

# Unitarity Bounds and Mass Generation Scale

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# **Beyond the Standard Model: Mass as a Clue**

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# Outline

- ★ Shape of the Standard Model (SM)
- ★ Scales of Mass Generations: Universal Limits
- ★ Higgs, Hierarchy Problem & “Little Higgs”
- ⊗ Beyond 4 Dimensions ?
- ★ Summary & Outlook

# Outline

## ★ Shape of the Standard Model (SM)

- Symmetries, Particles, Mass Spectrum & Exp Data
- Looking beyond: Lesson from Columbus ?
- Looking beyond: Two Essential Clues

## ★ Scales of Mass Generations: Universal Limits

- Scale of Mass Generation & Classic  $2 \rightarrow 2$  Unitarity Limits
- $2 \rightarrow n$  Scattering: Puzzle, Resolution & Surprise
- Scales for Weak Bosons, Quarks/Leptons & Majorana Neutrinos

## ★ Higgs, Hierarchy Problem & “Little Higgs”

- Higgs Mechanism & Hierarchy Problem
- Little Higgs Solves Little Hierarchy
- Unitarity of Little Higgs vs UV Completion

## ⊗ Beyond 4 Dimensions ?

## ★ Summary & Outlook

# Shape of the Standard Model

## Classical Mechanics

- angular momentum  $\gg \hbar$
- speed  $\ll c$

## Quantum Mechanics

- any angular momentum
- speed  $\ll c$

## Special Relativity

- angular momentum  $\gg \hbar$
- any speed

## Quantum field theory

- any angular momentum
- any speed

## Standard Model

- local quantum gauge theory
- $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$
- valid down to  $\sim 10^{-16}$  cm

# Shape of the Standard Model

Elementary Particles discovered by experiments:

$$\begin{array}{l}
 \text{leptons} \\
 \text{quarks}
 \end{array}
 \left\{ \begin{array}{ccc}
 \begin{pmatrix} \nu_L^e \\ e_L \\ e_R \end{pmatrix} & \begin{pmatrix} \nu_L^\mu \\ \mu_L \\ \mu_R \end{pmatrix} & \begin{pmatrix} \nu_L^\tau \\ \tau_L \\ \tau_R \end{pmatrix} \\
 \begin{pmatrix} u_L \\ d_L \\ u_R \\ d_R \end{pmatrix} & \begin{pmatrix} c_L \\ s_L \\ c_R \\ s_R \end{pmatrix} & \begin{pmatrix} t_L \\ b_L \\ t_R \\ b_R \end{pmatrix}
 \end{array} \right. \Rightarrow \left( \text{spin } \frac{1}{2} \right)$$

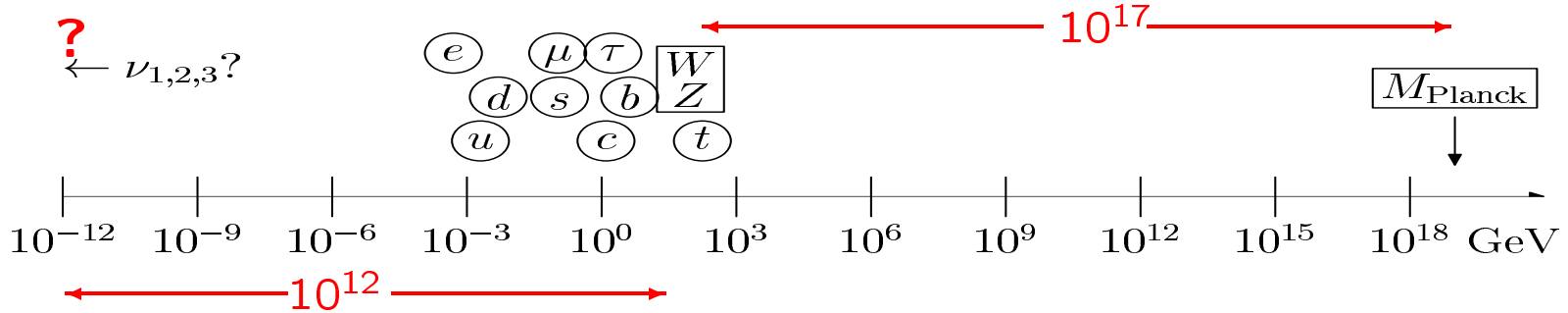
$SU(3)_C \times SU(2)_W \times U(1)_Y$  gauge bosons (spin 1)  
 8 gluons +  $W^\pm, Z, \gamma$

longitudinal  $W^\pm, Z$  (spin 0)

$\oplus$  Higgs  $H^0$  (spin 0)  $\Rightarrow$  not yet discovered!

“God Particle” gives masses to W/Z & Quarks/Leptons!

► SM Mass Spectrum:

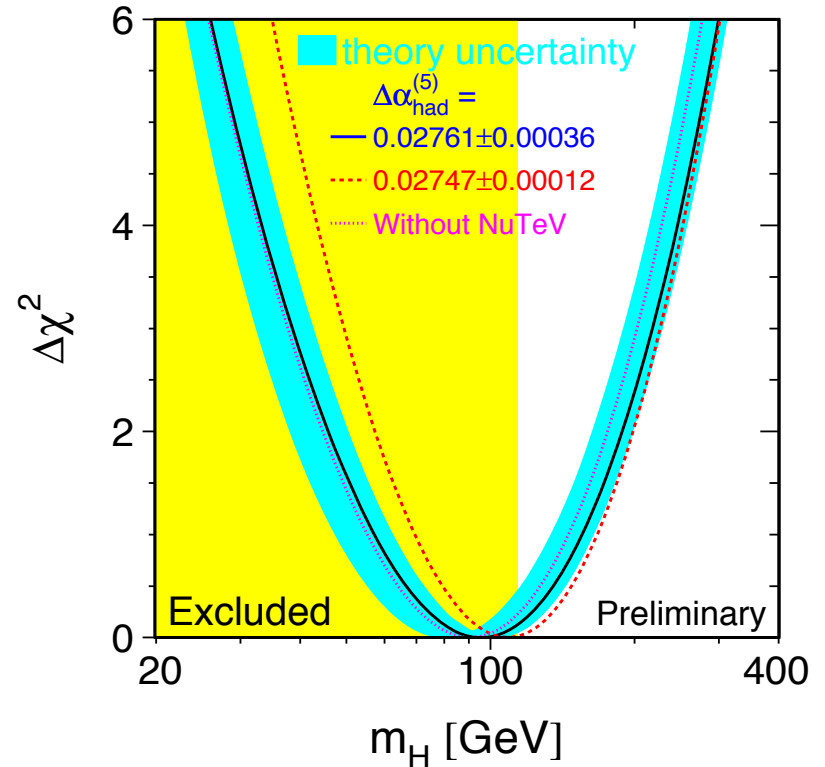


► Precision data fit SM well and favor a Light Higgs:

$114.4 \text{ GeV (direct)} < M_H < 219 \text{ GeV (precision)} \quad (95\% \text{ C.L.})$

