粒子物理

12. 深度非弹散射和简单部分子模型

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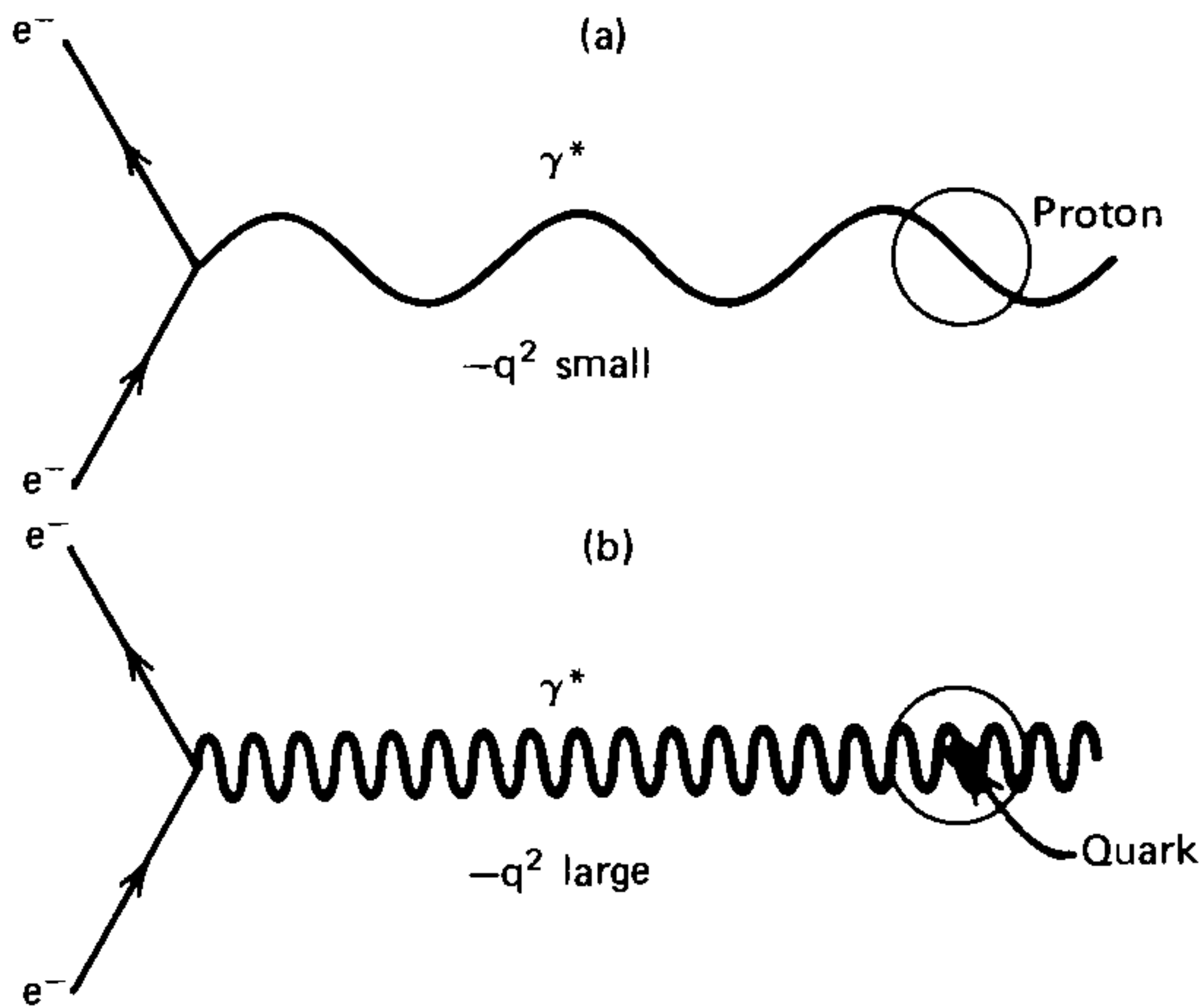
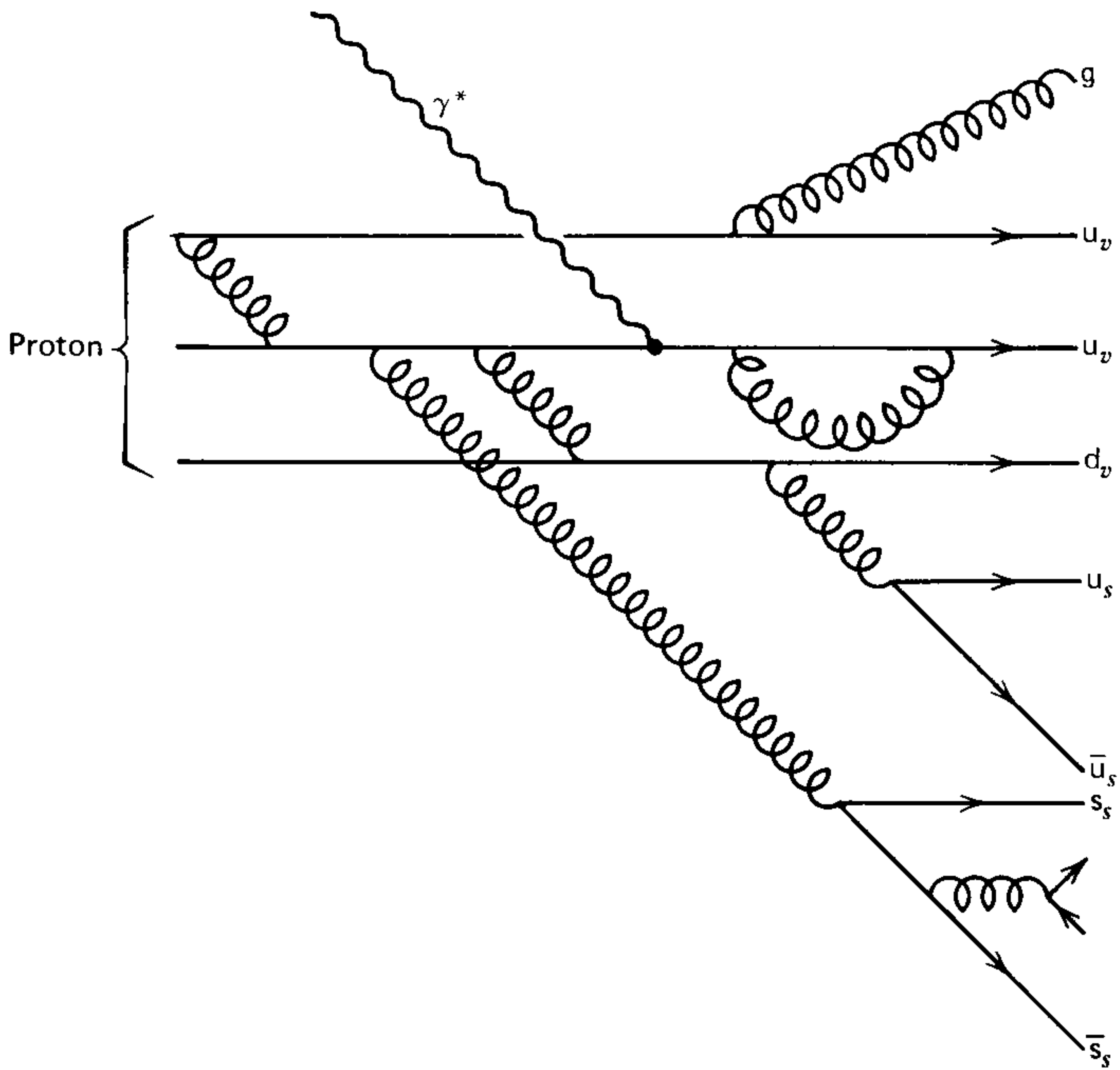
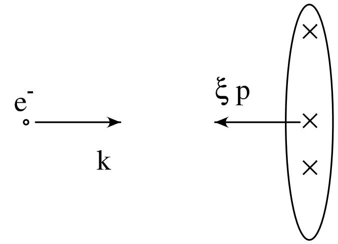

(a)
e⁻
γ*
Proton
−q² small
e⁻
e⁻
(b)
γ*
Quark
−q² large
e⁻



g
γ*
Proton
u_v
u_v
d_v
u_s
$\bar{u}_s$
s_s
$\bar{s}_s$

假设核子内部的部分子
具有特征时间长度

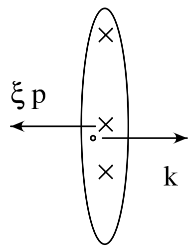
$$\tau > \tau_0$$



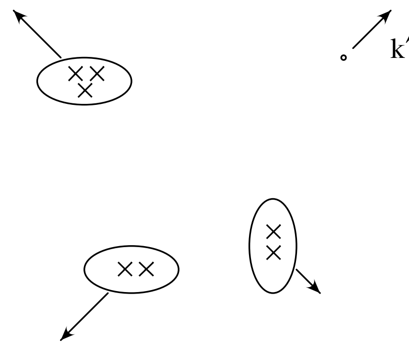
(a)

