粒子物理

21. 轻子弱相互作用和中微子散射实验

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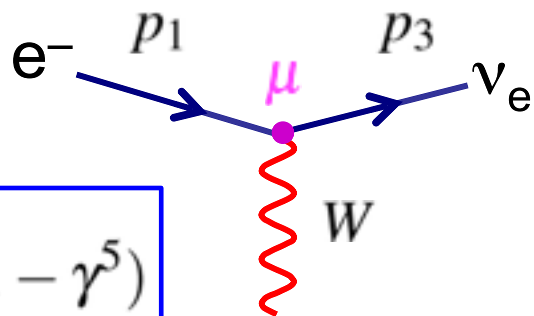
Lecture 20 Review

★ **Weak interaction is of form Vector – Axial-vector (V-A)**



$$\boxed{\frac{-ig_w}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)}$$

$$j^\mu \propto \overline{u}_{\nu_e}(\gamma^\mu - \gamma^\mu\gamma^5)u_e$$

V – A

★ **Consequently only left-handed chiral particle states and right-handed chiral anti-particle states participate in the weak interaction**

➡ **MAXIMAL PARITY VIOLATION**

★ **Weak interaction also violates Charge Conjugation symmetry**

★ **At low q^2 weak interaction is only weak because of the large W-boson mass**

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}}$$

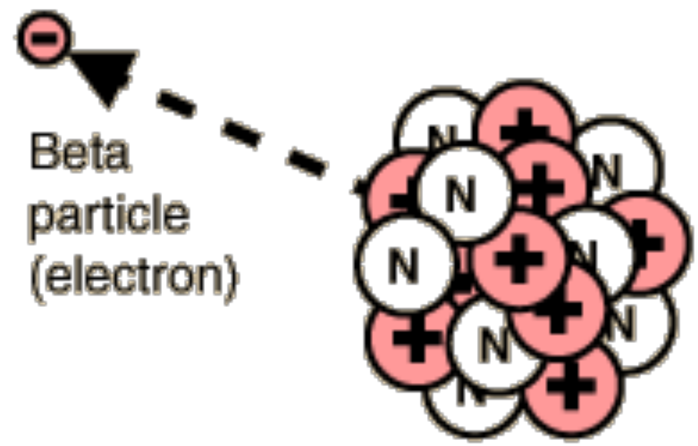
Intrinsic strength of weak interaction is similar to that of QED

The Birth: Beta Decay

Neutron not discovered yet

$$A \rightarrow B + e^{-}$$

$$(Z, N) \rightarrow (Z+1, N-1) + e^{-}$$



$m_n = 939.5656 \ MeV$

$m_p = 938.2723 \ MeV$

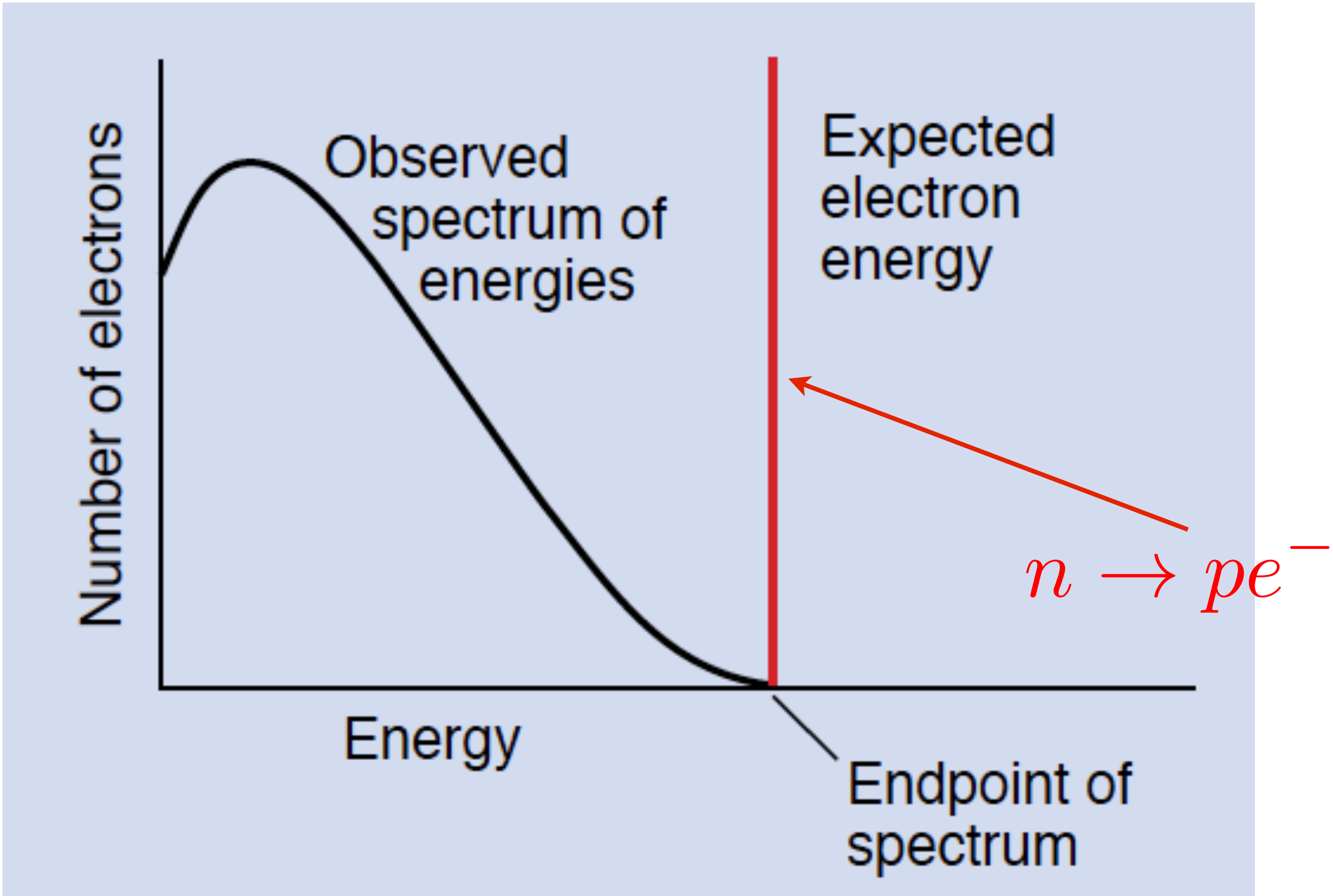
$m_e = 0.510999 \ MeV$

$0.7823 \ MeV = Q \ \ for \ \ n \rightarrow p + e^{-}$

The conservation of Energy and momentum
requires the electron have a single value of energy.

Beta Decay

1914, Chadwick



What is Wrong?

Something to loose

or

Something to add

Neil Bohr

1929

- ready to abandon the law of conservation of ene
- propose a statistical version of the conservation laws of energy, momentum, angular momentum

1924, Borh, Kramers, Slater, “辐射的量子理论”:
能量和动量在单个微观相互作用过程中不必守恒，
而只需要在统计意义上守恒。

1925年，康普顿电子-光子散射验证了微观散射过程中能动量守恒。

Neutrino

Wolfgang Pauli 1930

Letter to the physical Institute of the Federal Institute of Technology, Zurich

The Desperate Remedy

4 December 1930
Gloriastr.
Zürich

Physical Institute of the
Federal Institute of Technology (ETH)
Zürich

Dear radioactive ladies and gentlemen,

to save the "exchange theorem"* of statistics and the energy theorem. Namely [there is] the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons,** which have spin 1/2 and obey the exclusion principle, and additionally differ from light quan-

Neutrino

In 1932 Chadwick discovered a neutral nuclear constituent. By studying the properties of the neutral radiation n emitted in the process

$$^{9}Be + \alpha \rightarrow {}^{12}C + n$$

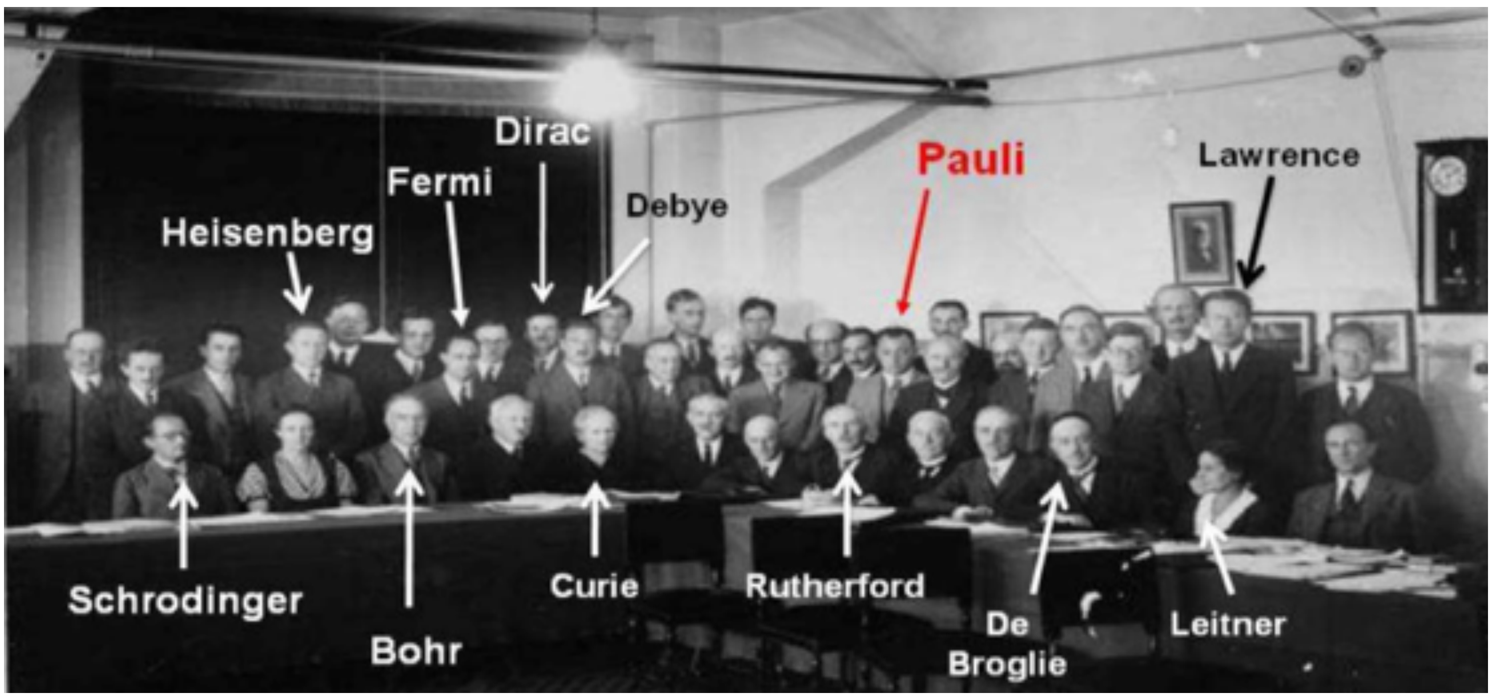
He found out that n was a deeply penetrating neutral particle slightly heavier than the proton, quite distinct from gamma-rays.

Pauli 说的“neutron”被Fermi改成“little neutral one”，成为今天常说的“Neutrino”

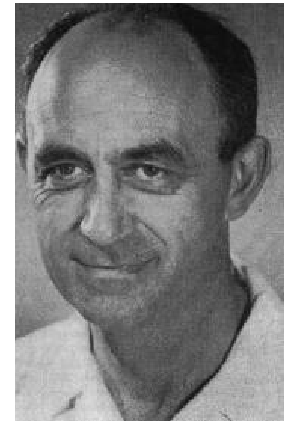
Neutrino

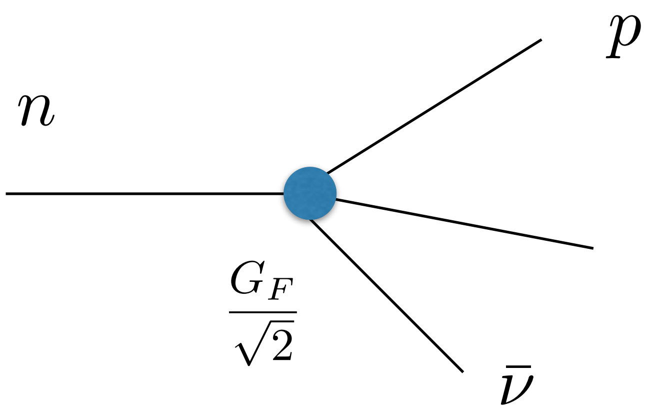
Solvay 1933 Physics Conference (Brussels, Belgium)

Pauli 报告了他的中微子设想



Fermi Theory





$$M_{fi} = G_F \left[\overline{\psi_n}\gamma^\mu \psi_p\right] \left[\overline{\psi_e}\gamma^\mu \psi_\nu\right]$$
$$G_F \sim 10^{-5}\,(\mathrm{GeV})^{-2}$$

Loosely like QED, but zero range and non-diagonal

The interaction behind beta decay remains unknown in Fermi's time.
It took some 20 years of work to figure out a detailed model fitting the observation.