Summer 2003

	Measurement	Fit	$10^{\text{meas}} - O_{\text{fit}} / \sigma_{\text{meas}}$
$\Delta\alpha_{\text{had}}^{(5)}(m_Z)$	$0.02761 \pm 0.00036$	0.02767	
$m_Z$ [GeV]	$91.1875 \pm 0.0021$	91.1875	
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	2.4960	
$\sigma_{\text{had}}^0$ [nb]	$41.540 \pm 0.037$	41.478	
$R_l$	$20.767 \pm 0.025$	20.742	
$A_{\text{fb}}^{0,l}$	$0.01714 \pm 0.00095$	0.01636	
$A_l(P_{\tau})$	$0.1465 \pm 0.0032$	0.1477	
$R_b$	$0.21638 \pm 0.00066$	0.21579	
$R_c$	$0.1720 \pm 0.0030$	0.1723	
$A_{\text{fb}}^{0,b}$	$0.0997 \pm 0.0016$	0.1036	
$A_{\text{fb}}^{0,c}$	$0.0706 \pm 0.0035$	0.0740	
$A_b$	$0.925 \pm 0.020$	0.935	
$A_c$	$0.670 \pm 0.026$	0.668	
$A_l(\text{SLD})$	$0.1513 \pm 0.0021$	0.1477	
$\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{\text{fb}})$	$0.2324 \pm 0.0012$	0.2314	
$m_W$ [GeV]	$80.426 \pm 0.034$	80.385	
$\Gamma_W$ [GeV]	$2.139 \pm 0.069$	2.093	
$m_t$ [GeV]	$174.3 \pm 5.1$	174.3	
$\sin^2\theta_W(\nu N)$	$0.2277 \pm 0.0016$	0.2229	
$Q_W(\text{Cs})$	$-72.84 \pm 0.46$	-72.90	



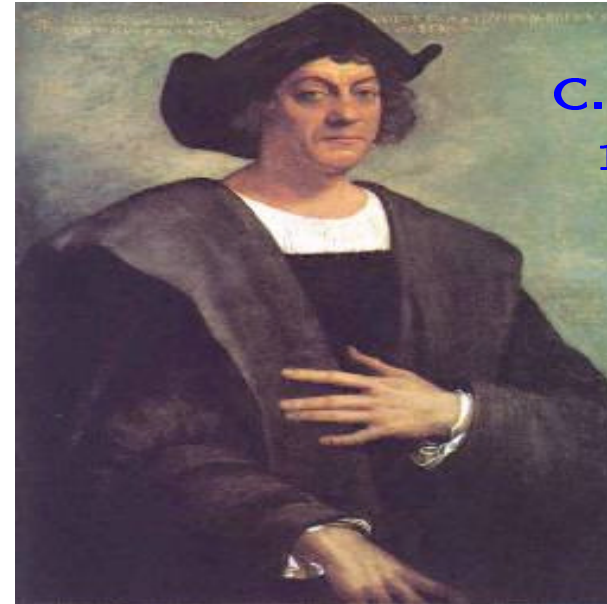
# Looking Beyond: Lesson from Columbus ?

► **1492-1504 A.D.:** Columbus carried out heroic **experimental** probes to prove **Pythagoras theory** (~500 B.C.) that **the Earth is round** (and to search for **Mainland China & Gold**).....But, he discovered **Something Else !!!**

**Voyage**  
**1492-1504**



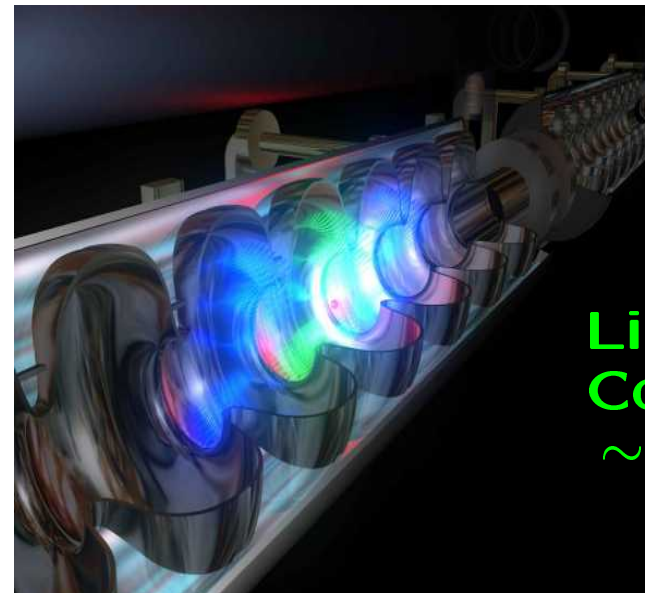
**C. Columbus**  
**1451-1506**



**CERN**  
**LHC**  
**2007**



**Linear**  
**Colliders**  
**~2012**





# Looking Beyond: Two Essential Clues

## ★ Clue-1:

**What's Wrong with just Bare Mass terms for All particles ??**

## ★ Clue-2:

**What's Wrong with having a Fundamental Higgs Boson ??**

# Looking Beyond: Two Essential Clues

## ★ Clue-1:

What's Wrong with just Bare Mass terms for All particles ??

▶ **Unitarity Violation**  $\Rightarrow$  Limit on Scale of Mass Generation!

$\Rightarrow$  e.g.  $WW \rightarrow WW$  Scattering justifies TeV Scale for LHC Energy! .....

## ★ Clue-2:

What's Wrong with having a Fundamental Higgs Boson ??

▶ **Hierarchy Problem**  $\Rightarrow$  Little Higgs, Extra Dim, DSB, SUSY...!

## Looking Beyond: Two Essential Clues (1)

★ Clue-1:

What's Wrong with just Bare Mass terms for All particles ??

# Scales of Mass Generations

## Define Scale of Mass Generation

- ▶ In Standard Model (SM), Weak Bosons  $W/Z$  and all Fermions obtain masses from a Hypothetic Fundamental Higgs  $H^0$ . But, the masses can of course be generated by **Something Else!** Without assuming  $H^0$ , all masses must be put in by hand for SM, which can be gauge-invariant only under Nonlinear realization of SM gauge group.  $\Rightarrow$  **Non-renormalizability!** This causes a **Unitarity Violation Scale**  $\Lambda_U$  in High Energy Scatterings at which **New Physics** must enter to **restore unitarity!**
- ▶ Define scale  $\Lambda_x$  for generating a mass  $m_x$  to be the **Minimal Energy** above which the Bare Mass Term for  $m_x$  has to be replaced by a **Renormalizable Interaction** involving **New State(s)**.
- ▶ **Unitarity Violation Scale**  $\Lambda_U$  provides a **Model-Independent Universal Upper Limit** on the Scale of Mass-Generation  $\Lambda_x$  (for mass  $m_x$ ):

$$\Lambda_x \leq \Lambda_U$$

# Nonlinear Realization of SM Gauge Symmetry

- ▶ **Without** assuming Higgs  $H^0$ , SM gauge symmetry must be **nonlinearly** realized, and 3 “eaten” Goldstones  $\{\pi^a\}$  are formulated by

$$U = \exp [i\pi^a \tau^a / v], \quad (v \simeq 250\text{GeV} \Rightarrow \text{recall: } f_\pi^{\text{QCD}} \simeq 92\text{MeV})$$

- ▶ **Gauge Boson** bare mass terms  $M_W^2 W^+ W^- + \frac{1}{2} M_Z^2 Z^2$  can be written as **dim-2 gauge-invariant operator**:

$$\mathcal{L}_{\text{mass}}^V = \frac{v^2}{4} |D_\mu U|^2,$$

- ▶ **For Dirac Fermions (Quarks/Leptons)**  $F = (f, f')^T$ , the **bare mass terms**  $-m_f \bar{f} f - m_{f'} \bar{f}' f'$  can be written as **gauge-invariant dim-3 operator**:

$$\mathcal{L}_{\text{mass}}^f = -m_f \bar{F}_L U \begin{pmatrix} 1 \\ 0 \end{pmatrix} f_R - m_{f'} \bar{F}_L U \begin{pmatrix} 0 \\ 1 \end{pmatrix} f'_R + \text{H.c.}$$

