequency oscillations have been a focus. Further investigations concluded that the stream field and temperature field of low-frequency oscillations exhibit prominent baroclinic structure, whose zonal structure is prominent with wave-number one, which exhibits two-dimensional Rossby wave-trains off the equator and its amplitude does not decay much during its propagating and its periodicity focuses on 30-60 days. The tropical low-frequency oscillations have been regarded as a system to be widely studied. Yasunai (1979, 1980) pointed out that the cloudiness over monsoon regions of Indian Ocean possesses the variation of 30-40 days through the analysis of satellite data of cloudiness. Krishnamurti (1982) confirmed that there exist 30-50 days oscillations in the variations of monsoon troughs and ridges over Southern Asia. Zhang (1996) thought that atmospheric low-frequency oscillations in the lower troposphere exist not only in the surface pressure and wind stresses, but there are also prominent low-frequency oscillations in the variations of relative humidity, surface sea temperature, sensible heat and latent heat fluxes. The investigations of Brantzer and Wallace (1996) showed that there also exist prominent 30-60 days variations in the mean magnitude, such as tropical troposphere temperature and precipitation. Chao and Lin (1994) and Lau et al (1989) among other authors thought that intraseasonal
oscillations over convective regions contain one or several super-cloud-clusters, every super-cloud-cluster also contains several cloud-clusters. The temporal and spatial scales and active periodicity of intraseasonal oscillations, super-cloud-clusters and cloud-clusters are different, their interaction makes intra-seasonal oscillations generating and developing.

All investigations show that low-frequency oscillations cannot be considered only as an isolated phenomenon to study. In fact, it is an interactive system, which contains different temporal scales and spatial scales. There exists broadband phenomenon in the oscillation periodicity and phase velocity of low-frequency oscillations. Based on OLR and atmospheric circulation data analysis, Kiladis and Weickmann (1992) pointed out multi-scale intraseasonal oscillations with periodicity of 6-14, 14-30 and 30-70 days and proposed that atmospheric circulation patterns associated with these different temporal scales are different and their corresponding mechanisms are also different. Zhang (1996) also thought that the dominant periods of intraseasonal oscillations range from 30 to 90 days with a strong spectral peak at 50-60 days. The broadbands are also discovered in the phase velocity rather than that the phase velocity is uniformly slow eastward (for example, \(5 \text{m} \cdot \text{s}^{-1} \sim 10 \text{m} \cdot \text{s}^{-1}\)). In fact, investigations based on simulations and observations have shown that the phase velocity of low-frequency oscillations over different regions is different. Madden and Julian (1972, 1994) found that the surface pressure variations associated with Madden Julian Oscillation (MJO) exhibit fast moving eastward over east Pacific regions (approximately 40 \(\text{m} \cdot \text{s}^{-1}\)). Milliff and Madden (1996) detected eastward phase propagation (at speeds faster than 30 \(\text{m} \cdot \text{s}^{-1}\)) of a signal with a first baroclinic mode vertical structure in the equatorial troposphere of the eastern Pacific in historical meteorological observations and recent data. They thought that this rapid eastward signal is a far field dispersion product of strong convection associated with the intraseasonal tropical oscillation in the Indian Ocean and western Pacific. The simulation carried by Sui and Lau (1989) identified two distinct intraseasonal modes: (1) a fast eastward propagating mode (18 \(\text{m} \cdot \text{s}^{-1}\)); (2) a slower eastward propagating mode (9 \(\text{m} \cdot \text{s}^{-1}\)). And these two modes correspond to the different maximum convective heating of troposphere. Lau et al. (1994) further pointed out that atmospheric general circulation variability is associated with a wide range of spatial and temporal scales and the mechanism of low-frequency oscillations is different for the different spatial and temporal scales.