Parity Violation

Parity conservation had been assumed, almost without question

$\theta - \tau$ puzzle (1950's)

$$\boldsymbol{\theta} \longrightarrow \underbrace{\boldsymbol{\pi}^{+}\boldsymbol{\pi}^{0}}_{\boldsymbol{P=+1}}$$

$$\boldsymbol{\tau} \longrightarrow \underbrace{\boldsymbol{\pi}^{+}\boldsymbol{\pi}^{+}\boldsymbol{\pi}^{0}}_{\boldsymbol{P=-1}}$$

Two particles with same mass, charge, spin, lifetime, but different decay modes and parity

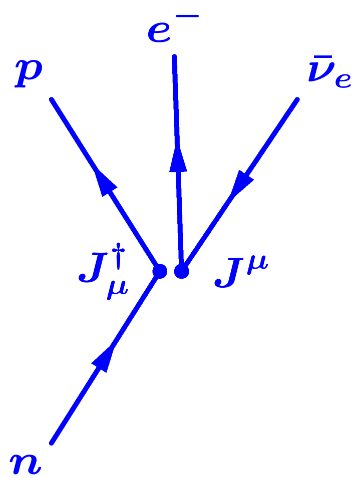
Lee, Yang (1956)

Need a pseudo-scalar to measure the parity violation effects.

$$\vec{\sigma} \cdot \vec{p}$$

V-A Theory

(maximal violation of parity and charge conjugation)



$$\mathcal{H} = \frac{G_F}{\sqrt{2}} J_\mu^\dagger J^\mu \qquad J_\mu = \underbrace{J_\mu^\ell}_{\text{leptonic}} + \underbrace{J_\mu^h}_{\text{hadronic}}$$

$$\begin{aligned} J_\mu^\ell &= \bar{e}^- \gamma_\mu \left(1 - \gamma^5\right) \nu_e + \bar{\mu} \gamma_\mu \left(1 - \gamma^5\right) \nu_\mu \\ &= 2 \left[\bar{e}_L \gamma_\mu \nu_{eL} + \bar{\mu}_L \gamma_\mu \nu_{\mu L}\right] \end{aligned}$$

- $\psi_L = P_L \psi = \frac{1-\gamma^5}{2} \psi$ **(vector - axial)**
- $G_F \simeq 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$ **(Fermi constant)**
- **Can extend third family, neutrino masses**
- **Will display hadronic current later**

V-A Theory

Feynman & Gell-man; Sudarshan, Marshak (1958)

Theory of the Fermi Interaction

R. P. Feynman and M. Gell-Mann
California Institute of Technology, Pasadena, California
(Received September 16, 1957)

The representation of Fermi particles by two-component Pauli spinors satisfying a second order differential equation and the suggestion that in β decay these spinors act without gradient couplings leads to an essentially unique weak four-fermion coupling. It is equivalent to equal amounts of vector and axial vector coupling with two-component neutrinos and conservation of leptons. (The relative sign is not determined theoretically.) It is taken to be "universal"; the lifetime of the μ agrees to within the experimental errors of 2%. The vector part of the coupling is, by analogy with electric charge, assumed to be not renormalized by virtual mesons. This requires, for example, that pions are also "charged" in the sense that there is a direct interaction in which, say, a π^0 goes to π^- and an electron goes to a neutrino. The weak decays of strange particles will result qualitatively if the universality is extended to include a coupling involving a Λ or Σ fermion. Parity is then not conserved even for those decays like $K \to 2\pi$ or 3π which involve no neutrinos. The theory is at variance with the measured angular correlation of electron and neutrino in He^6, and with the fact that fewer than 10^{-4} pion decay into electron and neutrino.

[5] A universal V, A interaction has also been proposed by E. C. G. Sudarshan and R. E. Marshak (to be published).

The authors have profited by conversations with F. Boehm, A. H. Wapstra, and B. Stech. One of us (M. G. M.) would like to thank R. E. Marshak and E. C. G. Sudarshan for valuable discussions.

V-A Theory

HEP **3** records found Search took 0.13 seconds.

1. **Group U(6) x U(6) generated by current components**
R.P. Feynman, Murray Gell-Mann, G. Zweig (Caltech, Kellogg Lab). 1964.
Published in **Phys.Rev.Lett. 13 (1964) 678-680**
DOI: 10.1103/PhysRevLett.13.678
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; Phys. Rev. Lett. Server; Link to Fulltext
Detailed record - Cited by 71 records 50+

2. **Theory of Fermi interaction**
R.P. Feynman, Murray Gell-Mann (Caltech). 1958.
Published in **Phys.Rev. 109 (1958) 193-198**
DOI: 10.1103/PhysRev.109.193
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Phys.Rev. Server; Link to Fulltext
Detailed record - Cited by 1283 records 1000+

3. **Strange particles and weak interactions**
J.R. Oppenheimer (chairman) (Princeton, Inst. Advanced Study) *et al.*. 1957. 51 pp.
Conference: C57-04-15, p.IX.1-52 Proceedings
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Link to Fulltext
Detailed record

V-A Theory

12. **Chirality invariance and the universal Fermi intera**
E.C.G. Sudarshan (Harvard U.), R.e. Marshak (Rochester U.). Ja
Published in **Phys.Rev. 109 (1958) 1860-1860**
DOI: 10.1103/PhysRev.109.1860

Chirality Invariance and the Universal Fermi Interaction*

E. C. G. SUDARSHAN, *Harvard University, Cambridge, Massachusetts*

AND

R. E. MARSHAK, *University of Rochester, Rochester, New York*
(Received January 10, 1958)

WE have shown[1] that the imposition of the requirement of chirality invariance[2] on each covariant in the four-fermion interaction Hamiltonian leads to the essentially unique expression[3]:

$$G\{[\bar{A}\gamma_\mu(1+\gamma_5)B]^\dagger[\bar{C}\gamma_\mu(1+\gamma_5)D]+\text{H.c.}\} \qquad \text{(I)}$$

G is the coupling constant and A, B, C, D are ... irac *particle* fields. In the standard terminology of parity-conserving interactions, (I) represents the ... $V-A$ (V is vector, A is axial vector). We ... (I) holds between any two of the pairs[4] ... $e^-\nu$ (i.e., the *particles* are taken to be the

* Supported in part by the U. S. Atomic Energy Commission.
[1] E. C. G. Sudarshan and R. E. Marshak, *Proceedings of Padua-Venice Conference on Mesons and Newly Discovered Particles, September, 1957* [Suppl. Nuovo cimento (to be published)].

[3] R. P. Feynman and M. Gell-Mann [Phys. Rev. **109**, 193 (1958)] have independently arrived at this expression on the basis of a two-component spinor theory for all spin ½ particles and no gradients in the interaction Hamiltonian; however, this theory requires the two-component wave function to satisfy a second order equation which raises certain difficulties of principle (quantization according to the usual rules, effect of strong interactions, etc.). More recently, J. J. Sakurai has obtained expression (I) by requiring the invariance of the four-fermion Hamiltonian under separate reversal of the sign of the mass in the Dirac equation for each fermion; this condition is completely equivalent to the condition of chirality invariance. We are indebted to the authors for sending us preprints of their papers.

邢老师科学网博客

http://blog.sciencenet.cn/blog-3779-689389.html

博文

不走运的苏达山 精选

老达尔文的儿子弗朗西斯·达尔文（Francis Darwin）曾经说过一句颇具争议的名言："在科学上，荣誉属于让全世界信服的人，而不属于先有想法的人"。事实上，最先提出某种科学思想的人往往最具有原创力和洞察力，他们的思想往往成为新突破的火种。如果一个年轻人率先提出了新想法，却因为种种原因而没有得到学界的充分认可，或者与本应属于他的荣誉失之交臂，那么他一定会很郁闷、很不甘的。印度裔的美国物理学家乔治·苏达山（George Sudarshan）就曾有过这种很不走运的经历。

What we learned …

http://blog.sciencenet.cn/blog-3779-873055.html

会“抢”才能赢 精选

美国理论物理学家默里·盖尔曼（Murray Gell-Mann）作为一代宗师，在上世纪60至70年代曾经主宰国际粒子物理学的走向长达10余年。这位天才的科学家，1929年生于纽约一个普通的犹太家庭，14岁被耶鲁大学录取，22岁在麻省理工学院获得博士学位，25岁成为加州理工学院最年轻的终身教授。盖尔曼深邃的洞察力与旺盛的创造力也许是与生俱来的，而这使得同时代的许多物理学家相形见绌，黯然失色。他24岁发现了基本粒子的一个新量子数---奇异数，28岁建立了正确描述弱相互作用的V-A理论，32岁提出了强子分类的八重法（相当于介子和重子的门捷列夫周期表），35岁创立了夸克模型，40岁荣获诺贝尔物理学奖…。然而给许多人留下深刻印象的却是盖尔曼的好斗性格，以及他那与众不同的思辨才能和抢占发现先机的特殊本事。

苏达山的不走运经历当然是受很多因素的影响所致。不过年轻的研究者们从中至少可以得到一条重要的启示：如果你有了崭新的科学思想和学术成果，那么无论如何尽快发表它才是硬道理。

What we know today : Neutrino Flavours

★ 2015 Nobel Prize ➡ neutrinos have mass (albeit very small)

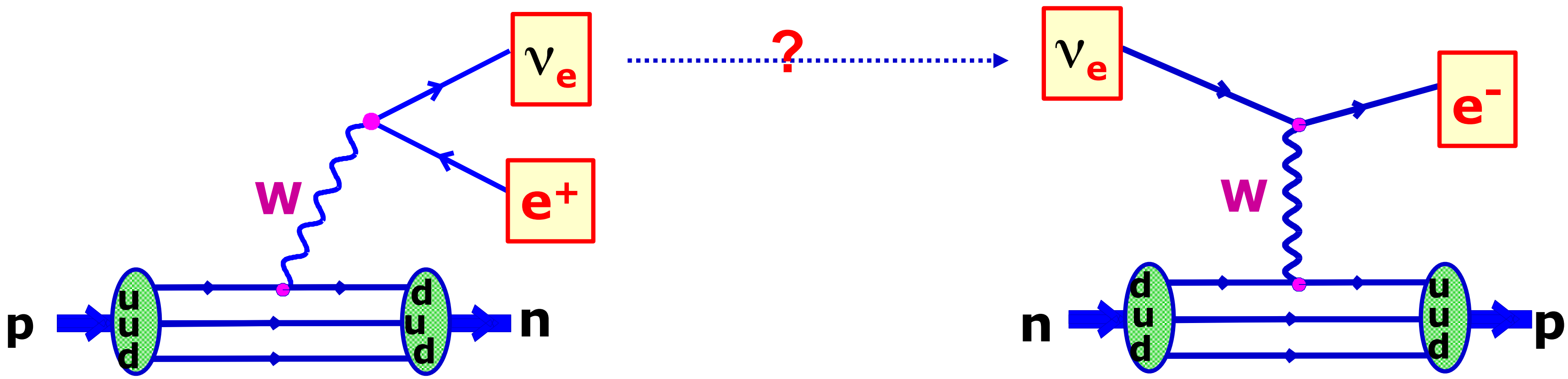
★ The textbook neutrino states, ν_e, ν_μ, ν_τ, are not the fundamental particles; these are ν_1, ν_2, ν_3

★ Concepts like "electron number" conservation are now known not to hold.

