Since the beginning of physics, symmetry considerations have provided us with an extremely powerful and useful tool in our effort to understand nature. Gradually they become the backbone of our theoretical formulation of physical laws.

T.D. Lee, Particle Physics and an Introduction to Field Theory.

8
Discrete Symmetries

8.1
Time-Reversal Invariance

According to Section 7.10, the symmetry transformations leaving the Hamiltonian invariant give rise to conservation laws. For discrete symmetries, the unitary operator of the transformation, $\hat{U} = \exp(i\hat{G})$, commutes with the Hamiltonian, (7.153), and the Hermitian observable G is conserved. Time-reversal operation $\mathcal{T}$ is exceptional since it *does not lead to a conservation law*. However, its consequences for quantum systems are indispensable.

In *classical mechanics*, the equations of motion in constant potential fields are *invariant with respect to time reversal*. This statement is to be understood as follows. The solution of classical equations requires initial conditions – we give the values of coordinates $q(0)$ and momenta $p(0)$ at some initial moment taken, for example, as $t = 0$. By solving the equations up to the time $t > 0$, we determine the phase space trajectory $q(t)$, $p(t)$. The system is time-reversal invariant if, for any "forward" solution, we can find the "backward" solution starting at the final point with the *reversed final velocities* and going all the way back to the initial point through all intermediate points within the same time intervals, Figure 8.1.

The classical Hamiltonian $H(q, p)$ of a closed system is $\mathcal{T}$*-invariant* if $H(q, p) = H(q, -p)$. This is the point where the coordinates and momenta which appear on equal footing in the Hamiltonian formalism can be distinguished. Although in classical mechanics, this difference is not significant (recall however the arguments used for the states of finite motion in Section 7.7).

The $\mathcal{T}$-invariance is violated in an *external magnetic field*. Through the *Lorentz force*, the magnetic field discerns two opposite directions of the velocity. The difference between the static electric and magnetic fields is due to their origin: electric fields are generated by charges while magnetic fields are generated by currents which follow the direction of motion of charges. If we include the source of the magnetic field as a part of our system, the extended system will again be

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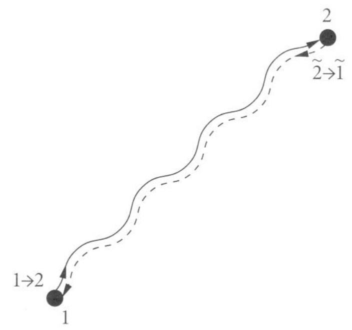

Figure 8.1 Forward and backward trajectories.

$\mathcal{T}$-invariant since the time reversal implies that all motions in the system should be reversed. The reversed current will create the reversed magnetic field and cause the reversed motion in the system, restoring the $\mathcal{T}$-invariance of the entire big complex. The same arguments are applicable to the rotating system where the angular velocity is similar to the magnetic field.

The non-relativistic *quantum mechanics* allows one to formulate the time reversal in a way close to classical mechanics, but the form of the result depends on the representation. Let us consider the Schrödinger equation for the wave function $\Psi(t)$ in some representation (the number of variables in the wave function is arbitrary),

$$i\hbar \frac{\partial \Psi(t)}{\partial t} = \hat{H}\Psi(t) . \tag{8.1}$$

Assuming that the system is closed, $\partial \hat{H}/\partial t = 0$, let us formally reverse time, $t \to -t$:

$$-i\hbar \frac{\partial \Psi(-t)}{\partial t} = \hat{H}\Psi(-t) . \tag{8.2}$$

To bring the equation back to the form of (8.1), we make complex conjugation and come to

$$i\hbar \frac{\partial \Psi^*(-t)}{\partial t} = \hat{H}^*\Psi^*(-t) . \tag{8.3}$$