Many investigations, e.g. Miyahara (1987), Lau and Peng (1987), Sui and Lau (1989) et al., have shown that the eastward propagating speed of the simulated MJO is larger than the observed and it is very sensitive to the vertical distribution of diabatic heating. Lau and Peng (1987), Sui and Lau (1989) found that the phase velocity of simulated MJO approximately coincides with the observed if the maximum heating lies in the lower level of troposphere. In fact, the observed maximum heating lies in the middle level of troposphere, this may be the main cause to bring about the different results between the simulation and observation. Slingo and Slingo (1988) found that in the tropical atmosphere, net diabatic heating results from a delicate balance between two large diabatic components, namely, latent heating and long-wave radiative cooling. The preceding investigations have indicated that the phase velocity of MJO is sensitive to diabatic heating profile, so there is a possibility that convective heating may not be the sole control on phase speeds. If radiative cooling can modify the convective heating profile, it may also partially control the phase speed of intraseasonal waves. Evidence of this can be found in Slingo and Madden's (1991) study. Mehta and Smith (1997) analyzed the cloud-modulated radiative cooling rates and found that the intraseasonal variability of the cooling rate profiles can much perturb convective heating profiles. Using a one-dimensional radiative-convective equilibrium model, they speculated that the phase speeds of Kelvin waves excited by convective heating can be altered due to the radiative damping effect. The results is a range of time-scales of propagation
providing a possible explanation for the broadband of periodicity that has been associated with the modulation of radiative cooling. At the same time, they calculated the effects of radiative cooling on the heating profiles under the radiative-convective equilibrium and found that the effects of cloud-radiative anomalies are predominant in the upper and lower troposphere, where latent heating is least significant. For convective regions, it is the middle troposphere where radiative cooling is least and latent heating largest. So, this means that perturbative effects of cloud-induced radiative cooling do not necessarily change the fundamental nature of the diabatic heating profiles. However, the vertical heating gradient changes due to radiative cooling anomalies, which in turn affect the stability of the troposphere and the phase velocity of perturbations.

However, we think that the omission of rotation effects of the earth in the Mehta and Smith’s (1997) studies may change the phase velocity or wave period of low-frequency oscillations. At the same time, they only investigate the effects of radiative cooling on the Kelvin waves, which does not provide enough cogency for low-frequency oscillations over global tropics. Based on the investigations of Mehta and Smith (1997) and Fu and Liu (1998), we further investigate the effects of radiative cooling on the broadband of low-frequency oscillations under the equatorial \( \beta \)-plane and long-wave approximation framework. We only consider the modulation of radiative cooling on the waves triggered by convective heating, for radiative cooling only provides a way to gauge the magnitude of the feedback effect of cloud-radiative cooling, not a mechanism that actually generates the intraseasonal waves, just as Mehta and Smith (1997) have proceeded.

2 BASIC EQUATIONS.

In the equatorial \( \beta \)-plane and long-wave approximation framework, the pure baroclinic model containing diabatic heating can be written as:

\[
\frac{\partial u}{\partial t} - \beta y \nu = -\frac{\partial \phi}{\partial x} \tag{1}
\]

\[
\beta y \nu = -\frac{\partial \phi}{\partial y} \tag{2}
\]

\[
\frac{\partial \phi}{\partial t} + c^2 \left( \frac{\partial u}{\partial x} + \frac{\partial \nu}{\partial y} \right) = -Q \tag{3}
\]

Eqs. (1)-(3) can be taken as the formulas about pure baroclinic components in the two-layer model, details of derivation can be got from Fu and Liu (1998), where \( u = (u_1 - u_3)/2 \), \( \nu = (u_1 - u_3)/2 \) and \( \phi = (\phi_1 - \phi_3)/2 \), with subscripts 1 and 3 referring to the upper and lower levels, respectively. The symbol \( c \) denotes the baroclinic wave speed, the term containing \( Q \) on the right side of Eq.(3) represents the diabatic one, in the context of present study, it is expressed:

\[
Q = -\epsilon_c \frac{\partial u}{\partial x} + \epsilon_r \phi \tag{4}
\]

where \( \epsilon_c \) is the coefficient of convective heating, \( \epsilon_r \) is the radiative damping time-scale.