时间因子:

$$\tau(1 - v^{*2}/c^2)^{-1/2} \gg \tau$$



(b)



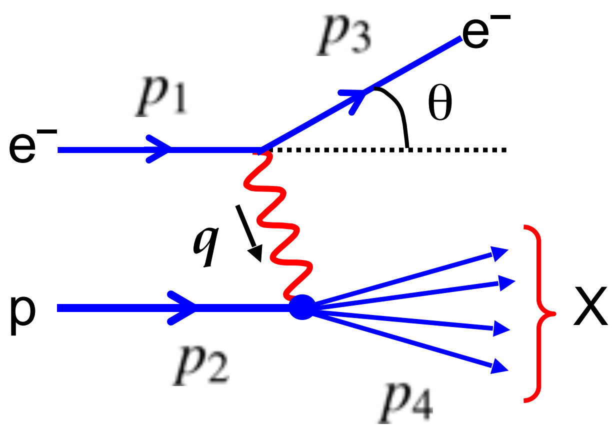
(c)

虚光子看到额外部分子的几率:

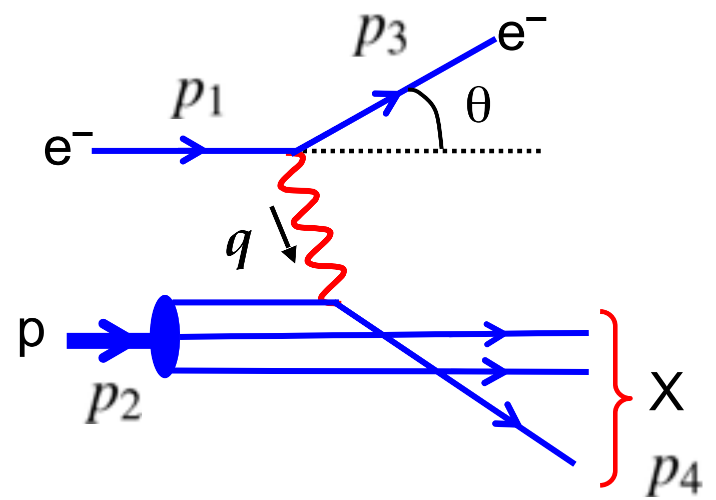
$$\frac{1/Q^2}{\pi R_0^2}$$

The Quark-Parton Model

- Before quarks and gluons were generally accepted Feynman proposed that the proton was made up of point-like constituents "partons"
- Both Bjorken Scaling and the Callan-Gross relationship can be explained by assuming that Deep Inelastic Scattering is dominated by the scattering of a single virtual photon from point-like spin-half constituents of the proton.



Scattering from a proton with structure functions



Scattering from a point-like quark within the proton

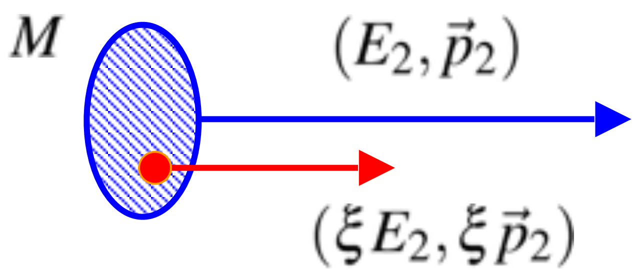
★ How do these two pictures of the interaction relate to each other?

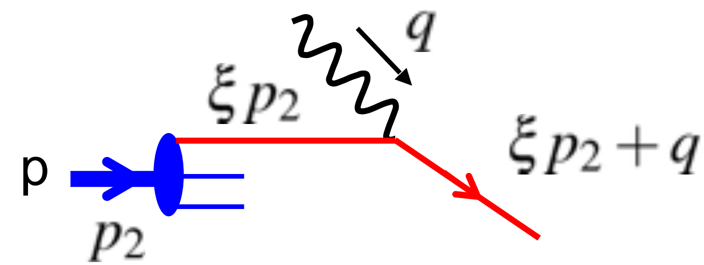
- In the parton model the basic interaction is **ELASTIC** scattering from a **"quasi-free"** spin-½ quark in the proton, i.e. treat the quark as a free particle!

- The parton model is most easily formulated in a frame where the proton has very high energy, often referred to as the **"infinite momentum frame"**, where we can neglect the proton mass and $p_2 = (E_2, 0, 0, E_2)$

- In this frame can also neglect the mass of the quark and any momentum transverse to the direction of the proton.

- Let the quark carry a fraction ξ of the proton's four-momentum.





- After the interaction the struck quark's four-momentum is $\xi p_2 + q$

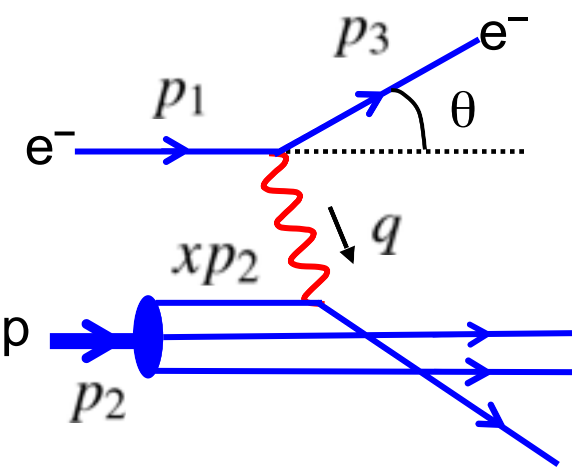
$$(\xi p_2 + q)^2 = m_q^2 \approx 0 \quad \Rightarrow \quad \cancel{\xi^2 p_2^2} + q^2 + 2\xi p_2 . q = 0 \qquad (\xi^2 p_2^2 = m_q^2 \approx 0)$$

$$\Rightarrow \quad \xi = \frac{Q^2}{2 p_2 . q} = x$$

Bjorken x can be identified as the fraction of the proton momentum carried by the struck quark (in a frame where the proton has very high energy)

• In terms of the proton momentum

$$s = (p_1 + p_2)^2 \simeq 2p_1.p_2 \qquad y = \frac{p_2.q}{p_2.p_1} \qquad x = \frac{Q^2}{2p_2.q}$$



• But for the underlying quark interaction

$$s^q = (p_1 + xp_2)^2 = 2xp_1.p_2 = xs$$

$$y_q = \frac{p_q.q}{p_q.p_1} = \frac{xp_2.q}{xp_2.p_1} = y$$

$x_q = 1$ **(elastic, i.e. assume quark does not break up)**

• Previously derived the Lorentz Invariant cross section for $e^-\mu^- \rightarrow e^-\mu^-$ elastic scattering in the ultra-relativistic limit (handout 4 + Q10 on examples sheet).

Now apply this to $e^-q \rightarrow e^-q$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 e_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s_q}\right)^2\right]$$

e_q **is quark charge, i.e.** $e_u = +2/3; \quad e_d = -1/3$

• Using $-q^2 = Q^2 = (s_q - m^2)x_q y_q \quad \Rightarrow \quad \frac{q^2}{s_q} = -y_q = -y$

$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1 + (1-y)^2\right]$$

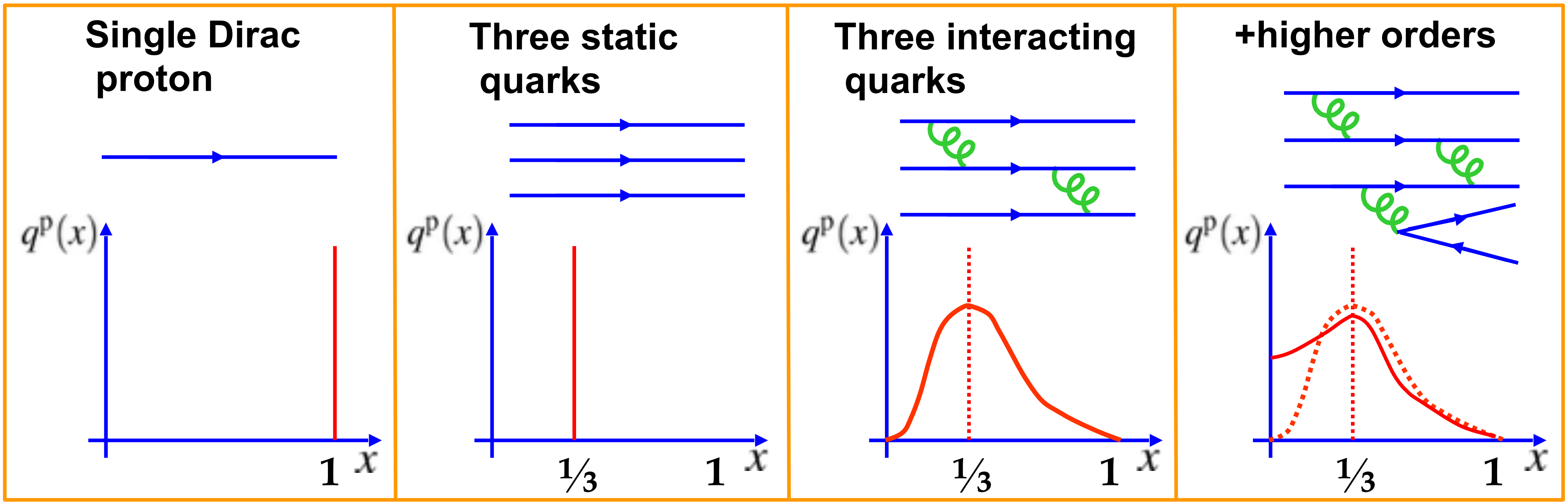
$$\boxed{\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]} \qquad (3)$$

★This is the expression for the differential cross-section for **elastic** e⁻q scattering from a quark carrying a fraction x of the proton momentum.

