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New Soliton-like Solutions for Combined KdV and mKdV Equation*

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Abstract *By the application of the extended tanh method and the symbolic computation system Mathematica, new soliton-like solutions are obtained for the combined KdV and mKdV (KdV-mKdV) equation.*

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The KdV-mKdV equation

$$u_t - 6uu_x + 6u^2u_x + u_{xxx} = 0\,, \tag{1}$$

arise in a variety of applications, among which are the solid-states physics, plasma physics, fluid dynamics, and quantum field theory.[1−4] Equation (1) has been payed much attention by many authors[1−11] and some types of the soliton, multisoliton, periodic-wave solutions have been obtained by means of the inverse scattering transform method, Bäcklund transformation, Malfliet's method,[5] Jacobi elliptic function method, and so on. The aim of this letter is to obtain the new soliton-like solutions to Eq. (1). We seek a travelling wave solution of the following form

$$u(x,t) = u(\xi)\,, \qquad \xi = kx - \omega t\,, \tag{2}$$

where k and ω are the wave number and angular frequency, respectively. Without loss of generality, we can define $k > 0$. Substituting Eq. (2) into Eq. (1) and integrating it with respect to ξ and taking the integration constant as zero yields

$$-\omega u - 3ku^2 + 2ku^3 + k^3\frac{\mathrm{d}^2u}{\mathrm{d}\xi^2} = 0\,. \tag{3}$$

Recently, to obtain new soliton-like solutions for the nonlinear partial differential equations, some authors introduced a further extension of the tanh method using a Riccati equation,[12−14] or a generalized Riccati equation.[15] Other extension of the tanh method has been made by using another auxiliary ordinary differential equation instead of Riccati equation or generalized Riccati equation.[16] In this paper, we will use the extended tanh method[16] to investigate the KdV-mKdV equation. The aim of this paper is to present other new soliton-like solutions for Eq. (1). Then we introduce the following auxiliary ordinary equation:

$$\Big(\frac{\mathrm{d}E}{\mathrm{d}\xi}\Big)^2 = aE^2(\xi) + bE^3(\xi) + cE^4(\xi)\,, \tag{4}$$

where a, b, and c are constants. We shall seek the exact solutions of Eq. (1) by using the following solutions of Eq. (4),

$$E(\xi) = \begin{cases} \dfrac{-ab\,\mathrm{sech}^2(\sqrt{a}\,\xi/2)}{b^2 - ac(1-\tanh(\sqrt{a}\,\xi/2))^2}\,, & a > 0 \\[2ex] \dfrac{2a\,\mathrm{sech}(\sqrt{a}\,\xi)}{\sqrt{b^2-ac} - b\,\mathrm{sech}(\sqrt{a}\,\xi)}\,, & a > 0\,, \quad b^2 - ac > 0\,. \end{cases} \tag{5}$$

By balancing the highest order derivative term u_{xxx} with the highest power nonlinear term u^2u_x in Eq. (1) yields $n = 1$. Therefore by the use of the extended tanh method we may choose the solution of Eq. (3) in the form

$$u(\xi) = a_0 + a_1E(\xi)\,, \tag{6}$$

where a_0 and a_1 are constants to be determined later. Substituting Eqs. (6) and (4) into Eq. (3) and collecting coefficients of polynomials of $E(\xi)$ with the aid of *Mathematica*, then equating the coefficients of various powers of $E(\xi)$ to zero, we get a set of algebraic equations for a_0, a_1, k, and ω.

$$\begin{aligned} E^3:&\quad 2ka_1(a_1^2 + k^2c) = 0\,,\\ E^2:&\quad 3ka_1\Big(\frac{bk^2}{2} - 2a_0a_1 - a_1\Big) = 0\,,\\ E^1:&\quad a_1(k^3a + 6ka_0^2 - 6ka_0 - \omega) = 0\,,\\ E^0:&\quad a_0(2ka_0^2 - 3ka_0 - \omega) = 0\,. \end{aligned} \tag{7}$$

Solving them consistently we obtain relations among the parameters a_0, a_1, k, and ω. If any of the parameters are left unspecified, they are regarded as being arbitrary

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constants. Making use of these relations we can find a final expression for $u(\xi)$, which leads to an expression for the travelling wave solutions for Eq. (1). Therefore, equation (5) establishes an algebraic mapping relation between the solution to Eq. (4) and that of Eq. (1). From the solution of Eqs. (7) under condition $a_1 \neq 0$, $k \neq 0$, and $\omega \neq 0$, the coefficients in Eq. (6) are determined

$$a_0 = 0\,, \quad a_1 = \pm\sqrt{-c}k\,, \quad k = \pm\frac{2}{b}\sqrt{-c}\,, \quad \omega = k^3 a\,, \tag{8}$$

which demands that $c < 0$. Substituting Eq. (8) into Eq. (6) yields the following new soliton-like solution to Eq. (1),

$$u(x,t) = \pm\sqrt{-c}\,k\frac{2a\,\mathrm{sech}[\sqrt{a}\,(kx-\omega t)]}{\sqrt{b^2-ac} - b\,\mathrm{sech}[\sqrt{a}\,(kx-\omega t)]}\,, \qquad a > 0\,, \quad c < 0\,. \tag{9}$$

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