粒子物理

第五节
1) 量纲分析：散射和衰变振幅、相空间;
2) 共振态

曹庆宏

北京大学物理学院理论物理所

PDG 综述: <u>KINEMATICS (运动学)</u>

1. 散射振幅和相空间的量纲

$2 \to n$ 体散射

散射截面（采用 PDG 约定——π 因子约定）：

$$d\sigma_{a+b\to n} = \frac{(2\pi)^4 |\mathcal{M}_{a+b\to n}|^2}{4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}} d\Phi_n(p_1, \cdots, p_n; p_a + p_b)$$

其中 n 体相空间为

$$d\Phi_n = \delta^4\left(p_a + p_b - \sum_i^n p_i\right) \prod_{j=1}^n \frac{d^3 p_j}{(2\pi)^3 2E_j}$$

这里 δ 函数保证能动量守恒。如果粒子具有自旋，我们还需要对末态自旋求和，对初态自旋求平均（对初态自旋求和后乘上 $1/(2s+1)$ 因子）。

量纲分析

相空间量纲：

$$\delta^4\left(p_a + p_b - \sum_i^n p_i\right) \quad : \quad [E]^{-4}$$

$$\prod_{j=1}^{n} \frac{d^3 p_j}{(2\pi)^3 2E_j} \quad : \quad [E]^{2n}$$

$$d\Phi_n = \delta^4(\cdots)\prod_{j=1}^{n} \frac{d^3 p_j}{(2\pi)^3 2E_j} \quad : \quad [E]^{2n-4}$$

$$\frac{1}{Flux} \quad : \quad [E]^{-2}$$

量纲分析

因为散射截面量纲为

$$[\sigma] = [E]^{-2}, \qquad d\sigma = |\mathcal{M}|^2 \times \frac{d\Phi_n}{F}$$

所以散射振幅的量纲是

$$[\mathcal{M}_{a+b\to n}] = [E]^{2-n}$$

$2 \to 2$ 散射过程的散射振幅量纲：

$$[\mathcal{M}_{a+b\to c+d}] = [d\Phi_2] = [E]^0$$

$2 \to 2$ 散射振幅

因为 $2 \to 2$ 散射过程的散射振幅量纲为 0，所以散射振幅一定是无量纲变量的函数。

其中 g 是无量纲的耦合常数（非常适合于规范相互作用）。

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$$\mathcal{M} = g^2 f(\theta, \phi) = g^2 f'\left(\frac{t}{s}, \frac{s}{m^2}, \frac{t}{m^2}\right)$$

在第一阶近似中，我们期望 f 和 f' 函数的数值大小在 1 左右，并且具体的函数形式可以告诉我们散射过程中的相互作用形式。例如，初态粒子是标量粒子或初态无横向极化，那么散射振幅一定不依赖于 ϕ。

$2 \to 2$ 散射振幅

$$\left.\frac{d\sigma}{d\cos\theta}\right|_{a+b\to1+2} = \frac{\left|\mathcal{M}_{a+b\to1+2}\right|^2}{32\pi s}\frac{p'_{cm}}{p_{cm}}$$

$$\sim \frac{g^4}{32\pi s}\frac{p'_{cm}}{p_{cm}}\left|f(\cos\theta)\right|^2$$

因为

$$t = m_a^2 + m_1^2 - 2\left(E_a E_1 - p'_{cm}p_{cm}\cos\theta_{cm}\right)$$

$$d\cos\theta_{cm} = \frac{dt}{2p_{cm}p'_{cm}},$$

所以

$$\frac{d\sigma}{dt} = \frac{\left|\mathcal{M}_{a+b\to1+2}\right|^2}{64\pi s p_{cm}^2} \xrightarrow{s\gg m^2} \frac{|\mathcal{M}|^2}{16\pi s^2} \sim \frac{g^4}{16\pi s^2}\left|f\left(\frac{t}{s}\right)\right|^2$$