★ So what are ν_e, ν_μ, ν_τ ?

★ Never directly observe neutrinos – can only detect them by their weak interactions. Hence by definition ν_e is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state ν_e produce an electron

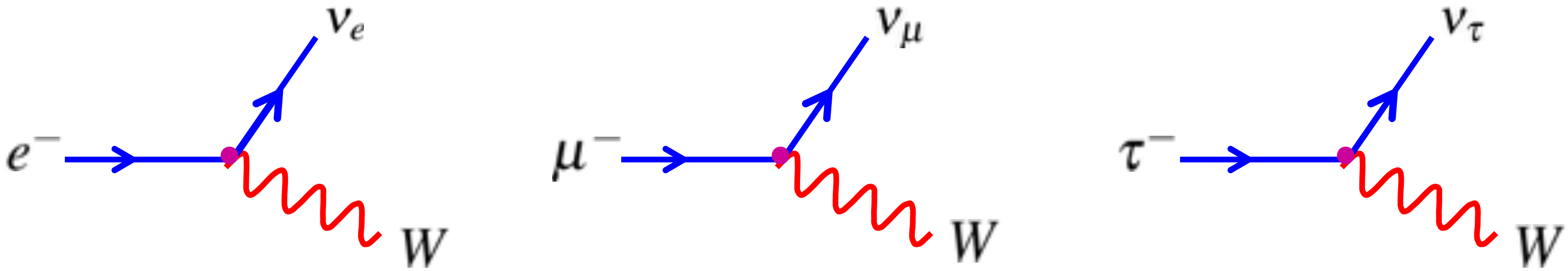
➡ $\boxed{\nu_e, \nu_\mu, \nu_\tau \text{ = weak eigenstates}}$



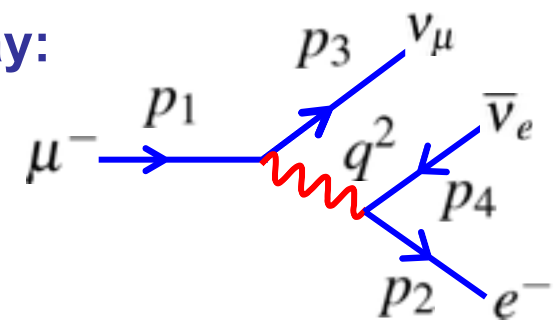
★ Unless dealing with <u>very large</u> distances: the neutrino produced with a positron will interact to produce an electron. For the discussion of the weak interaction continue to use ν_e, ν_μ, ν_τ as if they were the fundamental particle states.

Muon Decay and Lepton Universality

★ The leptonic **charged current** ($W^{\pm}$) interaction vertices are:



★ Consider muon decay:



It is straight-forward to write down the matrix element

$$M_{fi} = \frac{g_W^{(e)} g_W^{(\mu)}}{8m_W^2}[\overline{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)]g_{\mu\nu}[\overline{u}(p_2)\gamma^\nu(1-\gamma^5)v(p_4)]$$

Note: for lepton decay $q^2 \ll m_W^2$ so propagator is a constant $1/m_W^2$
i.e. in limit of Fermi theory

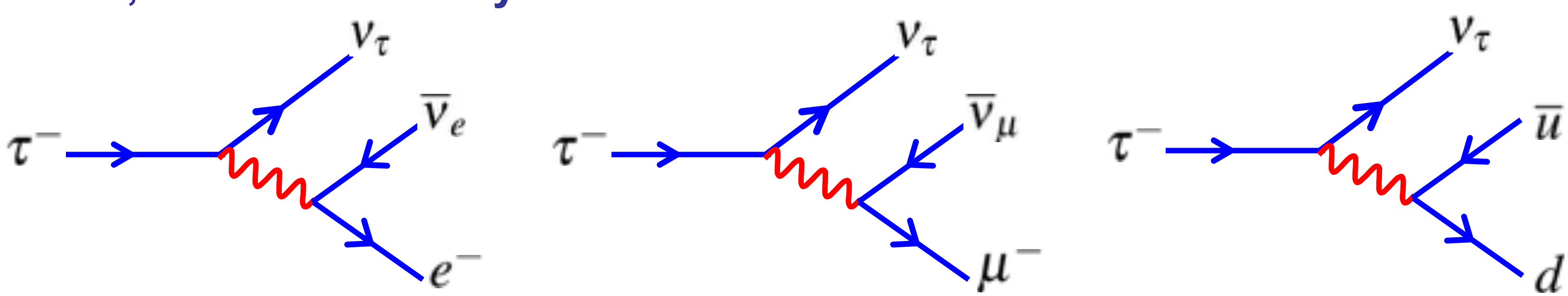
Its evaluation and subsequent treatment of a three-body decay is rather tricky (and not particularly interesting). Here will simply quote the result

•The muon to electron rate

$$\Gamma(\mu \to e\nu\nu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu} \quad \text{with} \quad G_F = \frac{g_W^2}{4\sqrt{2}m_W^2}$$

Similarly for tau to electron $\Gamma(\tau \to e\nu\nu) = \dfrac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$

•However, the tau can decay to a number of final states:



$Br(\tau \to e\nu\nu) = 0.1784(5)$ $\quad$ $Br(\tau \to \mu\nu\nu) = 0.1736(5)$

•Recall total width (total transition rate) is the sum of the partial widths

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau}$$

It can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \to e\nu\nu) = \Gamma_\tau Br(\tau \to e\nu\nu) = Br(\tau \to e\nu\nu)/\tau_\tau$$

Therefore predict

$$\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5} \qquad \tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \to e\nu\nu)$$

All these quantities are precisely measured:

$$m_\mu = 0.1056583692(94)\,\mathrm{GeV} \qquad \tau_\mu = 2.19703(4)\times 10^{-6}\,\mathrm{s}$$

$$m_\tau = 1.77699(28)\,\mathrm{GeV} \qquad \tau_\tau = 0.2906(10)\times 10^{-12}\,\mathrm{s}$$

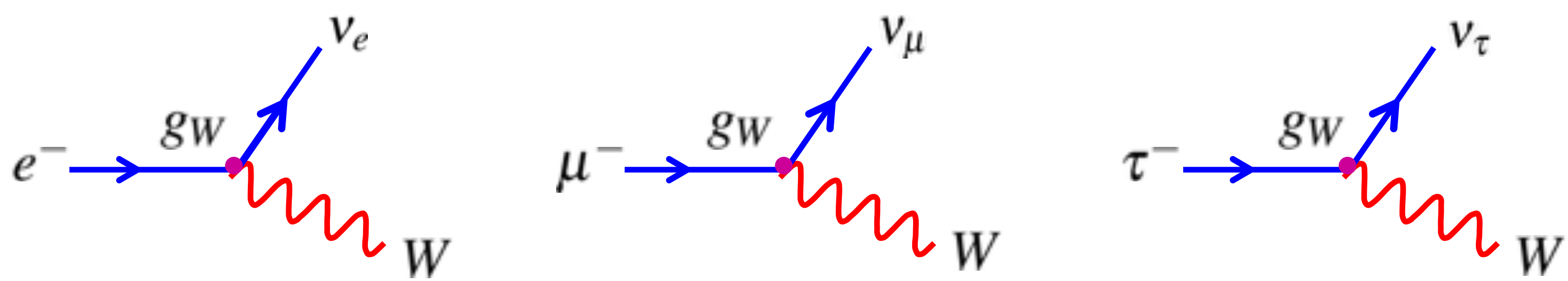
$$Br(\tau \to e\nu\nu) = 0.1784(5)$$

$$\Rightarrow \quad \boxed{\frac{G_\mathrm{F}^\tau}{G_\mathrm{F}^\mu} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau \to e\nu\nu) = 1.0024 \pm 0.0033}$$

Similarly by comparing $Br(\tau \to e\nu\nu)$ **and** $Br(\tau \to \mu\nu\nu)$

$$\boxed{\frac{G_\mathrm{F}^e}{G_\mathrm{F}^\mu} = 1.000 \pm 0.004}$$

★ **Demonstrates the weak charged current is the same for all leptonic vertices**



➡ **Charged Current Lepton Universality**

Neutrino Scattering

- We have considered electron-proton **Deep Inelastic Scattering** where a virtual photon is used to probe nucleon structure

- Can also consider the weak interaction equivalent: **Neutrino Deep Inelastic Scattering** where a virtual W-boson probes the structure of nucleons

  ➡ additional information about parton structure functions

  + provides a good example of calculations of weak interaction cross sections.

★ **Neutrino Beams:**

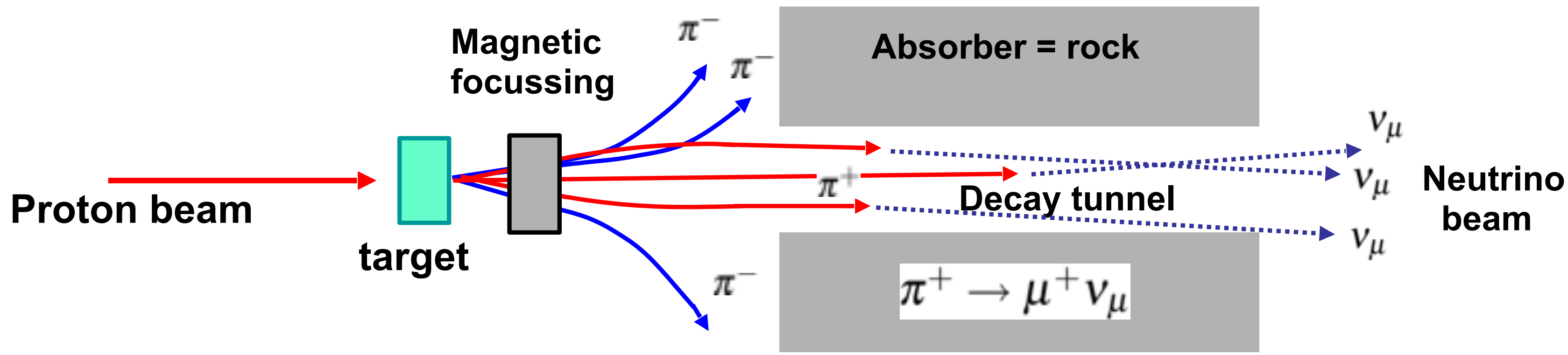
Smash high energy protons into a fixed target ➡ hadrons

Focus positive pions/kaons

Allow them to decay $\pi^+ \to \mu^+\nu_\mu$ + $K^+ \to \mu^+\nu_\mu \; (BR \approx 64\,\%)$

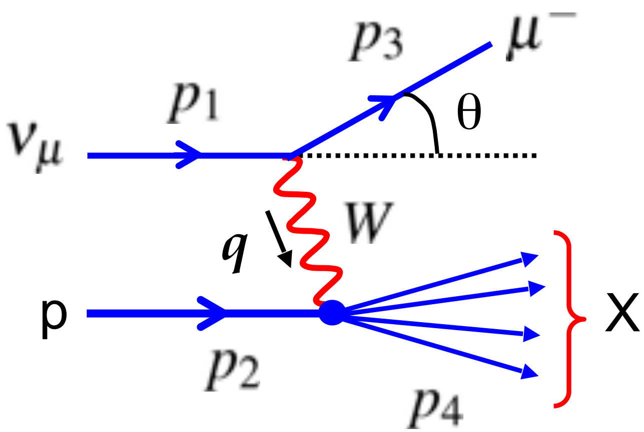
Gives a beam of "collimated" ν_μ

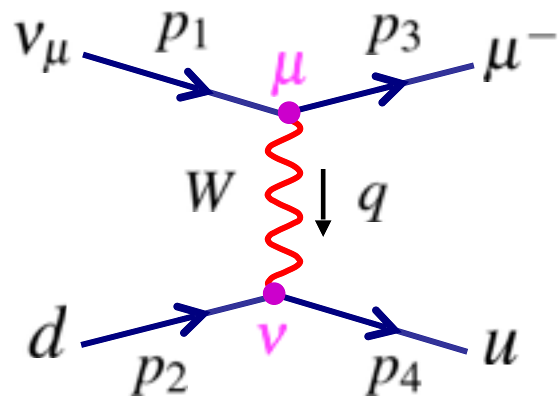
Focus negative pions/kaons to give beam of $\overline{\nu}_\mu$