Thus, the *time-reversed* function

$$\widetilde{\Psi}(t) = \Psi^*(-t) \tag{8.4}$$

satisfies the Schrödinger equation with the *time-reversed* Hamiltonian

$$\widetilde{\hat{H}} = \hat{H}^* . \tag{8.5}$$

The substitution $\Psi \to \widetilde{\Psi}$, $\hat{H} \to \widetilde{\hat{H}}$ describes the time reversal in quantum mechanics. Here, the complex conjugation has meaning similar to the reversal of trajectories in the classical case; for the plane wave $e^{i(\mathbf{k}\cdot\mathbf{r})}$, this is equivalent to $\mathbf{k} \to -\mathbf{k}$. Matrix elements of the Hamiltonian,

$$H_{12} = \langle \Psi_1(t)|\hat{H}|\Psi_2(t)\rangle \equiv \int d\tau \Psi_1^*(t)\hat{H}\Psi_2(t) , \tag{8.6}$$

where the Ψ-functions and the operator $\hat{H}$ are taken in a chosen representation with the volume element $d\tau$, are transformed into

$$\widetilde{H}_{12} = \int d\tau \Psi_1(-t)\hat{H}^*\Psi_2^*(-t) = \int d\tau(\hat{H}\Psi_2(-t))^*\Psi_1(-t) , \tag{8.7}$$

or, using the Hermiticity (6.62) of the Hamiltonian $\hat{H}$,

$$\widetilde{H}_{12} = \int d\tau \Psi_2^*(-t)\hat{H}\Psi_1(-t) . \tag{8.8}$$

We see that in the transformed matrix element $\widetilde{H}_{12}$, not only is the arrow of time reversed, but also the roles of the "initial", Ψ_2, and the "final", Ψ_1, states are interchanged compared to the original matrix element H_{12}, in keeping with the idea of interchanging the start and finish of the process.

The transformed Hamiltonian (8.5) coincides with the original one if it is real. Then, we say that the system is $\mathcal{T}$-invariant and both functions, $\Psi(t)$ and $\widetilde{\Psi}(t)$, (8.4), are the solutions of *the same* Schrödinger equation. For the stationary state,

$$\Psi(t) = e^{-(i/\hbar)Et}\psi , \quad \widetilde{\Psi}(t) = e^{(i/\hbar)E(-t)}\psi^* , \tag{8.9}$$

ψ and ψ^* satisfy the stationary Schrödinger equation with the same real energy E. If this eigenvalue is non-degenerate, there is only one independent solution with this value of E, and ψ and ψ^* coincide up to an irrelevant phase. If the eigenvalue is degenerate, then ψ and ψ^* can be independent and we can take their combinations, $\mathrm{Re}(\psi)$ and $\mathrm{Im}(\psi)$, as new functions with the same energy. Thus, in the case of $\mathcal{T}$-invariance, the stationary basis wave functions can be chosen to be *real*.

8.2
Time-Reversal Transformation of Operators

Time reversal is not a usual unitary transformation since it includes the complex conjugation $\hat{\mathcal{K}}$, (8.4). Its action on a superposition of states is not linear as it was defined in (6.46). It also changes the coefficients of the superposition to their complex conjugate (such transformations are sometimes called *antilinear*). We can define the time-reversal operation as

$$\hat{\mathcal{T}} = \hat{U}_T\hat{\mathcal{K}}\hat{O}_t , \tag{8.10}$$

where $\hat{O}_t$ changes $t \to -t$ in the explicit time dependence, $\hat{\mathcal{K}}$ is the complex conjugation, and $\hat{U}_T$ is an additional unitary operator that is needed to ensure the correct behavior of physical observables under time reversal.

The operator $\hat{U}_T$ depends on the representation. The similarity transformation (6.101) defines the time-reversed operator,

$$\hat{\tilde{A}} = \hat{\mathcal{T}} \hat{A} \hat{\mathcal{T}}^{-1} . \tag{8.11}$$

On the other hand, in many cases, we know the result of time reversal from our classical experience (correspondence principle). The position operator does not change, while the momentum has to change its sign,

$$\hat{\tilde{\mathbf{r}}} = \hat{\mathbf{r}} , \quad \hat{\tilde{\mathbf{p}}} = -\hat{\mathbf{p}} . \tag{8.12}$$