- ▶ **Light Neutrinos** can form **bare Majorana Mass term**  $-\frac{1}{2} m_\nu^{ij} \nu_{Li}^T \hat{C} \nu_{Lj} + \text{H.c.}$   
 $\Rightarrow$  **Gauge-invariant form**, with  $\Phi = U(0, v/\sqrt{2})^T$ ,  $F_{Lj} = L_j$ ,

$$\mathcal{L}_{\text{mass}}^\nu = -\frac{m_\nu^{ij}}{v^2} L_i^{\alpha T} \hat{C} L_j^\beta \Phi^{\alpha'} \Phi^{\beta'} \epsilon^{\alpha\alpha'} \epsilon^{\beta\beta'} + \text{H.c.}$$

## Classic Limits on Scales of Mass Generations

- ▶ **Scattering  $W_L W_L \rightarrow W_L W_L$  on Electroweak Symmetry Breaking Scale:**  
(Dicus & Mathur, Phys.Rev.1973; Lee, Quigg, Thacker, Phys.Rev.1977)

$$\Lambda_U \simeq \sqrt{8\pi} v \simeq 1.2 \text{ TeV} \quad \Rightarrow \quad \boxed{\text{TeV Scale for LHC !!!}}$$

- ▶ **Scattering  $f\bar{f} \rightarrow W_L W_L$  on Dirac Fermion Mass Generation:**  
(Appelquist & Chanowitz, Phys.Rev.Lett.1987)

$$\Lambda_U \simeq \frac{8\pi v^2}{\sqrt{2N_c} m_f} \simeq (3.6, 2 \times 10^5; 605, 2 \times 10^6) \text{ TeV} \quad \text{for } f = (t, u; \tau, e)$$

- ▶ **Scattering  $\nu_L \nu_L \rightarrow W_L W_L$  on Majorana Neutrino Mass Generation:**  
(Willenbrock et al, Phys.Rev.Lett.2001)

$$\Lambda_U \simeq \frac{2\pi v^2}{m_\nu} \simeq 10^{16} \text{ GeV} \quad (m_\nu \simeq 0.05 \text{ eV})$$

$\Rightarrow$  **Seesaw/GUT Scale!**

# High Energy Scattering: $W_L W_L, f\bar{f} \rightarrow nW_L$ ( $n \geq 2$ )

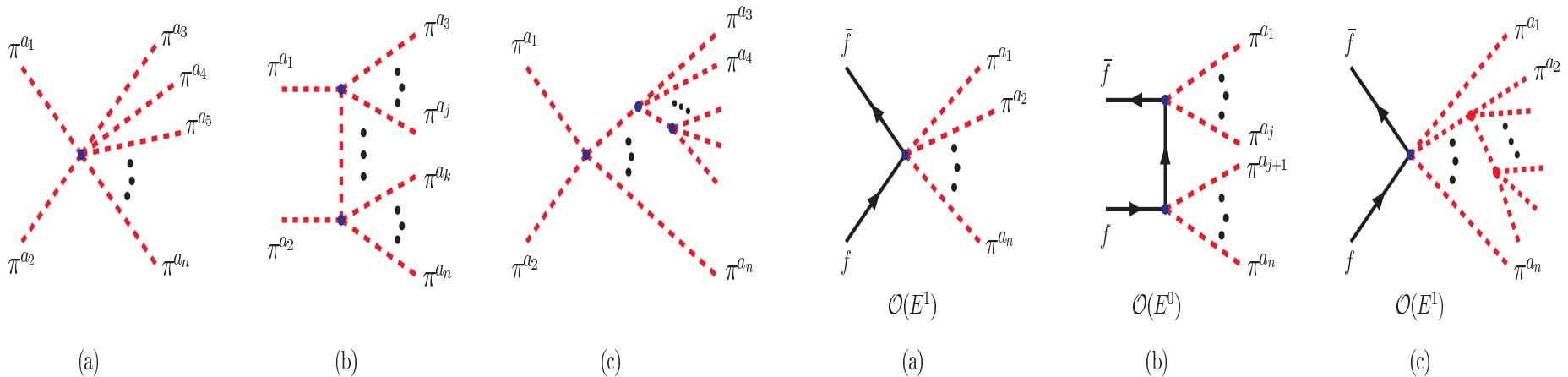
★ **Equivalence Theorem gives,** (for review, He, et al, hep-ph/9704276)

$$T[W_L^{a_1}, \dots, W_L^{a_n}; \Phi_{\text{phys}}] = (-i)^n T[\pi^{a_1}, \dots, \pi^{a_n}; \Phi_{\text{phys}}] + \mathcal{O}\left(\frac{M_W}{E_j}\right)$$

★ **Power Counting of high energy Scattering Amplitudes:**

$$T[f\bar{f}, \nu\nu \rightarrow nW_L^a] \simeq T[f\bar{f}, \nu\nu \rightarrow n\pi^a] = \mathcal{O}(1) \frac{m_{f,\nu}}{v^n} E,$$

$$T[W_L^{a_1} W_L^{a_2} \rightarrow nW_L^a] \simeq T[\pi^{a_1} \pi^{a_2} \rightarrow n\pi^a] = \mathcal{O}(1) \frac{E^2}{v^n},$$



# Puzzle: $2 \rightarrow n$ Scattering, E-Counting & Unitarity Limit

► General  $2 \rightarrow n$  Unitarity Condition by requiring  $SS^\dagger = S^\dagger S = 1$ ,

$$\sigma[2 \rightarrow n] < \frac{4\pi}{s}$$

► **Puzzle**: Energy Power Counting shows: ( $s = E^2$ )

$$\frac{1}{\mathcal{J}_{\text{in}}} \int_{\text{PS}_n} \sim s^{n-3},$$

$$\sigma[2 \rightarrow n] \propto \frac{1}{s} \left(\frac{s}{v^2}\right)^{n-2+\delta} \left(\frac{m_f}{v}\right)^{2(2-\delta)}, \quad (n \geq 2),$$

$$\Lambda_{\text{U}} \sim v \left[ c_0 \left(\frac{v}{m_f}\right)^{2(2-\delta)} \right]^{\frac{1}{2(n-2+\delta)}} \longrightarrow v, \quad (\text{for } n \rightarrow \text{large}),$$

Here  $\delta = 1$  &  $2$  for  $f\bar{f}/\nu\nu$  &  $W_L W_L$  scattering. ► **No New Scale for  $m_f$  ?!**

► BUT, we have **Kinematic Condition**:

$$\sqrt{s} > nM_W \simeq \frac{n}{3}v, \quad \longrightarrow \quad \Lambda_{\text{U}} > v \frac{n}{3} \quad (\uparrow \text{ with } n)$$



# Resolution and Surprise

Dicus & He, 2003

## ★ Resolution

- **Computing Exact Phase Space:** (recall: Fermi's Golden rule in QM...)

$$\begin{aligned} \frac{1}{\mathcal{J}_{\text{in}}} \int_{\text{PS}_n} &= \frac{1}{\mathcal{J}_{\text{in}}} \int \frac{d^3k_1 \cdots d^3k_n}{2E_1 \cdots 2E_n} (2\pi)^{4-3n} \delta^{(4)}\left(p_1 + p_2 - \sum k_j\right) \\ &= \frac{s^{n-3}}{2^{4(n-1)} \pi^{2n-3} [(n-1)!(n-2)!]} \end{aligned}$$

- **Improved Estimates:** (  $c_0 = \mathcal{O}(1)$ ,  $e = 2.718\dots$ ,  $(n!)^{1/n} \rightarrow n/e$  )

$$\begin{aligned} \Lambda_U &= v \left[ c_0 2^{4n-2} \pi^{2(n-1)} [(n-1)!(n-2)!] \left(\frac{v}{m_f}\right)^{2(2-\delta)} \right]^{\frac{1}{2(n-2+\delta)}} \\ &\Rightarrow v \frac{4\pi n}{e} > v \frac{n}{3} \quad (\text{for } n \gg 1) \end{aligned}$$

- **Kinematic Condition**  $\Lambda_U > v \frac{n}{3}$  is satisfied due to Exact Phase Space!