- Now need to account for distribution of quark momenta within proton

★ Introduce parton distribution functions such that $q^{\mathrm{p}}(x)\mathrm{d}x$ is the number of quarks of type q within a proton with momenta between $x \rightarrow x+\mathrm{d}x$

Expected **form of the parton distribution function ?**



- The cross section for scattering from a **particular quark type** within the proton which in the range $x \rightarrow x + \mathrm{d}x$ is

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right] \times e_q^2 q^{\mathrm{p}}(x)\mathrm{d}x$$

- Summing over all types of quark within the proton gives the expression for the electron-proton scattering cross section

$$\boxed{\frac{\mathrm{d}^2\sigma^{\mathrm{ep}}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\sum_q e_q^2 q^{\mathrm{p}}(x)} \qquad (5)$$

- Compare with the electron-proton scattering cross section in terms of structure functions (equation (2)):

$$\boxed{\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]} \qquad (6)$$

- By comparing (5) and (6) obtain the parton model prediction for the structure functions in the general L.I. form for the differential cross section

$$\boxed{F_2^{\mathrm{p}}(x,Q^2) = 2xF_1^{\mathrm{p}}(x,Q^2) = x\sum_q e_q^2 q^{\mathrm{p}}(x)} \Longrightarrow$$

Can relate measured structure functions to the underlying quark distributions

The parton model predicts:

- **Bjorken Scaling** $F_1(x,Q^2) \rightarrow F_1(x) \quad F_2(x,Q^2) \rightarrow F_2(x)$

  Due to scattering from point-like particles within the proton

- **Callan-Gross Relation** $F_2(x) = 2xF_1(x)$

  Due to scattering from spin half Dirac particles where the magnetic moment is directly related to the charge; hence the "electro-magnetic" and "pure magnetic" terms are fixed with respect to each other.

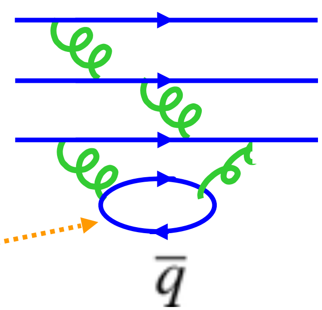
- **At present parton distributions cannot be calculated from QCD**

  Can't use perturbation theory due to large coupling constant

- **Measurements of the structure functions enable us to determine the parton distribution functions !**

- **For electron-proton scattering we have:**

$$F_2^{\mathrm{p}}(x) = x\sum_q e_q^2 q^{\mathrm{p}}(x)$$



- **Due to higher orders, the proton contains not only up and down quarks but also anti-up and anti-down quarks**

(will neglect the small contributions from heavier quarks)

For electron-proton scattering have:

$$F_2^{\mathrm{ep}}(x) = x\sum_q e_q^2 q^{\mathrm{p}}(x) = x\left(\frac{4}{9}u^{\mathrm{p}}(x) + \frac{1}{9}d^{\mathrm{p}}(x) + \frac{4}{9}\overline{u}^{\mathrm{p}}(x) + \frac{1}{9}\overline{d}^{\mathrm{p}}(x)\right)$$

For electron-neutron scattering have:

$$F_2^{\mathrm{en}}(x) = x\sum_q e_q^2 q^{\mathrm{n}}(x) = x\left(\frac{4}{9}u^{\mathrm{n}}(x) + \frac{1}{9}d^{\mathrm{n}}(x) + \frac{4}{9}\overline{u}^{\mathrm{n}}(x) + \frac{1}{9}\overline{d}^{\mathrm{n}}(x)\right)$$

Now assume "isospin symmetry", i.e. that the neutron (ddu) is the same as a proton (uud) with up and down quarks interchanged, i.e.

$$d^{\mathrm{n}}(x) = u^{\mathrm{p}}(x); \qquad u^{\mathrm{n}}(x) = d^{\mathrm{p}}(x)$$

and define the neutron distributions functions in terms of those of the proton

$$u(x) \equiv u^{\mathrm{p}}(x) = d^{\mathrm{n}}(x); \qquad d(x) \equiv d^{\mathrm{p}}(x) = u^{\mathrm{n}}(x)$$
$$\overline{u}(x) \equiv \overline{u}^{\mathrm{p}}(x) = \overline{d}^{\mathrm{n}}(x); \qquad \overline{d}(x) \equiv \overline{d}^{\mathrm{p}}(x) = \overline{u}^{\mathrm{n}}(x)$$

giving:

$$\boxed{F_2^{\mathrm{ep}}(x) = 2xF_1^{\mathrm{ep}}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x)\right)} \quad (7)$$

$$\boxed{F_2^{\mathrm{en}}(x) = 2xF_1^{\mathrm{en}}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\overline{d}(x) + \frac{1}{9}\overline{u}(x)\right)} \quad (8)$$

Integrating (7) and (8) :

$$\int_0^1 F_2^{ep}(x)\mathrm{d}x = \int_0^1 x\left(\frac{4}{9}[u(x)+\bar{u}(x)] + \frac{1}{9}[d(x)+\bar{d}(x)]\right)\mathrm{d}x = \frac{4}{9}f_u + \frac{1}{9}f_d$$

$$\int_0^1 F_2^{en}(x)\mathrm{d}x = \int_0^1 x\left(\frac{4}{9}[d(x)+\bar{d}(x)] + \frac{1}{9}[u(x)+\bar{u}(x)]\right)\mathrm{d}x = \frac{4}{9}f_d + \frac{1}{9}f_u$$

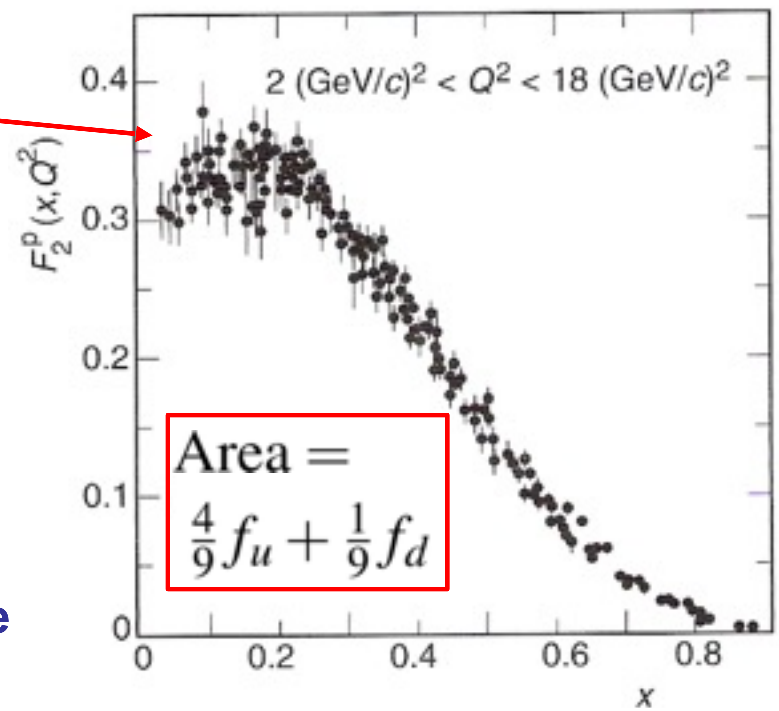
$$f_u = \int_0^1 [xu(x) + x\bar{u}(x)]\mathrm{d}x$$ is the fraction of the proton momentum carried by the up and anti-up quarks

Experimentally

$$\int F_2^{ep}(x)\mathrm{d}x \approx 0.18$$

$$\int F_2^{en}(x)\mathrm{d}x \approx 0.12$$

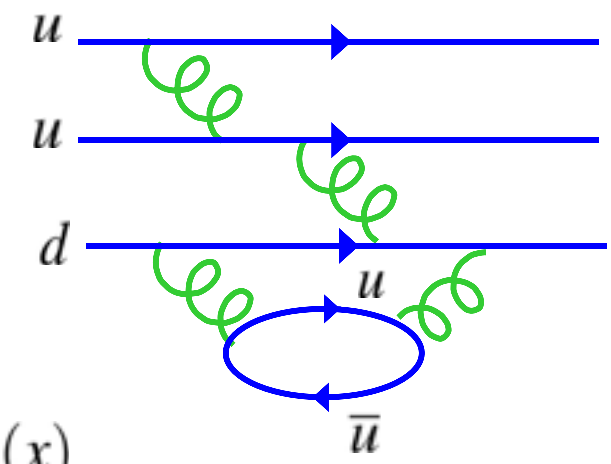
$$\Rightarrow \boxed{f_u \approx 0.36 \qquad f_d \approx 0.18}$$



- In the proton, as expected, the up quarks carry twice the momentum of the down quarks
- The quarks carry just over 50% of the total proton momentum. The rest is carried by gluons (which being neutral doesn't contribute to electron-nucleon scattering).