Using the coordinate representation (7.17), we see that the desired result is achieved without adding a special unitary operator in (8.10); it is sufficient to have the complex conjugation:

$$\hat{\tilde{\mathbf{p}}} = \hat{\mathcal{K}} \hat{\mathbf{p}} \hat{\mathcal{K}}^{-1} = \hat{\mathbf{p}}^* = (-i\hbar\nabla)^* = i\hbar\nabla = -\hat{\mathbf{p}} . \tag{8.13}$$

The orbital momentum also changes sign,

$$\hat{\tilde{\boldsymbol{\ell}}} = \frac{1}{\hbar} \hat{\mathcal{K}} [\hat{\mathbf{r}} \times \hat{\mathbf{p}}] \hat{\mathcal{K}}^{-1} = -\hat{\boldsymbol{\ell}} . \tag{8.14}$$

However, an additional unitary operation is needed when non-classical degrees of freedom are involved. The spin components, being a part of the total angular momentum, have to change their sign under time reversal precisely as the components of the orbital momentum do. This requires a definition of the time reversal $\hat{\mathcal{T}}$ with additional unitary operators acting in spin space, Section 20.5.

Summarizing these two subsections, we can say that the state vector $\tilde{\Psi}$ obtained from the initial vector Ψ by the $\hat{\mathcal{T}}$-operation describes the *final* state with all velocity-type characteristics reversed: in the state $\tilde{\Psi}$, all linear momenta and angular momenta become $\tilde{\mathbf{p}} = -\mathbf{p}$ and $\tilde{\mathbf{J}} = -\mathbf{J}$, respectively, if in the state Ψ, they were $\mathbf{p}$ and $\mathbf{J}$. If the process $i \to f$ develops according to the Schrödinger equation as

$$|\Psi_f\rangle = e^{-(i/\hbar)\hat{H}t} |\Psi_i\rangle , \tag{8.15}$$

the time-reversed process is $\tilde{f} \to \tilde{i}$,

$$|\tilde{\Psi}_i\rangle = e^{-(i/\hbar)\hat{\tilde{H}}t} |\tilde{\Psi}_f\rangle . \tag{8.16}$$

The time-reversal invariance (*reversibility* of quantum mechanics) means that $\hat{\tilde{H}} = \hat{H}$, so that for each direct process (8.15), there exists a reversed process (8.16) evolving according to the same laws. Currently, we know that $\mathcal{T}$-invariance is violated in nature. However, this violation was only observed as a small effect in specific processes of decay of neutral K- and B-mesons [15].

8.3 Inversion and Parity

Another discrete symmetry of the Hamiltonian is important for the search and classification of stationary states. The operation $\hat{\mathcal{P}}$ of *spatial inversion* changes the sign of spatial coordinates so that the localized state $|\mathbf{r}\rangle$ of a particle transforms as

$$|\mathbf{r}\rangle \Rightarrow \hat{\mathcal{P}}|\mathbf{r}\rangle = |-\mathbf{r}\rangle . \tag{8.17}$$

In the coordinate representation, for an arbitrary state $|\psi\rangle$, we have

$$\begin{aligned}\langle\mathbf{r}|\hat{\mathcal{P}}|\psi\rangle &= \int d^3x\,\langle\mathbf{r}|\hat{\mathcal{P}}|\mathbf{x}\rangle\langle\mathbf{x}|\psi\rangle = \int d^3x\,\langle\mathbf{r}|-\mathbf{x}\rangle\langle\mathbf{x}|\psi\rangle \\ &= \int d^3x\,\delta(\mathbf{r}+\mathbf{x})\psi(\mathbf{x}) = \psi(-\mathbf{r}) .\end{aligned} \tag{8.18}$$

Therefore, the action of inversion onto a coordinate wave function is simply

$$\hat{\mathcal{P}}\psi(\mathbf{r}) = \psi(-\mathbf{r}) . \tag{8.19}$$

When applied to the plane wave $\psi_{\mathbf{p}}(\mathbf{r}) = \exp[(i/\hbar)(\mathbf{p}\cdot\mathbf{r})]$, this is equivalent to the reversal of the direction of motion, $\mathbf{p} \to -\mathbf{p}$, as expected for spatial inversion.