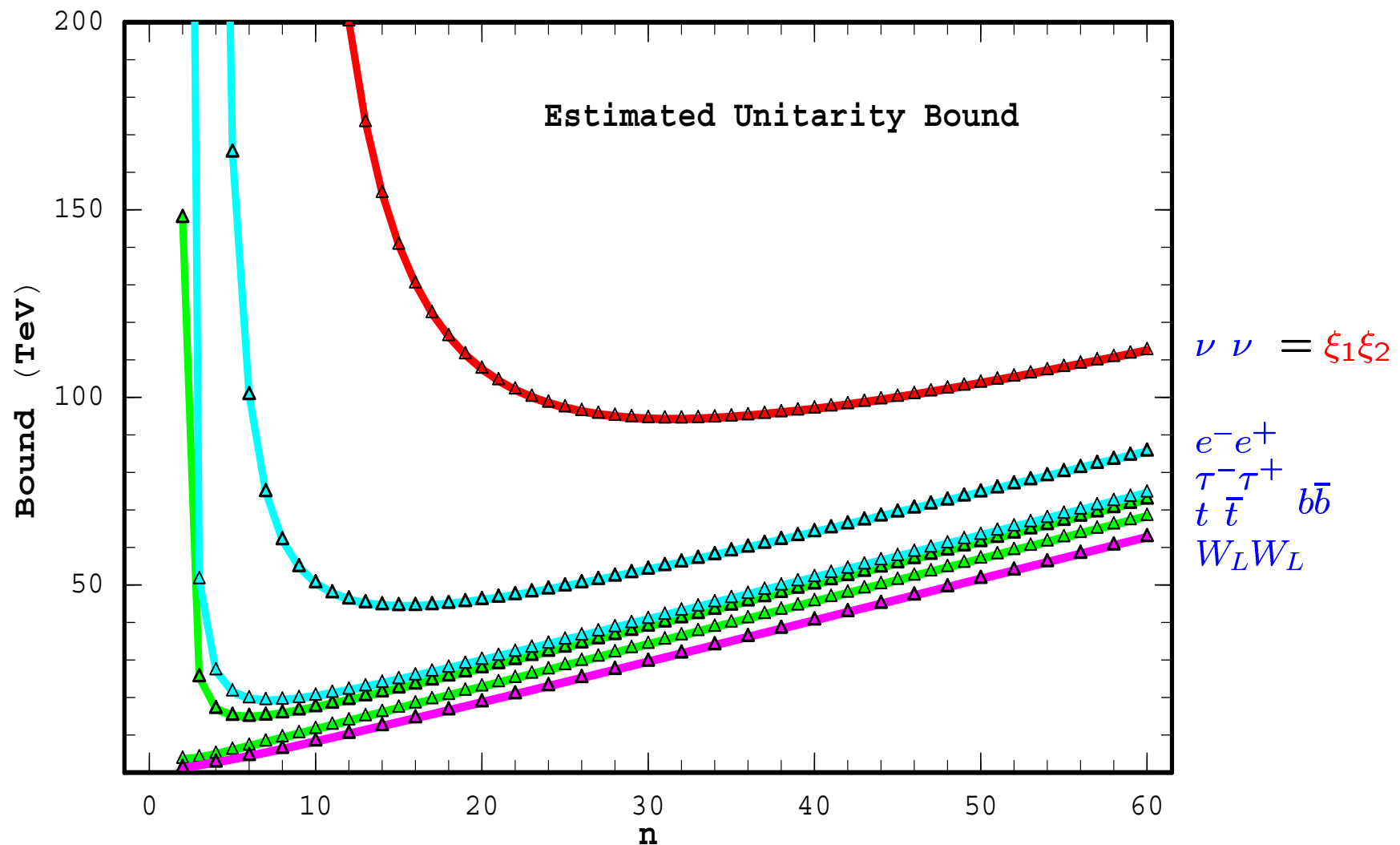
- **As  $n \uparrow$ , limit  $\Lambda_U$  exhibits competition between factors  $(\dots)^{1/2n}$  and  $(n-1)!(n-2)!$ .  $\Rightarrow$  **Minimal Bound**  $\Lambda_U^{\text{min}}$  occurs at a moderate value  $n = n_s$ .**

# Resolution and Surprise

Dicus & He, 2003

★ Surprise

► Improved Estimate of Unitarity Bound  $\Lambda_U$  from  $\xi_1 \xi_2 \rightarrow n W_L^a$ .



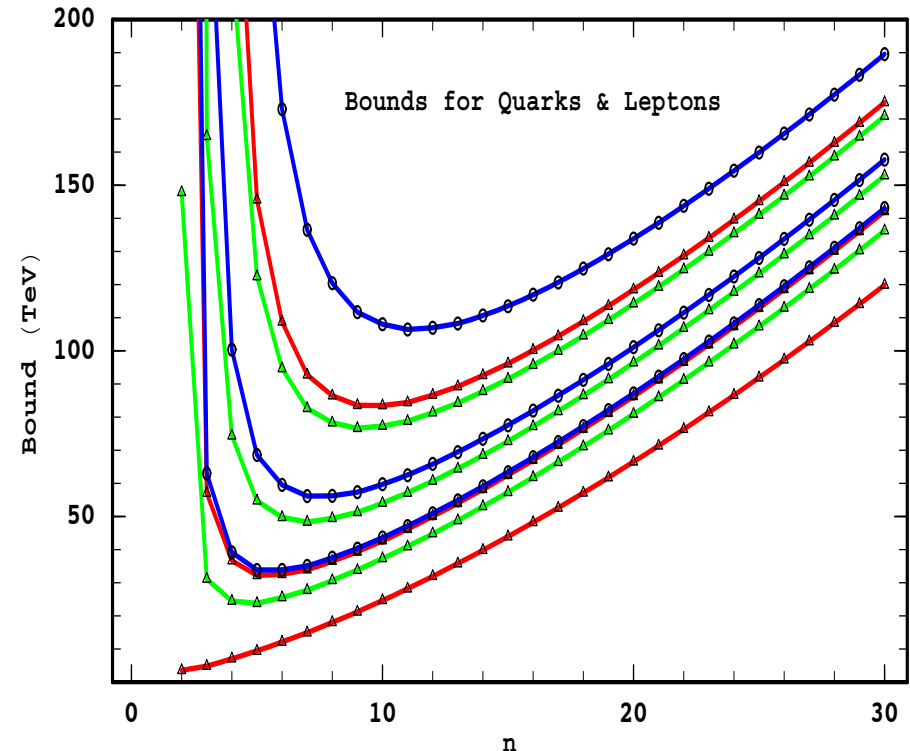
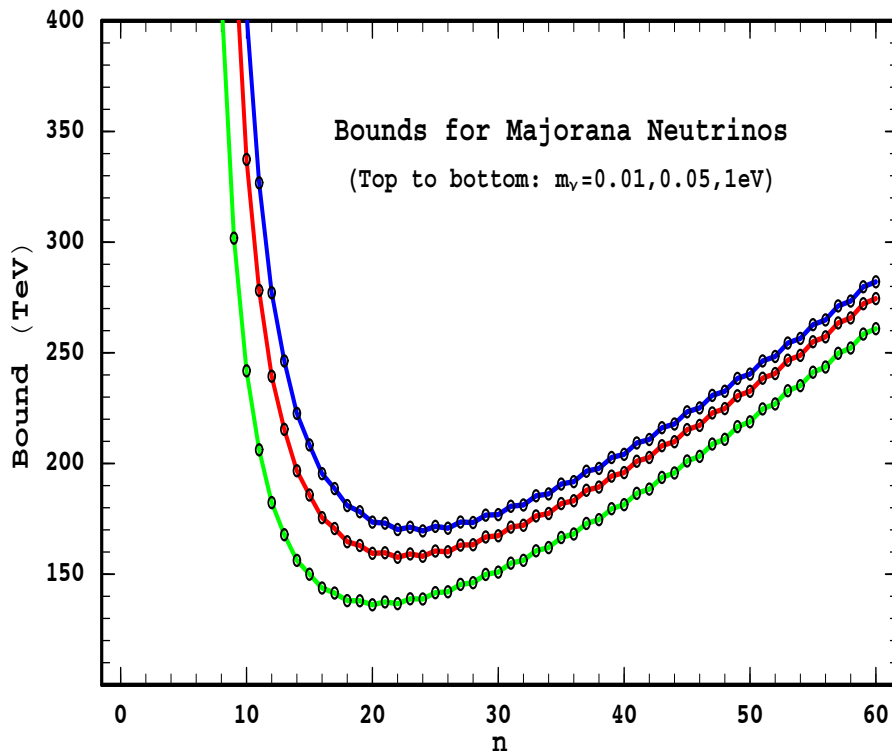
# Limits on Scales of Fermion Mass Generation