Valence and Sea Quarks

- As we are beginning to see the proton is complex…
- The parton distribution function $u^{p}(x) = u(x)$ includes contributions from the **"valence"** quarks and the virtual quarks produced by gluons: the **"sea"**



- Resolving into valence and sea contributions:

$$u(x) = u_V(x) + u_S(x) \qquad d(x) = d_V(x) + d_S(x)$$
$$\overline{u}(x) = \overline{u}_S(x) \qquad \overline{d}(x) = \overline{d}_S(x)$$

- The proton contains two valence up quarks and one valence down quark and would expect:

$$\int_0^1 u_V(x)\mathrm{d}x = 2 \qquad \int_0^1 d_V(x)\mathrm{d}x = 1$$

- But no *a priori* expectation for the total number of sea quarks !

  Sea quarks arise from gluon quark/anti-quark pair production and with $m_u = m_d$ it is reasonable to expect

$$u_S(x) = d_S(x) = \overline{u}_S(x) = \overline{d}_S(x) = S(x)$$

- With these relations (7) and (8) become

$$F_2^{ep}(x) = x\left(\frac{4}{9}u_V(x) + \frac{1}{9}d_V(x) + \frac{10}{9}S(x)\right) \qquad F_2^{en}(x) = x\left(\frac{4}{9}d_V(x) + \frac{1}{9}u_V(x) + \frac{10}{9}S(x)\right)$$

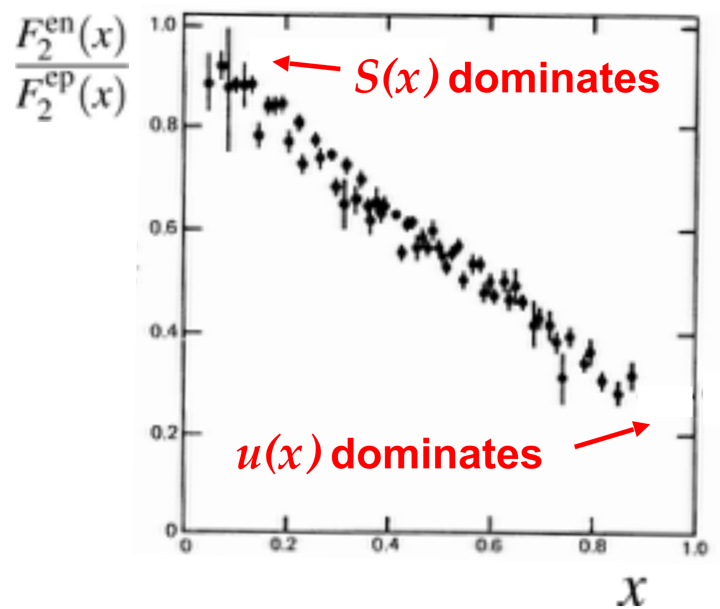
Giving the ratio $$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} = \frac{4d_V(x)+u_V(x)+10S(x)}{4u_V(x)+d_V(x)+10S(x)}$$

- The sea component arises from processes such as $g \longrightarrow \overline{u}u$. Due to the $1/q^2$ dependence of the gluon propagator, much more likely to produce low energy gluons. Expect the sea to comprise of low energy $q/\overline{q}$

- Therefore at low x expect the sea to dominate:

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} \to 1 \quad \text{as} \quad x \to 0$$

Observed experimentally



- At high x expect the sea contribution to be small

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} \to \frac{4d_V(x)+u_V(x)}{4u_V(x)+d_V(x)} \quad \text{as} \quad x \to 1$$

Note: $u_V = 2d_V$ would give ratio 2/3 as $x \to 1$

Experimentally $F_2^{\mathrm{en}}(x)/F_2^{\mathrm{ep}}(x) \to 1/4 \quad \text{as} \quad x \to 1$

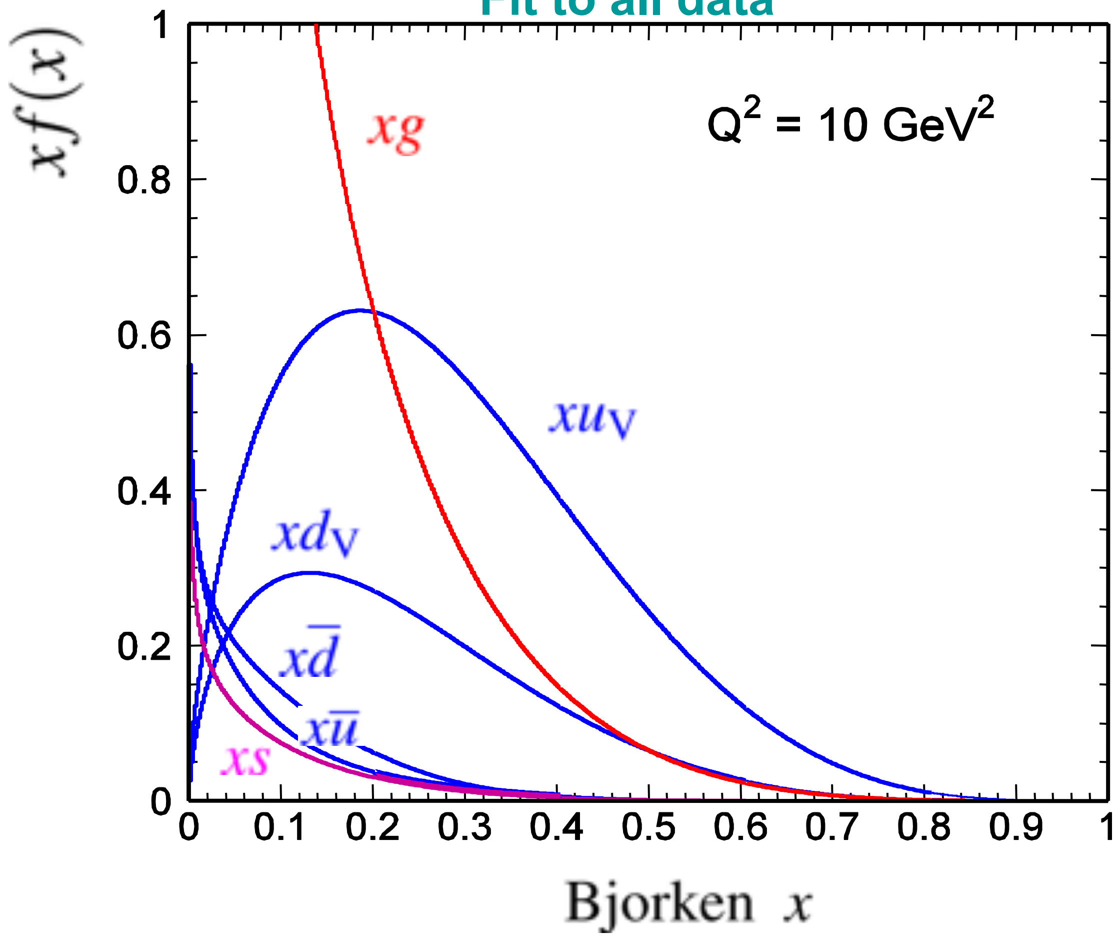
$\Rightarrow$ $d(x)/u(x) \to 0 \quad \text{as} \quad x \to 1$

This behaviour is not understood.