It is easy to check that the inversion operator is *linear*, in contrast to time reversal. The operator $\hat{\mathcal{P}}$ is Hermitian and satisfies an obvious geometric relation

$$\hat{\mathcal{P}}^2 = 1 , \tag{8.20}$$

which shows that $\hat{\mathcal{P}} = \hat{\mathcal{P}}^{-1}$, and therefore the Hermitian operator $\hat{\mathcal{P}}$ is also unitary. This operator only has two different eigenvalues $\Pi = \pm 1$. These eigenvalues define the quantum number of *parity* that distinguishes between *even* functions,

$$\hat{\mathcal{P}}\psi(\mathbf{r}) = \psi(-\mathbf{r}) = \psi(\mathbf{r}) , \quad \Pi = +1 , \tag{8.21}$$

and *odd* functions,

$$\hat{\mathcal{P}}\psi(\mathbf{r}) = \psi(-\mathbf{r}) = -\psi(\mathbf{r}) , \quad \Pi = -1 . \tag{8.22}$$

Any function can be uniquely presented as a superposition of the parity eigenfunctions,

$$\psi(\mathbf{r}) = \psi_{\text{even}}(\mathbf{r}) + \psi_{\text{odd}}(\mathbf{r}) = \frac{\psi(\mathbf{r}) + \psi(-\mathbf{r})}{2} + \frac{\psi(\mathbf{r}) - \psi(-\mathbf{r})}{2} . \tag{8.23}$$

Problem 8.1

Show that parity of a wave function (with respect to its corresponding argument) is the same in the coordinate and momentum representations.

8.4 Scalars and Pseudoscalars, Vectors and Pseudovectors

The inversion transformation of the operators is, in accordance with the general rule (6.101),

$$\hat{Q}' = \hat{\mathcal{P}}\hat{Q}\hat{\mathcal{P}}^{-1} . \tag{8.24}$$

An invariant quantity, $\hat{Q}' = \hat{Q}$, commutes with the inversion operator. This gives a criterion for classifying observables.

It is useful to combine this classification with the one based on the rotational properties (later, they will be studied in much more detail). The quantities invariant under rotations are called *scalars*. The mathematical definition of a scalar $\hat{S}$ follows from the meaning of the angular momentum $\hat{\mathbf{J}}$ as a generator of rotations,

$$[\hat{\mathbf{J}}, \hat{S}] = 0 . \tag{8.25}$$

However, a scalar may not commute with the inversion. We can divide scalar observables that obey (8.25) into genuine scalars and *pseudoscalars* that change sign under inversion. To give a physical example, let us first consider *vectors*.

We have already discussed that the general definition of a vector $\hat{\mathbf{V}}$ can be based on its behavior under infinitesimal rotations, Section 6.10. Now, we can distinguish two classes of vector observables: genuine (*polar*) vectors $\mathbf{V}$, whose Cartesian components $\hat{V}_i$ change sign under spatial inversion,

$$\hat{\mathcal{P}}\hat{V}_i\hat{\mathcal{P}}^{-1} \equiv \hat{\mathcal{P}}\hat{V}_i\hat{\mathcal{P}} = -\hat{V}_i , \tag{8.26}$$

and *pseudovectors*, or *axial vectors*, $\mathbf{A}$, whose components $\hat{A}_i$ do not change under inversion,

$$\hat{\mathcal{P}}\hat{A}_i\hat{\mathcal{P}} = \hat{A}_i . \tag{8.27}$$

The operators of coordinates, $\mathbf{r}$, and momentum, $\mathbf{p}$, are genuine vectors. However, the components of the orbital momentum operator, $[\mathbf{r} \times \mathbf{p}]$, built as a *cross product of two polar vectors* do not change under inversion and therefore the orbital momentum is an axial vector. The formal proof states, for example, for the position operator, that by using the Hermiticity of $\hat{\mathcal{P}}$, an arbitrary matrix element of the transformed operator $\hat{\mathcal{P}}\hat{\mathbf{r}}\hat{\mathcal{P}}$ is given by

$$\begin{aligned}\int d^3r\,\psi_1^*(\mathbf{r})\hat{\mathcal{P}}\hat{\mathbf{r}}\hat{\mathcal{P}}\psi_2(\mathbf{r}) &= \int d^3r\,\psi_1^*(-\mathbf{r})\hat{\mathbf{r}}\psi_2(-\mathbf{r}) \\ &= \int d^3r\,\psi_1^*(\mathbf{r})(-\hat{\mathbf{r}})\psi_2(\mathbf{r}) ,\end{aligned} \tag{8.28}$$

which means that $\hat{\mathbf{r}}$ is a genuine vector,

$$\hat{\mathcal{P}}\hat{\mathbf{r}}\hat{\mathcal{P}} = -\hat{\mathbf{r}} . \tag{8.29}$$