Neutrino-Quark Scattering

★ For ν_μ-proton Deep Inelastic Scattering the underlying process is $\nu_\mu d \to \mu^- u$





★ In the limit $q^2 \ll m_W^2$ the W-boson propagator is $\approx ig_{\mu\nu}/m_W^2$

The Feynman rules give:

$$-iM_{fi} = \left[-i\frac{g_W}{\sqrt{2}}\overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1)\right]\frac{ig_{\mu\nu}}{m_W^2}\left[-i\frac{g_W}{\sqrt{2}}\overline{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]$$

$$\boxed{M_{fi} = \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1)\right]\left[\overline{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]}$$

Evaluate the matrix element in the extreme relativistic limit where the muon and quark masses can be neglected

In this limit the helicity states are equivalent to the chiral states and

$$\tfrac{1}{2}(1-\gamma^5)u_\uparrow(p_1)=0 \qquad \tfrac{1}{2}(1-\gamma^5)u_\downarrow(p_1)=u_\downarrow(p_1)$$

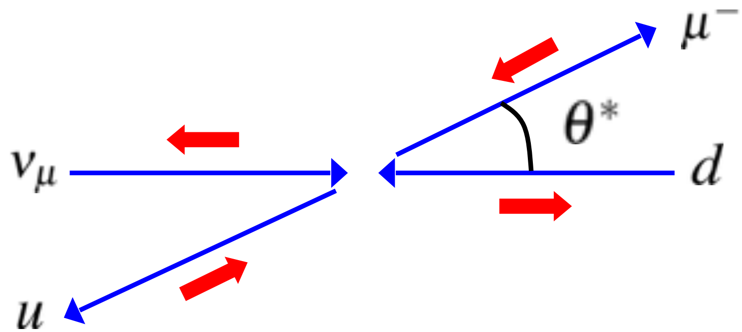
$\Rightarrow$ $M_{fi}=0$ **for** $u_\uparrow(p_1)$ **and** $u_\uparrow(p_2)$

Since the weak interaction "conserves the helicity", the only helicity combination where the matrix element is non-zero is

$$M_{fi}=\frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right]\left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

NOTE: we could have written this down straight away as in the ultra-relativistic limit only LH helicity particle states participate in the weak interaction.

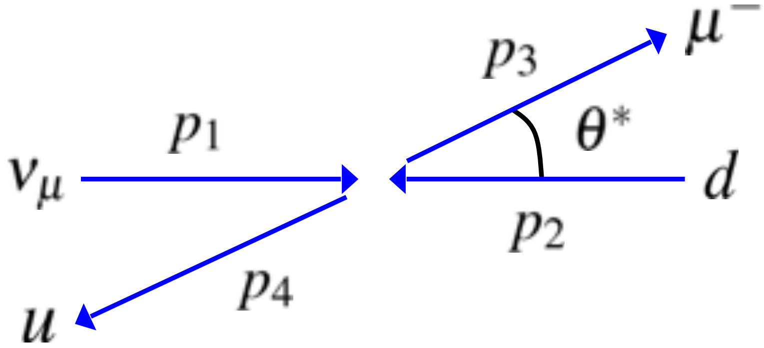
★ **Consider the scattering in the C.o.M frame**



Evaluation of Neutrino-Quark Scattering ME

- **Go through the calculation in gory detail (fortunately only one helicity combination)**

In the $\nu_\mu d$ CMS frame, neglecting particle masses:



$$p_1 = (E,0,0,E),$$
$$p_2 = (E,0,0,-E)$$
$$p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$$
$$p_4 = (E, -E\sin\theta^*, 0, -E\cos\theta^*)$$

Dealing with LH helicity particle spinors. For a massless particle traveling in direction (θ,ϕ):

$$u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix} \qquad c = \cos\frac{\theta}{2}; \quad s = \sin\frac{\theta}{2}$$

Here $(\theta_1,\phi_1) = (0,0);\ (\theta_2,\phi_2) = (\pi,0);\ (\theta_3,\phi_3) = (\theta^*,0);\ (\theta_4,\phi_4) = (\pi-\theta^*,\pi)$

giving:

$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix};\ u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix};\ u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix};\ u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

To calculate

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right] \left[\bar{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

need to evaluate two terms of form

$$\begin{aligned}
\overline{\psi}\gamma^0\phi &= \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 \\
\overline{\psi}\gamma^1\phi &= \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \\
\overline{\psi}\gamma^2\phi &= \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1) \\
\overline{\psi}\gamma^3\phi &= \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2
\end{aligned}$$

Using

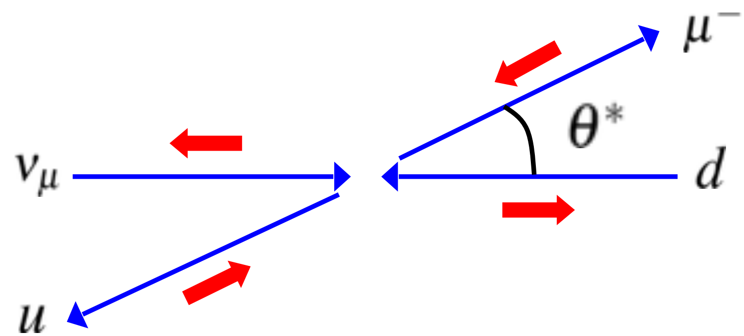
$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix};\; u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}-1\\0\\1\\0\end{pmatrix};\; u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix}-s\\c\\s\\-c\end{pmatrix};\; u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix}-c\\-s\\c\\s\end{pmatrix}$$

$$\boxed{\begin{aligned}
\bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) &= 2E(c, s, -is, c) \\
\bar{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2) &= 2E(c, -s, -is, -c)
\end{aligned}}$$

$$M_{fi} = \frac{g_W^2}{2m_W^2} 4E^2(c^2 + s^2 + s^2 + c^2) = \frac{g_W^2 \hat{s}}{m_W^2} \qquad \hat{s} = (2E)^2$$

★ Note the Matrix Element is isotropic

$$\boxed{M_{fi} = \frac{g_W^2}{m_W^2}\hat{s}}$$



we could have anticipated this since the helicity combination (spins anti-parallel) has $S_z = 0$ ➡ no preferred polar angle

★ As before need to sum over all possible spin states and average over all possible initial state spin states. Here only one possible spin combination (LL→LL) and only 2 possible initial state combinations (the neutrino is always produced in a LH helicity state)

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2} \cdot \left| \frac{g_W^2}{m_W^2}\hat{s} \right|^2$$

The factor of a half arises because half of the time the quark will be in a RH states and won't participate in the charged current Weak interaction

★ In the extreme relativistic limit, the cross section for any 2→2 body scattering process is

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} \langle |M_{fi}|^2 \rangle$$

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2\hat{s}}\langle|M_{fi}|^2\rangle = \frac{1}{64\pi\hat{s}}\frac{1}{2}\left(\frac{g_W^2\hat{s}}{m_W^2}\right)^2 = \left(\frac{g_W^2}{8\sqrt{2}\pi m_W^2}\right)^2\hat{s}$$

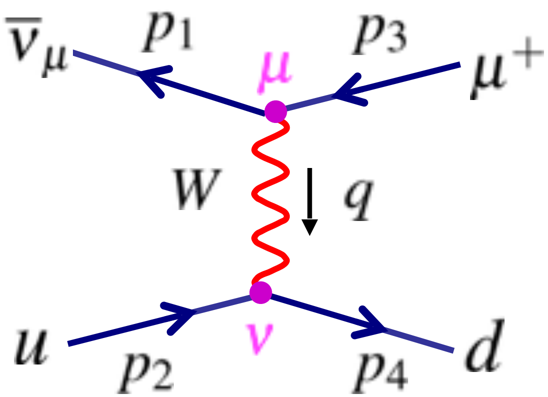
using $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$ $\Rightarrow$ $\boxed{\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}}$

★ **Integrating this isotropic distribution over** $d\Omega^*$

$$\Rightarrow \boxed{\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}} \qquad \text{(1)}$$

cross section is a Lorentz invariant quantity so this is valid in any frame

Antineutrino-Quark Scattering



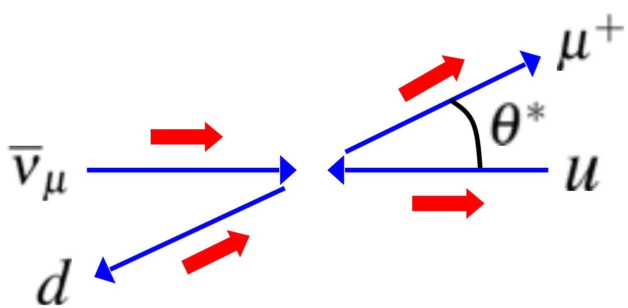
In the ultra-relativistic limit, the charged-current interaction matrix element is:

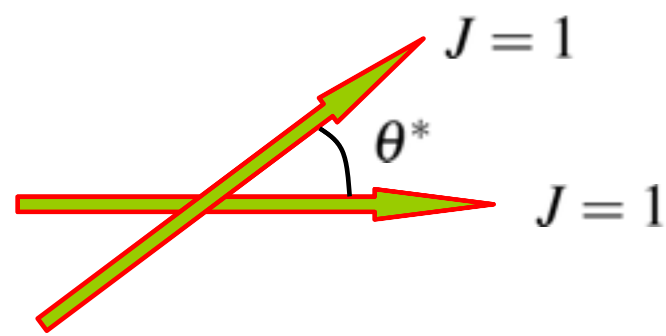
$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{v}(p_1)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_3)\right] \left[\overline{u}(p_4)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)u(p_2)\right]$$

★ **In the extreme relativistic limit only LH Helicity particles and RH Helicity anti-particles participate in the charged current weak interaction:**

$$\Rightarrow \quad M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{v}_\uparrow(p_1)\gamma^\mu v_\uparrow(p_3)\right] \left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

★ **In terms of the particle spins it can be seen that the interaction occurs in a total angular momentum 1 state**

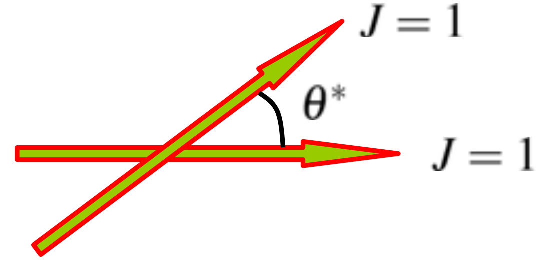




★ In a similar manner to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu q}}{d\Omega^*}\frac{1}{4}(1+\cos\theta^*)^2$$

The factor $\frac{1}{4}(1+\cos\theta^*)^2$ can be understood in terms of the overlap of the initial and final angular momentum wave-functions