D. A. Dicus, H.-J. He, 2003

## ★ Surprise

- ▶ **Unitarity Bound  $\Lambda_U^{\text{new}}$  for Quarks/Leptons (right plot) and Majorana Neutrinos (left plot) from  $f\bar{f} \rightarrow n\pi^a$ .** [In right plot, curves from bottom to top:  $f = t, b, c, \tau, s, \mu, d, u, e.$ ]
- ▶ **Best bounds  $\Lambda_U^{\text{min}}$  always occur at  $n > 2$  for ALL light quarks/leptons and neutrinos.  $\Rightarrow$  A robust **Upper Bound for ALL fermions:****

$$\Lambda_U^{\text{min}} \lesssim 170 \text{ TeV}$$



# Scales for Mass Generations: Summary

D. A. Dicus, H.-J. He, 2003

- **Summary of Classic Unitarity Limits**  $\Lambda_U^{\text{old}}$  ( $n = 2$ ) **vs New Unitarity Limits**  $\Lambda_U^{\text{new}}$  ( $n = n_s$ ) **for Scattering**  $\xi_1 \xi_2 \rightarrow n \pi^a$  ( $n W_L^a$ ). ( $\xi_1 \xi_2 = \pi^{a_1} \pi^{a_2}$ , or,  $f\bar{f}$ , and  $n_s$  is # of final state  $\pi^a$ 's ( $W_L^a$ 's) corresponding to **best limit**  $\Lambda_U^{\text{new}}$ .)

$\xi_1 \xi_2$	$\pi^{a_1} \pi^{a_2}$	$t\bar{t}$	$b\bar{b}$	$c\bar{c}$	$s\bar{s}$	$d\bar{d}$	$u\bar{u}$	$\tau^-\tau^+$	$\mu^-\mu^+$	$e^-e^+$	$\nu\nu$
$\Lambda_U^{\text{old}}$ (TeV)	1.2	3.6	148	497	$4 \times 10^3$	$10^5$	$2 \times 10^5$	605	$10^4$	$2 \times 10^6$	$10^{13}$
$\Lambda_U^{\text{new}}$ (TeV)	1.2	3.6	25	33	49	77	84	34	56	107	158
$n_s$	2	2	4	6	8	10	10	6	8	12	22

- ★ These limits are **Universal & Independent** of any detail of the Mechanism of Mass Generation.
- ★ **Strong Non-Decoupling** of  $\Lambda_U^{\text{new}}$  for fermions is essentially due to the **Chiral Structure** of fermion bare mass term – all **left-handed** fermions are weak-**doublet** but **right-handed** chiral partners are **singlet**.

## Looking Beyond: Two Essential Clues (2)

★ Clue-2:

**What's Wrong with having a Fundamental Higgs Boson ??**

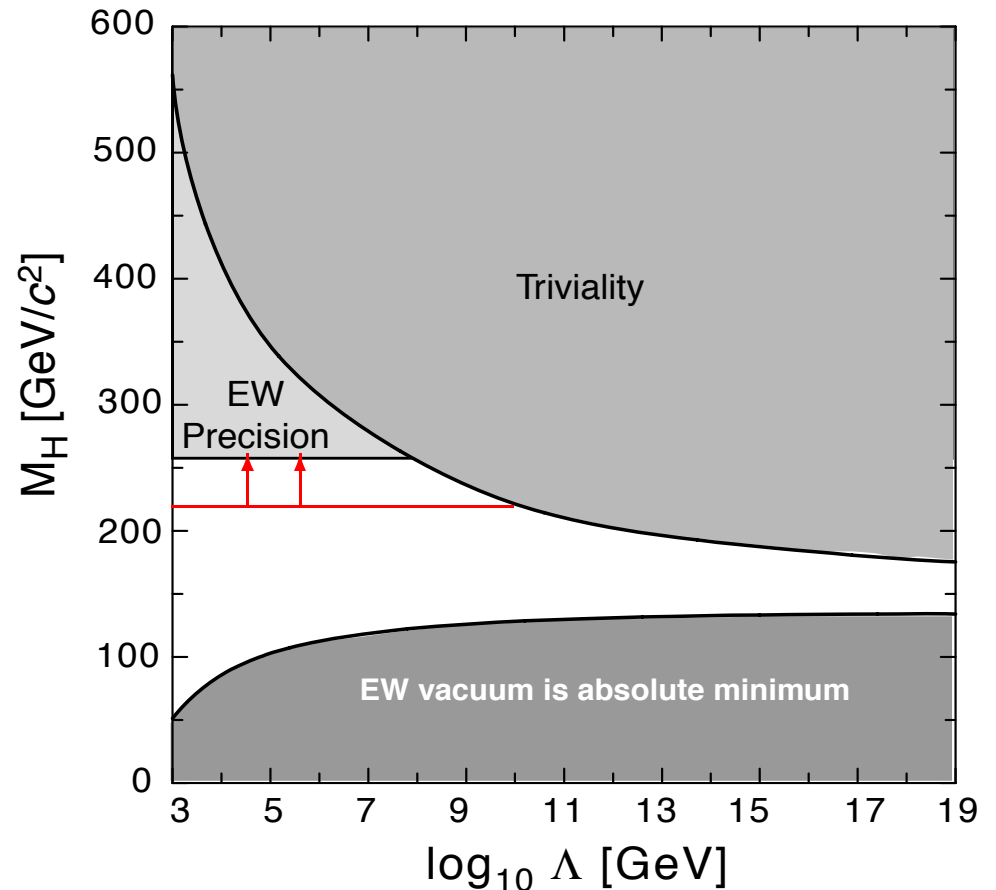
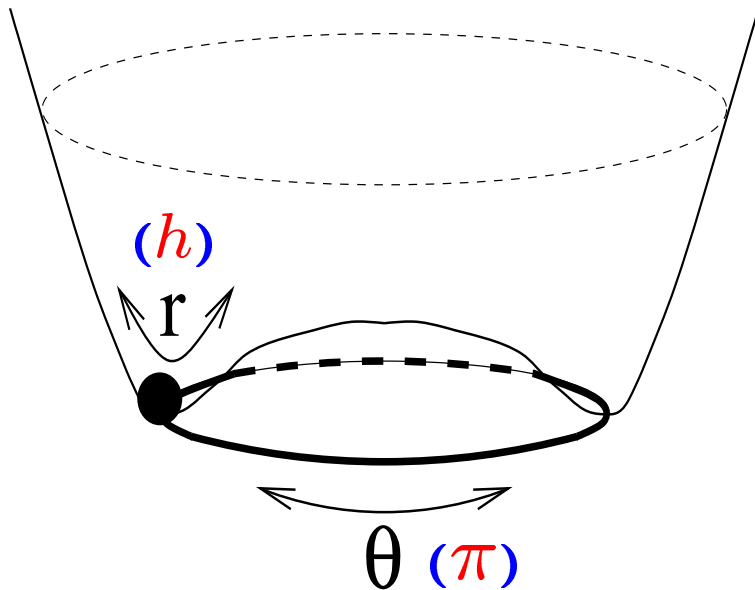
# Higgs Mechanism & Higgs Mass in SM

