

11. 深度非弹散射





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部分子模型

*) 强3内部结构 升级版的Rutherfuld 乾射实验(eP乾射) (低能:弹性散射——虚充3和质3整体相互作用 探测质3的整体性质(例如质3半径) (高能:深度非弹散射——虚光3和质计夸克的弹频射 探测质3中夸克的动量分布



x) eP 弹性散射截面 $P_{1} \xrightarrow{e^{-}} 3 \xrightarrow{\beta_{3}} 2^{-}$ り Rutherfold 乾年 低能, 无核子反冲 (Mp=10), 非相对论电子 $\left(\frac{d\sigma}{dQ}\right)_{R} = \frac{d^{2}}{16E_{k}^{2}} \sin \frac{4\theta}{2} = \frac{p^{2}}{2Me} \ll M_{e}$ (仅有电荷相互作用)

2) Mott 散射 无质子反冲,但e-是相对论性 (me Exermp) (e-Helicity conservation)

 $\left(\frac{d\Omega}{d\Omega}\right)_{\text{Mott}} = \frac{\sqrt{2}}{4E^2 \sin^4\theta} \cos^2\frac{\theta}{2}$ 初奏电子自这和表电。 EK->E 自旋波函数重叠 (,E>>me) $d_{V_{2}}^{V_{2}}(\theta)$ (仅有电荷相互作用)

3)
$$f_{0} = f_{0} = \int P(\vec{r}) e^{-i\vec{r}\cdot\vec{r}} d^{3}\vec{r}$$

 $\vec{r} = \vec{r} \cdot \vec{r}_{3}$
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相对论性电子+质子反冲 4,



$$P_{1} = (E_{1}, 0, 0, E_{1})$$

$$P_{2} = (M, 0, 0, 0)$$

$$P_{3} = (E_{3}, 0, E_{3} \sin \theta, E_{3} \sin \theta)$$

$$P_{4} = (E_{4}, \vec{P}_{4})$$

$$\left| m \right|^{2} \propto \frac{5^{2} + u^{2}}{t^{2}}$$

$$= \frac{8e^{4}}{(P_{1} - P_{3})^{4}} \left[(P_{1} \cdot P_{2})(P_{3} \cdot P_{4}) + (P_{1} \cdot P_{4})(P_{2} \cdot P_{3}) - M^{2}(P_{1} \cdot P_{3}) \right]$$

$$(E_1 - E_3)^2 = -4E_1E_3 \sin^2 \frac{1}{2} < 0 \qquad E_1 - E_3 = -\frac{g^2}{2M} > 0 \qquad (E_1 > E_3)$$

代代建

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{recoil}} = \frac{\chi^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \xrightarrow{E_3}_{E_1} \left(\cos^2 \frac{\theta}{2} - \frac{\theta^2}{2M^2} \sin^2 \frac{\theta}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{moth}} = \frac{\chi^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \cos^2 \frac{\theta}{2} \xrightarrow{\text{Proton}} \xrightarrow{\text{ZZZZE}}_{Vecoil} \xrightarrow{\text{ZZZZE}}_{Vecoil} \xrightarrow{\text{Sin}}_{Vecoil} \xrightarrow{\text{Sin}}_{Vecoil}$$

注意: 弹性敬肆裁局仅依赖于单-变量0

$$\frac{E_5}{E_1} = \frac{M}{M + E_1(1 - \cos\theta)}, \quad g^2 = -\frac{2ME_1^2(1 - \cos\theta)}{M + E_1(1 - \cos\theta)}$$

5)形状闭子 一般而善,我们需要两个结构函数束描述有限大小质子 地荷分布 (GE) 和 石油石市 (GM) 量子场论: $e - e - \gamma$: $A_{\mu}j^{\mu} = A_{\mu}(-e) \overline{U}(k') \mathcal{Y}^{\mu}U(k)$ P - P - Y: $A_{\mu} J^{\mu} = A_{\mu} (+e) \overline{\mathcal{U}}(P') \underbrace{1}_{\cdots} \underbrace{1}_{\mathcal{U}(P)}$ 因为JM是 Loventy 矢量, 所叫 JM中仅有两项 $\begin{cases} -\frac{1}{2} = \begin{cases} F_1(g^2) \gamma^{\mu} + \frac{k}{2m} F_2(g^2) i \sigma^{\mu\nu} g_{\nu} \\ g_{\nu} = \rho - \rho' \end{cases}$ 其中F、(5)和F、(5)是形状团8, K-反常磁频巨