Parton Distribution Functions

- Ultimately the parton distribution functions are obtained from a fit to all experimental data including neutrino scattering
  - Hadron-hadron collisions give information on gluon pdf $g(x)$

Fit to all data

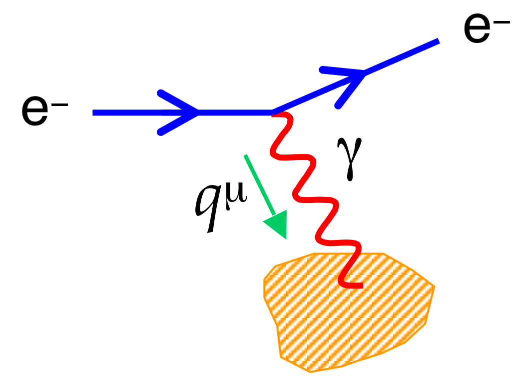


Note:

- Apart from at large x
  $u_V(x) \approx 2d_V(x)$
- For $x < 0.2$
  gluons dominate
- In fits to data assume
  $u_s(x) = \bar{u}(x)$
- $\bar{d}(x) > \bar{u}(x)$
  not understood – exclusion principle?
- Small strange quark component $s(x)$

Scaling Violations

- In last 40 years, experiments have probed the proton with virtual photons of ever increasing energy

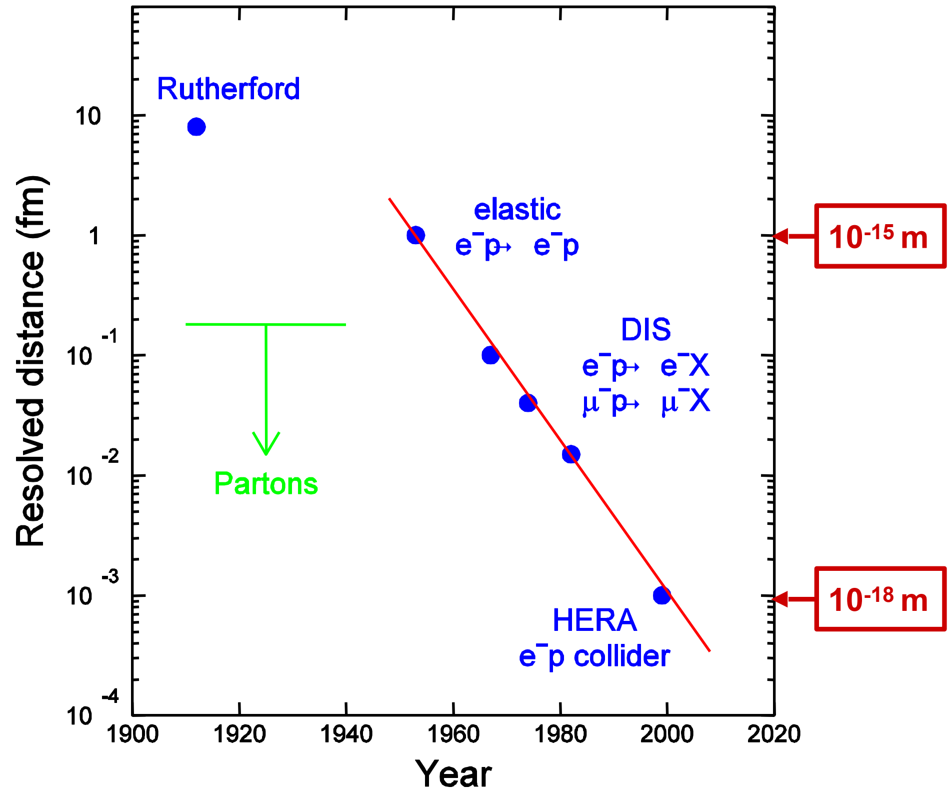


- Non-point like nature of the scattering becomes apparent when λ_γ ~ size of scattering centre

$$\lambda_\gamma = \frac{h}{|\vec{q}|} \sim \frac{1\,\mathrm{GeV\,fm}}{|\vec{q}|(\mathrm{GeV})}$$

- Scattering from point-like quarks gives rise to **Bjorken scaling**: no q^2 cross section dependence
- IF quarks were not point-like, at high q^2 (when the wavelength of the virtual photon ~ size of quark) would observe rapid decrease in cross section with increasing q^2.
- To search for quark sub-structure want to go to highest q^2