## ★ SM Higgs Potential & Theory/Exp Constraints:

$$V(H) = -\mu^2 H^2 + \lambda H^4, \quad \Rightarrow \quad \langle H \rangle = \frac{v}{\sqrt{2}} = \sqrt{\mu^2/2\lambda}$$

$$M_H = \sqrt{2\lambda} v$$

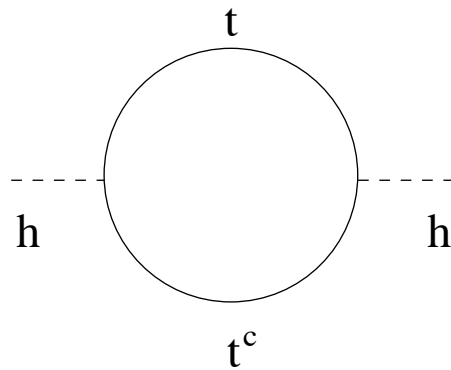
$$v \simeq 250 \text{ GeV}$$



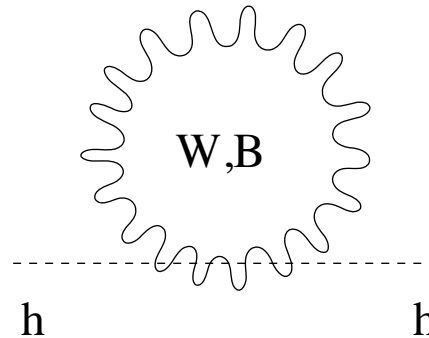
★ SM with a **light** Higgs ( $M_H \sim 160 \text{ GeV}$ ) could be an **Effective Theory** valid up to Ultraviolet (UV) Cutoff  $\Lambda \sim M_{\text{Planck}}$  !!  $\Rightarrow$  **What's WRONG ?!**

# Higgs Mass & Big Hierarchy Problem

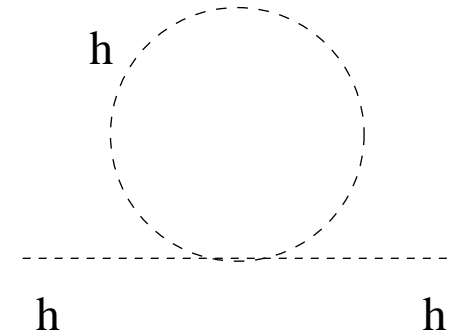
► BUT, SM Quantum Corrections are quadratically sensitive to  $\Lambda^2$ :



(a)



(b)



(c)

$$M_H^2 = M_{H0}^2 - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \frac{1}{16\pi^2} g^2 \Lambda^2 + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

$$(200 \text{ GeV})^2 = M_{H0}^2 + [-(2 \text{ TeV})^2 + (700 \text{ GeV})^2 + (500 \text{ GeV})^2] \left(\frac{\Lambda}{10 \text{ TeV}}\right)^2$$

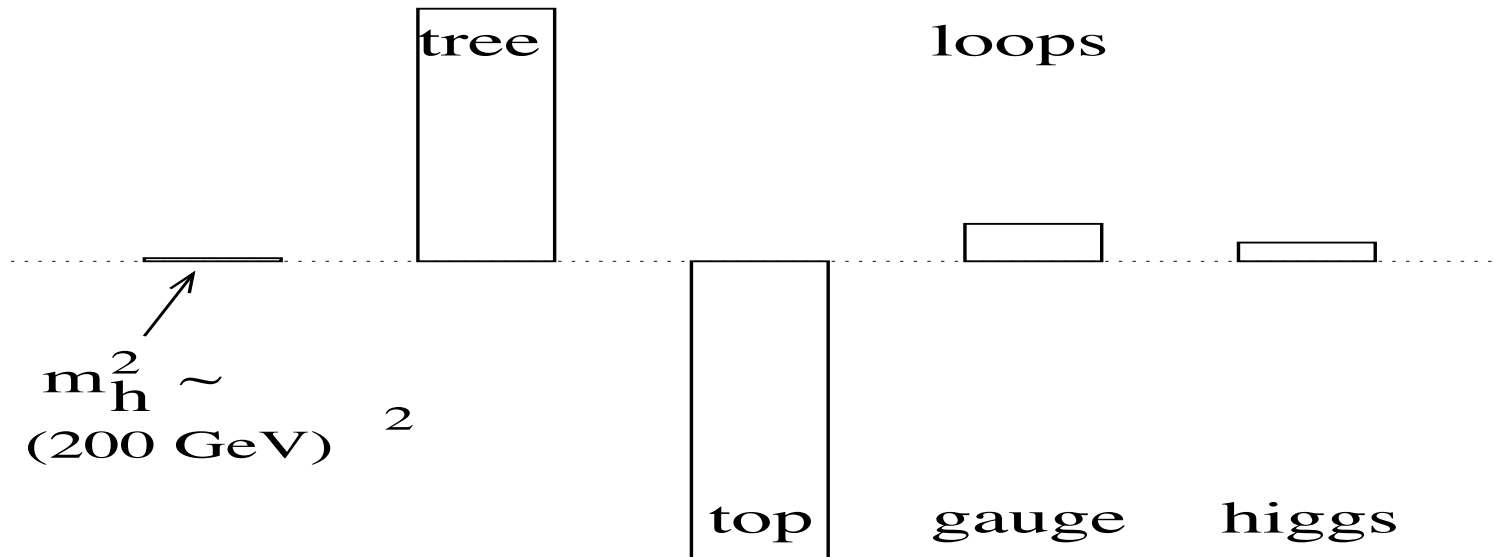
► **Big Hierarchy Problem:**

$$(M_{\text{Planck}} \simeq 10^{19} \text{ GeV})$$

$\Lambda \sim M_{\text{Planck}}$  would require a **fine-tuned cancellation** down to  $10^{-30}$  !!

# Higgs Mass & Little Hierarchy Problem

$$(200 \text{ GeV})^2 = M_{H0}^2 + [-(2 \text{ TeV})^2 + (700 \text{ GeV})^2 + (500 \text{ GeV})^2] \left(\frac{\Lambda}{10 \text{ TeV}}\right)^2$$



## ▶ Little Hierarchy Problem:

Demanding the **fine-tuning**  $\gtrsim 10\%$  in  $M_H^2$  gives

$$\Lambda_t \lesssim 3 \text{ TeV}, \quad \Lambda_W \lesssim 9 \text{ TeV}, \quad \Lambda_H \lesssim 12 \text{ TeV}$$

$\Rightarrow$  **New Phys** below  $\sim 3 \text{ TeV}$  for SM holding up to  $\sim 10 \text{ TeV}$  !!!