Combining Eqs. (4) and (3) yields:
\[
\frac{\partial \phi}{\partial t} + c^2 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = \varepsilon \frac{\partial u}{\partial x} - \varepsilon \phi
\]  
(5)

Thus, assuming the solutions to Eqs. (1), (2) and (5) take the form:

\[
(u, v, \phi) = (U(y), V(y), \Phi(y)) \exp(ikx - i\omega t)
\]  
(6)

and rewrite Eqs.(1), (2) and (5) by using Eq.(6), we have:

\[
i\omega U + \beta y V = ik\Phi
\]  
(7)

\[
\beta y U = -\frac{d\Phi}{dy}
\]  
(8)

\[
(\varepsilon, -i\omega)\Phi + (c^2 - \varepsilon)ikU + c^2 \frac{dV}{dy} = 0
\]  
(9)

Rewriting Eq.(7) yields:

\[
\Phi = \frac{i\omega U + \beta y V}{ik}
\]  
(10)

and substitute it into Eqs.(8) and (9) by using Eq.(10), we have:

\[
U = \frac{(\varepsilon, -i\omega)\beta y V / ik + c^2 dV / dy}{(\varepsilon, -i\omega)\omega / k + ik(c^2 - \varepsilon)}
\]  
(11)

\[
\frac{dU}{dy} = \beta V + \beta y \frac{dV}{dy}
\]  
(12)

Substituting Eq.(11) into Eq.(12) yields:

\[
\frac{d^2 V}{dy^2} + \frac{\varepsilon, k\beta y}{\omega c^2} \frac{dV}{dy} - \frac{(c^2 - \varepsilon)c^2 + (\omega + i\varepsilon)\beta^2 y^2}{\omega c^2} = 0
\]  
(13)

Transforming

\[
W(y) = V(y) \exp\left(\frac{\varepsilon, k\beta y^3}{4\omega c^2}\right)
\]  
(14)

and using Eq.(14) to rewrite Eq.(13) gives the Weber-type Equation:

\[
\frac{d^2 W}{dy^2} + \frac{2(c^2 - \varepsilon)c^2 + 4\omega c^2 + i4\varepsilon, \omega c^2} {4\omega^2 c^4} \beta^2 y^2 = 0
\]  
(15)

Letting \( \varepsilon = 0 \), Eq.(15) is transformed into:

\[
\frac{d^2 W}{dy^2} + \frac{-k\beta}{\omega} - \frac{1}{\omega} \frac{\beta^2 y^2}{c^2} = 0
\]  
(16)

Eq.(16), when meeting the condition of \( y \to \pm \infty \), \( W \to 0 \), gives the eigenvalue of:

\[
\left(\frac{k\beta}{\omega}\right)^2 = 2m + 1 \quad (m = 0, 1, 2, \ldots)
\]  
(17)

and its corresponding eigenfunction:
\[ W = B_w \exp\left(-\frac{y^2}{2L^2}\right) H_m\left(\frac{y}{L}\right) \]

(18)

where \( B_w \) is random constant, \( H_m\left(\frac{y}{L}\right) \) is Hermite polynomial and \( 1/L^4 = (1 + i\varepsilon_r / \omega) \beta^2 / c^2 \).

Eq.(17) is the same as Eq.(29) given in Fu and Liu (1998).

While Eq.(15) meets the condition of \( y \to \pm \infty, \ W \to 0 \), we have the eigenvalue of:

\[ \left(2c^2 - \varepsilon_e \right) k_0^L / 2\omega c^2 \right) L_0^2 = 2m + 1 \quad (m = 0, 1, 2, \ldots) \]

(19)

and its corresponding eigenfunction:

\[ W = B_w \exp\left(-\frac{y^2}{2L^2}\right) H_m\left(\frac{y}{L_0}\right) \]

(20)

where \( B_{w0} \) is random constant, \( H_m\left(\frac{y}{L_0}\right) \) is Hermite polynomial and

\[ 1/L_0^4 = (\varepsilon_e k^2 + 4\omega^2 c^2 + i4\varepsilon_e \omega c^2) / (4\omega^2 c^4) \]

Eq.(19) is the same as Eq.(17) when \( \varepsilon_e = 0 \).