HERA

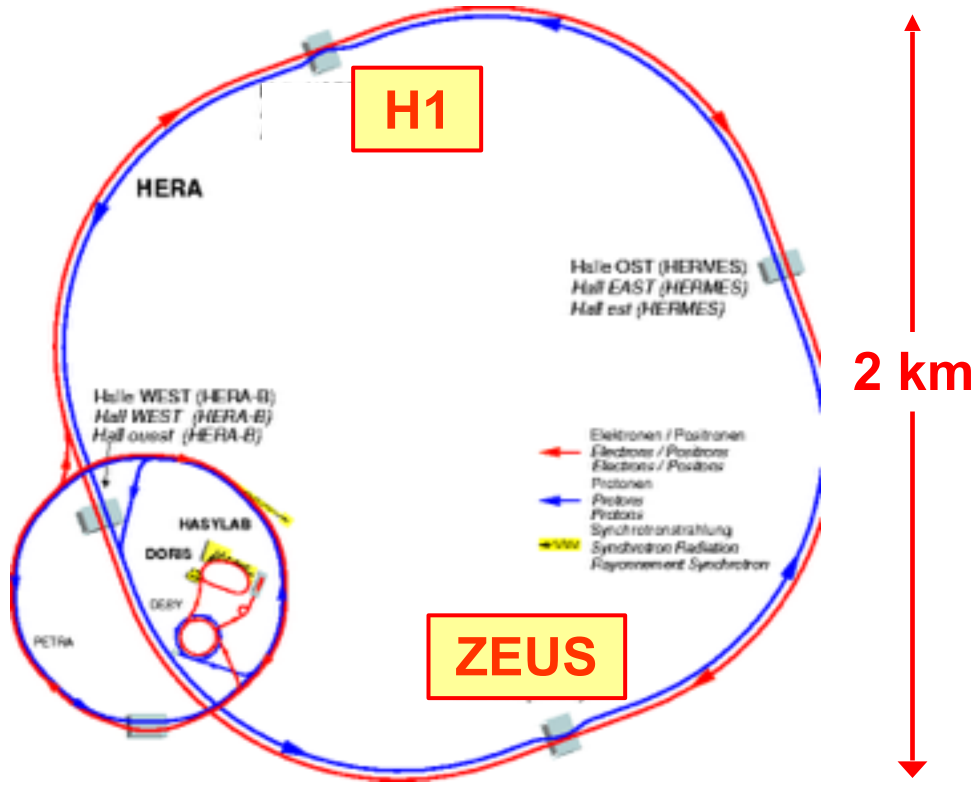


HERA $e^{\pm}p$ Collider : 1991-2007

★ **DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany**

$e^{\pm}$ **27.5 GeV** → ← **820 GeV** p

$\sqrt{s} = 300$ **GeV**

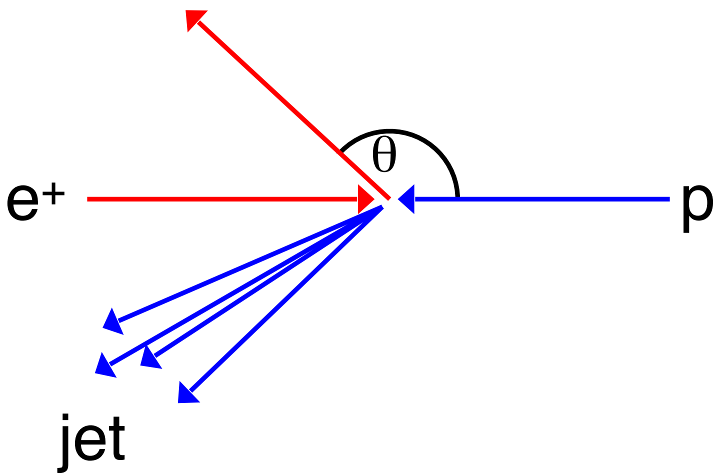


★ **Two large experiments : H1 and ZEUS**

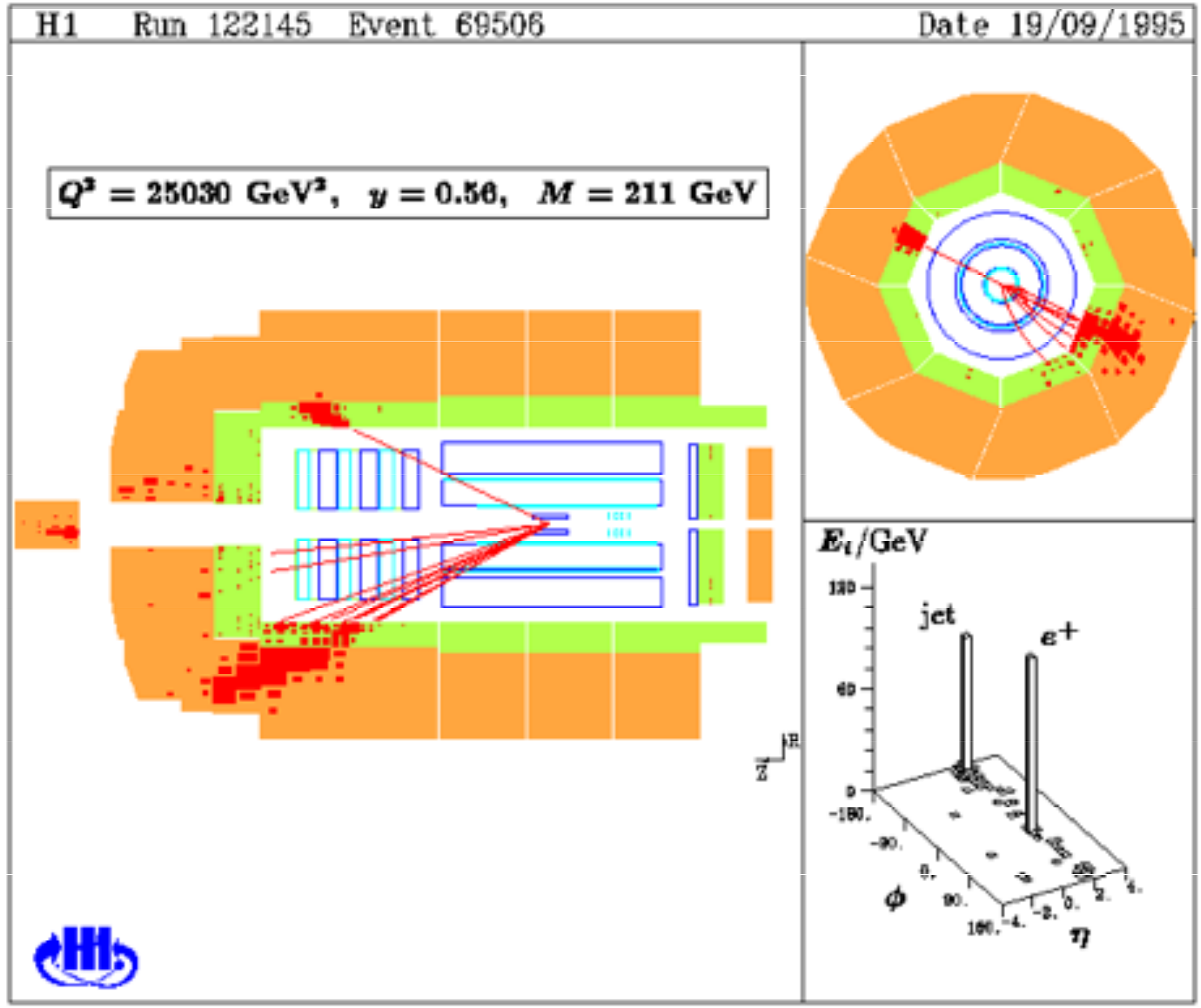
★ **Probe proton at very high Q^2 and very low x**

Example of a High Q^2 Event in H1

✶Event kinematics determined from electron angle and energy



✶Also measure hadronic system (although not as precisely) – gives some redundancy



$F_2(x,Q^2)$ Results

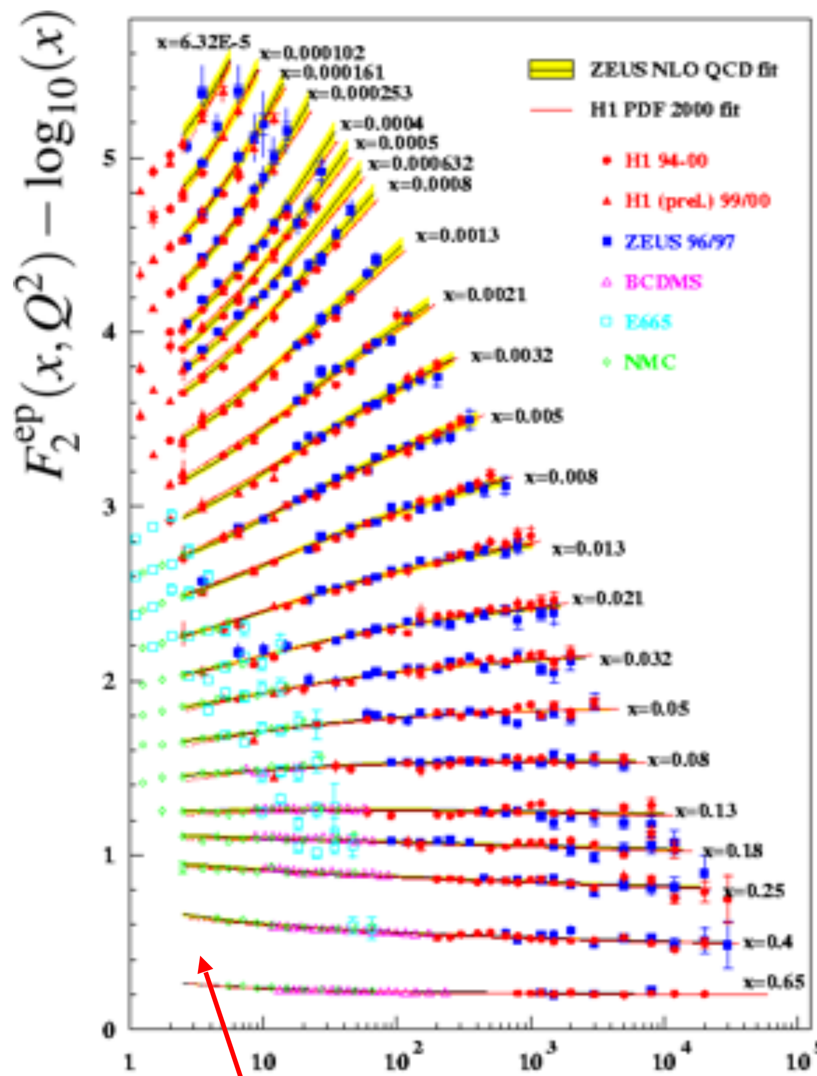
★ No evidence of rapid decrease of cross section at highest Q^2

➡ $\boxed{R_{\text{quark}} < 10^{-18}\,\text{m}}$

★ For $x > 0.05$, only weak dependence of F_2 on Q^2 : consistent with the expectation from the quark-parton model

★ But observe clear scaling violations, particularly at low x

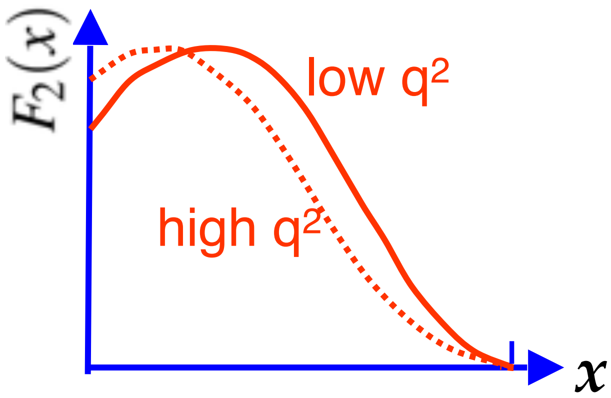
$$F_2(x,Q^2) \neq F_2(x)$$

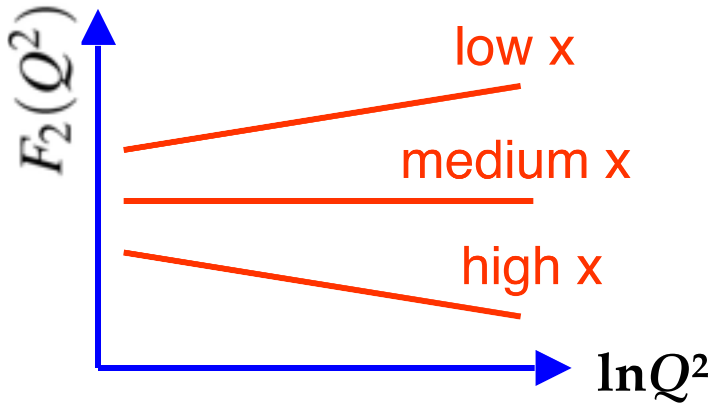


Earlier fixed target data

Origin of Scaling Violations

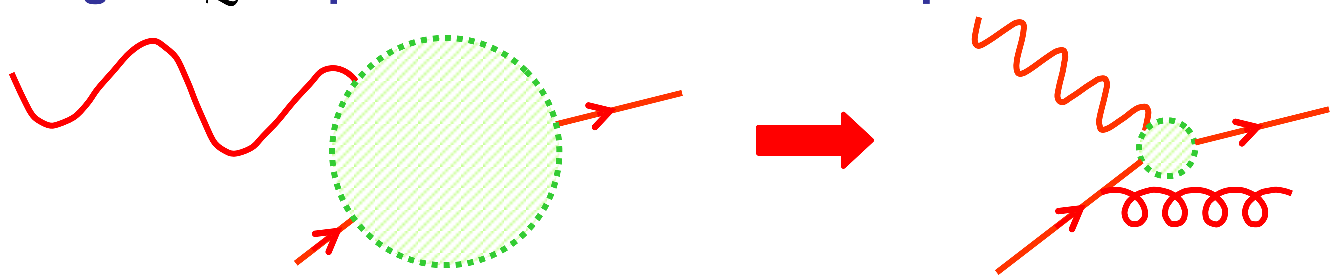
★ Observe "small" deviations from **exact Bjorken scaling** $F_2(x) \rightarrow F_2(x, Q^2)$