# “Little Higgs” solves “Little Hierarchy”

- ▶ “Little Higgs” (LH) opens up a **New Avenue** for

## Natural Electroweak Symmetry Breaking!

*Arkani-Hamed, Cohen, Georgi, hep-ph/0105239*

*Arkani-Hamed, et al, hep-ph/0206020*

*Arkani-Hamed, et al, hep-ph/0206021*

followed up by  $O(100)$  papers since 2002-2003..... (*cf. Spires*)

- ▶ “Little Higgs” is an **Effective Theory**:

**No Fine-tuning** & **Weakly Coupled** up to  $\sim 10\text{TeV}$

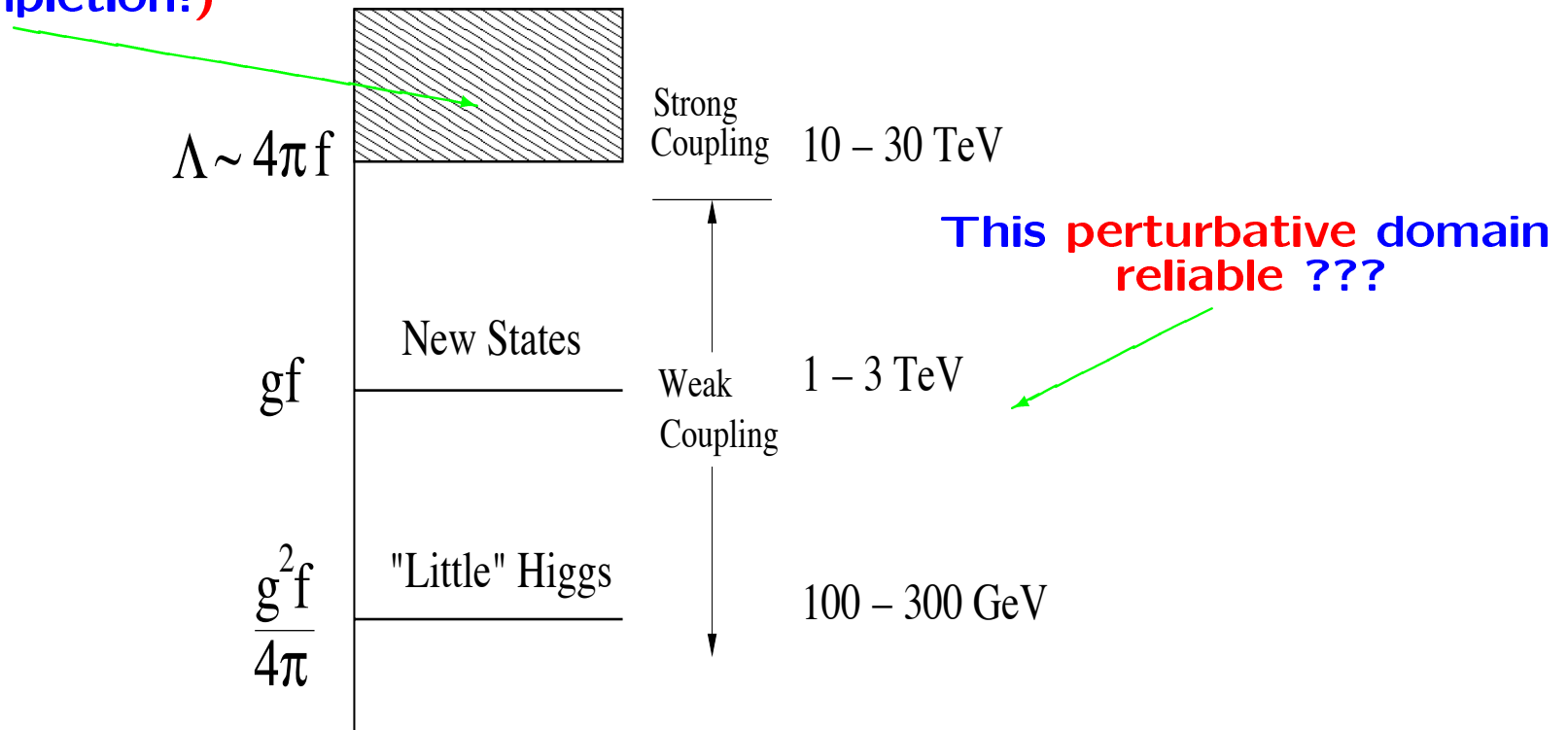
- ▶ Today I'll discuss something **new** about this direction ...

# “Little Higgs” solves “Little Hierarchy”

- ▶ Higgs is a pseudo-Goldstone boson due to Global Symmetry Breaking,  $\mathcal{G} \rightarrow \mathcal{H}$ , at Ultraviolet (UV) Scale  $\Lambda \sim 4\pi f$ . (Georgi & Pais, 1974)
- ▶ Higgs is naturally Light as protected by Goldstone theorem; Higgs acquires small mass radiatively. Quadratic Divergence cancelled at 1-Loop by New States with **Same Spin**: (Arkani-Hamed, et al, 2001,2002)

$$W, Z, B \leftrightarrow W', Z', B', \quad t \leftrightarrow t', \quad H \leftrightarrow \chi, \Phi.$$

(UV Completion!)



# Unitarity of Little Higgs vs UV Completion

S. Chang, H.-J. He, hep-ph/0311177

- ▶ Unitarity of  $S$ -matrix:  $S^\dagger S = 1$ , ( $S = 1 + iT$ ),  $\Rightarrow T^\dagger T = 2\text{Im}T$
- ▶ Partial Wave Expansion:  $T = 16\pi \sum (2j+1) P_j(\cos\theta) a_j$
- ▶ Unitarity Condition on Partial Wave:  $|\text{Re} a_j| < 1/2$
- ▶ Coupled Channel Analysis for All Goldstones in each LH model:
- ★ Our observation is that Global Symmetry Breaking of LH Model results in a Large Multiplet of Many Goldstones (including SM  $H^0$ ). The collective effect of Goldstone Scatterings will much enhance  $S$ -matrix via Coupled channels, and thus strengthen Unitarity Limit  $\Lambda_U$  to be significantly below naive cutoff scale  $\Lambda \sim 4\pi f$ , ie,  $\Lambda_U \ll \Lambda$ .
- ▶ Roughly,  $\Lambda_U$  scales like: (cf. also GDA, Chivukula et al, 1992, Georgi, 1993)

$$\sqrt{s} < \Lambda_U \propto 4\pi f \frac{O(1)}{\mathcal{N}^{1/4}} < 4\pi f$$

where  $\mathcal{N}$  is the number of Goldstones.

# Unitarity of Little Higgs vs UV Completion

S. Chang, H.-J. He, hep-ph/0311177

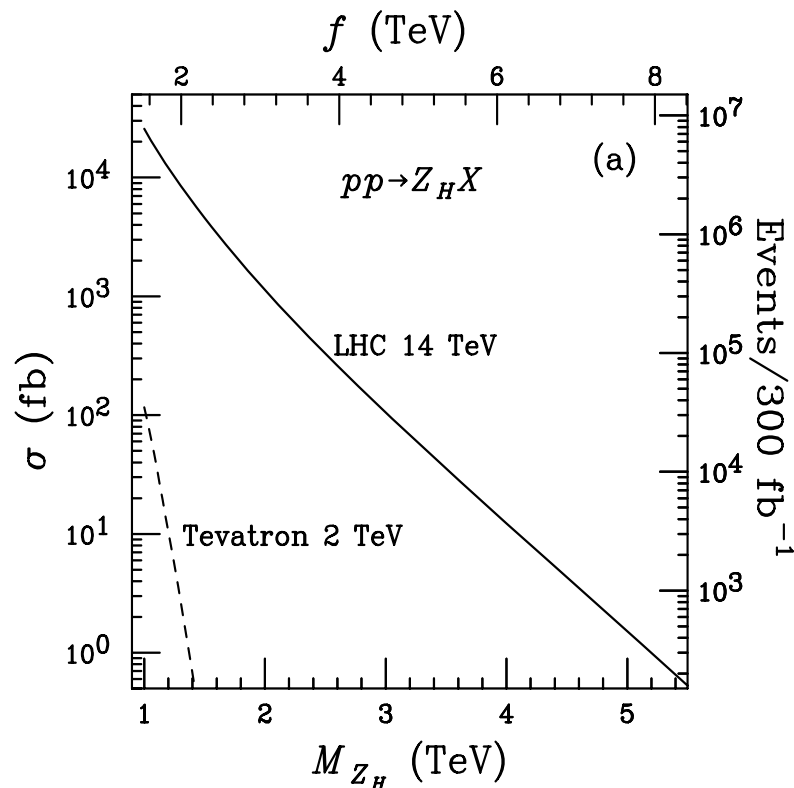
- ▶ We found **Unitarity Violation Scale**  $\Lambda_U \sim (3 - 4)f \ll \Lambda \sim 4\pi f \sim 10 \text{ TeV}$ .
- ▶  $\Lambda_U$  is comparable with  $W'$  mass  $M_{W'}$   $\Rightarrow$  **Crucial to do UV completion** – additional **New States** must be included in **Multi-TeV** range to restore Unitarity!  $\Rightarrow$  **More New Signals for LHC !!**
- ▶ **New States** reflect **Little Higgs UV dynamics**, and should be included in a way consistent with  $\Lambda^2$ -Cancellation in Higgs Mass at 1-Loop.
- ▶ **Summary of Unitarity Bounds** in various Models. (Note:  $N \gg 1$ )