Eq.(19) can be rewritten as:

\[ \omega^2 + i\varepsilon_e \omega + \frac{\varepsilon_e^2 k^2 m(m+1)}{(2m+1)^2 c^2} + \frac{k^2 \varepsilon_e}{(2m+1)^2} - \frac{k^2 c^2}{(2m+1)^2} = 0 \quad (m = 0, 1, 2, \ldots) \]

(21)

Eq.(21) is the same as Eq.(17) when \( \varepsilon_e = 0 \):

\[ \omega^2 + i\varepsilon_e \omega - \frac{k^2 c^2}{(2m+1)^2} = 0 \quad (m = 0, 1, 2, \ldots) \]

(22)

Letting:

\[ \omega = \omega_r + i\omega_i \]

(23)

where \( \omega_r \) is the wave frequency, \( \omega_i \) is the wave growth.

Using Eq.(23) to rewrite Eq.(21), we have:

\[ \omega_r^2 - \varepsilon_e \omega_r + \frac{\varepsilon_e^2 k^2 m(m+1)}{(2m+1)^2 c^2} + \frac{k^2 \varepsilon_e}{(2m+1)^2} - \frac{k^2 c^2}{(2m+1)^2} = 0 \]

(24)

\[ 2\omega_r \omega_i + \varepsilon_e \omega_r = 0 \]

(25)

Eq.(25) produces \( \omega_r = 0 \) or \( \omega_i = -\varepsilon_e / 2 \).

When \( \omega_r = 0 \), Eq.(24) yields:

\[ \omega_i = \frac{1}{2} \left[-\varepsilon_e \pm \sqrt{\varepsilon_e^2 - 4 \left[\varepsilon_e^2 k^2 m(m+1) / (2m+1)^2 c^2 - \frac{k^2 \varepsilon_e}{(2m+1)^2} + \frac{k^2 c^2}{(2m+1)^2}\right]}\right] \]

when \( \omega_i = -\varepsilon_e / 2 \), Eq.(24) yields:
$$\omega_r = \pm \left[ \frac{\varepsilon_r^2 k^2 m(m+1)}{(2m+1)^2 c^2} - \frac{k^2 \varepsilon_r}{(2m+1)^2} + \frac{k^2 c^2}{(2m+1)^2} - \varepsilon_r \right]^{1/2}$$

When $Q = 0$ i.e. $\varepsilon_r = \varepsilon_c = 0$, Eq.(13) gives the relation about adiabatic waves:

$$\frac{d^2 V}{dy^2} + \left[ -\frac{k\beta}{\omega} - \frac{\beta^2}{c^2} y^4 \right] V = 0$$  (26)

Eq.(26), when meeting the condition of $y \to \pm \infty$, $V \to 0$, gives the eigenvalue:

$$-\frac{k\beta}{\omega} = (2m+1) \frac{\beta}{c}$$  (27)

i.e.

$$\omega_0 = -\frac{k\beta}{2m+1}$$  (28)

when $m = -1$, Eq.(28) gives the dispersion relation of adiabatic Kelvin waves; when $m \geq 0$, it gives the dispersion relation of adiabatic Rossby waves.

So, when $\varepsilon_r = \varepsilon_c = 0$ the condition that $\omega_r = 0$, $\omega_r = \omega_0$ must be met. Thus,

$$\omega_r = -\frac{\varepsilon_r}{2}$$  (29)

$$\omega_r = -\frac{k\beta}{2m+1} \left[ 1 - \frac{\varepsilon_c^2}{c^4} \left( 1 - \frac{\varepsilon_r^2 m(m+1)}{4k^2 c^2} \right)^2 \right]^{1/2}$$  (30)

i.e.

$$\omega_r = \omega_0 \left[ 1 - \frac{\varepsilon_c^2}{c^4} \left( 1 - \frac{\varepsilon_r^2 m(m+1)}{4k^2 c^2} \right)^2 \right]^{1/2}$$  (31)

when $m = -1$, Eq.(31) gives the dispersion relation of diabatic Kelvin waves; when $m \geq 0$, it gives the dispersion relation of diabatic Rossby waves.