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将文述 P-P-r 有效相对作用代入到载射 探佛命中终

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}\right) \frac{E'}{E} \left\{ \left(\pi^2 - \frac{k^2 g^2}{4M^2} F_2^2\right) \cos^2 \frac{\theta}{2} - \frac{g^2}{2M^2} \left(F_1 + k F_2\right)^2 \sin^2 \frac{\theta}{2} \right\}$$
Rosenbluth
Formula = = $\left(\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}}\right) \frac{E'}{E} \left\{ \frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right\}$

 $\frac{1}{2M} G_E = F_1 + \frac{k g^2}{4M^2} F_2, \quad T = \frac{-g^2}{4M^2} > 0$

 $G_M = F_1 + k F_2$

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rosenbluth}} &= \left(\frac{\alpha^{2}}{4E^{2}\sin^{2}\theta_{Z}}\right)\frac{E'}{E}\cos^{2}\theta_{Z}\left(\frac{G_{E}^{2}+\tau G_{M}^{2}}{l+\tau}+2\tau G_{M}^{2}\tan^{2}\theta_{Z}\right) \\ &= \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}\left(\frac{G_{E}^{2}+\tau G_{M}^{2}}{l+\tau}+2\tau G_{M}^{2}+2\tau G_{M}^{2}\tan^{2}\theta_{Z}\right) \\ At \log \theta_{z}^{2}: \tau = -\frac{\theta^{2}}{4M^{2}}\approx 0 \qquad At high \theta_{z}^{2}: \tau >> 1 \\ \frac{d\sigma}{d\Omega}\frac{d\sigma}{d\Omega}\sum_{\mu \text{ott}} \approx G_{E}^{2}(\theta^{2}) \qquad \frac{d\sigma}{d\Omega}\sum_{\mu \text{ott}} \approx \tau\left(\frac{l+2\tan^{2}\theta}{2}\right)G_{M}^{2}(\theta^{2}) \end{aligned}$$

e p Elastic Scattering at Very High q^2

\starAt high q^2 the Rosenbluth expression for elastic scattering becomes

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{elastic} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\frac{q^2}{2M^2} G_M^2 \sin^2 \frac{\theta}{2}\right) \qquad \tau = -\frac{q^2}{4M^2} \gg 1$$

•From e⁻ p elastic scattering, the proton magnetic form factor is





基于量纲分析,我们只能 比较具有相同量纲的物理 量。

深度非弹散射过程(DIS) 的结构函数具有标度无关 性,意味着结构函数都是 无量纲的物理量的函数。

这些无量纲物理量要依赖 于具有量纲的物理量,因 此无量纲物理量是有量纲 物理量的比值或组合。



Kinematics of Inelastic Scattering



For inelastic scattering the mass of the final state hadronic system is no longer the proton mass, *M*The final state hadronic system must contain at least one baryon which implies the final state invariant mass *M_X* > *M*

$$M_X^2 = p_4^2 = (E_4^2 - |\vec{p}_4|^2)$$

***** For inelastic scattering introduce four new kinematic variables:

$$x, y, v, Q^2$$

★ Define:

$$x \equiv \frac{Q^2}{2p_2.q}$$
Bjorken x (Lorentz Invariant)
where $Q^2 \equiv -q^2$ $Q^2 > 0$
• Here $M_X^2 = p_4^2 = (q+p_2)^2 = -Q^2 + 2p_2.q + M^2$
 $\Rightarrow Q^2 = 2p_2.q + M^2 - M_X^2 \Rightarrow Q^2 \le 2p_2.q$
Note: in many text
books W is often
used in place of M_X
hence $0 < x < 1$ inelastic $x = 1$ elastic $M_X = M$



$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$$
 (Lorentz Invariant)

•In the Lab. Frame:

$$p_1 = (E_1, 0, 0, E_1) \quad p_2 = (M, 0, 0, 0)$$

$$q = (E_1 - E_3, \vec{p}_1 - \vec{p}_3)$$

$$\Rightarrow \quad y = \frac{M(E_1 - E_3)}{ME_1} = 1 - \frac{E_3}{E_1}$$



So *y* is the fractional energy loss of the incoming particle 0 < y < 1

•In the C.o.M. Frame (neglecting the electron and proton masses):

$$p_1 = (E, 0, 0, E); \quad p_2 = (E, 0, 0, -E); \quad p_3 = (E, E \sin \theta^*, 0, E \cos \theta^*)$$

$$\rightarrow \quad y = \frac{1}{2}(1 - \cos \theta^*) \quad \text{for} \quad E \gg M$$

$$\bigstar \text{Finally Define:} \quad v \equiv \frac{p_2 \cdot q}{M} \text{ (Lorentz Invariant)}$$

•In the Lab. Frame: $v = E_1 - E_3$ v is the energy lost by the incoming particle

Relationships between Kinematic Variables

•Can rewrite the new kinematic variables in terms of the squared centre-of-mass energy, s, for the electron-proton collision $s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 = 2p_1 \cdot p_2 + M^2 + p_2^2$ Neglect mass $2p_1 \cdot p_2 = s - M^2$ •For a fixed centre-of-mass energy, it can then be shown that the four kinematic variables $Q^2 \equiv -q^2$ $x \equiv \frac{Q^2}{2p_2 \cdot q}$ $y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$ $v \equiv \frac{p_2 \cdot q}{M}$

are not independent.