Similarly,

$$\hat{\mathcal{P}}\hat{\mathbf{p}}\hat{\mathcal{P}} = -\hat{\mathbf{p}}\,, \tag{8.30}$$

in agreement with the classical definition $\mathbf{p} = m\dot{\mathbf{r}}$ which is also valid for quantum equations of motion (7.89 and 7.90). In addition to the difference between electric and magnetic fields with respect to time reversal, they behave differently under inversion as well: the electric field vector $\mathcal{E}$ and electric current $\mathbf{j}$ are polar vectors, while the magnetic field vector $\mathcal{B}$ is axial. Then, the Lorentz force $\sim [\mathbf{v} \times \mathcal{B}]$, a cross-product of a polar and an axial vector, is again a polar (and time-even) vector.

Note that we use the picture of an *active* transformation that acts on the object while the coordinate unit vectors $\mathbf{e}^{(i)}$ are kept intact. The polar vector $\mathbf{V}$ is transformed to $-\mathbf{V}$; then, its coordinates V_i with respect to the same old set $\mathbf{e}^{(i)}$ change sign. The coordinates of axial vectors remain the same along with the object itself. In the *passive* formulation, we invert the coordinate frame, $\mathbf{e}^{(i)} \Rightarrow -\mathbf{e}^{(i)}$. This converts the right-handed triplet of coordinate axes defined according to $[\mathbf{e}^{(x)} \times \mathbf{e}^{(y)}] = \mathbf{e}^{(z)}$ into the left-handed one and therefore changes the sign of all quantities whose definition contains the reference to the *handedness* or the sense of rotation. The polar vectors are not touched while their coordinates V_i again change sign. The axial vectors, such as the orbital momentum, change their direction, while their coordinates, once again, do not change.

Now, we can construct a pseudoscalar as a scalar product of a polar and an axial vector. However, we cannot use the examples of the previous paragraph for this purpose. Indeed,

$$(\mathbf{r} \cdot \boldsymbol{\ell}) = (\mathbf{p} \cdot \boldsymbol{\ell}) = 0\,. \tag{8.31}$$

This property has a simple meaning: if $\boldsymbol{\ell}$ is the generator of rotation, it moves the particle perpendicularly to the radius-vector. A pseudoscalar called *helicity* can be constructed using another part of the angular momentum, namely, the spin momentum $\mathbf{s}$ which is, as any angular momentum, also an axial vector. The helicity h is the projection of the spin onto the direction of motion,

$$h = \mathbf{s} \cdot \frac{\mathbf{p}}{p}\,. \tag{8.32}$$

With respect to rotation, helicity is a scalar; though it does change sign under inversion,

$$\hat{\mathcal{P}}\hat{h}\hat{\mathcal{P}} = -\hat{h}\,. \tag{8.33}$$

8.5
Parity Conservation

If the potential in the Schrödinger equation is invariant under inversion,

$$U(\mathbf{r}) = U(-\mathbf{r})\,, \tag{8.34}$$

the Hamiltonian as a whole, $\hat{H} = \hat{K} + \hat{U}$, commutes with the inversion operator. Therefore, we can ascribe a quantum number of parity to the stationary states.