★ At high Q^2 observe more low x quarks

★ **"Explanation":** at high Q^2 (shorter wave-length) resolve finer structure: i.e. reveal quark is sharing momentum with gluons. At higher Q^2 expect to "see" more low x quarks



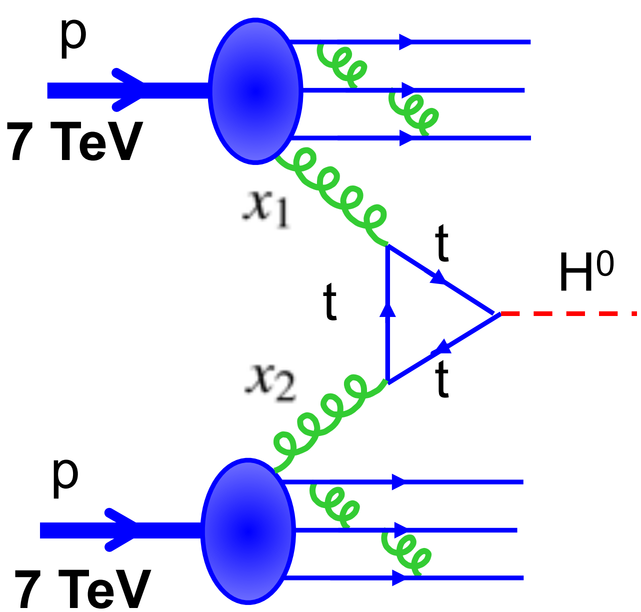
★ QCD cannot predict the x dependence of $F_2(x, Q^2)$

★ But QCD **can** predict the Q^2 dependence of $F_2(x, Q^2)$

Proton-Proton Collisions at the LHC

★ Measurements of structure functions not only provide a powerful test of QCD, the **parton distribution functions** are essential for the calculation of cross sections at pp and $p\bar{p}$ colliders.

- **Example:** Higgs production at the Large Hadron Collider LHC (2009-)
  - The LHC will collide 7 TeV protons on 7 TeV protons
  - However underlying collisions are between partons
  - Higgs production the LHC dominated by "gluon-gluon fusion"



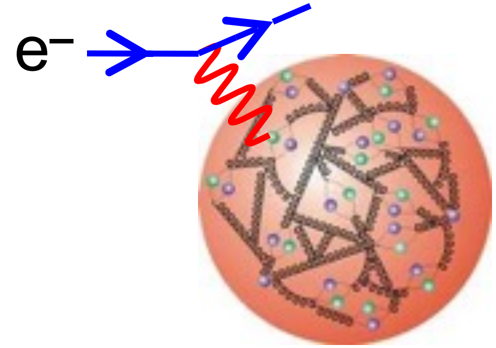
- Cross section depends on gluon PDFs

$$\sigma(pp \to HX) \sim \int_0^1 \int_0^1 g(x_1)g(x_2)\sigma(gg \to H)\mathrm{d}x_1\mathrm{d}x_2$$

- Uncertainty in gluon PDFs lead to a ±5 % uncertainty in Higgs production cross section

- Prior to HERA data uncertainty was ±25 %

Summary



- At very high electron energies $\lambda \ll r_p$:
  the proton appears to be a sea of quarks and gluons.

- Deep Inelastic Scattering = Elastic scattering from the quasi-free constituent quarks

  ➡ Bjorken Scaling $F_1(x,Q^2) \rightarrow F_1(x)$ point-like scattering

  ➡ Callan-Gross $F_2(x) = 2xF_1(x)$ Scattering from spin-1/2

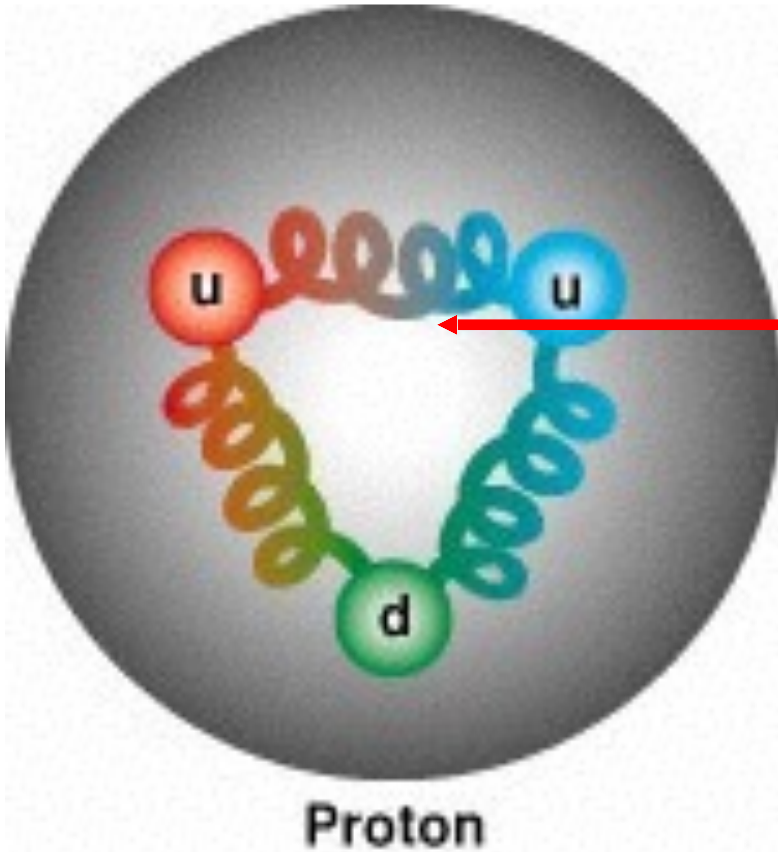
- Describe scattering in terms of parton distribution functions $u(x), d(x), \ldots$ which describe momentum distribution inside a nucleon

- The proton is much more complex than just uud - sea of anti-quarks/gluons

- Quarks carry only 50% of the protons momentum – the rest is due to low energy gluons

- We will come back to this topic when we discuss neutrino scattering…

Gluons bind quark together



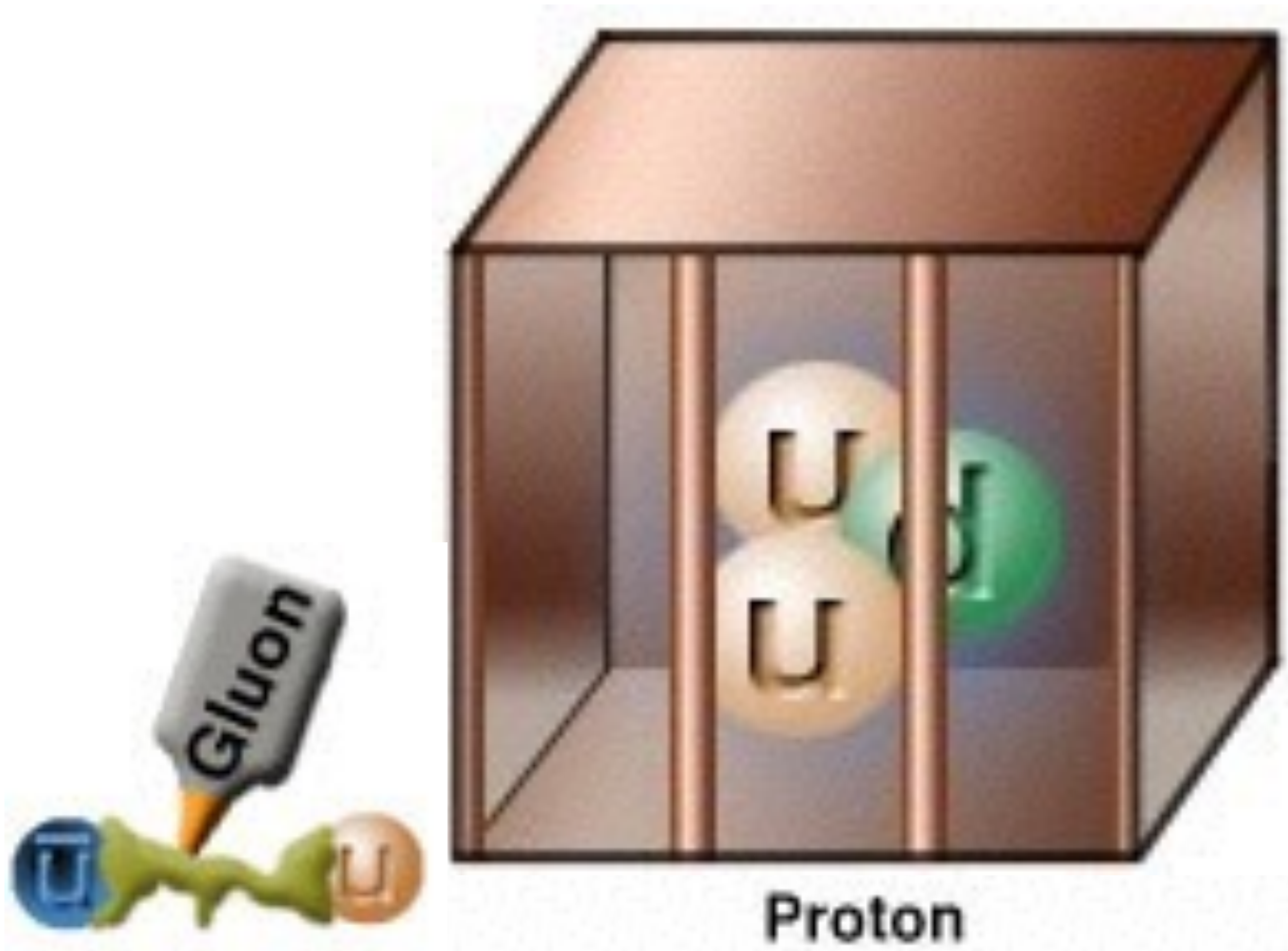
The quarks are stuck together by the exchange of gluon, which is finally understood by Quantum Chromodynamics (Asymptotic Freedom in high energy).