粒子的两体衰变（$1 \to 2$）

如果粒子的寿命为 τ，那么由 N 个相同粒子所组成的体系随时间变化行为遵从指数衰减定律

$$\Gamma \equiv -\frac{1}{N}\frac{dN}{dt} \implies N(t) = N(0)e^{-t\Gamma}$$

其中宽度 Γ 和粒子寿命 τ 的关系为 $\tau \equiv \dfrac{1}{\Gamma}$。

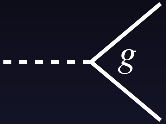
- 微分衰变宽度：$d\Gamma_{a\to n} = \dfrac{(2\pi)^4 \left|\mathcal{M}_{a\to n}\right|^2}{2m_a} d\Phi_n$
- 总宽度: $\Gamma_{\text{total}} = \sum_i \Gamma_{a\to i}$
- 分支比: $\text{Br}(a \to i) \equiv \dfrac{\Gamma_{a\to i}}{\Gamma_{\text{total}}}$

粒子的两体衰变的量纲分析

首先，$[\Gamma_{a\to 1+2}] = [E]^1$

$$d\Gamma_{a\to 1+2} = \frac{(2\pi)^4 \left|\mathcal{M}_{a\to 1+2}\right|^2}{2m_a} d\Phi_2$$

$$\xrightarrow{[d\Phi_2]=E^0} [\mathcal{M}_{a\to 1+2}] = [E]^1$$



$$\Longrightarrow \ \mathcal{M}_{a\to 1+2} \sim g\, m_a\, f(\theta) \quad \Longrightarrow \quad \Gamma_{a\to 1+2} \sim g^2 m_a$$

如果我们知道粒子的宽度和质量，那么就可以估算相互作用的强度。例如质量为 1GeV 的强子，其寿命是 $\sim 10^{-10}$s 或宽度为 $\sim 10^{-14}$GeV，那么

$$g \sim \sqrt{\frac{\Gamma}{m_a}} \sim 10^{-7}$$

n 体反应（$2 \to n-2$）的洛伦兹不变量

此过程的初末态涉及 n 个粒子，设其动量为

$$p_i^\mu, \quad i = 1, 2, \cdots, n$$

能动量守恒要求：仅有 $n-1$ 个动量是独立的，可构成的洛伦兹不变量是

$$\frac{(n-1)(n-2)}{2} + (n-1) = \frac{n(n-1)}{2}$$

其中有 n 个粒子的质壳条件，因此独立洛伦兹变量是

$$\frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$$

例：$2 \to 2$ 散射，$n = 4$，因此独立洛伦兹变量个数为 $4(4-3)/2 = 2$。

n 体末态的相空间

四维动量空间体积元 $d^4p = d^3pdE$ 是洛伦兹不变的。散射末态的粒子都是在质壳上的，满足

$$p^2 = E^2 - \vec{p}^2 = m^2$$

四维动量相空间体积元 d^4p 总是和质壳条件乘在一起，

$$d^4p\delta(p^2 - m^2)\theta(E)$$

对四维动量相空间积分时，dE 积分是确定的，

$$\begin{aligned}
\int_{\infty}^{\infty} dEd^3\vec{p}\delta(E^2 - \vec{p}^2 - m^2)\theta(E) &= \int_0^{\infty} \frac{dE^2}{2E}\delta(E^2 - \vec{p}^2 - m^2)d^3\vec{p} = \frac{d^3\vec{p}}{2\sqrt{m^2 + \vec{p}^2}} \\
&= \frac{d^3\vec{p}}{2E}
\end{aligned}$$

n 体末态的相空间

$$d\Phi_n = \delta^4\left(p_a + p_b - \sum_i^n p_i\right)\prod_{j=1}^n \frac{d^3p_j}{(2\pi)^3 2E_j}$$

这个相空间是 $3n$ 重积分，积掉 δ 函数后还有 $3n-4$ 重积分。

一般情况下，相空间积分非常复杂。但如果所有末态粒子质量都为零，相空间积分容易得到。

1 体相空间：Φ_1

一体末态：$a+b\to 1$ 过程

$$
\begin{aligned}
d\Phi_1 &= (2\pi)\frac{d^3\vec{p}_1}{2E}\delta^4(p_a+p_b-p_1) \\
&= (2\pi)d^4\delta^4(p_a+p_b-p_1)\delta^+(p_1^2-m_1^2) \\
&= (2\pi)\delta(s-m_1^2)
\end{aligned}
$$