The discussions above are carried out under the condition:

$$-\frac{\varepsilon_r^2 k^2 m(m+1)}{(2m+1)^2 c^2} - \frac{k^2 \varepsilon_r}{(2m+1)^2} + \frac{k^2 c^2}{(2m+1)^2} - \varepsilon_r \geq 0.$$  

when $-\frac{\varepsilon_r^2 k^2 m(m+1)}{(2m+1)^2 c^2} - \frac{k^2 \varepsilon_r}{(2m+1)^2} + \frac{k^2 c^2}{(2m+1)^2} - \varepsilon_r < 0$, we can obtain

$$\omega_r = 0$$  (32)

$$\omega_r = -\frac{\varepsilon_r}{2} \pm \frac{k\beta}{2m+1} \left[ \frac{\varepsilon_c^2}{c^4} + \frac{\varepsilon_r^2 m(m+1)}{4k^2 c^2} - \frac{\varepsilon_r^2 (2m+1)^2}{4k^2 c^2} \right]^{1/2}$$  (33)

in which the waves trend to be stationary.

Eqs. (29) - (33) are the basis we use to carry out some discussions in this paper.

3 ANALYSIS AND DISCUSSIONS
The wave growth rate can be obtained from Eq.(29) as \( \omega = -\varepsilon_2/2 \). When \( \varepsilon_2 > 0 \) (in this paper \( \varepsilon_2 > 0 \)), radiative cooling damps the development of waves.

Letting:

\[
c_p = \frac{\omega}{k}
\]

where \( c_p \) is the phase velocity of waves.

From Eq.(28), we have:

\[
c_{p0} = -\frac{c}{2m+1}
\]

where \( c_{p0} \) is the phase velocity of adiabatic waves. When \( m = -1 \), Eq. (35) gives the phase velocity of adiabatic Kelvin waves; when \( m \geq 0 \), it gives phase velocity of adiabatic Rossby waves.

From Eq.(31), we obtain:

\[
c_p = c_{p0}\left[1 - \frac{\varepsilon_2}{c^2} - \frac{\varepsilon_2^2 m(m+1)}{c^4} - \frac{\varepsilon_2^2 (2m+1)^2}{4k^2 c^2}\right]^{1/2}
\]

where \( c_{p0} \) is phase velocity of adiabatic waves. When \( m = -1 \), Eq.(36) gives the phase velocity of diabatic Kelvin waves; when \( m \geq 0 \), it gives the phase velocity of diabatic Rossby waves. Comparing Eqs.(35) and (36) yields \( |c_p| < |c_{p0}| \), which indicates that diabatic heating reduces the phase velocity of waves.

From Eq.(36)

\[
c_{pc} = c_{p0}\left[1 - \frac{\varepsilon_2^2 m(m+1)}{c^4}\right]^{1/2} (\varepsilon_2 = 0)
\]

\[
c_{pr} = c_{p0}\left[1 - \frac{\varepsilon_2^2 (2m+1)^2}{4k^2 c^2}\right]^{1/2} (\varepsilon_2 = 0)
\]

can be obtained. Comparing them yields \( |c_p| < |c_{pc}| < |c_{p0}| \) and \( |c_p| < |c_{pr}| < |c_{p0}| \).

Combining Eqs.(34) and (37) yields:

\[
c_p = c_{pc}\left[1 - \frac{\varepsilon_2^2 (2m+1)^2 c^2}{4k^2 c^4 - 4k^2 c^2 \varepsilon_2 - 4k^2 \varepsilon_2^2 m(m+1)}\right]^{1/2}
\]

Rewriting Eqs.(35), (36) and (37) gives:

\[
c_p^2 = c_{pc}^2 - \frac{\varepsilon_2^2}{4k^2}
\]

Because \( 4k^2 c^4 - 4k^2 c^2 \varepsilon_2 - 4k^2 \varepsilon_2^2 m(m+1) > 0 \), \( |c_p| < |c_{pc}| \) can be got. If assuming:

\[
T = \frac{2\pi}{|\omega_2|}
\]

where \( T \) is the wave period, the following formula is got when rewriting Eq.(28) by using Eq.(41):
\[ T_0 = \frac{2|m+1|\pi}{kc} \]  \hspace{1cm} (42)

where \( T_0 \) is the wave period of adiabatic waves. When \( m = -1 \), Eq.(42) gives wave period of adiabatic Kelvin waves; when \( m \geq 0 \), it gives wave period of adiabatic Rossby waves.