In other words, if $\psi(\mathbf{r})$ describes a stationary state of the $\hat{\mathcal{P}}$-invariant Hamiltonian with energy E, the reflected function $\psi(-\mathbf{r})$ also corresponds to a stationary state with the same energy. If this energy is not degenerate, the two functions can differ by a constant factor only, $\psi(-\mathbf{r}) = c\psi(\mathbf{r})$. By repeating the inversion, we obtain $\psi(\mathbf{r}) = c\psi(-\mathbf{r}) = c^2\psi(\mathbf{r})$, which means $c^2 = 1$, $c = \pm 1$. This is the same statement as obtained earlier in the operator language: with an even potential (8.34), the non-degenerate stationary solutions have certain parity. If the energy eigenvalue is degenerate, $\psi(\mathbf{r})$ and $\psi(-\mathbf{r})$ can be linearly independent. However then, any linear combination of them also describes a stationary state with the same energy and we can always construct even and odd superposition, $\psi(\mathbf{r}) \pm \psi(-\mathbf{r})$.

Thus, if (8.34), or, more generally,

$$[\hat{\mathcal{P}}, \hat{H}] = 0 \,, \tag{8.35}$$

is fulfilled, the stationary solutions of the Schrödinger equation can be classified by parity. In our simple examples of Chapter 2, the bound states in a box or a finite well (where we have only one-dimensional inversion) can acquire certain parity if we set the coordinate axis in such a way that the origin coincide with the middle point. In the continuum problems (reflection and transmission), the symmetry was violated by the boundary condition when we assumed that the source of the wave was located on one side of the observed region. The symmetry is restored by the existence of the equivalent, mirror-reflected solution with the same energy when the source is put on the opposite side.

Problem 8.2

Establish the correspondence between the stationary states of one-dimensional motion in a symmetric potential, $U_1(x) = U_1(-x)$, and in the potential $U_2(x)$ that coincides with $U_1(x)$ at $x > 0$ and cut off from the left half of the plane by an impenetrable wall at $x = 0$.

Parity of a complex system is a *multiplicative* quantum number being a product of parities of subsystems or constituents. In application to elementary particles, one can speak about their *intrinsic parity*: the internal wave function of a particle is also transformed in a certain way under spatial reflection. Intrinsic parity of the proton and the neutron is the same (and then it does not matter if we assume this parity even or odd because in all nuclear processes, where parity is conserved, the total number of *nucleons*, neutrons and protons, – the so-called *baryon charge* – is conserved as well). But relativistic theory shows that this parity is opposite to that of the antineutron and antiproton. Also, the intrinsic parity of the electron and the positron, quark and antiquark is opposite. In the case of *mesons*, for example pions or kaons, it makes sense to speak of absolute intrinsic parity since these particles can be created and absorbed one at a time changing therefore total parity of the state in a certain way. The wave function of these mesons is scalar under rotations, but pseudoscalar under inversion; this is determined by the internal structure of the meson built of the quark and antiquark which have opposite intrinsic parity.

In order to get information on intrinsic parity of particles, one needs to observe the processes of their creation, annihilation and mutual transformation and compare parity of the initial and final state, taking into account the intrinsic wave functions of the particles as well as the wave functions of their relative motion. The experiment shows that the majority of interactions of elementary particles are invariant under spatial inversion. As far as we know, only the Hamiltonian of the *weak interactions* does not conserve parity. The interactions of this type occur at very small distances between the particles, $\sim 10^{-16}$ cm; they are responsible for the slowest nuclear processes, such as *beta-decay* of the neutron into the proton, electron and electron antineutrino, or beta-decay of complex nuclei. The lifetime of the free neutron, $\sim 10^3$ s, is large compared to the typical time of nuclear processes of $10^{-(21\div 23)}$ s that can be estimated by the time of flight of a particle with velocity $\sim (0.1 \div 1)c$ through the nuclear radius $\sim 10^{-(12\div 13)}$ cm.

Roughly speaking, if the parity conservation in a process holds, then in a mirror-reflected laboratory, the process occurs in complete analogy and leads to the mirror-reflected result. It is not the case in beta-decay. In the famous experiment by *C.S. Wu et al.* [16], Figure 8.2, the beta-decay of polarized (having fixed orientation of angular momentum $\mathbf{J}$) nuclei of ^{60}Co was studied. The distribution of emitted electrons depends on the angle ϑ between the electron momentum $\mathbf{p}$, or velocity $\mathbf{v}$, and polarization direction $\mathbf{J}$. The number of electrons emitted at the angle ϑ turned out to be proportional to