其中 $s=(p_a+p_b)^2=m_1^2$。

Φ_1 的量纲：

$$[\Phi_1]=[E]^{-2}$$

2 体相空间：Φ_2

$$\begin{aligned}
\Phi_2 &= \int \delta(\sqrt{s}-p_1-p_2)\delta^3(\vec{p}_1+\vec{p}_2)\frac{p_1^2 dp_1 d\Omega_1 d^3\vec{p}_2}{(2\pi)^6 4p_1p_2} \\
&= \int \delta(\sqrt{s}-2p)\frac{dp\,d\cos\theta\, d\phi}{(2\pi)^6 4} \\
&= \frac{4\pi}{(2\pi)^6 8} = \frac{1}{4(2\pi)^5}
\end{aligned}$$

Φ_2 的量纲：

$$[\Phi_2] = [E]^0$$

多体相空间：Φ_3 和 Φ_2 比较

$$\Phi_1 = \frac{2\pi}{s}$$

$$\Phi_2 = \frac{1}{4(2\pi)^5}, \qquad \frac{\Phi_2}{\Phi_1} = \frac{s}{16\pi^2}$$

$$\Phi_3 = \frac{s}{32(2\pi)^7} \qquad \frac{\Phi_3}{\Phi_2} = \frac{s}{32\pi^2}$$

$$\frac{\Phi_{n+1}}{\Phi_n} = \frac{s}{16\pi^2 n(n-1)}$$

一般情况，

$$\Phi_n = \frac{1}{4(2\pi)^5(n-1)!(n-2)!}\left(\frac{s}{16\pi^2}\right)^{n-2}$$

无量纲化的 n 体相空间

$$\Phi_n = \frac{1}{4(2\pi)^5(n-1)!(n-2)!}\left(\frac{s}{16\pi^2}\right)^{n-2}$$

Φ_n 的量纲随 n 变化，人们定义无量纲化的相空间如下：

$$\Phi_n' = \frac{\Phi_n}{s^{n-2}}$$

末态粒子质量为零时，无量纲化的三体相空间约为无量纲化的两体相空间的 1/316。

一般而言，末态粒子每增加一个，相应的无量纲化的相空间要减少两个数量级。这正是多体衰变和散射过程中，产生的粒子数越多，概率越小的运动学原因。

2. 散射截面、积分亮度和事例数

散射截面（σ）

- 量纲：$[\sigma] = m^2$

$$1\text{barn} = 10^{-24}(\text{cm})^2 = 10^{-28}m^2 = 100(\text{fm})^2$$

$$1\text{mb} = 10^{-3}\text{b}$$

$$1\mu\text{b} = 10^{-6}\text{b}$$

$$1\text{nb} = 10^{-9}\text{b}$$

$$1\text{pb} = 10^{-12}\text{b}$$

$$1\text{fb} = 10^{-15}\text{b}$$

汤姆逊散射

低能光子-非相对论性（近乎静止）电子之间散射

经典物理反射和衍射

$$\sigma_{\mathrm{Tot}}^{\mathrm{classic}} = 2\pi r_e^2 \qquad\qquad r_e \text{ 电子经典半径}$$

$$\frac{e^2}{r_e} = m_e c^2 \Longrightarrow r_e = \frac{e^2}{m_e c^2} \sim \frac{1/137}{0.511\mathrm{MeV}} \sim 2.8\mathrm{fm}$$

量子力学计算给出

$$\sigma_{\mathrm{Tot}} = \frac{8}{3}\pi r_e^2 = 67(\mathrm{fm})^2 = 0.67\mathrm{barn}$$

高能对撞机

估算质子-质子对撞机

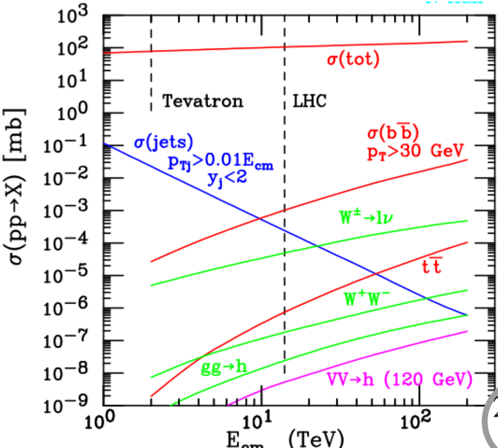
$$\sigma \sim \pi \lambda_p^2 \sim \frac{\pi}{\mathrm{GeV}^2} = \frac{\pi(\mathrm{fm})^2}{10^6(1/197)^2} \sim 10^{-3}\mathrm{barn} = \mathrm{mb}$$

14TeV大型强子对撞机

$$\sigma(pp)_{\mathrm{total}} \sim 110\mathrm{mb}$$

1.96TeV Tevatron

$$\sigma(p\bar{p})_{\mathrm{total}} \sim 60\mathrm{mb}$$



事例数

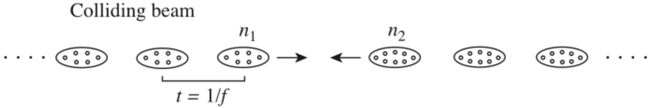
$$\text{Number of Event} = \sigma \cdot \mathcal{L}$$

实验学家

理论学家

加速器学家

亮度 (luminosity)



瞬时亮度

$$\mathcal{L} \propto f n_1 n_2 / \Sigma$$

$$[\mathcal{L}] = \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

# of particles passing each other per unit time through unit transverse area at the interaction point

$n_{1,2}$: particle in each bunch in beam 1,2

f : beam crossing frequency

Σ : transverse profile of the bream

$$1\mathrm{cm}^{-2}\mathrm{s}^{-1} = 10^{-33}\mathrm{nb}^{-1}\mathrm{s}^{-1}$$

积分亮度：$10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1} = 1\mathrm{nb}^{-1}\mathrm{s}^{-1} = 10\mathrm{fb}^{-1}/\mathrm{year}$

Past, current and Further collider

Name	Type	$\sqrt{s}$ (GeV)	$L_{\rm int}$ (pb^{-1})	Years of operation	Detectors	Location
LEP	e^+e^-	91.2 (LEP-1)	≈ 200 (LEP-1)	1989-95 (LEP-1)	ALEPH, OPAL,	CERN
		130-209 (LEP-2)	≈ 600 (LEP-2)	1996-2000 (LEP-2)	DELPHI, L3	
SLC	e^+e^-	91.2	20	1992-98	SLD	SLAC
HERA	$e^\pm p$	320	500	1992-2007	ZEUS, H1	DESY
Tevatron	$p\bar{p}$	1800 (Run-I)	160 (Run-I)	1987-96 (Run-I)	CDF, DØ	FNAL
		1960 (Run-II)	6 K (Run-II, 06/09)	2000-??? (Run-II)		
LHC	pp	14000	10 K/yr ("low-L")	2010? - 2013?	ATLAS, CMS	CERN
			100 K/yr ("high-L")	2013?? - 2016???		
ILC	e^+e^-	500-1000	1 M???	???	???	???

3. 共振态

Resonance: Ubiquitous phenomenon in Physics

Note Title 10/20/2014

- Mechanics $\longrightarrow$ Electrodynamics
- Accoustics $\longrightarrow$ optics
- Atomic $\longrightarrow$ Nuclear Physics
- $\longrightarrow$ Hadron Spectroscopy

$\Rightarrow$ common feature: Phase

$\omega \approx \omega_0 \Rightarrow$ Magnitude $\rightarrow$ Maximum

ω: Variation of the system

ω_0: characteristic frequency of the system

共振态特征：如何描述共振态？

- 电荷
- 角动量
- 宇称
- 同位旋（强、弱。。。）
- 质量（M）
- 寿命（τ 或 Γ）

问题：如何测量这些物理量？

例子

考虑通过强相互作用衰变的不稳定粒子。质量约为 1 GeV，寿命约为

$$\tau \sim 10^{-24}\mathrm{s} \implies \gamma \sim 300$$

在实验室系中，该粒子的衰变长度为

$$L = c\tau\gamma \approx 1\ \mathrm{fm}$$

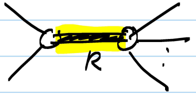
此粒子寿命奇短，产生后瞬间就在出生地衰变了，无法测量其衰变距离。应该如何测量此粒子呢？

- 散射截面依赖于入射粒子的能量，散射截面的局域最大峰值所对应的能量值就是共振态的质量
- 测量末态粒子的不变质量的峰值位置

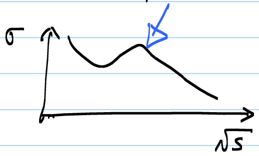
Resonances in Particle Physics

not strictly speaking, resonances appear in

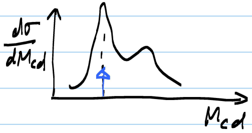
*) the process of "Formation"



a local maximum in the energy dependence of a cross-section



*) the production processes



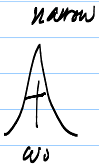
as a maximum in the invariant mass distribution of a few particles in the final state

Resonance: Breit-Wigner line-shape

Ex: damped oscillator.

The response function in the neighborhood of maximum

$$R(\omega) = \frac{\Gamma^2 \omega^2}{(\omega_0^2 - \omega^2)^2 + \omega^2 \Gamma^2}$$



If the width is small, $\Gamma \ll \omega_0$

then in the vinicity of $\omega \sim \omega_0$

$$(\omega_0^2 - \omega^2)^2 + \omega^2 \Gamma^2$$

$$= (\omega_0 - \omega)^2 (\omega_0 + \omega)^2 + \omega^2 \Gamma^2 \Rightarrow (\omega_0 + \omega_0)^2 (\omega_0 - \omega)^2 + \omega_0^2 \Gamma^2$$

$$= 4\omega_0^2 \left[(\omega_0 - \omega)^2 + \left(\frac{\Gamma}{2}\right)^2 \right]$$

$$\Rightarrow R(\omega) \approx L(\omega) = \frac{\left(\frac{\Gamma}{2}\right)^2}{(\omega_0 - \omega)^2 + \left(\frac{\Gamma^2}{2}\right)}$$

Resonance: Quantum description

Consider the elastic scattering $a + b \rightarrow R \rightarrow a' + b'$

The wave function of the unstable resonance R

$$\psi(t) = \psi(0)\, e^{-i\omega_R t}\, e^{-\frac{t}{2\tau}} = \psi(0)\, e^{-iE_R t}\, e^{-\frac{\Gamma}{2}t}$$

$E = E_R - i\frac{\Gamma}{2}$ $\qquad E_R = \hbar\omega_R \underset{NU}{=} \omega_R\,, \quad \tau = \frac{\hbar}{\Gamma} \underset{NU}{=} \frac{1}{\Gamma}$

Probability of finding the particle at a time t is

$$I(t) = \psi^*(t)\,\psi(t) = \psi(0)^2\, e^{-t/\tau} = I(0)\, e^{-t/\tau}$$

The Fourier transform of $\psi(t)$

$$\chi(E) = \frac{1}{\sqrt{2\pi}}\int \psi(t)\, e^{iEt}\, dt = \psi(0)\int_0^\infty e^{-t\left[\frac{\Gamma}{2} + iE_R - iE\right]}\, dt$$

$$= \frac{K}{(E_R - E) - i\Gamma/2}$$

K: normalization factor

Cross section of elastic scattering $a + b \rightarrow R \rightarrow a' + b'$

(首先考虑无极化情况)

Probality of finding particle R in energy E

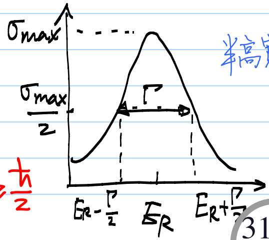
$$\sigma(E) = \sigma_0 \chi^*(E)\chi(E) = \sigma_0 \frac{k^2}{[(E_R - E)^2 + \Gamma^2/4]}$$

Normalization

$$\chi^*(E_R)\chi(E_R) = \frac{4k^2}{\Gamma^2} = 1 \Longrightarrow k^2 = \frac{\Gamma^2}{4}$$

$$\sigma_0 = \pi(2\lambda\!\!\!^-)^2 = 4\pi\lambda\!\!\!^-{}^2$$

$$\Longrightarrow \sigma_{el}(E,J) = 4\pi\lambda\!\!\!^-{}^2 \left[\frac{\Gamma^2/4}{(E_R - E)^2 + \Gamma^2/4}\right]$$



半高宽度

Γ 表征能量弥散大小、 $\Delta E \cdot \Delta \tau \geq \frac{\hbar}{2}$

Resonance: partial wave analysis

Refs. 曾谨言老师《量子力学》卷一 第13.4节 分波法

For spinless particle

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = |f(\theta,\phi)|^2$$

$$f(\theta) = \frac{1}{2ik} \sum_\ell (2\ell+1)\left(e^{2i\delta_\ell}-1\right) P_\ell(\cos\theta)$$

k: wave number (magnitude of incoming particle's momentum in the C.M.F.)

δ_ℓ: phase shift / phase change of ℓ^{th} partial wave

Integrate over angles $\Rightarrow$ total elastic scattering x-section

$$\sigma_{EL} = \frac{\pi}{k^2} \sum_\ell (2\ell+1)\left|e^{2i\delta_\ell}-1\right|^2 = \frac{4\pi}{k^2}\sum_\ell (2\ell+1)\sin^2\delta_\ell$$

Phase shift δ_ℓ depends on Energy E : $\delta_\ell(E)$

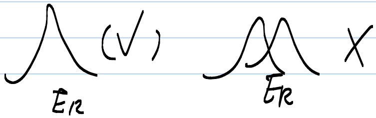
if $\delta_\ell(E_R) \approx (n+\frac{1}{2})\pi$, $n=0,1,2,\cdots$

then $\sin^2\delta_\ell(E_R) \approx 1$

$\Rightarrow$ the ℓ^{th} partial wave contributes maximally to X-section

(A resonance in the ℓ^{th} partial wave or, a particle of spin ℓ)

For simplicity, we assume only one resonance exist around E_R



To illustrate the resonant behavior more explicitly

$$e^{2i\delta_\ell}-1 = 2i\,e^{i\delta_\ell}\sin\delta_\ell = 2i\,\frac{\sin\delta_\ell}{\cos\delta_\ell - i\sin\delta_\ell}$$

The energy dependence of $f_\ell(E)$

$$f_\ell(E) = \frac{\sin\delta_\ell(E)}{\cos\delta_\ell(E) - i\sin\delta_\ell(E)}$$

let $E \to E_R$, $\delta_\ell(E_R) = \frac{\pi}{2}$, $\sin\delta_\ell(E_R) = 1$, $\cos\delta_\ell(E_R) = 0$

In the vinicity of E_R.

$$\sin\delta_\ell(E) = \sin\delta_\ell(E_R) + \left[\cos\delta_\ell(E)\cdot\frac{d\delta_\ell}{dE}\right]\Big|_{E_R}(E-E_R)+\cdots \simeq 1$$

$$\cos\delta_\ell(E) = \cos\delta_\ell(E_R) - \left[\sin\delta_\ell(E)\cdot\frac{d\delta_\ell}{dE}\right]\Big|_{E_R}(E-E_R)+\cdots$$

$$\simeq -\underbrace{\frac{d\delta_\ell}{dE}\Big|_{E_R}}_{\equiv \frac{2}{\Gamma_\ell}}\times(E-E_R)$$

Real number, width

$$f(\theta, E) = \frac{2l+1}{k} e^{i\delta_l} \sin\delta_l \, P_l(\cos\theta)$$

The energy dependence

$$f(E) = e^{i\delta_l} \sin\delta_l = \frac{\sin\delta_l(E)}{\cos\delta_l(E) - i\sin\delta_l(E)}$$

$$\overset{E \sim E_R}{\approx} \frac{1}{-\frac{2}{\Gamma_l}(E - E_R) - i}$$

$$= \frac{\Gamma_l / 2}{(E - E_R) + i\,\Gamma_l/2}$$

$$f(\theta, E) \approx \frac{(2l+1)}{k} P_l(\cos\theta) \frac{\Gamma_l/2}{E_R - E - i\frac{\Gamma_l}{2}}$$

$$\Rightarrow \sigma_{El} \simeq \frac{4\pi}{k^2}(2l+1) \frac{\Gamma_l^2/4}{(E - E_R)^2 + \Gamma_l^2/4}$$

Realistic scattering: $\left\{ \begin{array}{l} \text{spined initial/final states} \\ \text{colored resonance} \\ \text{Many decay channels} \end{array} \right.$

$$A + B \rightarrow R \rightarrow A' + B'$$
$$\qquad\qquad\quad \rightarrow C + D + \cdots$$
$$\qquad\qquad\quad \rightarrow \cdots\cdots$$

(1) <u>Spin</u>

$(2l+1)$ in the partial wave cross-section tells us that

$\Rightarrow$ After integrating over angles, the $(2l+1)$ states of different l_z all contribute equally

(1) If the final state particles carry spin "s", then all $(2s+1)$ states of each particle can be produced.

$\Rightarrow$ sum all spin state of final state particles $\sum_{\text{final}} (\cdots)$

(2) Averaging over initial state "spin"

Example: two spin-½ particles combine into a spin-0 resonance

$$\begin{pmatrix} \uparrow_1 \\ \downarrow_1 \end{pmatrix} \otimes \begin{pmatrix} \uparrow_2 \\ \downarrow_2 \end{pmatrix} = \begin{pmatrix} \uparrow\uparrow \\ \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ \downarrow\downarrow \end{pmatrix}_{\text{triplet}} + \underbrace{\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)_{\text{singlet}}}_{\text{yielding a spin-zero resonance}}$$

$$\Longrightarrow \boxed{\begin{array}{l} \text{only } 1/4 \text{ probability} \\ \dfrac{1}{4} = \dfrac{1}{(2S_a+1)(2S_b+1)} \end{array}}$$

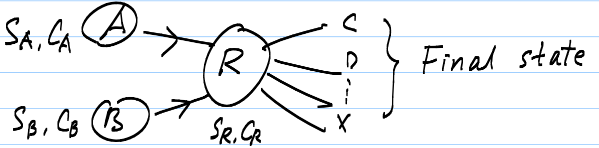
Question: two spin-½ particles $\Rightarrow$ spin-1 resonance

the possibility $= \frac{3}{4} = \frac{(2J+1)}{(2S_a+1)(2S_b+1)}$

A similar argument also holds for the color states

$$C_q = 3 \,, \quad C_g = 8 \,, \quad C_\gamma = 1$$

Resonance : inelastic scattering



$$\sigma(A+B \to R \to C+D+\cdots+X)$$

$$= \frac{\pi}{k^2}\left[\frac{(2S_R+1)\,C_R}{(2S_A+1)(2S_B+1)\,C_A C_B}\right]\frac{\Gamma(R\to AB)\,\Gamma(R\to f)}{(E_R-E)^2+\Gamma_R^2/4}$$

$\Gamma(R\to AB)$: partial width of $R\to AB$

$\Gamma(R\to f)$: partial width of $R\to f$

Γ_R = total width $\qquad \Gamma_R = \sum_f \Gamma(R\to f)$

For $E \approx E_R$, $\quad \left(\frac{E_R + E}{E_R + E} \right)^2 \approx \frac{4E_R^2}{(E+E_R)^2} = \frac{4s}{(E+E_R)^2}$

$$\Rightarrow \quad \sigma = \frac{\pi}{k^2} \left[\frac{(2S_R+1)\, C_R}{(2S_A+1)(2S_B+1)\, C_A C_B} \right] \frac{\Gamma(R\to AB)\,\Gamma(R\to f)}{(E-E_R)^2 + \frac{\Gamma_R^2}{4}} \times \frac{4s}{(E+E_R)^2}$$

$$= \frac{4\pi s}{k^2} [\cdots] \frac{\Gamma(R\to AB)\;\Gamma(R\to f)}{(E_R^2 - E^2)^2 + \frac{\Gamma_R}{4}(E+E_R)^2}$$

$E_R^2 = m_R^2$

$$= \frac{4\pi s}{k^2} [\cdots] \frac{\Gamma(R\to AB)\;\Gamma(R\to f)}{(E_R^2 - E^2)^2 + \frac{\Gamma_R}{4} \times 4m_R^2}$$

$$= \frac{4\pi s}{k^2} [\cdots] \frac{\Gamma(R\to AB)\;\Gamma(R\to f)}{(m_R^2 - s)^2 + m_R^2 \Gamma_R^2}$$

Lorentz invariant form

$$S = (P_A + P_B)^2$$

$$E_{tot}^{cm} = E_A + E_B$$

$$S = E_{tot}^2 = (E_A + E_B)^2$$

At the resonance $S_R = E_R^2 = m_R^2$

The incoming particle's wave #

$$k^2 = \frac{1}{4S}\left[\left(S - (m_A + m_B)^2\right)\left(S - (m_A - m_B)^2\right)\right]$$

(magnitude of three momentum in C.M.F)

X-section is small if $E \neq E_R$ $\Rightarrow$ $E \to E_R$ everywhere but not the denominator

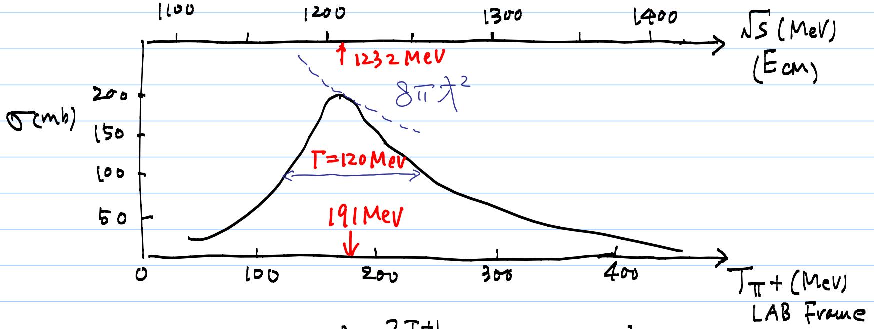
$$\Rightarrow \sigma(A + B \to R \to C + D + \cdots)$$

$$= \frac{4\pi S}{k^2}\left[\frac{(2S_R+1)\, C_R}{(2S_A+1)(2S_B+1)\; C_A C_B}\right]\frac{\Gamma(R\to AB)\,\Gamma(R\to f)}{(S - m_R^2)^2 + m_R^2 \Gamma_R^2}$$

Ex: the $\Delta^{++}(1232)$ Resonance

See Braibant's book Page-150, Fig 7.7

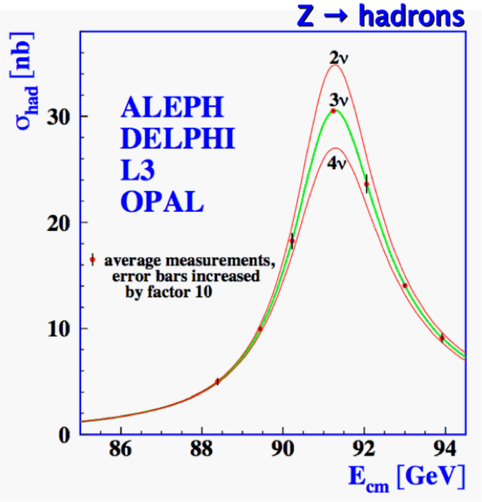
$$\pi^+ + p \to \Delta^{++}$$



$$\sigma_{max} = 4\pi\lambda\!\!\!^{-2} \frac{2J+1}{(2S_\pi+1)(2S_p+1)} = 2\pi\lambda\!\!\!^{-2}(2J+1)$$

$$\Rightarrow 2\pi\lambda\!\!\!^{-2}(2J+1) = 8\pi\lambda\!\!\!^{-2} \Rightarrow 2J+1 = 4 \Rightarrow \underline{\underline{J = 3/2}}$$

Z 共振态：$e^+e^- \to Z \to q\bar{q}$



总结

- 量纲分析：散射振幅和相空间
  - $2 \to n$ 散射过程

$$\left[\delta^4(\cdots)\prod_{j=1}^{n}\frac{d^3p_j}{(2\pi)^3 2E_j}\right] = [E]^{2n-4}, \quad [\mathcal{M}_{2\to n}] = [E]^{2-n}$$

  - $1 \to 2$ 衰变

$$[M_{a\to 1+2}] = [E]^1, \quad [\Phi_2] = [E]^0$$

- n 体相空间比较

$$\Phi'_n = \frac{\Phi_n}{s^{n-2}}, \qquad \frac{\Phi'_{n+1}}{\Phi'_n} \approx \frac{1}{100} - \frac{1}{1000}$$

- 事例数、散射截面和积分亮度

$$\text{事例数} = \sigma \times \mathcal{L}$$

总结

- 共振态：质量、宽度、自旋和颜色

$$
\begin{aligned}
\sigma_{\text{Elastic}}(E,J) &= 4\pi\lambda^2\left[\frac{\Gamma^2/4}{(E_R-E)^2+\Gamma^2/4}\right]\\
\sigma_{\text{Inelastic}}(E,J) &= \left[\frac{(2s_R+1)C_R}{(2s_A+1)(2s_B+1)C_AC_B}\right]\\
&\times\ \frac{\pi}{k^2}\,\frac{\Gamma(R\to AB)\Gamma(R\to f)}{(E_R-E)^2+\Gamma_R^2/4}\\
\sigma_{\text{Inelastic}}(s,J) &= \left[\frac{(2s_R+1)C_R}{(2s_A+1)(2s_B+1)C_AC_B}\right]\\
&\times\ \frac{4\pi s}{k^2}\,\frac{\Gamma(R\to AB)\Gamma(R\to f)}{(s-m_R^2)^2+m_R^2\Gamma_R^2}
\end{aligned}
$$