•i.e. the scaling variables *x* and *y* can be expressed as

$$x = \frac{Q^2}{2M\nu} \qquad \qquad y = \frac{2M}{s - M^2}\nu \qquad \qquad \text{Note the simple} \\ \text{relationship between} \\ y \text{ and } \nu \\ y \text$$

•For a fixed centre of mass energy, the interaction kinematics are completely defined by any <u>two</u> of the above kinematic variables (except y and v)

•For elastic scattering (x = 1) there is only one independent variable. As we saw previously if you measure electron scattering angle know everything else.

Inelastic Scattering

Example: Scattering of 4.879 GeV electrons from protons at rest

- Place detector at 10° to beam and measure the energies of scattered e⁻
- Kinematics fully determined from the electron energy and angle !
- e.g. for <u>this energy and angle</u>: the invariant mass of the final state hadronic system

$$W^2 = M_X^2 = 10.06 - 2.03E_3$$



Inelastic Cross Sections

•Repeat experiments at different angles/beam energies and determine q^2 dependence of elastic and inelastic cross-sections



- •Elastic scattering falls of rapidly with q^2 due to the proton not being point-like (i.e. form factors)
- •Inelastic scattering cross sections only weakly dependent on q^2
- •Deep Inelastic scattering cross sections almost independent of q^2 !
 - i.e. "Form factor" \rightarrow 1

Scattering from point-like objects within the proton !

M.Breidenbach et al., Phys. Rev. Lett. 23 (1969) 935 *) 林建不变性(伸缩不变性)

基于"量钢分析":我们仅能在相同量钢的物理量词做比较 深度非弹散射过程(DIS)的结构函数是无量物量(它们没是无量物数值。但它们保赖于有量物的变量,因此结构函数反称 依赖于这些有量纲变量的无量纲比值或细合。 通常选取了和V来到西DIS过程,另外个量钢物理量M [GeV]² [GeV] [Gev] 一 九 无量纲 如果一般。是唯的无量纲组合、那么也是结构还数唯一可 像颜的量 ——> 严格的标度无利性

问题:如何证明 型是描述 DIS过程的唯相关的话题量? 由了,V和M还那构造其他无量纲量能, 显然如果结构函数依赖于一或 前 那哈门北程 林度无关的 (杂本并随空和以变化) => 林度无关性假设: 当了和小都逐渐变大时,它们的比值 32 保持有限(比约肯权限)

Elastic →Inelastic Scattering

★<u>Recall:</u> Elastic scattering

•Only one independent variable. In Lab. frame express differential cross section in terms of the electron scattering angle (Rosenbluth formula)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \qquad \tau = \frac{Q^2}{4M^2}$$

Note: here the energy of the scattered electron is determined by the angle.

-In terms of the Lorentz invariant kinematic variables can express this differential cross section in terms of Q^2

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \left(1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

which can be written as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4} \left[f_2(Q^2) \left(1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

★ Inelastic scattering

•For Deep Inelastic Scattering have two independent variables. Therefore need a double differential cross section

Deep Inelastic Scattering

★ It can be shown that the most general Lorentz Invariant expression for e⁻p → e⁻X inelastic scattering (via a single exchanged photon is):

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2} \right) \frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2) \right]$$
(1)

INELASTIC SCATTERING

c.f.
$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2} \right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2) \right]$$
 ELASTIC SCATTERING

We will soon see how this connects to the quark model of the proton

• NOTE: The form factors have been replaced by the STRUCTURE FUNCTIONS $F_1(x,Q^2)$ and $F_2(x,Q^2)$ 结构函数

which are a function of x and Q^2 : can not be interpreted as the Fourier transforms of the charge and magnetic moment distributions. We shall soon see that they describe the momentum distribution of the quarks within the proton

★ In the limit of high energy (or more correctly $Q^2 \gg M^2 y^2$) eqn. (1) becomes:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2) \right]$$
(2)

• In the Lab. frame it is convenient to express the cross section in terms of the angle, θ , and energy, E_3 , of the scattered electron – experimentally well measured.





$$Q^2 = 4E_1E_3\sin^2\theta/2; \quad x = \frac{Q^2}{2M(E_1 - E_3)}; \quad y = 1 - \frac{E_3}{E_1}; \quad v = E_1 - E_3$$