Gross, Wilczek, Politzer, 1973

1973 DESY(PETRA): confirm gluon in $e^+ e^- \longrightarrow$ 3 jets

Gross, Wilczek, Politzer, 2004 Nobel Prize

Gluons bind quark together



In low energy, we can't see free quark. The quarks are confined inside the proton.

1 million dollars problem!

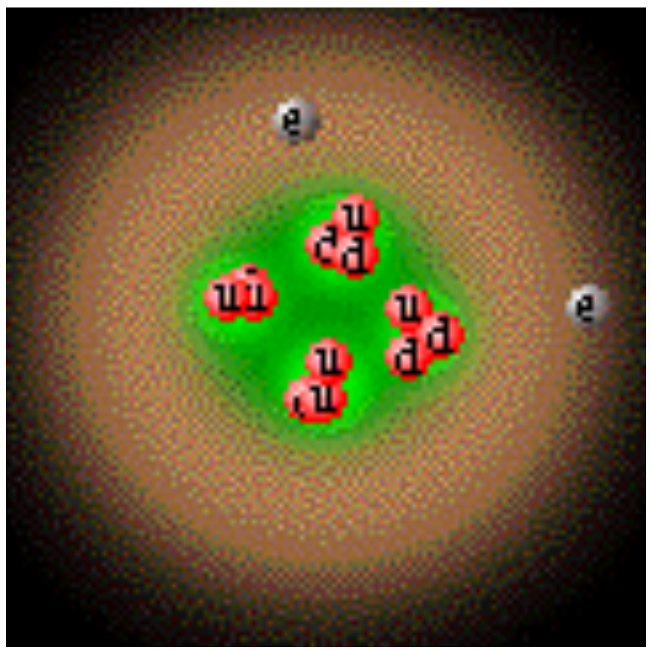
(infrared confinement)

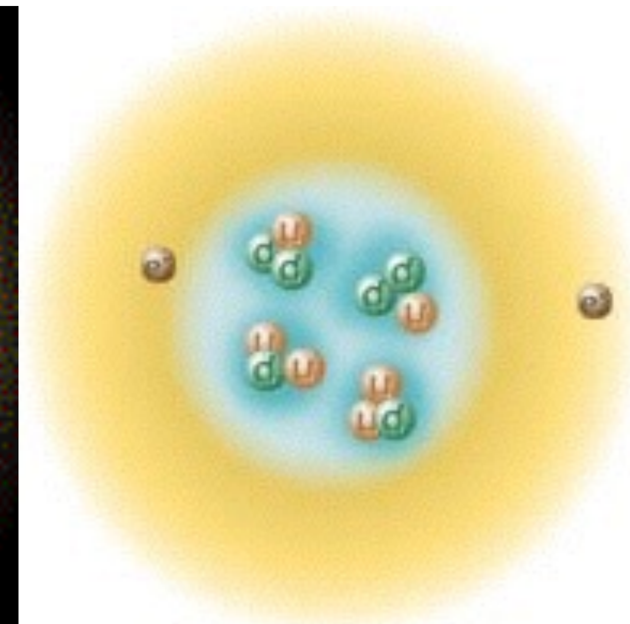
Gross, Wilczek, Politzer, 2004
Nobel Prize

1973 DESY(PETRA): confirm gluon in $e^+ e^- \longrightarrow$ 3 jets

The modern atom model

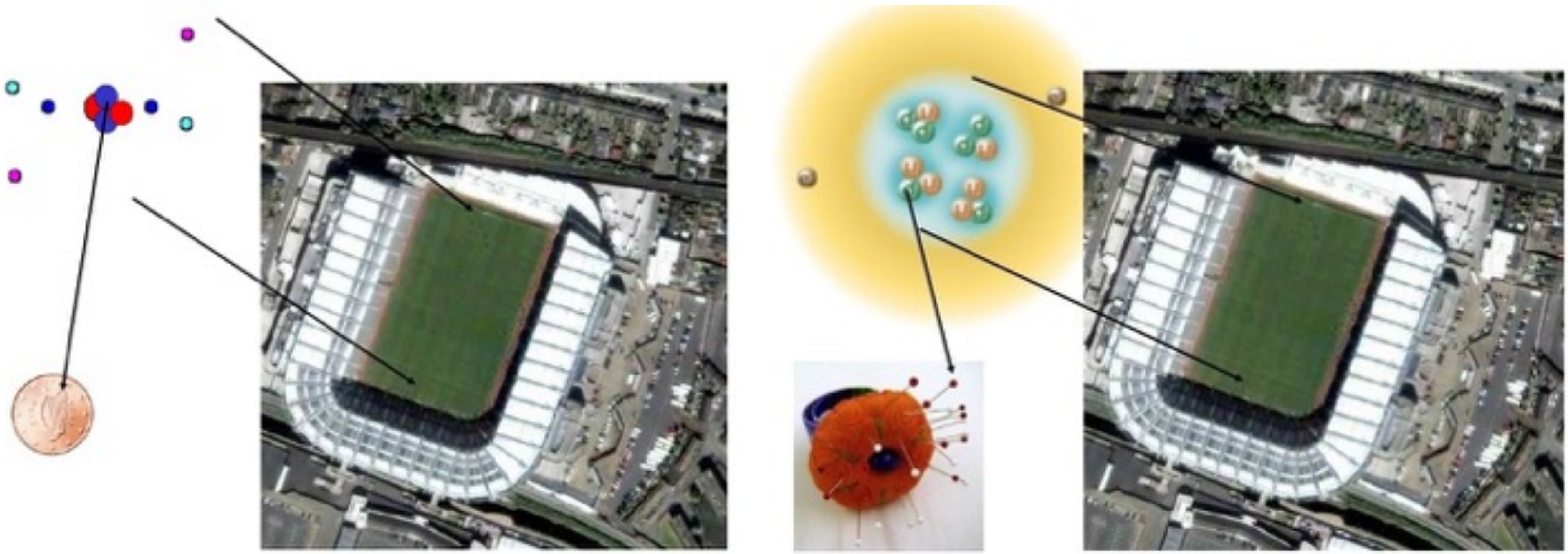
- Electrons are moving constantly around nucleus which is made of protons and neutrons which are the composites of quarks.





99.999999999999% of an atom's volume is just empty space!

- If we drew atom to scale and made protons and neutrons a centimeter in diameter, then electrons and quarks would be less than the diameter of hair, and the entire atom's diameter would be greater than the length of 30 football fields!



Empty Space?

- The "little universe" of atom is occupied mostly by "Emptiness".

- In fact, the 3 (u u d) quarks inside the proton only occupy one part in a billion (10^{-9}) of the proton's volume.

⇨ Lots of "Emptiness"

What is The True Nature of Emptiness?

Interactions Generate Mass

- Quarks themselves only contribute a very small part, about 1.3%, of the proton (uud) mass:
- 99.7% of proton mass originates from the interactions among the 3 quarks inside proton. (Mass is a form of kinetic and potential energy.)

Nambu (Nobel Prize 2008)

- These energies are converted into the mass of the proton as described by Einstein's equation that relates Energy (E) to Mass (M) by

$$E = M c^2$$

The Emptiness inside a Proton

- More than 99.9999999% of the proton's volume is empty space!
- The mass of proton is mainly generated by the contribution from the empty space (Emptiness) inside the proton.

⇨ "Emptiness" is not "void".

空非空

Mass is a form of Energy

色即是空