★ Similarly to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

$$d\Omega = d\phi\sin\theta d\theta \rightarrow d\phi d(\cos\theta)$$

★ Integrating over solid angle:

$$\int(1+\cos\theta^*)^2 d\Omega^* = \int(1+\cos\theta^*)^2 d(\cos\theta^*)d\phi = 2\pi\int_{-1}^{+1}(1+\cos\theta^*)^2 d(\cos\theta^*) = \frac{16\pi}{3}$$

$$\Rightarrow \boxed{\sigma_{\overline{\nu}q} = \frac{G_F^2\hat{s}}{3\pi}}$$

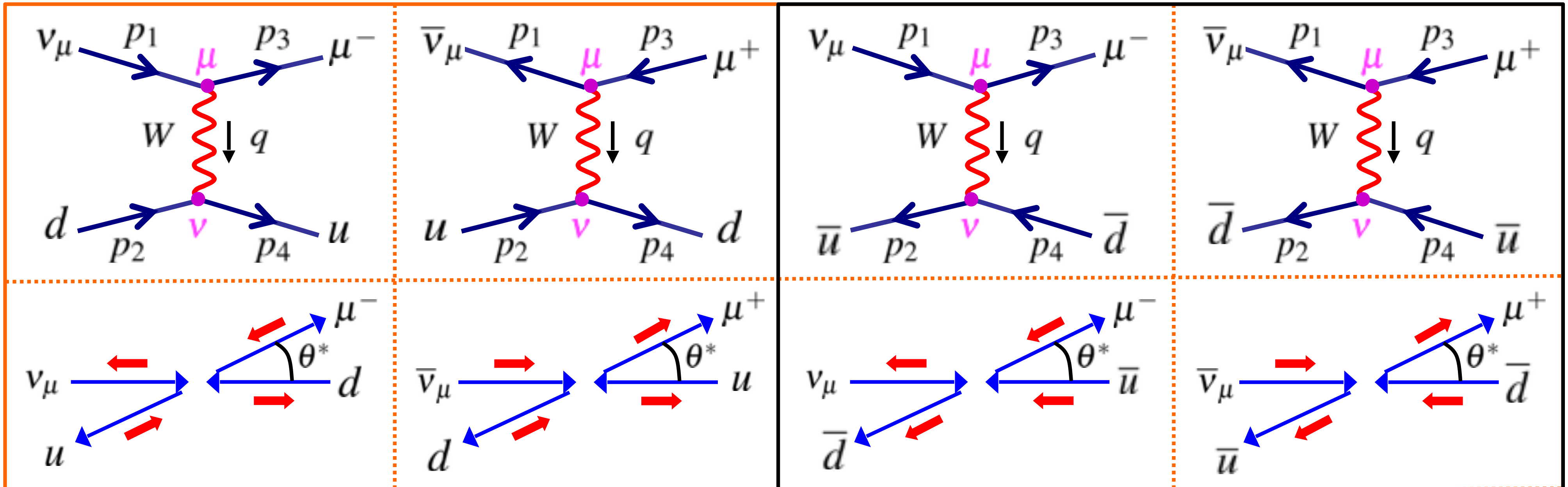
★ This is a factor three smaller than the neutrino quark cross-section

$$\boxed{\frac{\sigma_{\overline{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}}$$

(Anti)neutrino-(Anti)quark Scattering

Non-zero anti-quark component to the nucleon $\Rightarrow$ also consider scattering from $\bar{q}$

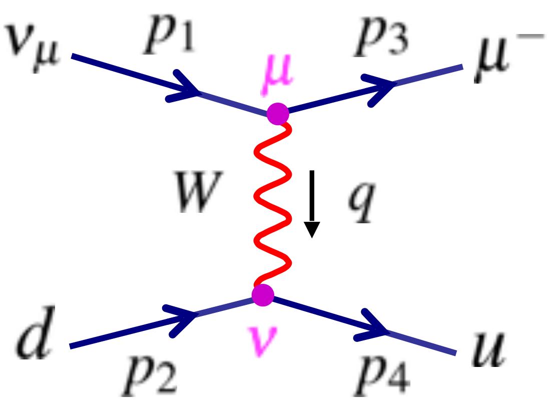
Cross-sections can be obtained immediately by comparing with quark scattering and remembering to only include LH particles and RH anti-particles



$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$	$\frac{d\sigma_{\bar{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\nu \bar{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\bar{\nu}\bar{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$
$\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}$	$\sigma_{\bar{\nu} q} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\nu\bar{q}} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\bar{\nu}\bar{q}} = \frac{G_F^2\hat{s}}{\pi}$

Differential Cross Section dσ/dy

★ **Derived differential neutrino scattering cross sections in C.o.M frame, can convert to Lorentz invariant form**



As for DIS use Lorentz invariant $y \equiv \dfrac{p_2 \cdot q}{p_2 \cdot p_1}$

In relativistic limit y can be expressed in terms of the C.o.M. scattering angle

$$y = \tfrac{1}{2}(1 - \cos\theta^*)$$

In lab. frame $\quad y = 1 - \dfrac{E_3}{E_1}$

★ **Convert from** $\dfrac{d\sigma}{d\Omega^*} \longrightarrow \dfrac{d\sigma}{dy}$ **using**

$$\frac{d\sigma}{dy} = \left|\frac{d\cos\theta^*}{dy}\right| \frac{d\sigma}{d\cos\theta^*} = \left|\frac{d\cos\theta^*}{dy}\right| 2\pi \frac{d\sigma}{d\Omega^*} = 4\pi \frac{d\sigma}{d\Omega^*}$$

Already calculated (1) $\quad \dfrac{d\sigma}{d\Omega^*} = \dfrac{G_F^2}{4\pi^2}\hat{s}$

Hence:

$$\boxed{\frac{d\sigma_{\nu q}}{dy} = \frac{d\sigma_{\overline{\nu}\overline{q}}}{dy} = \frac{G_F^2}{\pi}\hat{s}}$$

and
$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu\overline{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

becomes
$$\frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \frac{G_F^2}{4\pi}(1+\cos\theta^*)^2\hat{s}$$

from
$$y = \tfrac{1}{2}(1-\cos\theta^*) \rightarrow 1-y = \tfrac{1}{2}(1+\cos\theta^*)$$

and hence
$$\boxed{\frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}}$$

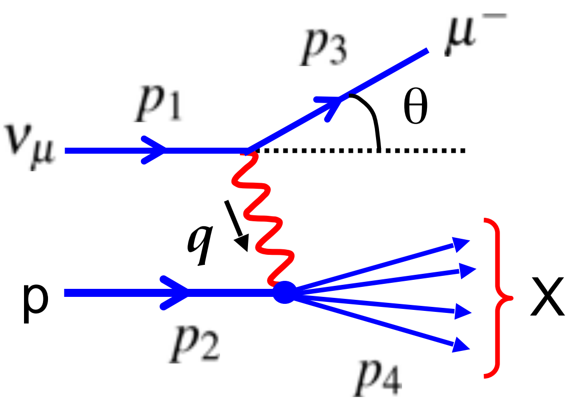
★ For comparison, the electro-magnetic $e^{\pm}q \rightarrow e^{\pm}q$ cross section is:

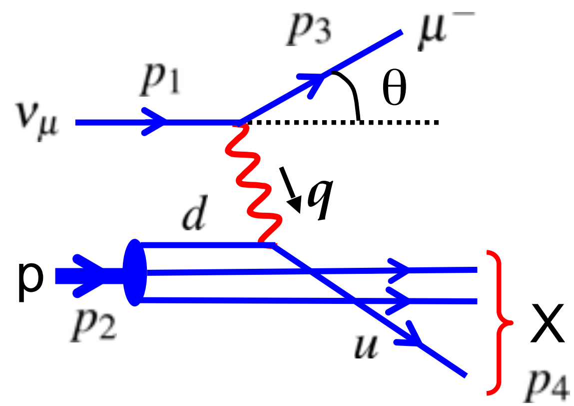
QED
$$\frac{d\sigma_{e^{\pm}q}}{dy} = \underbrace{\frac{2\pi\alpha^2}{q^4}e_q^2}_{\text{Interaction +propagator}}\underbrace{\left[1+(1-y)^2\right]}_{\text{Helicity Structure}}\hat{s}$$

DIFFERENCES:

WEAK
$$\frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \overbrace{\frac{G_F^2}{\pi}}^{\text{Interaction +propagator}}\overbrace{(1-y)^2}^{\text{Helicity Structure}}\hat{s}$$

Parton Model For Neutrino Deep Inelastic Scattering





Scattering from a proton with structure functions → **Scattering from a point-like quark within the proton**

★ **Neutrino-proton scattering can occur via scattering from a <u>down-quark</u> or from an <u>anti-up quark</u>**

In the parton model, number of down quarks within the proton in the momentum fraction range $x \to x + \mathrm{d}x$ **is** $d^p(x)\mathrm{d}x$. **Their contribution to the neutrino scattering cross-section is obtained by multiplying by the** $\nu_\mu d \to \mu^- u$ **cross-section derived previously**

$$\frac{\mathrm{d}\sigma^{\nu p}}{\mathrm{d}y} = \frac{G_F^2}{\pi}\hat{s}d^p(x)\mathrm{d}x$$

where $\hat{s}$ **is the centre-of-mass energy of the** $\nu_\mu d$

Similarly for the $\overline{u}$ contribution

$$\frac{d\sigma^{\nu p}}{dy} = \frac{G_F^2}{\pi}\hat{s}(1-y)^2\overline{u}^p(x)dx$$

★ Summing the two contributions and using $\hat{s} = xs$

$$\Rightarrow \quad \frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2}{\pi}sx\left[d^p(x) + (1-y)^2\overline{u}^p(x)\right]$$

★ The anti-neutrino proton differential cross section can be obtained in the same manner:

$$\frac{d^2\sigma^{\overline{\nu} p}}{dxdy} = \frac{G_F^2}{\pi}sx\left[(1-y)^2u^p(x) + \overline{d}^p(x)\right]$$

★ For (anti)neutrino – neutron scattering:

$$\frac{d^2\sigma^{\nu n}}{dxdy} = \frac{G_F^2}{\pi}sx\left[d^n(x) + (1-y)^2\overline{u}^n(x)\right]$$

$$\frac{d^2\sigma^{\overline{\nu} n}}{dxdy} = \frac{G_F^2}{\pi}sx\left[(1-y)^2u^n(x) + \overline{d}^n(x)\right]$$

As before, define neutron distributions functions in terms of those of the proton

$$u(x) \equiv u^{\mathrm{p}}(x) = d^{\mathrm{n}}(x); \qquad d(x) \equiv d^{\mathrm{p}}(x) = u^{\mathrm{n}}(x)$$

$$\overline{u}(x) \equiv \overline{u}^{\mathrm{p}}(x) = \overline{d}^{\mathrm{n}}(x); \qquad \overline{d}(x) \equiv \overline{d}^{\mathrm{p}}(x) = \overline{u}^{\mathrm{n}}(x)$$

$$\boxed{\begin{aligned} \frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} &= \frac{G_{\mathrm{F}}^2}{\pi} sx\left[d(x) + (1-y)^2\overline{u}(x)\right] \\ \frac{\mathrm{d}^2\sigma^{\overline{\nu} p}}{\mathrm{d}x\mathrm{d}y} &= \frac{G_{\mathrm{F}}^2}{\pi} sx\left[(1-y)^2 u(x) + \overline{d}(x)\right] \end{aligned}}$$

(2)

(3)

$$\frac{\mathrm{d}^2\sigma^{\nu n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_{\mathrm{F}}^2}{\pi} sx\left[u(x) + (1-y)^2\overline{d}(x)\right] \quad (4)$$

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_{\mathrm{F}}^2}{\pi} sx\left[(1-y)^2 d(x) + \overline{u}(x)\right] \quad (5)$$

★ **Because neutrino cross sections are very small, need massive detectors. These are usually made of Iron, hence, experimentally measure a combination of proton/neutron scattering cross sections**

★ **For an isoscalar target (i.e. equal numbers of protons and neutrons), the mean cross section per nucleon:**

$$\frac{d^2\sigma^{\nu N}}{dxdy} = \frac{1}{2}\left(\frac{d^2\sigma^{\nu p}}{dxdy} + \frac{d^2\sigma^{\nu n}}{dxdy}\right)$$

$$\Rightarrow \boxed{\frac{d^2\sigma^{\nu N}}{dxdy} = \frac{G_F^2}{2\pi} sx\left[u(x) + d(x) + (1-y)^2(\overline{u}(x) + \overline{d}(x))\right]}$$

Integrate over momentum fraction x

$$\boxed{\frac{d\sigma^{\nu N}}{dy} = \frac{G_F^2}{2\pi} s\left[f_q + (1-y)^2 f_{\overline{q}})\right]} \qquad \textbf{(6)}$$

where f_q and $f_{\overline{q}}$ are the total momentum fractions carried by the quarks and by the anti-quarks within a nucleon