• In the Lab. frame, Equation (2) becomes:

$$\frac{d^2\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \left[\frac{1}{\nu} F_2(x, Q^2) \cos^2 \frac{\theta}{2} + \frac{2}{M} F_1(x, Q^2) \sin^2 \frac{\theta}{2} \right]$$
(3)
Electromagnetic Structure Function Pure Magnetic Structure Function

Measuring the Structure Functions

★ To determine $F_1(x,Q^2)$ and $F_2(x,Q^2)$ for a given x and Q^2 need measurements of the differential cross section at several different scattering angles and incoming electron beam energies

Example: electron-proton scattering F_2 vs. Q^2 at fixed x



• Experimentally it is observed that both F_1 and F_2 are (almost) independent of Q^2

Bjorken Scaling and the Callan-Gross Relation

★The near (see later) independence of the structure functions on Q² is known as Bjorken Scaling, i.e.

$$F_1(x,Q^2) \rightarrow F_1(x)$$
 $F_2(x,Q^2) \rightarrow F_2(x)$

• It is strongly suggestive of scattering from point-like constituents within the proton

★It is also observed that $F_1(x)$ and $F_2(x)$ are not independent but satisfy the Callan-Gross relation

$$F_2(x) = 2xF_1(x)$$

• As we shall soon see this is exactly what is expected for scattering from spin-half quarks.

<u>Note:</u> if quarks were spin zero particles we would expect the purely magnetic structure function to be zero, i.e. $F_1(x) = 0$



The Quark-Parton Model

- Before quarks and gluons were generally accepted Feynman proposed that the proton was made up of point-like constituents "partons"
- Both Bjorken Scaling and the Callan-Gross relationship can be explained by assuming that Deep Inelastic Scattering is dominated by the scattering of a single virtual photon from point-like spin-half constituents of the proton.



★ How do these two pictures of the interaction relate to each other?

费曼的部分3模型

电子可视作为无结构的定程子,而质子有结构







十强相距布用修道





强耦合常数很大,所有阶扰动都要考虑 (无穷多的复杂项) 一日场地中的质子具有内在的复杂结构的有短寿命彩子和对于新有短寿命彩子和对于新闻和美国 => 质子外级着作为粒子之

家金无防计杯

费曼认为: 预运(和所有强运)都是由无数,超运物成的文 四介了一核了的地中, 了由介了, 核子和反核子细成 ② 夸克理论中, 了里包含夸克和反夸克 费受简单地假定料路育量。最不鲜的实体 - 部分子 这听起来太平庸了!但贵曼的天才构践。 高能的两位子碰撞 每一夜31%中 的重载鲜桃物 因为强相互的制度短程的 两盘之间肉有很短时间的相对作用 弃近,在比期间它们实际看到的是-张冻结的第分的快照



高能强强道着我是发生在两个盘子的个别新分子之间 由于相互作用时间非常短,所则每个盘子内部的 部分已间的相互作用可以忽略不计。 一分发生高能散射的瞬间,每663内的部分都是独的 准自由的实体。

注意: 相对论性质和纪行为冻结的圆盘走指 "无穷大动量坐标系" 郭分子视师防闭中教子是指"冲涨近何"

家王模型:刘子中心的家族及弥漫其周围的电话 新分子模型: 质子电单-的无定开的新分子组成的 费曼的部分模型避开。最困难的强相互作用 一下有强相互作用的影响都包含在新分子的动量分布之内 一如果能够知道部分动量分布,那城市以特部分和作 监治了,从而电子-质静射抗可以按电子-郭分子;间的QE 相至作用。

一个上午地纪60年代,猜测部分的性质及其量强是热门银行。 流行的猜测:部分3具有与参轧相同的量3数 夸克-部分子模型:将部分子认同为夸克 困难:标度无利性要求夸克在和电子相互作的财为自由教子 自由彩子就可以任于中打击,当现在来受约中 個人表现測到自由夸克,这意味着我们需要假设 自由夸克必须持续经历软的,低动量转移的与伴随部分; 之间的强相互作用,并且要求这些软性作用不破坏样度硬性。

新分支模型中的DIS:



深度非弹鼓射可视作为· - 行入射电3发射-- 行光子,而后 这个光子与单个自由部分子发生 相互作用 - 1 电3-5支柱2的部分3-2间的

弹性散射

 结构函数 P系标度无关性之外,其大小和形状均可以在各种 测量过程中被测得。每个部分子对总结构函数的成就都可按 QZD 计标,并且其关南式 仅与自适和 电奇相关
 → 测量 Al可的函数 可得到 部分子量子数的信息