And by rewriting Eq.(30), we get:

\[ T = T_0 \left[ 1 - \frac{\epsilon_c}{c^2} - \frac{\epsilon_r^2 m(m+1)}{c^4} - \frac{\epsilon_r^2 (2m+1)^2}{4k^2 c^2} \right]^{1/2} \]  \hspace{1cm} (43)

\[ T_r = T_0 \left[ 1 - \frac{\epsilon_r}{c^2} - \frac{\epsilon_r^2 m(m+1)}{c^4} \right]^{1/2} \hspace{1cm} (\epsilon_r = 0) \]  \hspace{1cm} (44)

\[ T_c = T_0 \left[ 1 - \frac{\epsilon_c^2 (2m+1)^2}{4k^2 c^2} \right]^{1/2} \hspace{1cm} (\epsilon_c = 0) \]  \hspace{1cm} (45)

when \( m = -1 \). Eq.(43) gives the wave period of diabatic Kelvin waves; when \( m \geq 0 \), it gives the wave period of diabatic Rossby waves. And there exist the following relations: \( T > T_r > T_0 \) and \( T > T_r > T_c \).

Comparisons above show that: as a modulator, radiative cooling contributes to the development of low-frequency waves. According to Eq.(43), low-frequency waves can be excited by convective heating, and Eq.(44) indicates that radiative cooling contributes to these excited low-frequency waves developing further toward low-frequency and reducing their phase velocity.

There are two possible ways in which the phase velocity or wave period of a wave excited by convective heating can be altered by radiative cooling factor, if Eq.(33) is also considered. If the radiative cooling factor is moderate, it can reduce the phase velocity and enlarge the value of wave period; when the radiative cooling factor exceeds a certain value, it will turn the low-frequency waves excited by convective heating to be stationary waves. Using the radiative-convective equilibrium model, the radiative damping time-scale can be identified. Mehta and Smith found that it is approximately 3 days, or an \( \epsilon_c \) of \( 3.85 \times 10^{-6} \) m·s\(^{-1}\) for the equatorial Indian Ocean regions. This is smaller than the radiative damping time-scale of 5 days that was previously used by Chang (1977) and Lau (1981) among others. As we know, the value of radiative damping time-scale varies much over different regions or under different conditions. So, this will contribute to the broadband of phase velocity or wave period of low-frequency waves excited by convective heating or other diabatic heating.

Letting \( \epsilon_c = \frac{1}{86400} \), then \( \frac{\epsilon_c^2}{4k^2} = \frac{1}{4 \times 86400^2 k^2 \tau^2} \), where \( \tau \) is radiative damping time-scale and its unit is day. So using Eq.(38) can draw Fig.1a & b. Here some notes should be given: In the figure, the vertical coordinate is phase velocity \( c_p \) and the horizontal coordinate is radiative damping time-scale \( \tau \). For equatorial waves with zonal one-wave, the values of different terms or parameters are \( c = 35 \) m/s and \( \frac{1}{4 \times 86400^2 k^2} = 1364.4 \) m\(^2\). Fig.1a represents the response of Rossby waves \( (m = 1) \) to the variations of radiative damping time-scale \( \tau \) under the weakly convective heating \( (\epsilon_c = 50 \) m\(^2\)/s\(^2\)\); Fig.1b represents the response of Rossby
Fig.1 Variations of phase velocity $c_p$ with radiative damping time-scale $\tau$; (a) for Rossby waves ($m = 1$); (b) for Rossby waves ($m = 0$, dashed line) and Kelvin waves ($m = -1$, solid line).

wave ($m = 0$, curve 2) and Kelvin wave ($m = -1$, curve 1), whose wave period is 30 days, to the variations of radiative damping time-scale $\tau$ under the moderate convective heating ($\epsilon_c = 984.75m^2/s^2$). The result of Fig.1 indicates that weak heating takes the phase velocity of Rossby waves ($m = 1$) to be that of low-frequency wave. The calculation confirms that the Rossby waves ($m = 1$) will turn to be decaying waves, when the intensity of convective heating is a little larger than a certain value (for example, $\epsilon_c = 100m^2/s^2$). Fig.1b gives a different result that the moderate heating ($\epsilon_c = 984.75m^2/s^2$) makes the phase velocity of Rossby waves ($m = 0$) and Kelvin waves ($m = -1$) (with different directions) similar to that of low-frequency waves with 30-day periods. The broadband of phase velocity will be present, if radiative cooling is involved. The comparison between Fig.1(a & b) indicates that the responses of Rossby waves ($m = 1$) and Rossby waves ($m = 0$, including Kelvin waves) are different to the diabatic heating and Rossby waves ($m = 1$) is more sensitive.