$$N(\vartheta) \propto 1 + \alpha \cos\vartheta \,, \tag{8.36}$$

with the asymmetry coefficient $\alpha \approx -v/c$ (the relativistic electrons are emitted mostly opposite to the nuclear polarization). In this specific case, the result is due to the property of the so-called left current responsible for weak interactions including the beta-decay; the antineutrino are practically completely longitudinally polarized along the motion (see Vol. 2 Section 14.6). Then, the recoil electron with the same spin polarization (total spin has to be conserved) is forced by the momentum conservation to move in the opposite direction. Since $\cos\vartheta$ is determined by the scalar product of the polar vector $\mathbf{p}$ and axial vector $\mathbf{J}$, this is a pseudoscalar quantity. Therefore, in the mirror-reflected laboratory, this quantity would change sign and we would obtain the different angular distribution, $\propto 1 - \alpha\cos\vartheta$. The presence of the scalar *and* the pseudoscalar in the experimental result (8.36) makes the results in two laboratories non-equivalent and corresponds to *parity non-conservation* in weak interactions.

For parity-conserving systems, the operators with certain behavior under spatial inversion reveal specific *selection rules*: their matrix elements with a given initial state can only connect to a certain class of final states. The operators changing sign under inversion change parity of the state so that parity of the final state should be opposite to that of the initial state. Take, for example, transitions induced by the *dipole moment* of the charge distribution, recall Problem 7.9. A dipole transition,

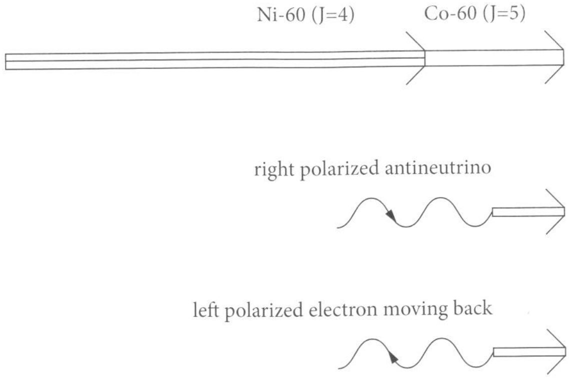

Figure 8.2 Scheme of the experiment by Wu *et al.*

described by the matrix element $\langle f|\hat{\mathbf{d}}|i\rangle$, is only possible between the states of opposite parity. If parity is conserved, the expectation value of $\mathbf{d}$ in any state of certain parity is forbidden. The presence of the non-vanishing dipole moment in a *stationary state* reveals parity non-conservation. The existence of polar molecules (like water or NH_3) shows that either the state is not stationary, although maybe with a long lifetime, or the orientation of the molecule is fixed by external fields. In a free stationary state, the molecule has a certain angular momentum and the rotation averages out the intrinsic dipole.

Until now, the long-going experimental search for the electric dipole moment (EDM) of elementary particles, atoms and nuclei did not provide certain results. It did, however, push the upper boundary if the EDM lower and lower. Meanwhile, parity non-conservation in weak interactions is a well established fact. The problem with the dipole moment is aggravated by the vector character of this operator. Its expectation value in a stationary state has to be directed along the only conserved vector characterizing the system, namely, that of its total angular momentum. However, the angular momentum changes sign under time reversal while the dipole moment does not. Therefore, the discovery of the dipole moment would also contradict the $\mathcal{T}$-invariance [17]. The forces which are *simultaneously* $\mathcal{P}$- and $\mathcal{T}$-violating are much weaker than "normal" weak interactions.

Another polar vector, the so-called *anapole moment*, proportional to the cross product $[\mathbf{r} \times \mathbf{s}]$ where $\mathbf{s}$ is a spin operator, changes parity of the state, though its existence does not contradict $\mathcal{T}$-invariance. The parity violating anapole moment was measured in cesium atoms [18]. On the other hand, magnetic dipole moment $\boldsymbol{\mu}$ is an axial vector proportional to the orbital or spin moment of a particle, (1.74). The nonzero expectation of the magnetic moment agrees with both $\mathcal{P}$- and $\mathcal{T}$-invariance.