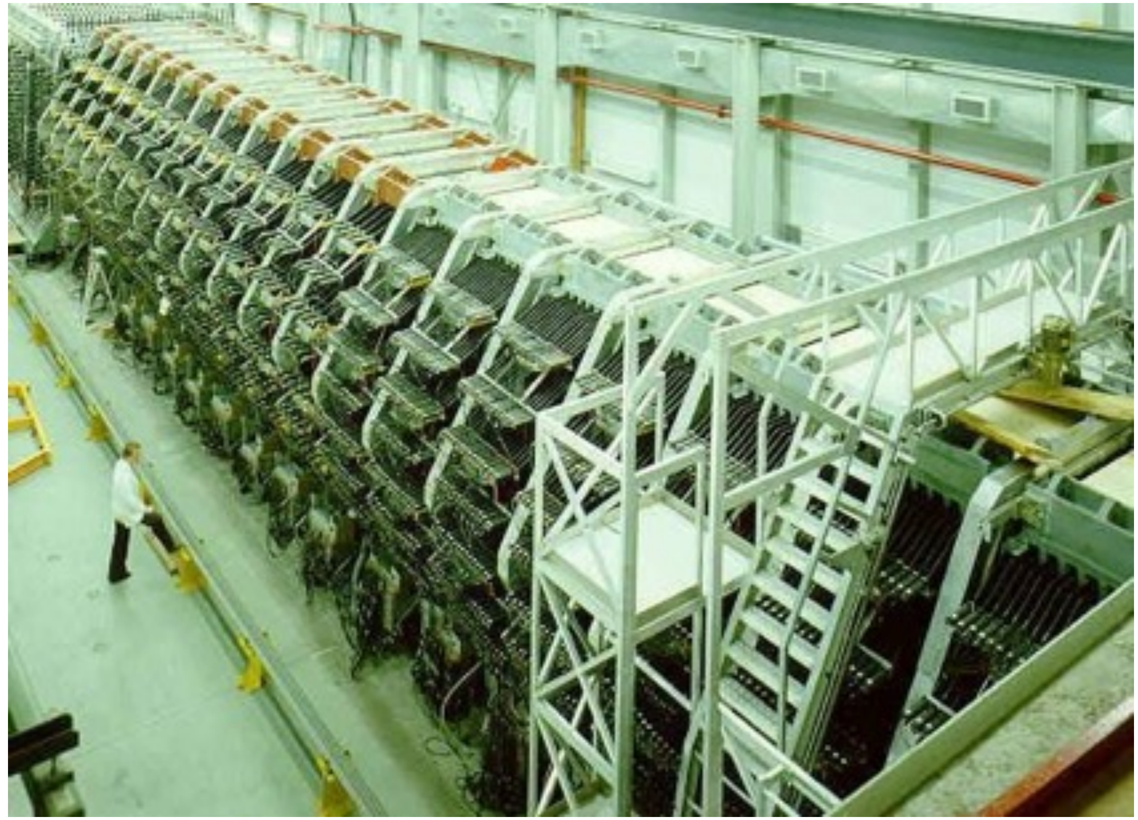
$$f_q \equiv f_d + f_u = \int_0^1 x\left[u(x) + d(x)\right] dx; \quad f_{\overline{q}} \equiv f_{\overline{d}} + f_{\overline{u}} = \int_0^1 x\left[\overline{u}(x) + \overline{d}(x)\right] dx$$

Similarly

$$\boxed{\frac{d\sigma^{\overline{\nu} N}}{dy} = \frac{G_F^2}{2\pi} s\left[(1-y)^2 f_q + f_{\overline{q}})\right]} \qquad \textbf{(7)}$$

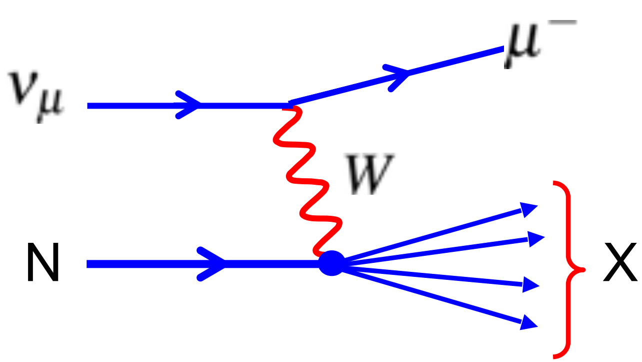
e.g. CDHS Experiment (CERN 1976-1984)

CERN, Dortmund,
Heidelberg, Saclay
+ Warsaw

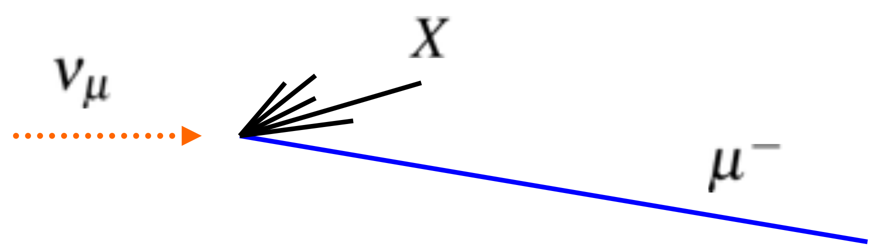


- **1250 tons**
- **Magnetized iron modules**
- **Separated by drift chambers**

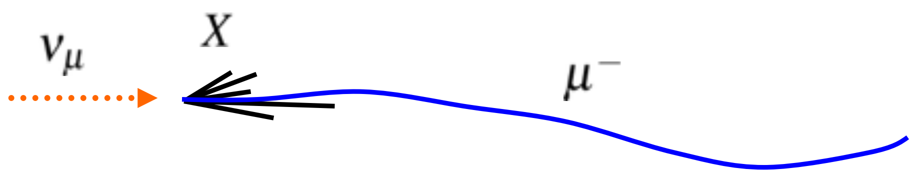
Study Neutrino Deep Inelastic Scattering

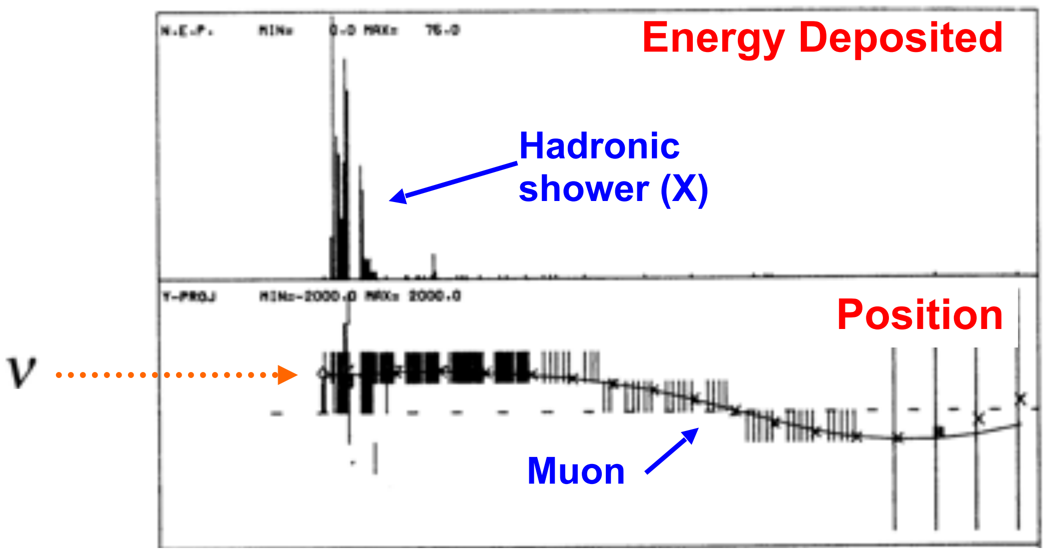


Experimental Signature:



Example Event:





Measure energy of X

$$E_X$$

Measure muon momentum from curvature in B-field

$$E_\mu$$

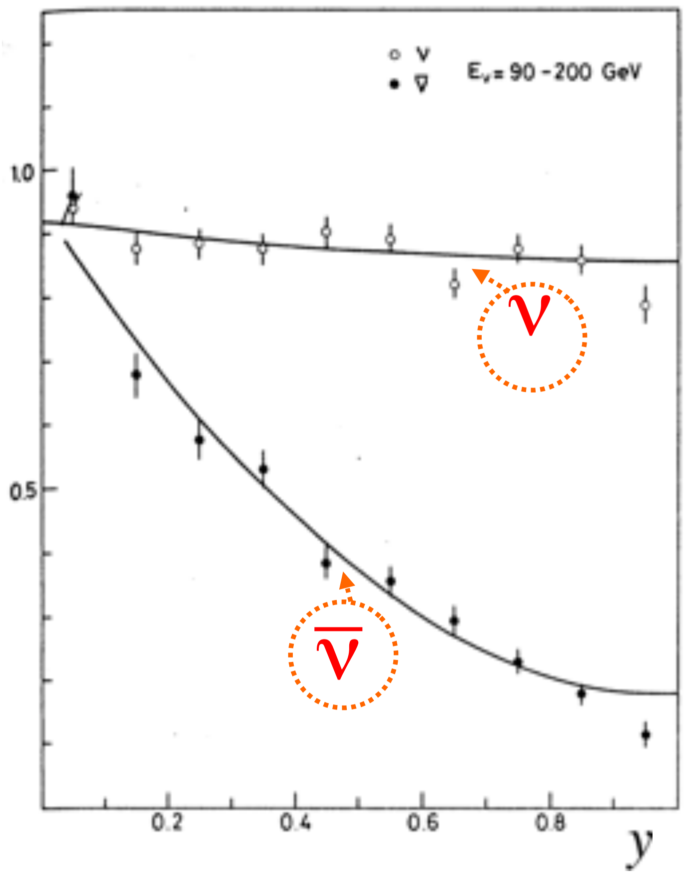
★ For each event can determine neutrino energy and y !

$$E_\nu = E_X + E_\mu$$

$$E_\mu = (1-y)E_\nu \quad \longrightarrow \quad y = \left(1 - \frac{E_\mu}{E_\nu}\right)$$

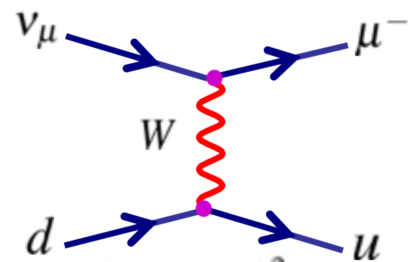
Measured y Distribution

CDHS measured y distribution

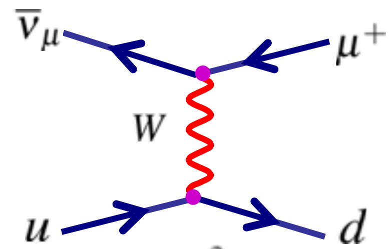


J. de Groot et al., Z.Phys. C1 (1979) 143

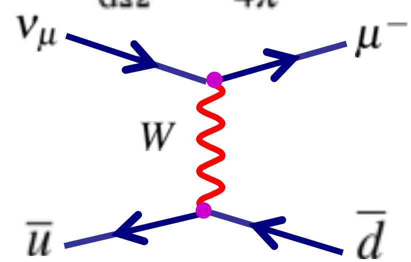
Shapes can be understood in terms of (anti)neutrino – (anti)quark scattering



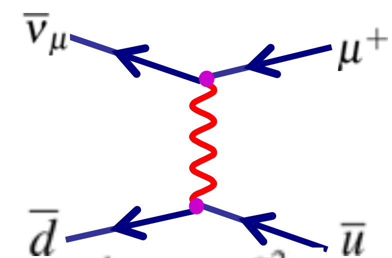
$$\frac{d\sigma_{\nu d}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$$



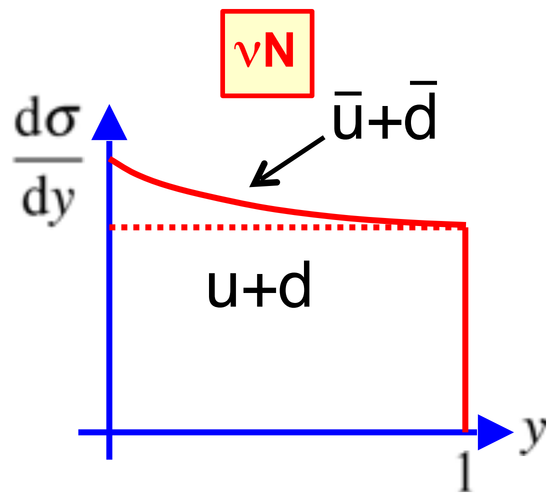
$$\frac{d\sigma_{\overline{\nu} u}}{d\Omega^*} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}$$

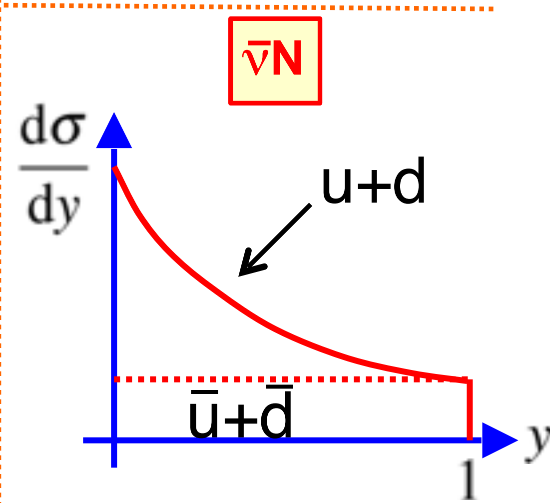


$$\frac{d\sigma_{\nu\overline{u}}}{d\Omega^*} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}$$



$$\frac{d\sigma_{\overline{\nu}\overline{d}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$$





Measured Total Cross Sections

★ Integrating the expressions for $\frac{d\sigma}{dy}$ (equations (6) and (7))

$$\boxed{\sigma^{\nu N} = \frac{G_F^2 s}{2\pi}\left[f_q + \frac{1}{3} f_{\bar{q}}\right]} \qquad \boxed{\sigma^{\bar{\nu} N} = \frac{G_F^2 s}{2\pi}\left[\frac{1}{3} f_q + f_{\bar{q}}\right]}$$

$$\underset{(E_\nu, 0, 0, +E_\nu)}{\nu} \longrightarrow \underset{(m_p, 0, 0, 0)}{p} \qquad s = (E_\nu + m_p)^2 - E_\nu^2 = 2E_\nu m_p + m_p^2 \approx 2E_\nu m_p$$

➡ **DIS cross section $\propto$ lab. frame neutrino energy**

★ Measure cross sections can be used to determine fraction of protons momentum carried by quarks, f_q , and fraction carried by anti-quarks, $f_{\bar{q}}$

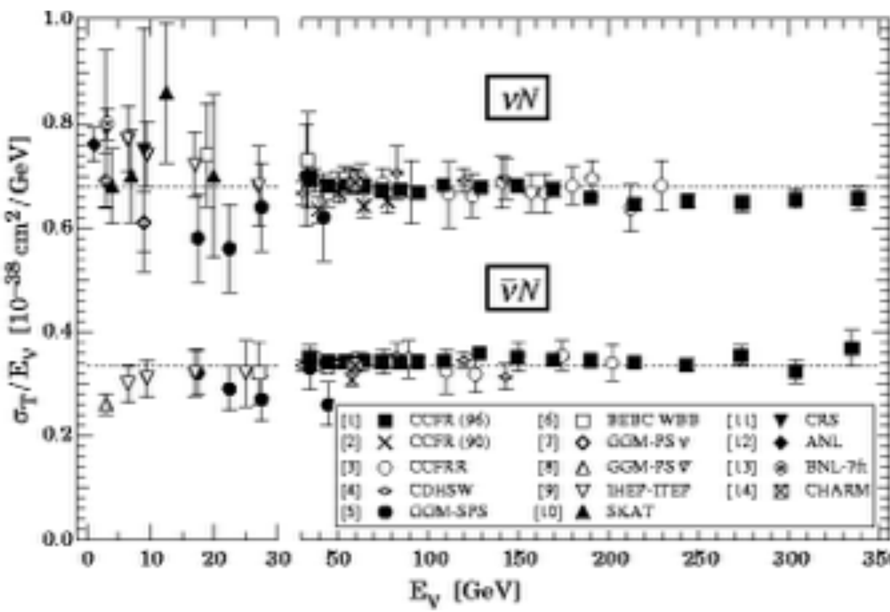
Find: $f_q \approx 0.41; \qquad f_{\bar{q}} \approx 0.08$

- ~50% of momentum carried by gluons (which don't interact with virtual W boson)
- If no anti-quarks in nucleons expect

$$\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} = 3$$

- Including anti-quarks

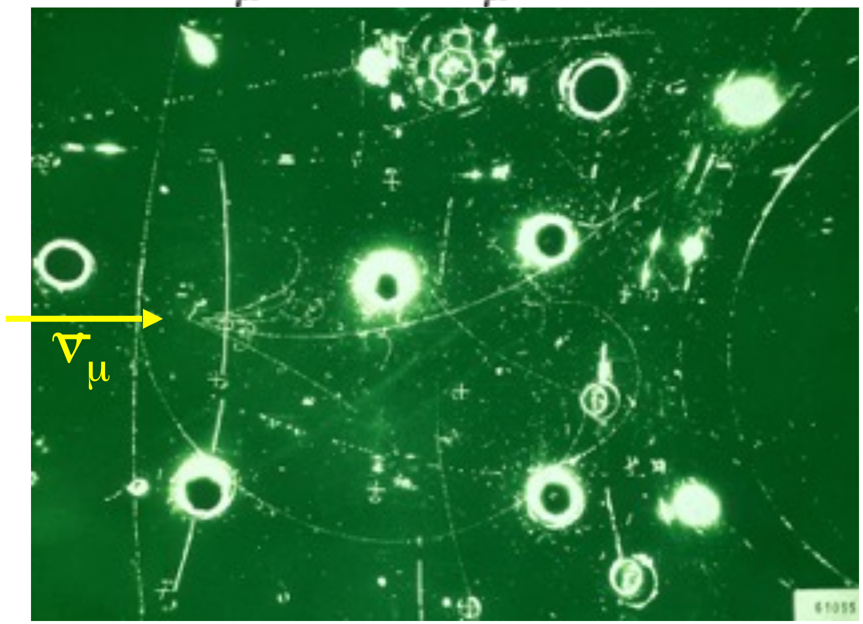
$$\boxed{\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} \approx 2}$$



Weak Neutral Current

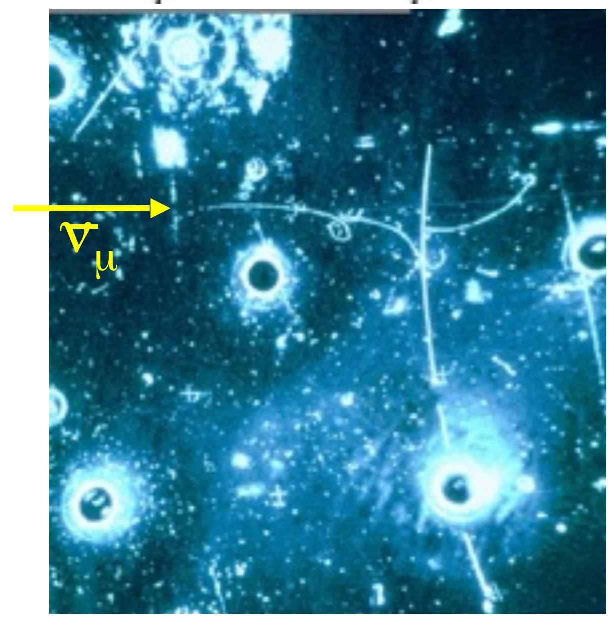
★ Neutrinos also interact via the Neutral Current. First observed in the Gargamelle bubble chamber in 1973. Interaction of muon neutrinos produce a final state muon

$$\overline{\nu}_\mu + N \rightarrow \overline{\nu}_\mu + \text{hadrons}$$



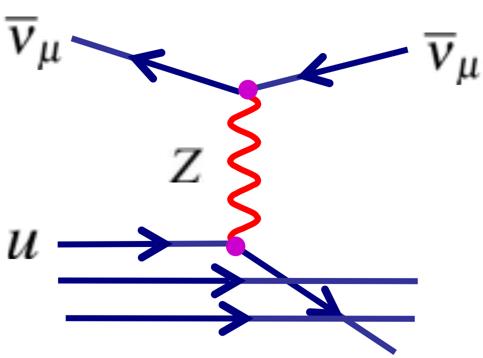
F.J. Hasert et al., Phys. Lett. 46B (1973) 138

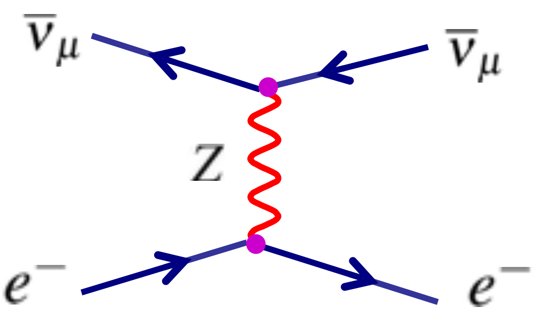
$$\overline{\nu}_\mu + e^- \rightarrow \overline{\nu}_\mu + e^-$$



F.J. Hasert et al., Phys. Lett. 46B (1973) 121

★ Cannot be due to W exchange - first evidence for Z boson





Summary

★ Derived neutrino/anti-neutrino – quark/anti-quark weak charged current (CC) interaction cross sections

★ Neutrino – nucleon scattering yields extra information about parton distributions functions:

- ν couples to d and $\overline{u}$; $\overline{\nu}$ couples to u and $\overline{d}$
  ➡ investigate flavour content of nucleon
- can measure anti-quark content of nucleon

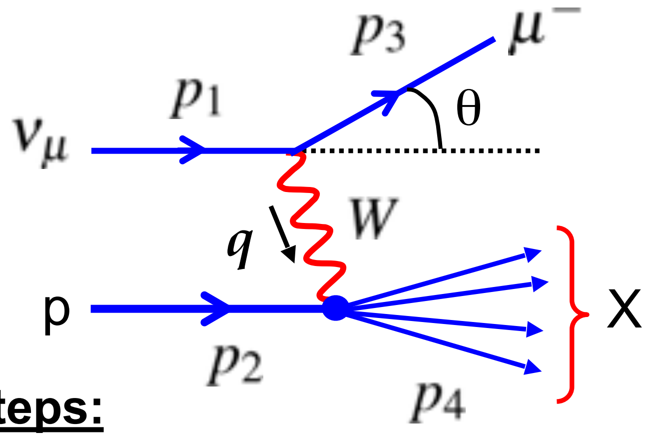
$\nu\overline{q}$ suppressed by factor $(1-y)^2$ compared with νq

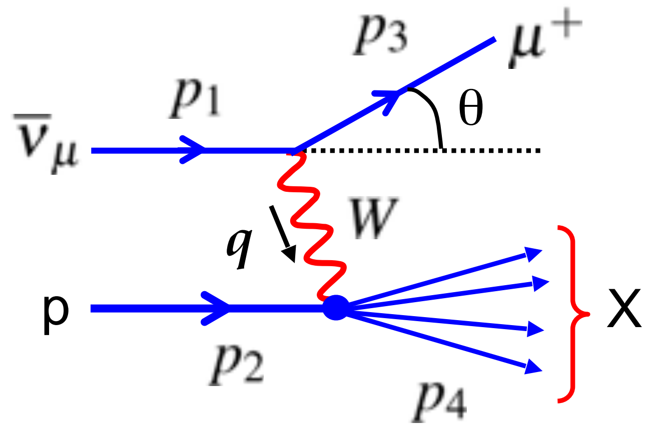
$\overline{\nu}q$ suppressed by factor $(1-y)^2$ compared with $\overline{\nu}\overline{q}$

★ Further aspects of neutrino deep-inelastic scattering (expressed in general structure functions) are covered in **Appendix II**

★ Finally observe that neutrinos interact via weak neutral currents (NC)

Appendix: Deep-Inelastic Neutrino Scattering





Two steps:

- **First write down most general cross section in terms of structure functions**
- **Then evaluate expressions in the quark-parton model**

QED Revisited

★**In the limit** $s \gg M^2$ **the most general electro-magnetic deep-inelastic cross section (from single photon exchange) can be written**

$$\frac{d^2\sigma_{e^\pm p}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

- **For neutrino scattering typically measure the energy of the produced muon** $E_\mu = E_\nu(1-y)$ **and differential cross-sections expressed in terms of** $dxdy$

- **Using** $Q^2 = (s-M^2)xy \approx sxy$ $\Rightarrow$ $\dfrac{d^2\sigma}{dxdy} = \left|\dfrac{dQ^2}{dy}\right|\dfrac{d^2\sigma}{dxdQ^2} = sx\dfrac{d^2\sigma}{dxdQ^2}$

♦ In the limit $s \gg M^2$ the general Electro-magnetic DIS cross section can be written

$$\frac{d^2\sigma^{e^\pm p}}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}\left[(1-y)F_2(x,Q^2) + y^2 x F_1(x,Q^2)\right]$$

NOTE: This is the most general **Lorentz Invariant** parity conserving expression

★ For neutrino DIS parity is violated and the general expression includes an additional term to allow for parity violation. New structure function $F_3(x,Q^2)$

$\nu_\mu p \to \mu^- X$

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu p}(x,Q^2) + y^2 x F_1^{\nu p}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x,Q^2)\right]$$

- For anti-neutrino scattering new structure function enters with opposite sign

$\overline{\nu}_\mu p \to \mu^+ X$

$$\frac{d^2\sigma^{\overline{\nu} p}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\overline{\nu} p}(x,Q^2) + y^2 x F_1^{\overline{\nu} p}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\overline{\nu} p}(x,Q^2)\right]$$

- Similarly for neutrino-neutron scattering

$\nu_\mu n \to \mu^- X$

$$\frac{d^2\sigma^{\nu n}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu n}(x,Q^2) + y^2 x F_1^{\nu n}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu n}(x,Q^2)\right]$$

$\overline{\nu}_\mu n \to \mu^+ X$

$$\frac{d^2\sigma^{\overline{\nu} n}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\overline{\nu} n}(x,Q^2) + y^2 x F_1^{\overline{\nu} n}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\overline{\nu} n}(x,Q^2)\right]$$

Neutrino Interaction Structure Functions

★In terms of the parton distribution functions we found (2) :

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2}{\pi} sx\left[d(x) + (1-y)^2 \overline{u}(x)\right]$$

Compare coefficients of y with the general Lorentz Invariant form and assume Bjorken scaling, i.e. $F(x,Q^2) \rightarrow F(x)$

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu p}(x) + y^2 x F_1^{\nu p}(x) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x)\right]$$

Re-writing (2)

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2}{2\pi} s\left[2xd(x) + 2x\overline{u}(x) - 4xy\overline{u}(x) + 2xy^2\overline{u}(x)\right]$$

and equating powers of **y**

$$\left.\begin{aligned} 2xd + 2x\overline{u} &= F_2 \\ -4x\overline{u} &= -F_2 + xF_3 \\ 2\overline{u} &= F_1 - xF_3/2 \end{aligned}\right\}$$

gives:

$$\boxed{\begin{aligned} F_2^{\nu p} = 2xF_1^{\nu p} &= 2x[d(x) + \overline{u}(x)] \\ xF_3^{\nu p} &= 2x[d(x) - \overline{u}(x)] \end{aligned}}$$

NOTE: again we get the Callan-Gross relation $F_2 = 2xF_1$

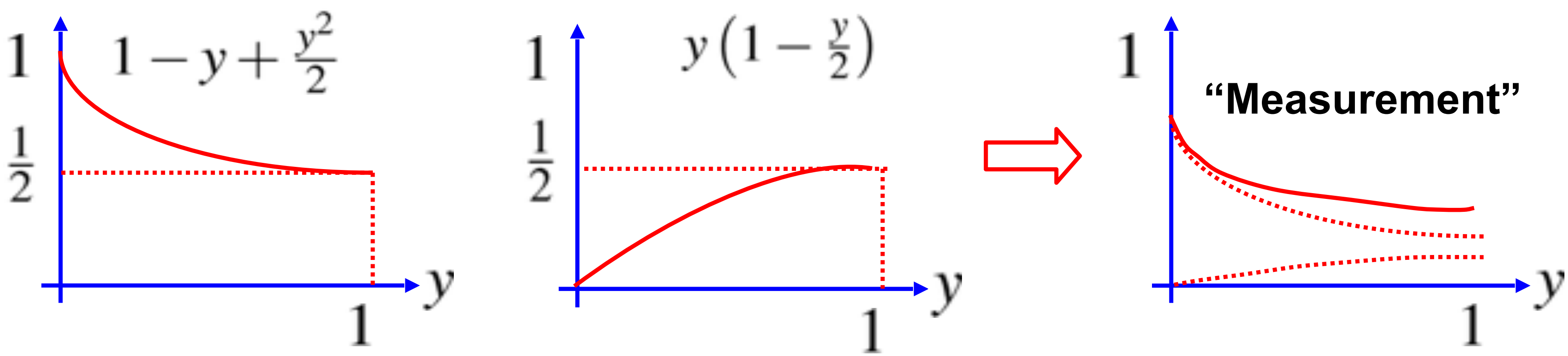
No surprise, underlying process is scattering from point-like spin-1/2 quarks

★ Substituting back in to expression for differential cross section:

$$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2 s}{2\pi}\left[\left(1 - y + \frac{y^2}{2}\right)F_2^{\nu p}(x) + y\left(1 - \frac{y}{2}\right)xF_3^{\nu p}(x)\right]$$

★ Experimentally measure F_2 and F_3 from y distributions at fixed x

- Different **y** dependencies (from different rest frame angular distributions) allow contributions from the two structure functions to be measured



★ Then use $F_2^{\nu p} = 2x[d(x) + \overline{u}(x)]$ and $F_3^{\nu p} = 2[d(x) - \overline{u}(x)]$

➡ Determine $d(x)$ and $\overline{u}(x)$ separately

★ Neutrino experiments require large detectors (often iron) i.e. isoscalar target

$$\boxed{\begin{aligned} F_2^{\nu N} = 2xF_1^{\nu N} &= \tfrac{1}{2}\left(F_2^{\nu p} + F_2^{\nu n}\right) = x[u(x) + d(x) + \overline{u}(x) + \overline{d}(x)] \\ xF_3^{\nu N} &= \tfrac{1}{2}\left(xF_3^{\nu p} + xF_3^{\nu n}\right) = x[u(x) + d(x) - \overline{u}(x) - \overline{d}(x)] \end{aligned}}$$

★ For electron – nucleon scattering:

$$F_2^{ep} = 2xF_1^{ep} = x[\tfrac{4}{9}u(x) + \tfrac{1}{9}d(x) + \tfrac{4}{9}\overline{u}(x) + \tfrac{1}{9}\overline{d}(x)]$$

$$F_2^{en} = 2xF_1^{en} = x[\tfrac{4}{9}d(x) + \tfrac{1}{9}u(x) + \tfrac{4}{9}\overline{d}(x) + \tfrac{1}{9}\overline{u}(x)]$$

For an isoscalar target

$$\boxed{F_2^{eN} = \tfrac{1}{2}\left(F_2^{ep} + F_2^{en}\right) = \tfrac{5}{18}x[u(x) + d(x) + \overline{u}(x) + \overline{d}(x)]}$$

$$\Rightarrow \boxed{F_2^{\nu N} = \tfrac{18}{5}F_2^{eN}}$$

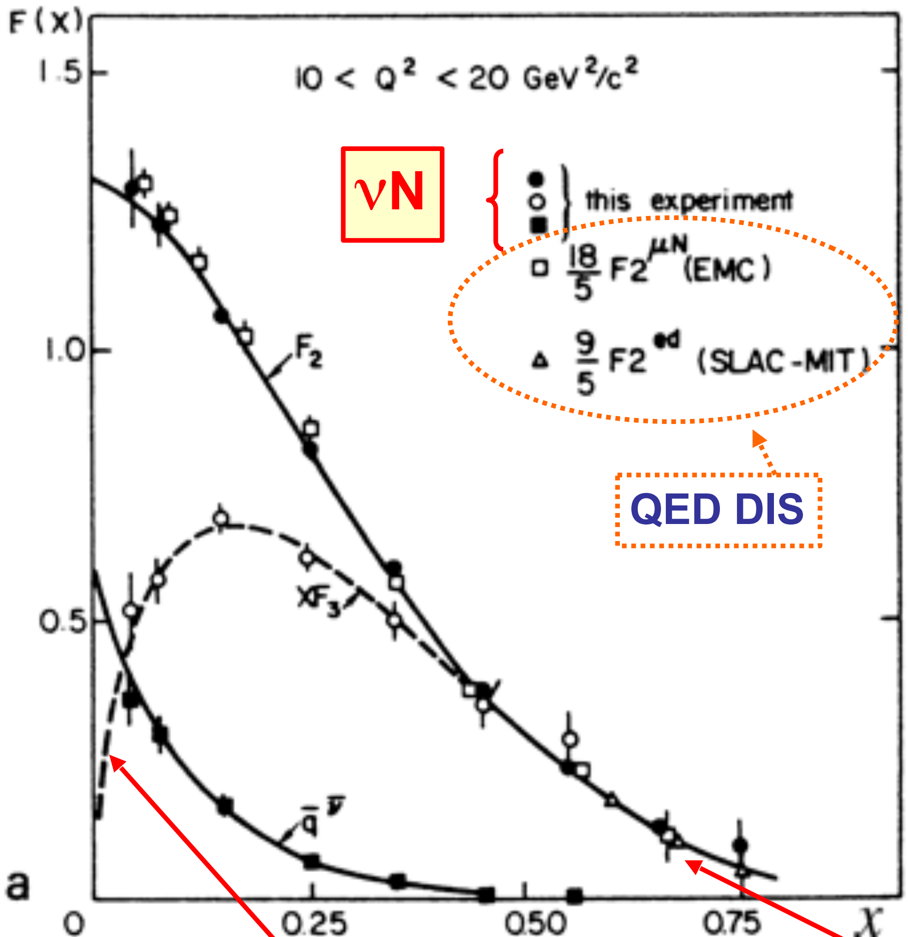
Note that the factor $\frac{5}{18} = \frac{1}{2}\left(q_u^2 + q_d^2\right)$ and by comparing neutrino to electron scattering structure functions measure the sum of quark charges

Experiment: 0.29 ± 0.02

5/18 = 0.278

Measurements of $F_2(x)$ and $F_3(x)$

CDHS Experiment $\nu_\mu + \mathrm{Fe} \to \mu^- + X$



H. Abramowicz et al., Z.Phys. C17 (1983) 283

$$F_2^{\nu N} = x[u(x)+d(x)+\overline{u}(x)+\overline{d}(x)]$$
$$xF_3^{\nu N} = x[u(x)+d(x)-\overline{u}(x)-\overline{d}(x)]$$

$$\Rightarrow \quad F_2^{\nu N} - xF_3^{\nu N} = 2x[\overline{u}+\overline{d}]$$

✶ **Difference in neutrino structure functions measures anti-quark (sea) parton distribution functions**

Sea dominates so expect xF_3 to go to zero as $q(x) = \overline{q}(x)$

Sea contribution goes to zero

Valence Contribution

★ Separate parton density functions into sea and valence components

$$u(x) = u_V(x) + u_S(x) = u_V(x) + S(x)$$
$$d(x) = d_V(x) + d_S(x) = d_V(x) + S(x)$$
$$\overline{u}(x) = \overline{u}_S(x) = S(x)$$
$$\overline{d}(x) = \overline{d}_S(x) = S(x)$$

➡ $$\boxed{F_3^{\nu N} = [u(x) + d(x) - \overline{u}(x) - \overline{d}(x)] = u_V(x) + d_V(x)}$$

➡ $$\int_0^1 F_3^{\nu N}(x)\mathrm{d}x = \int_0^1 (u_V(x) + d_V(x))\,\mathrm{d}x = N_u^V + N_d^V$$

★ Area under measured function $F_3^{\nu N}(x)$ gives a measurement of the total number of valence quarks in a nucleon !

expect $$\int_0^1 F_3^{\nu N}(x)\mathrm{d}x = 3$$ "Gross – Llewellyn-Smith sum rule"

Experiment: 3.0±0.2

Note: $F_2^{\overline{\nu}p} = F_2^{\nu n}$; $F_2^{\overline{\nu}n} = F_2^{\nu p}$; $F_3^{\overline{\nu}p} = F_3^{\nu n}$; $F_3^{\overline{\nu}n} = F_3^{\nu p}$ and anti-neutrino structure functions contain same pdf information