Combining (43) and (44) yields:

$$T = T_c \sqrt{\frac{1 - \frac{\epsilon_c^2}{c^2} m(m+1)}{1 - \frac{\epsilon_c^2}{c^2} m(m+1) + \frac{\epsilon_c^2}{c^4} (2m+1)^2 \frac{4k^2 c^2}{4k^2 c^2}}}$$  \hspace{1cm} (46)

Fig.2 (a & b) can be got by using Eq.(46). Here some notes also should be given: In the figure, the vertical coordinate is wave period $T$ and the horizontal coordinate is $\tau$, radiative damping time-scale is $\tau$. For equatorial waves with zonal one-wave, the values of different terms or parameters are $\frac{1}{4 \times 86400^2 k^2} = 1364.4m^2$ and $c = 35m/s$. Fig.2a represents the responses of Rossby waves ($m = 1$) to the variations of radiative damping time-scale $\tau$ under the weakly ($\epsilon_c = 50m^2/s^2$, curve 2) and moderately ($\epsilon_c = 984.75m^2/s^2$, curve 1) convective
heating, respectively. Fig.2b represents the responses of Rossby waves \( (m = 0) \), curve 2) and Kelvin waves \( (m = -1) \), curve 1), whose wave period is 30 days, to the variations of radiative damping time-scale \( \tau \) under the moderate convective heating \( (C_c = 984.75 \text{m}^2/\text{s}^2) \). The results of Fig.2 (a & b) indicates that Rossby waves excited by the weak heating \( (C_c = 50 \text{m}^2/\text{s}^2) \) is easier to result in the broadband phenomenon under the effect of radiative cooling; while Rossby waves excited by stronger heating \( (C_c = 984.75 \text{m}^2/\text{s}^2) \) are numb to the modulation of radiative cooling, for they have been turned into decaying waves. Two curves of Rossby waves \( (m = 0) \), curve 2) and Kelvin waves \( (m = -1) \), curve 1) overlap in Fig.2b., indicating that the periods of Rossby waves \( (m = 0) \), curve 2) and Kelvin waves \( (m = -1) \), curve 1) are the same under the same diabatic heating, although their propagating directions are different. At the same time, the periods of Rossby waves \( (m = 0) \), curve 2) and Kelvin waves \( (m = -1) \), curve 1) have the broadband phenomenon under the modulation of radiative cooling.

![Diagram](image)

Fig.2 Variations of wave period \( T \) with radiative damping time-scale \( \tau \): (a) for Rossby waves \( (m = 1) \), solid line and dashed line separately means the slightly strong heating and weak heating; (b) for Rossby waves \( (m = 0) \) and Kelvin waves \( (m = -1) \)

4 CONCLUSIONS

Based on the analyses above, some conclusions can be reached as follows: cloud-radiative cooling, as a passive response, can adjust the intensity of convective heating through the response to convective heating. This brings about the delicate equilibrium between the convective heating and radiative cooling. We know that convective heating contributes to the generation of low-frequency waves. The change of the vertical profile of diabatic heating due to radiative cooling leads to the modification of the phase speed and period of low-frequency waves, which at last brings about the broadband of the phase speed and period of low-frequency waves. The indirect contributions of cloud radiative cooling to the broadband of low-frequency waves make its damping effects unimportant. The diabatic heating contributes to the generation and development of low-frequency waves only when its intensity is appropriate, not the larger, the better. The
investigations also indicate that the moderate intensity of diabatic heating associated with different low-frequency types is different. The moderate intensity of different low-frequency waves contributes only to their broadband phenomenon, the stronger intensity of diabatic heating just makes low-frequency waves decaying and trend to be stationary waves.

REFERENCES:


