粒子物理

17. 量子色动力学
(Quantum Chromodynamics)

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From QED to QCD

★ Suppose there is another fundamental symmetry of the universe, say

"invariance under SU(3) local phase transformations"

- i.e. require invariance under $\psi \rightarrow \psi' = \psi e^{ig\vec{\lambda}.\vec{\theta}(x)}$ where

$\vec{\lambda}$ are the eight 3x3 Gell-Mann matrices introduced in handout 7

$\vec{\theta}(x)$ are 8 functions taking different values at each point in space-time

⇒ 8 spin-1 gauge bosons

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

wave function is now a vector in COLOUR SPACE

⇒ QCD !

★ QCD is fully specified by require invariance under SU(3) **local** phase transformations

Corresponds to rotating states in colour space about an axis whose direction is different at every space-time point

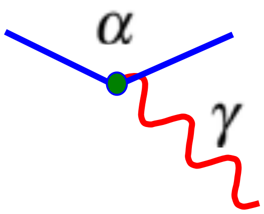
⇒ interaction vertex: $-\frac{1}{2}ig_s\lambda^a\gamma^\mu$

★ Predicts 8 massless gauge bosons – the gluons (one for each λ)

★ Also predicts exact form for interactions between gluons, i.e. the 3 and 4 gluon vertices – the details are beyond the level of this course

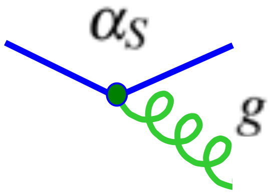
Colour in QCD

★ The theory of the strong interaction, Quantum Chromodynamics (QCD), is very similar to QED but with 3 conserved "colour" charges



In QED:

- the electron carries one unit of charge $-e$
- the anti-electron carries one unit of anti-charge $+e$
- the force is mediated by a massless "gauge boson" – the photon



In QCD:

- quarks carry colour charge: r, g, b
- anti-quarks carry anti-charge: $\bar{r}, \bar{g}, \bar{b}$
- The force is mediated by massless gluons

★ In QCD, the strong interaction is invariant under rotations in colour space

$$r \leftrightarrow b;\ r \leftrightarrow g;\ b \leftrightarrow g$$

i.e. the same for all three colours

➡ **SU(3) colour symmetry**

- This is an **exact** symmetry, unlike the approximate **uds** flavour symmetry discussed previously.

★ Represent r, g, b SU(3) colour states by:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

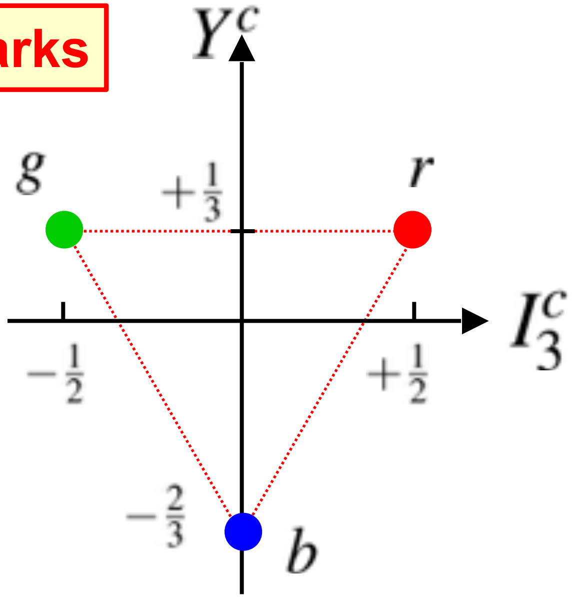
★ Colour states can be labelled by two quantum numbers:

- I_3^c colour isospin
- Y^c colour hypercharge

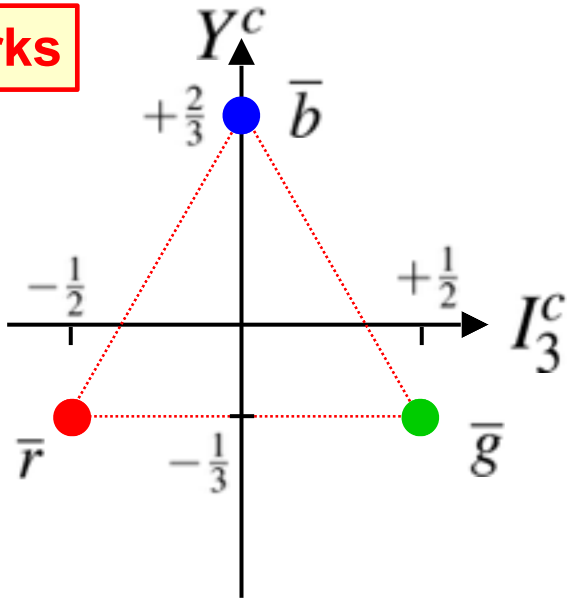
Exactly analogous to labelling u,d,s flavour states by I_3 and Y

★ Each quark (anti-quark) can have the following colour quantum numbers:

quarks



anti-quarks



Colour Confinement

★ It is believed (although not yet proven) that all observed free particles are "colourless"
- i.e. never observe a free quark (which would carry colour charge)
- consequently quarks are always found in bound states colourless hadrons

★ **Colour Confinement Hypothesis:**

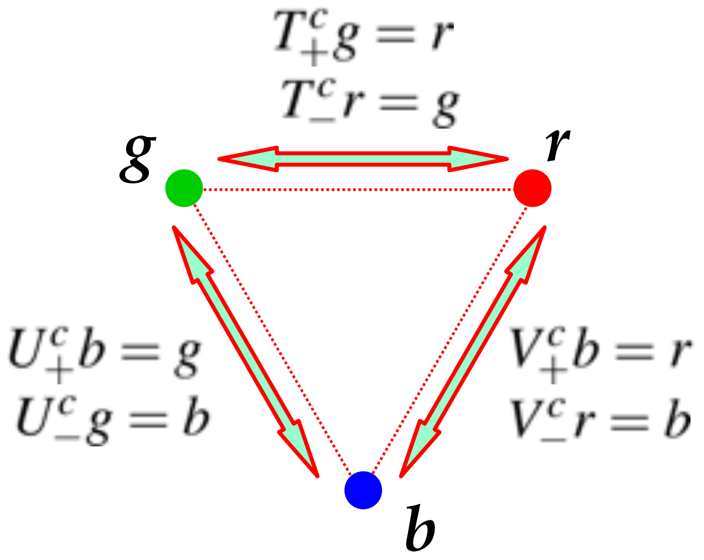
only colour singlet states can exist as free particles

★ All hadrons must be "colourless" i.e. colour singlets

★ To construct colour wave-functions for hadrons can apply results for SU(3) flavour symmetry to SU(3) colour with replacement

$$\boxed{\begin{aligned} u &\rightarrow r \\ d &\rightarrow g \\ s &\rightarrow b \end{aligned}}$$

★ just as for uds flavour symmetry can define colour ladder operators



Colour Singlets

★ It is important to understand what is meant by a singlet state

★ Consider spin states obtained from two spin 1/2 particles.

- Four spin combinations: $\uparrow\uparrow,\ \uparrow\downarrow,\ \downarrow\uparrow,\ \downarrow\downarrow$
- Gives four eigenstates of $\hat{S}^2,\ \hat{S}_z$ $\qquad (2\otimes 2 = 3\oplus 1)$

$$\begin{aligned}|1,+1\rangle &= \uparrow\uparrow\\ |1,0\rangle &= \tfrac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\\ |1,-1\rangle &= \downarrow\downarrow\end{aligned}$$

spin-1 triplet

$$\oplus\ |0,0\rangle = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)$$

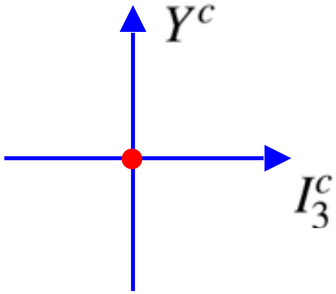
spin-0 singlet

★ The singlet state is "spinless": it has zero angular momentum, is invariant under SU(2) spin transformations and spin ladder operators yield zero

$$S_{\pm}|0,0\rangle = 0$$

★ In the same way COLOUR SINGLETS are "colourless" combinations:

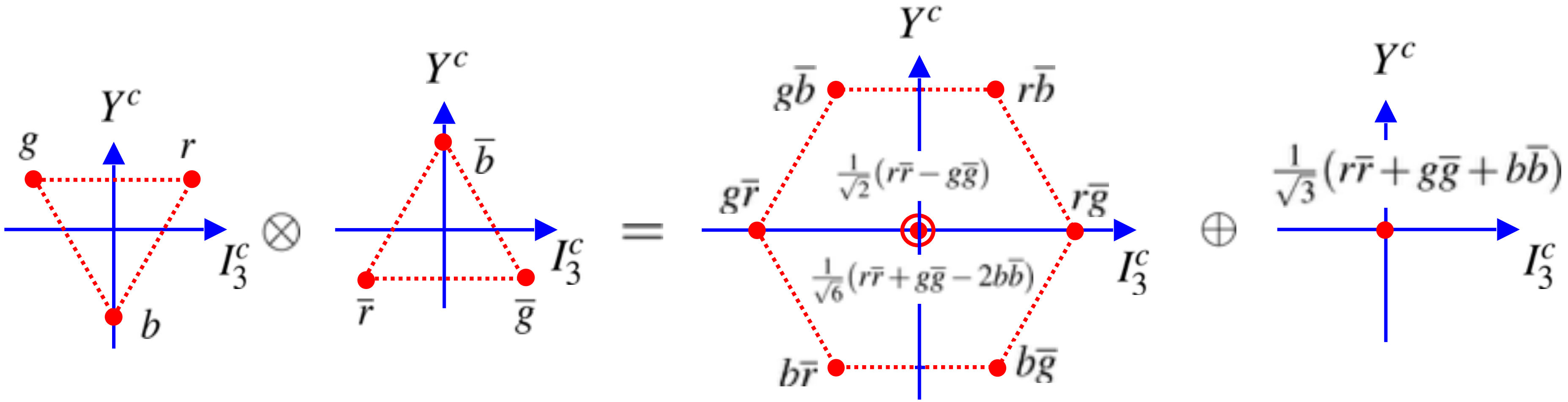
- ♦ they have zero colour quantum numbers $I_3^c = 0,\ Y^c = 0$
- ♦ invariant under SU(3) colour transformations
- ♦ ladder operators $T_{\pm},\ U_{\pm},\ V_{\pm}$ all yield zero



★ NOT sufficient to have $I_3^c = 0,\ Y^c = 0$: does not mean that state is a singlet

Meson Colour Wave-function

- ★ Consider colour wave-functions for $q\bar{q}$
- ★ The combination of colour with anti-colour is mathematically identical to construction of meson wave-functions with uds flavour symmetry



→ **Coloured octet and a colourless singlet**

- •Colour confinement implies that hadrons only exist in colour singlet states so the colour wave-function for mesons is:

$$\boxed{\psi_c^{q\bar{q}} = \frac{1}{\sqrt{3}}(r\bar{r}+g\bar{g}+b\bar{b})}$$

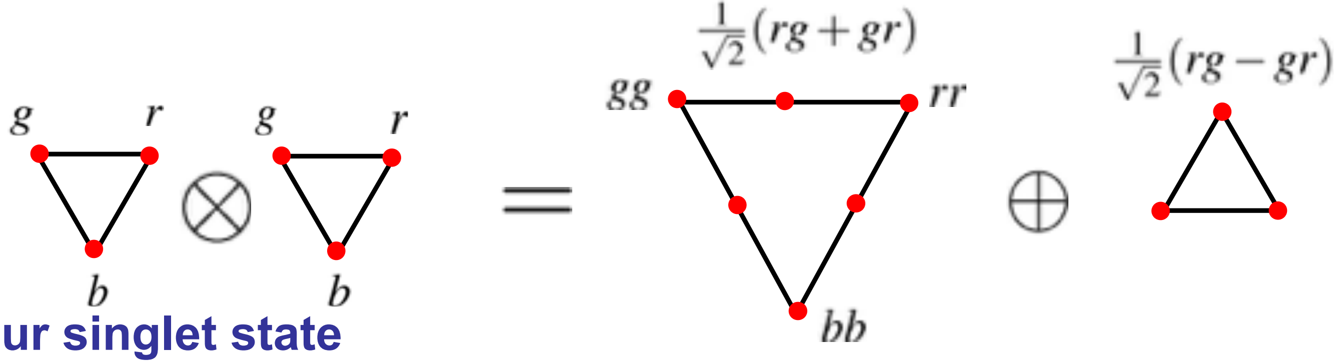
- ★ Can we have a $qq\bar{q}$ state ? i.e. by adding a quark to the above octet can we form a state with $Y^c = 0;\ I_3^c = 0$. The answer is clear no.

→ $qq\bar{q}$ bound states do not exist in nature.

Baryon Colour Wave-function

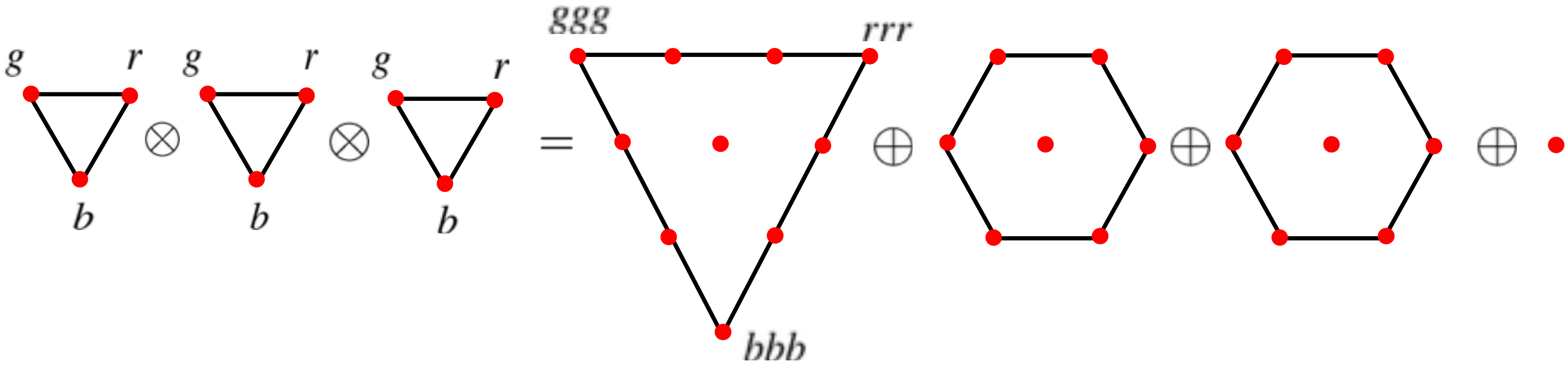
★ Do **qq** bound states exist ? This is equivalent to asking whether it possible to form a colour singlet from two colour triplets ?

- Following the discussion of construction of baryon wave-functions in SU(3) flavour symmetry obtain



- No **qq** colour singlet state
- Colour confinement ➡ bound states of **qq** do not exist

★ BUT combination of three quarks (three colour triplets) gives a colour singlet state (pages 235-237)



★The singlet colour wave-function is:

$$\boxed{\psi_c^{qqq} = \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr)}$$

Check this is a colour singlet…

- It has $I_3^c = 0,\ Y^c = 0$: a necessary but not sufficient condition
- Apply ladder operators, e.g. T_+ (recall $T_+ g = r$)

$$T_+ \psi_c^{qqq} = \frac{1}{\sqrt{6}}(rrb - rbr + rbr - rrb + brr - brr) = 0$$

- Similarly $T_- \psi_c^{qqq} = 0;\quad V_\pm \psi_c^{qqq} = 0;\quad U_\pm \psi_c^{qqq} = 0;$

★ Colourless singlet - therefore **qqq** bound states exist !

⇒ **Anti-symmetric colour wave-function**

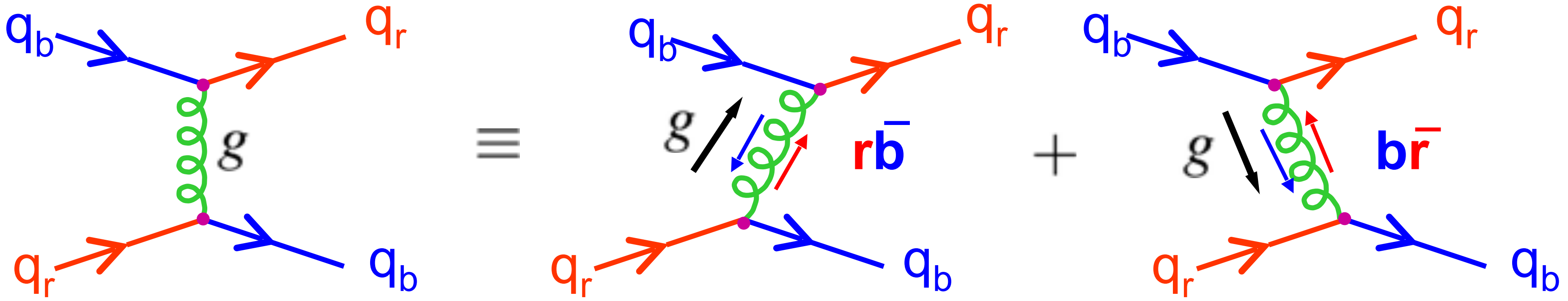
Allowed Hadrons i.e. the possible colour singlet states

- ● $q\bar{q},\ qqq$ **Mesons and Baryons**
- ● $q\bar{q}q\bar{q},\ qqqq\bar{q}$ **Exotic states, e.g. pentaquarks**

To date all confirmed hadrons are either **mesons** or **baryons**. However, some recent (but not entirely convincing) "evidence" for pentaquark states

Gluons

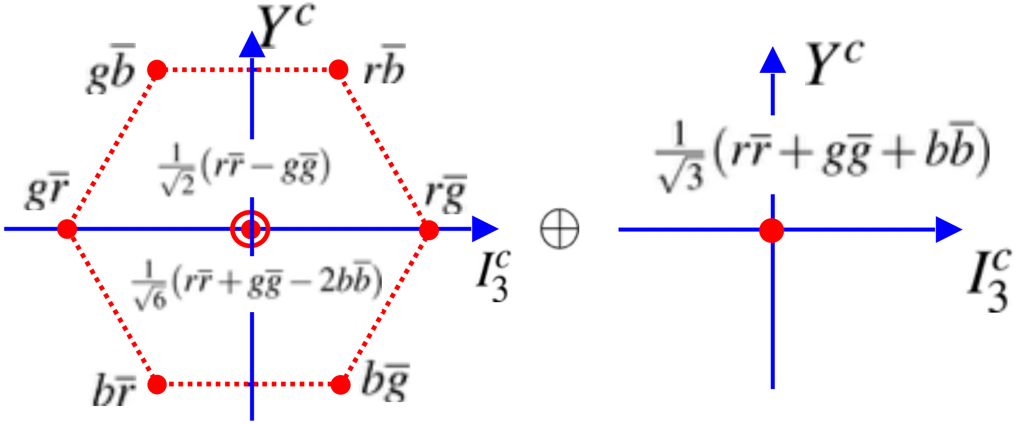
★ In QCD quarks interact by exchanging virtual massless gluons, e.g.



★ Gluons carry colour and anti-colour, e.g.

★ Gluon colour wave-functions (colour + anti-colour) are the same as those obtained for mesons (also colour + anti-colour)

⇨ OCTET + "COLOURLESS" SINGLET



- ★ So we might expect 9 physical gluons:

OCTET: $r\bar{g},\ r\bar{b},\ g\bar{r},\ g\bar{b},\ b\bar{r},\ b\bar{g},\ \frac{1}{\sqrt{2}}(r\bar{r}-g\bar{g}),\ \frac{1}{\sqrt{6}}(r\bar{r}+g\bar{g}-2b\bar{b})$

SINGLET: $\frac{1}{\sqrt{3}}(r\bar{r}+g\bar{g}+b\bar{b})$

- ★ **BUT**, colour confinement hypothesis:

only colour singlet states can exist as free particles ⇒ **Colour singlet gluon** would be unconfined. It would be behave like a strongly interacting photon ➡ infinite range Strong force.

- ★ **Empirically**, the strong force is short range and therefore know that the physical gluons are confined. The colour singlet state does not exist in nature !

NOTE: this is not entirely ad hoc. In the context of gauge field theory (see minor option) the strong interaction arises from a fundamental SU(3) symmetry. The gluons arise from the generators of the symmetry group (the Gell-Mann λ matrices). There are 8 such matrices ➡ 8 gluons. Had nature "chosen" a U(3) symmetry, would have 9 gluons, the additional gluon would be the colour singlet state and QCD would be an unconfined long-range force.

NOTE: the "gauge symmetry" determines the exact nature of the interaction ➡ FEYNMAN RULES

Gluon-Gluon Interactions

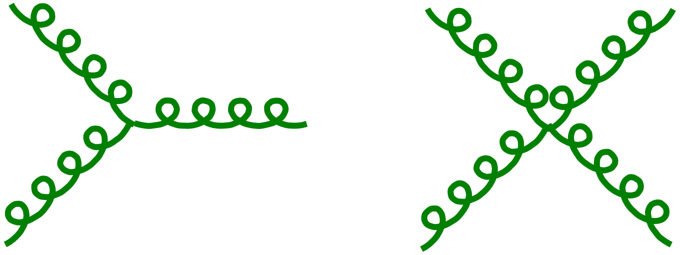
★ In QED the photon does not carry the charge of the EM interaction (photons are electrically neutral)

★ In contrast, in QCD the gluons do carry colour charge

⇨ **Gluon Self-Interactions**

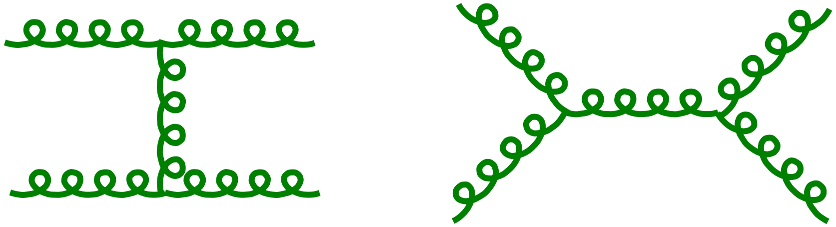
★ Two new vertices (no QED analogues)

triple-gluon vertex

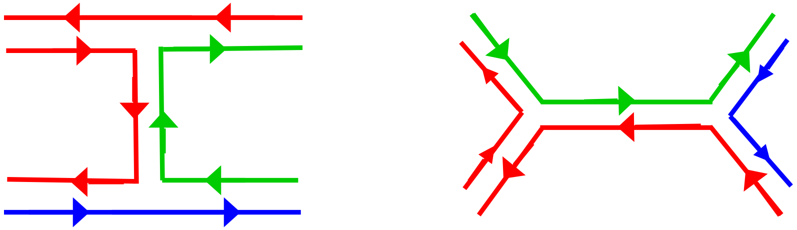


quartic-gluon vertex

★ In addition to quark-quark scattering, therefore can have gluon-gluon scattering



e.g. possible way of arranging the colour flow

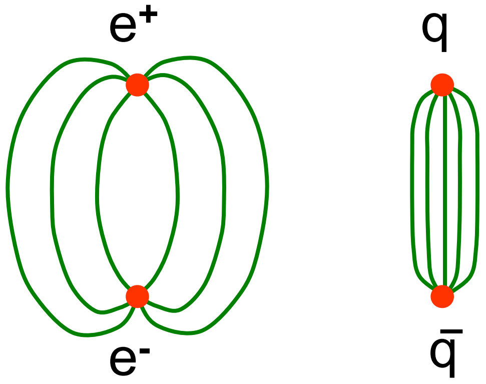


Gluon self-Interactions and Confinement

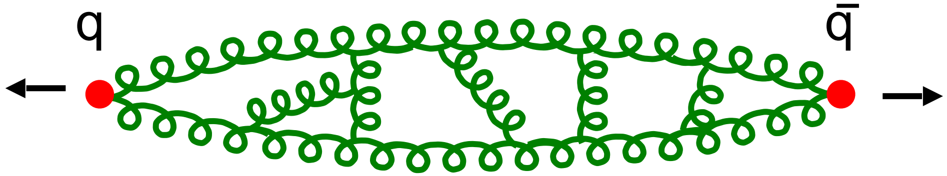
★ **Gluon self-interactions are believed to give rise to colour confinement**

★ **Qualitative picture:**

- **Compare QED with QCD**
- **In QCD “gluon self-interactions squeeze lines of force into a flux tube”**



★ **What happens when try to separate two coloured objects e.g.** $q\bar{q}$



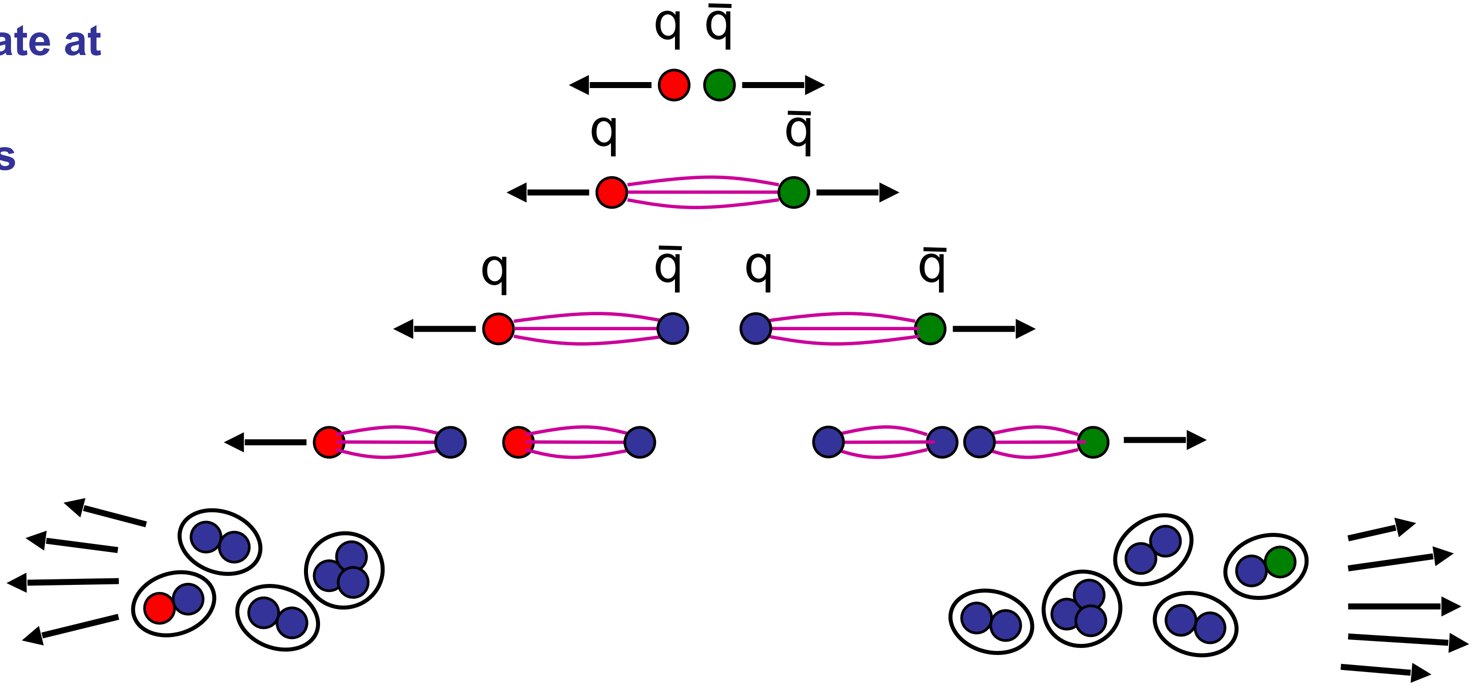
- **Form a flux tube of interacting gluons of approximately constant energy density** $\sim 1\,\mathrm{GeV/fm}$

➡ $V(r) \sim \lambda r$

- **Require infinite energy to separate coloured objects to infinity**
- **Coloured quarks and gluons are always confined within colourless states**
- **In this way QCD provides a plausible explanation of confinement – but not yet proven (although there has been recent progress with Lattice QCD)**

Hadronisation and Jets

★Consider a quark and anti-quark produced in electron positron annihilation



i) Initially Quarks separate at high velocity

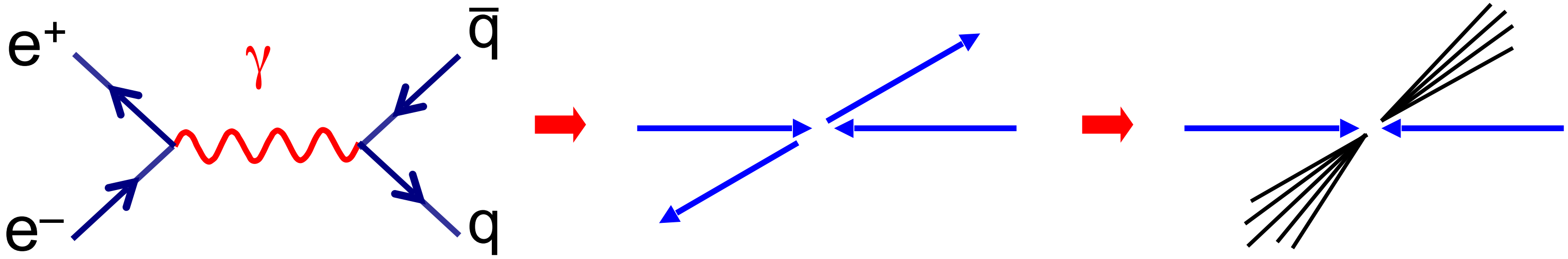
ii) Colour flux tube forms between quarks

iii) Energy stored in the flux tube sufficient to produce **q$\bar{\text{q}}$** pairs

iv) Process continues until quarks pair up into jets of colourless hadrons

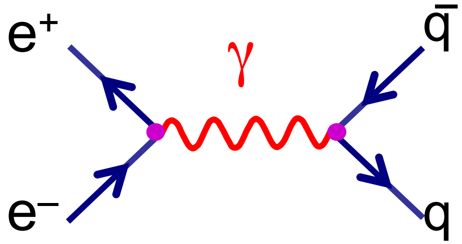
★ This process is called hadronisation. It is not (yet) calculable.

★ The main consequence is that at collider experiments **quarks** and **gluons** observed as jets of particles



QCD and Colour in e^+e^- Collisions

★ e^+e^- colliders are an excellent place to study QCD

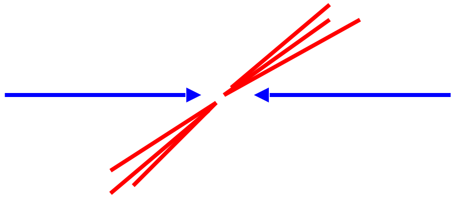


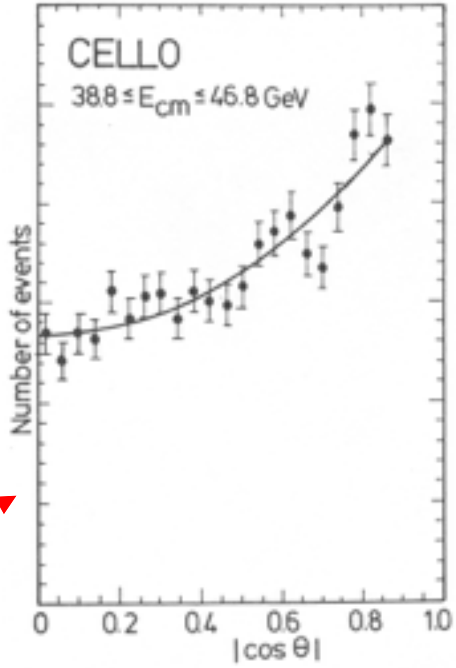
★ **Well defined production of quarks**
- QED process well-understood
- no need to know parton structure functions
- \+ experimentally very clean – no proton remnants

★ In handout 5 obtained expressions for the $e^+e^- \to \mu^+\mu^-$ cross-section

$$\sigma = \frac{4\pi\alpha^2}{3s} \qquad \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

- In e^+e^- collisions produce all quark flavours for which $\sqrt{s} > 2m_q$
- In general, i.e. unless producing a $q\bar{q}$ bound state, produce jets of hadrons
- Usually can't tell which jet came from the quark and came from anti-quark





H.J.Behrend et al., Phys Lett 183B (1987) 400

★ Angular distribution of jets $\propto (1+\cos^2\theta)$

➡ **Quarks are spin ½**

- ★ Colour is conserved and quarks are produced as $r\bar{r}, g\bar{g}, b\bar{b}$
- ★ For a **single quark flavour** and **single colour**

$$\sigma(e^+e^- \to q_i\bar{q}_i) = \frac{4\pi\alpha^2}{3s}Q_q^2$$

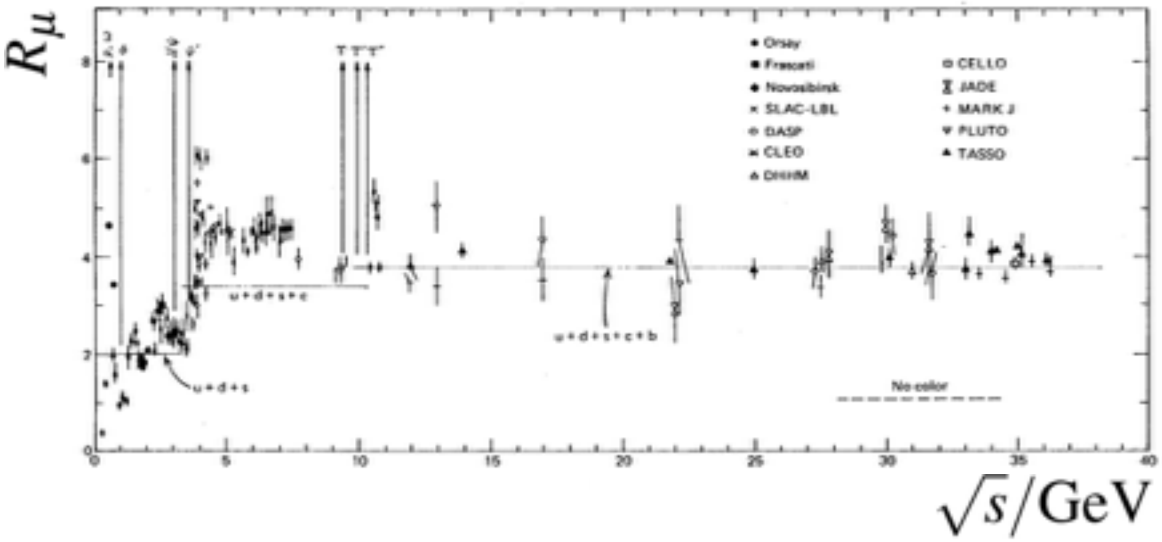
- Experimentally observe jets of hadrons:

$$\sigma(e^+e^- \to \text{hadrons}) = 3\sum_{u,d,s,..}\frac{4\pi\alpha^2}{3s}Q_q^2$$

Factor 3 comes from colours

- Usual to express as ratio compared to $\sigma(e^+e^- \to \mu^+\mu^-)$

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_{u,d,s,..}Q_q^2$$



u,d,s: $R_\mu = 3 \times (\frac{1}{9} + \frac{4}{9} + \frac{1}{9}) = 2$

u,d,s,c: $R_\mu = \frac{10}{3}$

u,d,s,c,b: $R_\mu = \frac{11}{3}$

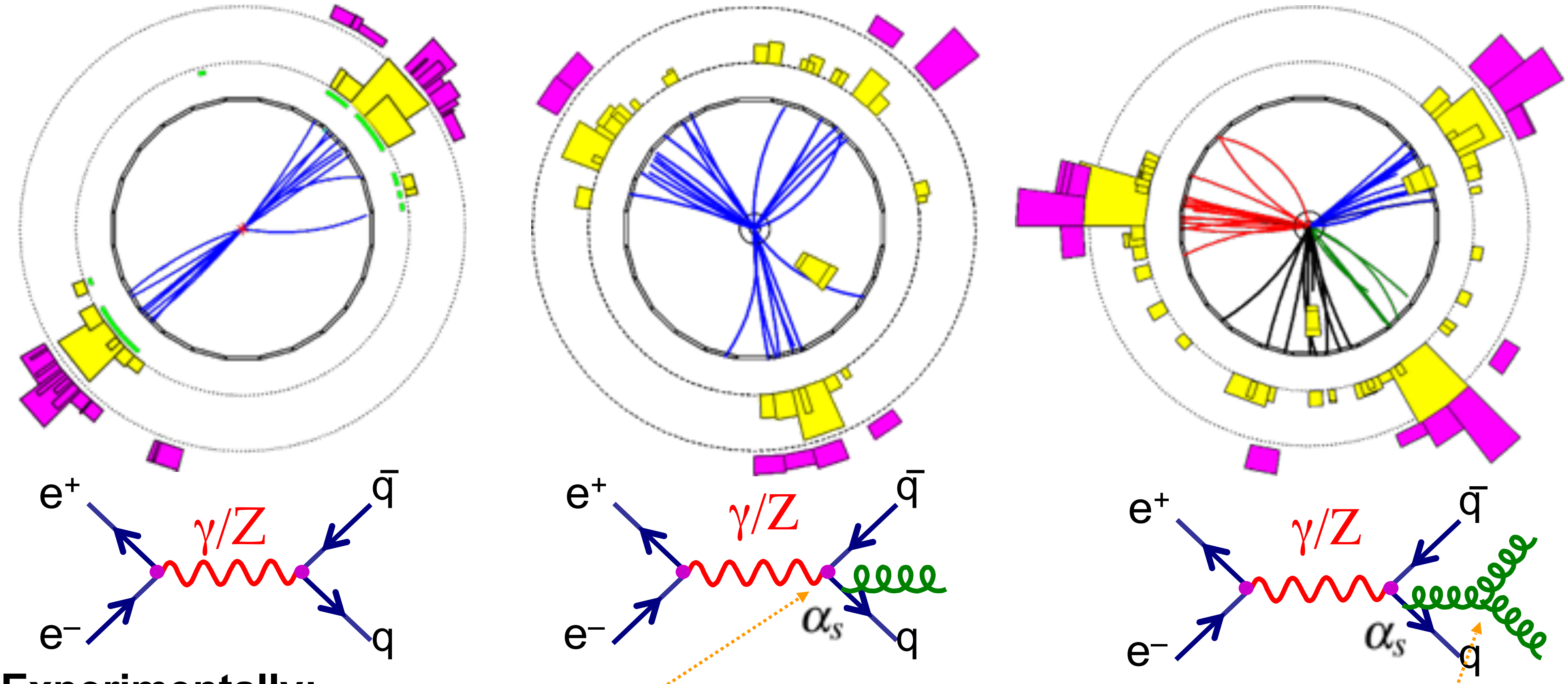
★Data consistent with expectation with factor 3 from colour

Jet production in e+e- Collisions

★e^+e^- colliders are also a good place to study gluons

$e^+e^- \rightarrow q\bar{q} \rightarrow 2\text{jets}$ $e^+e^- \rightarrow q\bar{q}g \rightarrow 3\text{jets}$ $e^+e^- \rightarrow q\bar{q}gg \rightarrow 4\text{jets}$

OPAL at LEP (1989-2000)

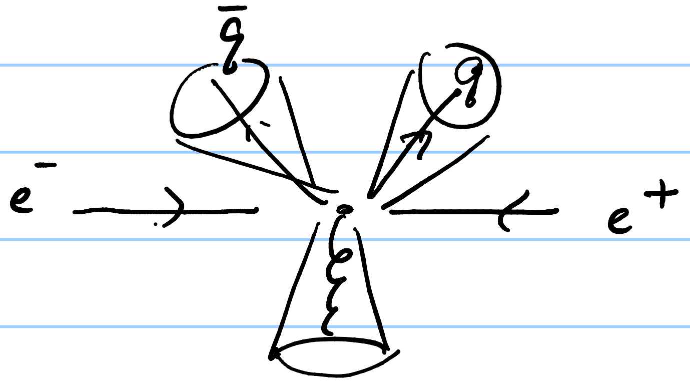


Experimentally:

- Three jet rate → measurement of α_s
- Angular distributions → gluons are spin-1
- Four-jet rate and distributions → QCD has an underlying SU(3) symmetry

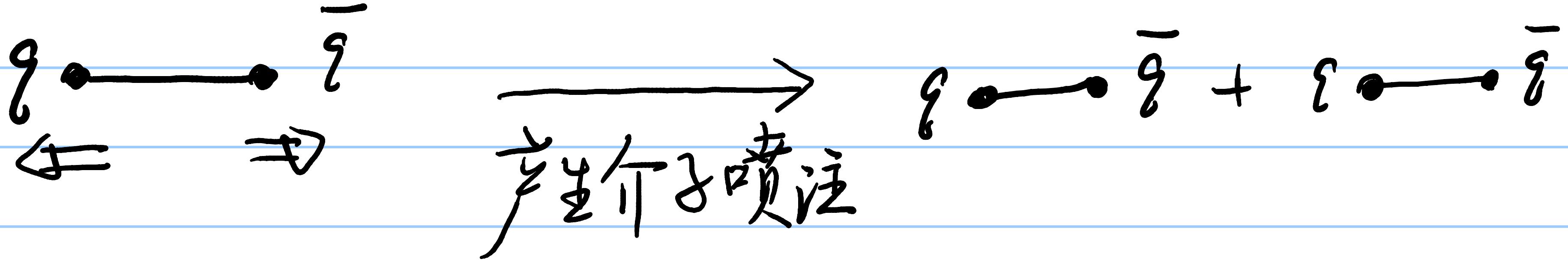
*) 胶子的发现及其属性检验

在正负电子对撞机上人们首先观测到强子喷注(jet)并通过和 $\mu^+\mu^-$ 产生截面相比，人们验证了夸克的色。传播强相互作用的媒介粒子——胶子——首先是在所谓的“三喷注事例”中观测到。



ANDERSSON, Gustafson
Sjostrand (1980)

⊛ Lund string model：夸克和胶子被囚禁于类似弦的结构中
喷注的形成是由于这些弦的不断的断裂



① 对撞过程所提供的能量都转化为产生强子。

色禁闭意味着带色粒子(quark和gluon)最终都转化为强子

② 实验上，　10 GeV 夸克 $\xrightarrow[\text{fragment}]{\text{碎裂}}$ ~ 7 强子

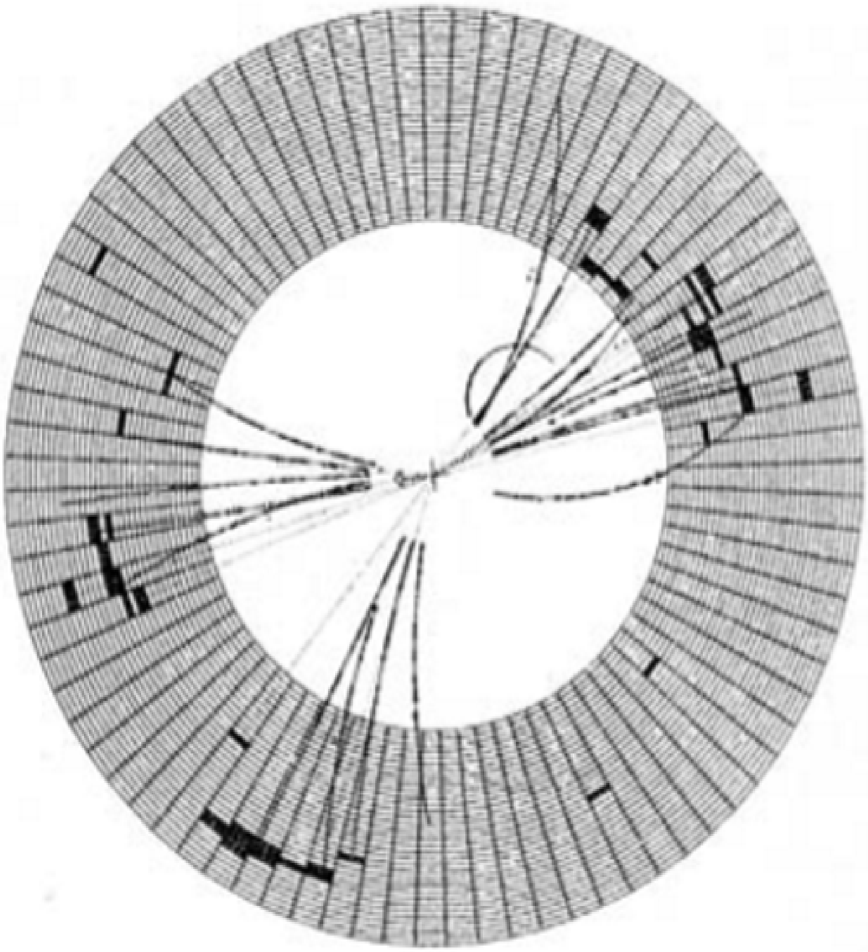
100 GeV 夸克 $\longrightarrow$ ~ 15 强子

π介子是最轻的强子，因此强子喷注的主要成分是π介子

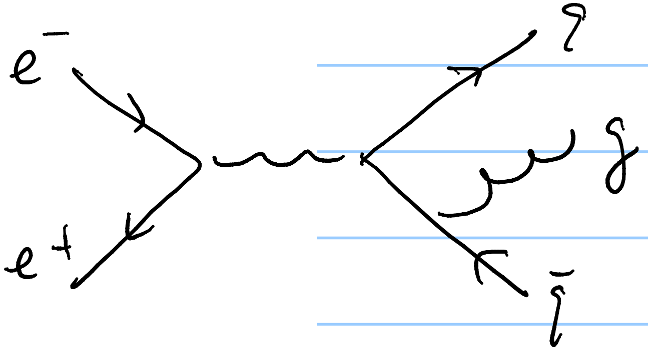
胶子强子化生成粒子数目（multiplicity）和夸克强子化完全不同。此二者的比值为

$$\sim \frac{C_A}{C_F} = \frac{N_c}{\frac{N_c^2-1}{2N_c}} = \frac{3}{4/3} = \frac{9}{4}$$

*) 三喷注事例 (three-jets) 1979年



three-jet event
at JADE detector
at Petra collider
at DESY



1976年 Ellis, Gaillard and Ross, NPB111, 253

"Search for gluons in e^+e^- annihilation"

提出：按照量子色动力学，当原有夸克之一放射出"硬"胶子（即携带大横动量的胶子），就会有三喷注事例

此硬胶子转变为第三道胶子喷注

三喷注事例首次于1979年8月在美国费米国家实验室举办的高能会议上公布。按照流行的描述，这一事例被视作是胶子存在的确凿证据，被认为是人类努力了解物质的细结构方面的重大进展……（理论上10年前就预言胶子）

⟹ 狂欢的美国

但欧洲同行的反映平淡

——这有什么值得大惊小怪的呢？

研究人员并没有在仪器真正地“看到”胶子

虽然人们在PETRA上发现一小部分正负电子碰撞产生了三个粒子喷注，而且这些喷注都在同一平面，

但这些结果并不代表胶子存在的确凿证据。

为什么要大力鼓吹这些薄弱的证据？

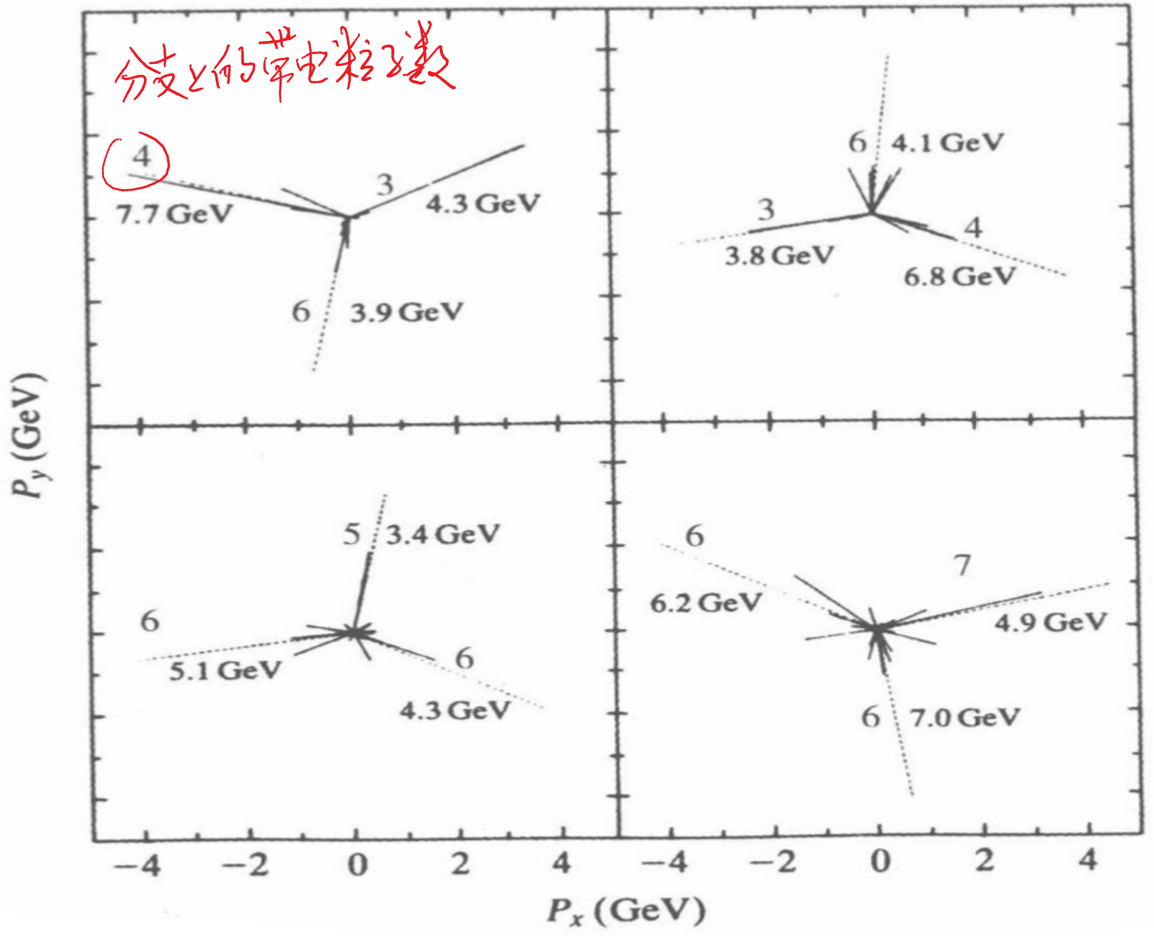
腹黑的同行得到一个答案：

美国的粒子物理学家正力图促使美国政府保持对他们实验研究的资助力度。这场竞争已经延续、到下一代更高能量加速度的建设上。因此，美国同行需要证明，未来的投入是正确的，极具潜力的。

1979年PETRA公布三喷注事例，引起人们注意

下面是4个早期的三喷注候选事例

Phys. Lett. 86B, 243 (1979)


分支上的带电粒子数

实线 — 离开正负电子作用区的带电粒子轨迹
每根线的长度正比于粒子测得动量

虚线 — 实验家计标绘出的喷注的轴

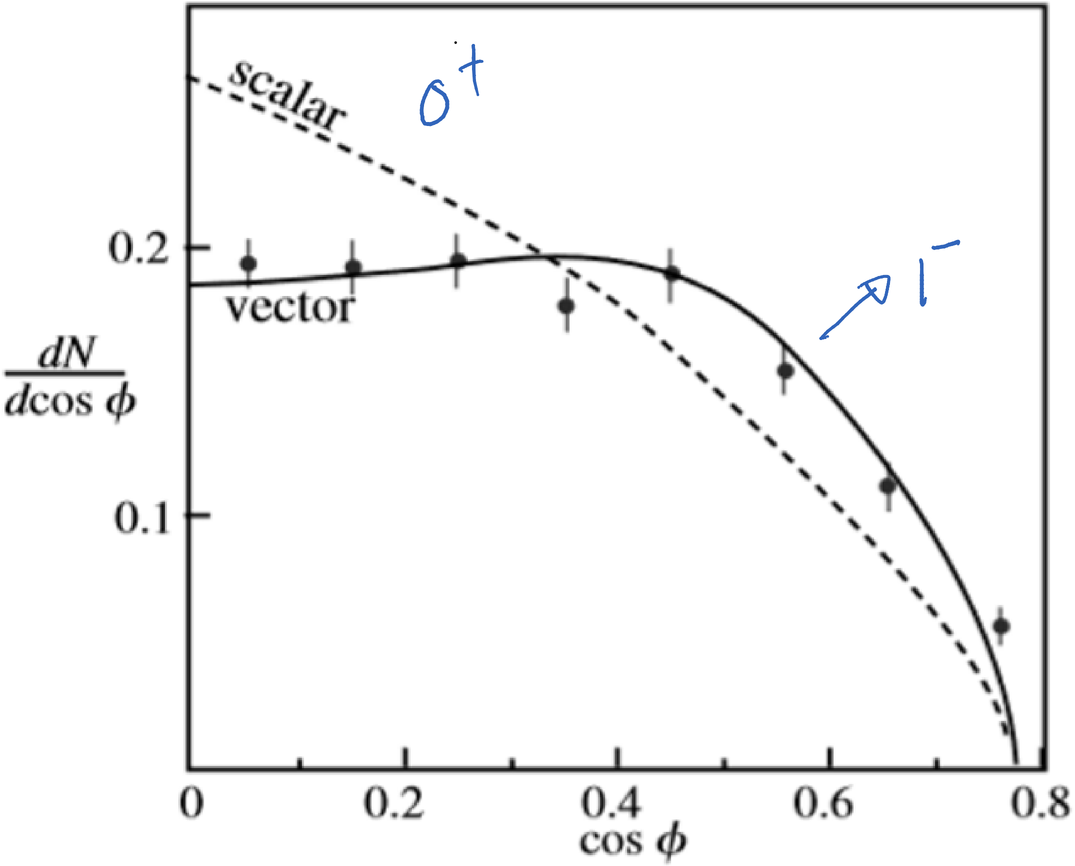
PETRA实验 ⇒ 支持QCD的硬胶子发射图像

胶子自旋测量

TASSO collabration

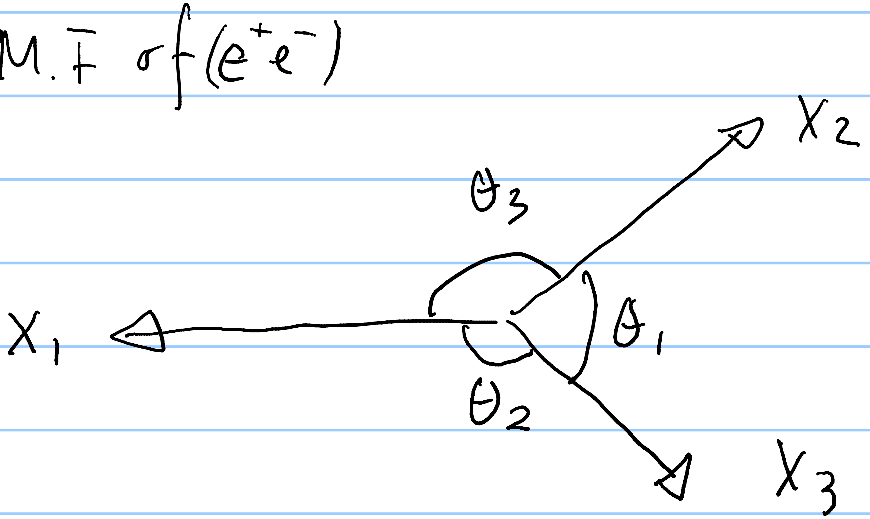
Phys. Lett. B 97, 453 (1980)

6.2 Scattering experiments

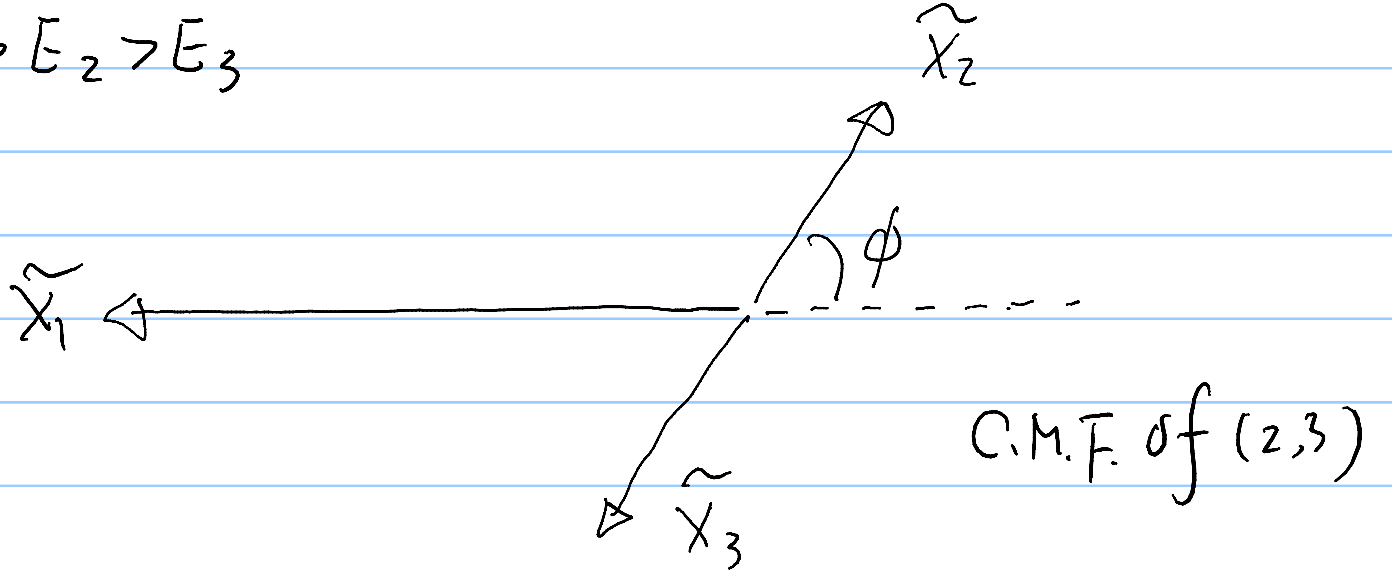


角动量守恒 $\Longrightarrow$ 末态三个喷注总处于同一平面

(a) C.M.F of (e^+e^-)



(b) $E_1 > E_2 > E_3$



历史：虽然当时QCD理论尚未完善，但人们已经从QED中学到许多经验，可以采用微扰论来计算。

De Rújula, Ellis, Petronzio, Preparata and Scott 等人于1979年在意大利的国际暑期学校的讲稿中给出

保证你薪水的绝对可靠的方法

(1) 随机地做一个量子电动力学的计算

如果没有题目或不知道怎么做，不妨参考苏联人 Gribov 和 Lipatov (1972) SJNP 15, 438

（为 QED 有关技术的开创性文献）

(2) 将 $\alpha \longrightarrow \alpha_s(Q^2)$

(3) 将标题中的量子电动力学 ⟶ 量子色动力学

(!!!) 别忘了将作者名字改为你自己的名字（别抄错了！）

(4) 投稿发表

(5) 子循环结束，返回(1)处

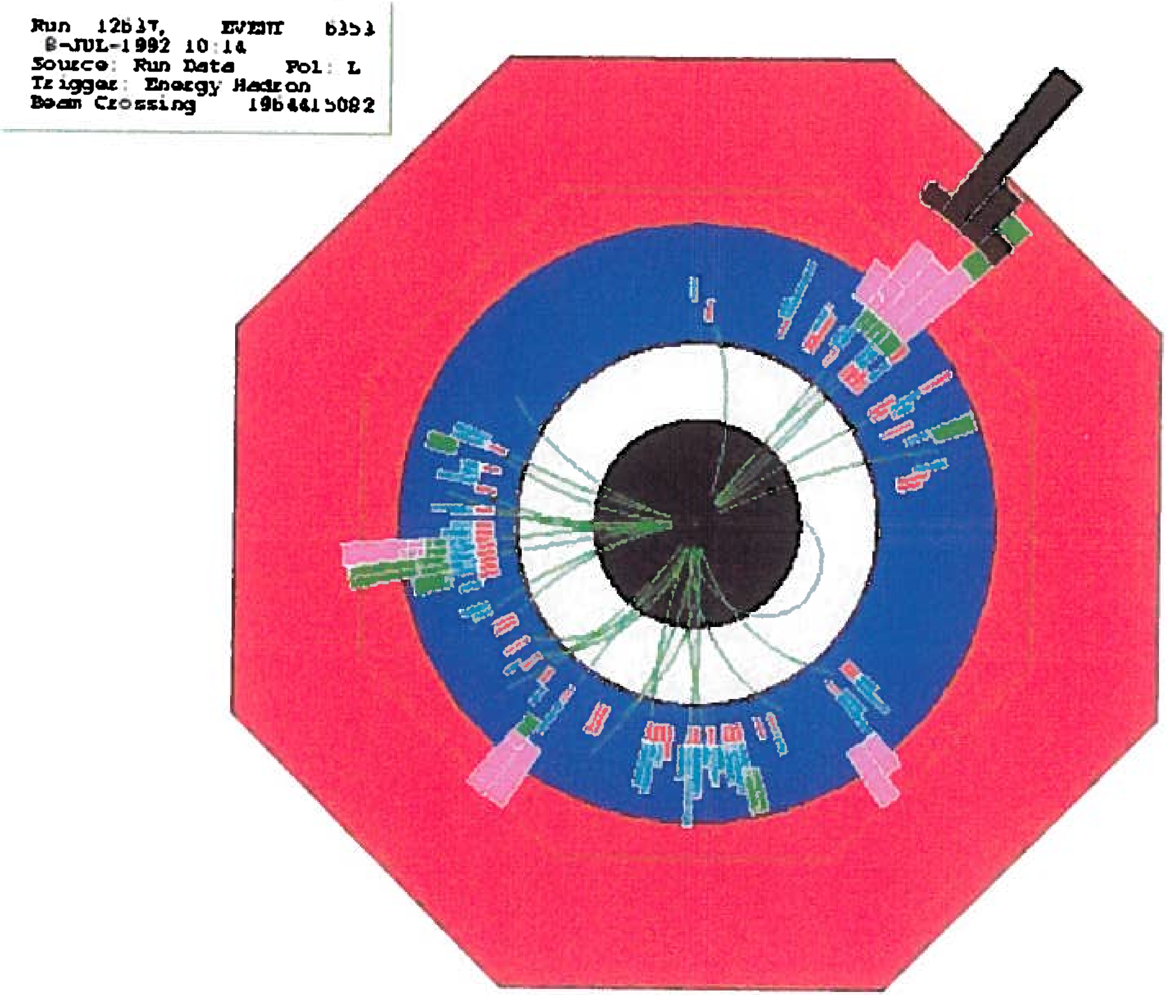


Fig 3 3-jet event at 91 GeV

from www.slac.stanford.edu/exp/sld/figure/intro.html

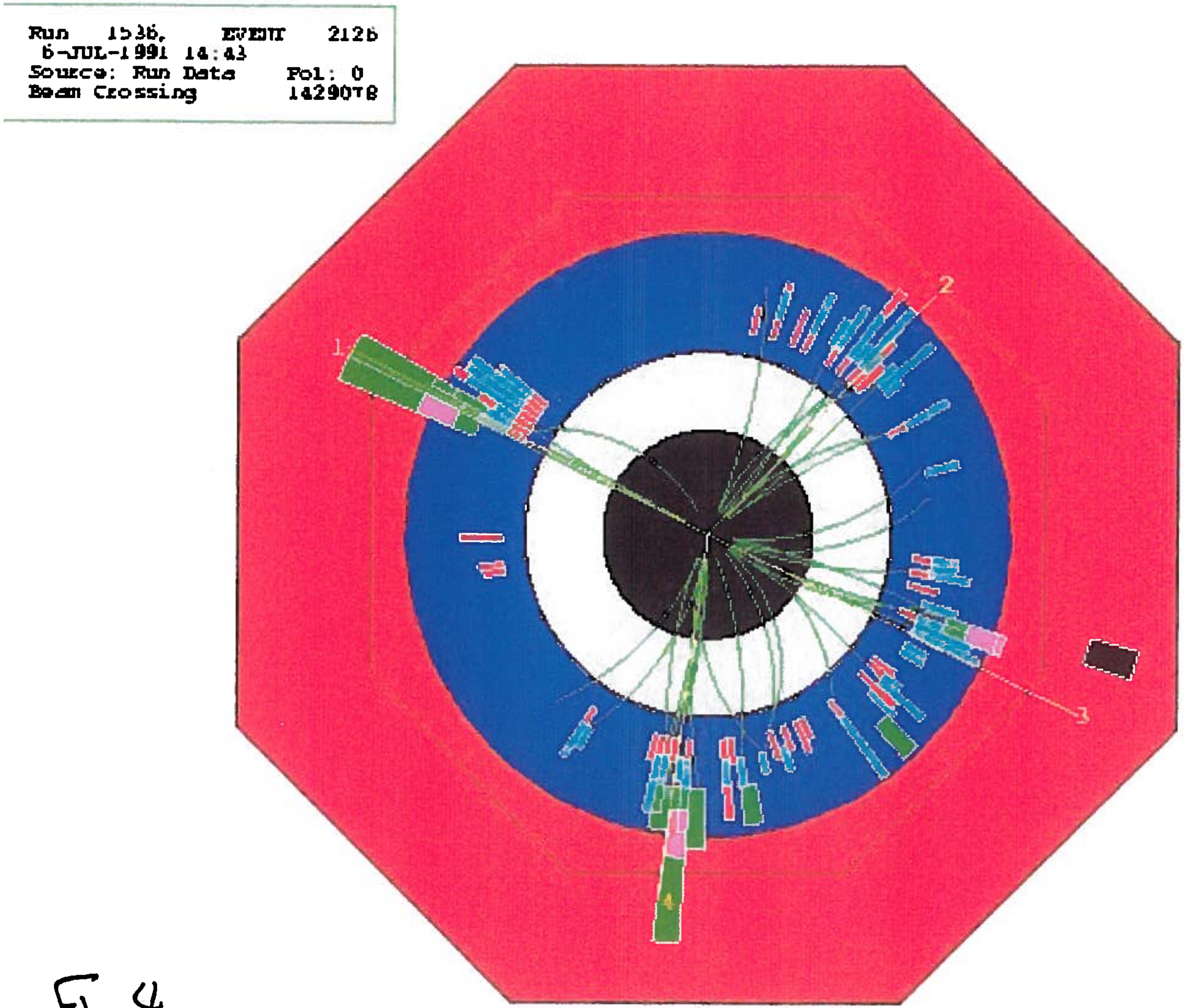


Fig. 4

4-jet event at 91 GeV, from

www.slac.stanford.edu/exp/sld/figure/intro.html

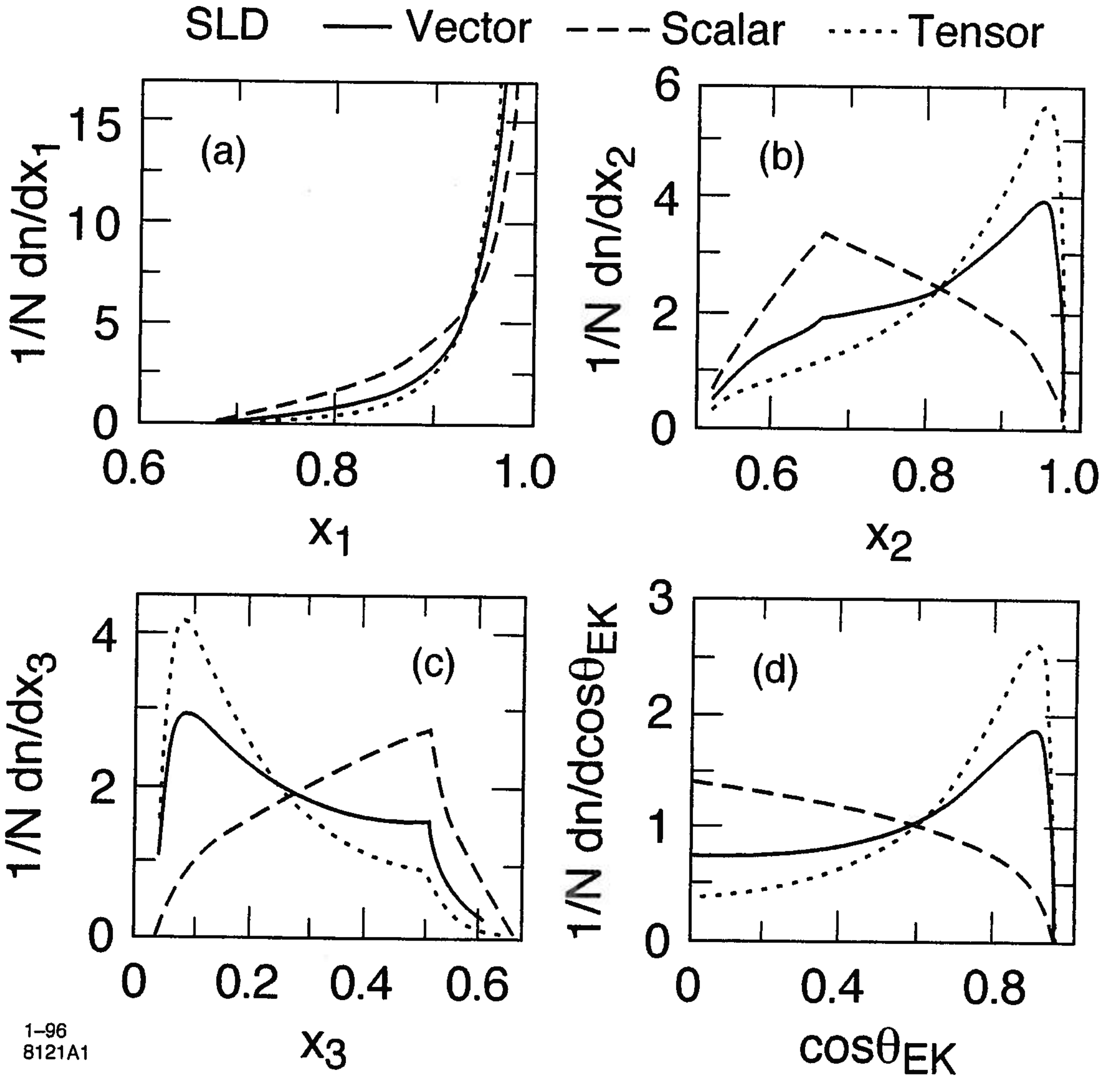


Fig. 6

Expections for x_1 x_2 x_3 in 3-jet events at 91 GeV, from K. Abe et al Phys. Rev D55, 2533 (1997)

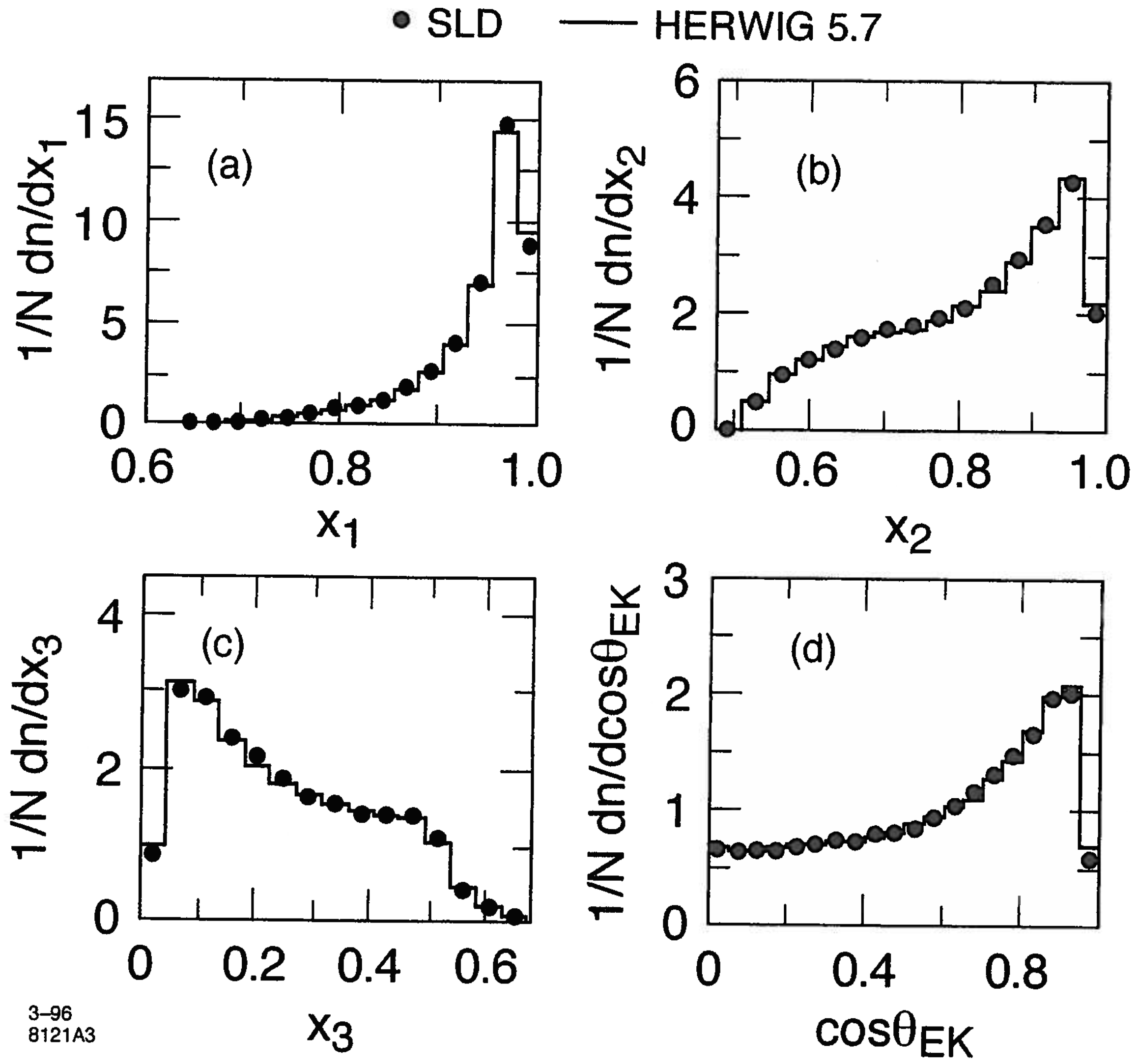


Fig. 7 Measurements of x_1 x_2 x_3 in 3-jet events at 91 GeV, from K. Abe et al Phys. Rev. D<u>55</u>, 2533 (1997)

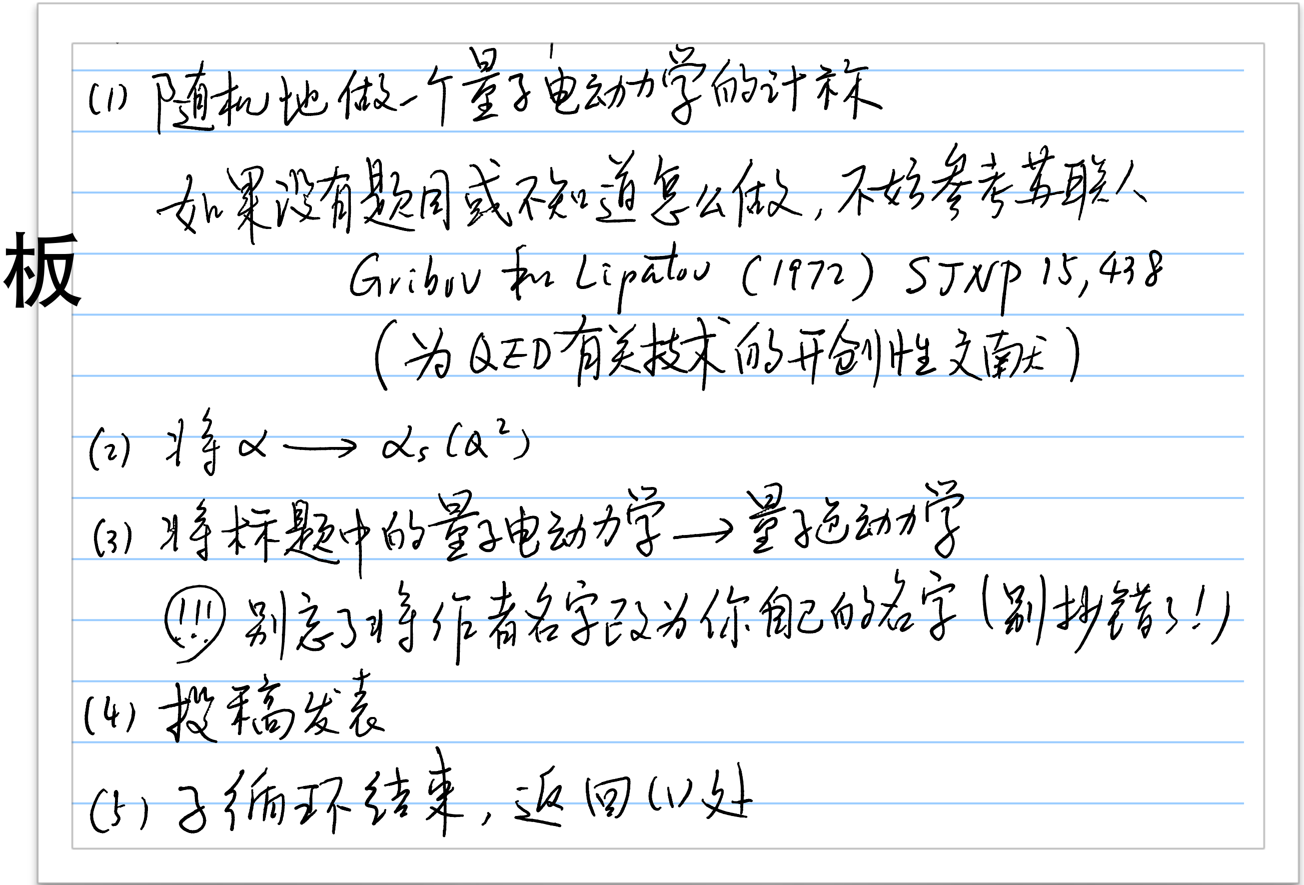
小结

1）色紧闭：色单态——色空间波函数

2）胶子：正反色荷

3）多喷注事例

4）发文章的模板



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