Little Higgs Model	G	H	N	$\Lambda_U/f$	$M_{W'}/f$
Minimal Moose	$SU(3)^2$	$SU(3)$	8	2.89	2.29
Littlest Higgs	$SU(5)$	$SO(5)$	14	3.17	1.62
Antisymm Condensate	$SU(6)$	$Sp(6)$	14	3.68	1.62
$SO(5)$ Moose	$SO(5)^2$	$SO(5)$	10	4.09	3.24
$SO(9)$ Littlest Higgs	$SO(9)$	$SO(5) \otimes SO(4)$	20	3.79	2.29

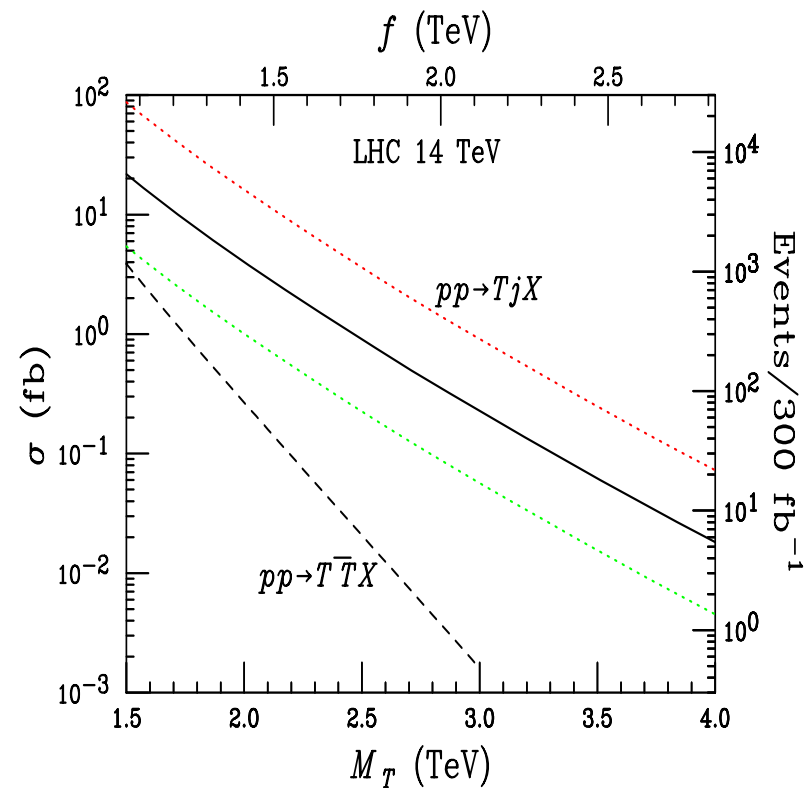
# Little Higgs Collider Signals

- ▶ Typical Collider Signals of heavy New States in the Littlest Higgs Model  $SU(5)/SO(5)$ . (cf. Han et al, hep-ph/0301040, Peskin et al, hep-ph/0310039,...)
- ▶ LC can test anomalous  $ZZH$  coupling... More studies are upcoming!.....

$Z'$  Production:



$t'$  Production:



## Summary and Outlook ...

★ The Standard Model (SM) is a Local Quantum Gauge Theory, successfully describing the Nature down to  $\sim 10^{-16}$  cm. SM contains 19 free parameters; but 16 are due to our lack of knowledge about the **Origin of Mass Generations**.

★ With all Bare Masses putting in by hand, SM is plagued with **Unitarity Violation**, which puts **Upper Limits on Scale of Mass Generation**.  
★ By assuming a single fundamental Higgs  $H^0$  for giving masses to ALL particles, SM is then plagued with **Hierarchy Problem**.

★ Unitarity provides **Universal Upper Limits on the Scales of Mass Generation** for **ALL** SM particles. Our limit  $\Lambda_U^{\text{new}}$  from  $2 \rightarrow n$  ( $n > 2$ ) Scatterings revealed strong **Non-Decoupling** nature:

$$\Lambda_U^{\text{new}} < (3.6, 84, 107, 170) \text{ TeV} \quad (\text{for } f = t, u, e, \nu)$$

$$\Lambda_U^{\text{old}} < (3.6, 2 \times 10^5, 2 \times 10^6, 10^{13}) \text{ TeV} \quad (\text{for } f = t, u, e, \nu)$$

★ “Little Higgs” is an elegant idea, realizing Higgs as a Pseudo-Goldstone boson (–naturally light). It solves a Little Hierarchy Problem, but we showed the UV Completion is crucial and additional **New States** in **Multi-TeV Range** are forced by Unitarity.

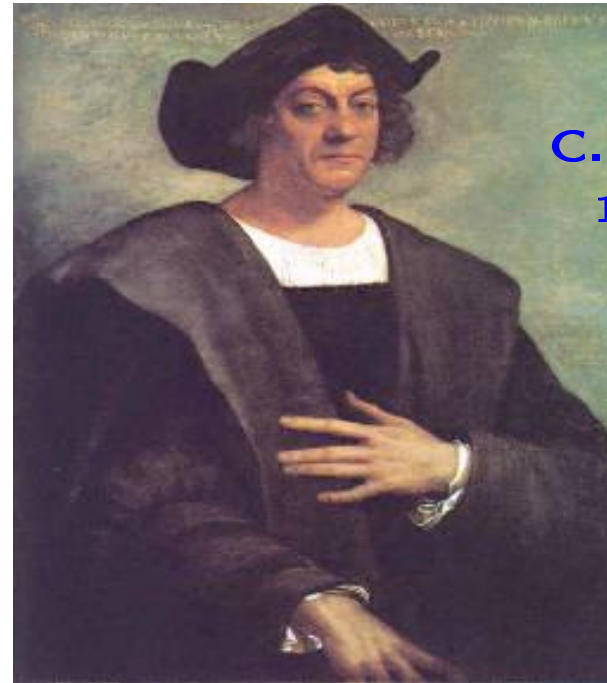
## Looking back ... The Past ...

- ▶ During **1492-1504**, **Columbus** carried out heroic **EXPERIMENTAL** probes to prove **Pythagoras Theory** (~500 B.C.) that **the Earth is Round** (and to search for **Mainland China & Gold**).....
- ▶ He had the **Correct Theory** in mind, but he had **not** finally proven it before his death.....**instead**, he discovered **Something Else !!!**

**Voyage**  
**1492-1504**



**C. Columbus**  
**1451-1506**



★ **Lessons:** (1) **A Good Theory!** (2) **Vigorous Experiments!!**

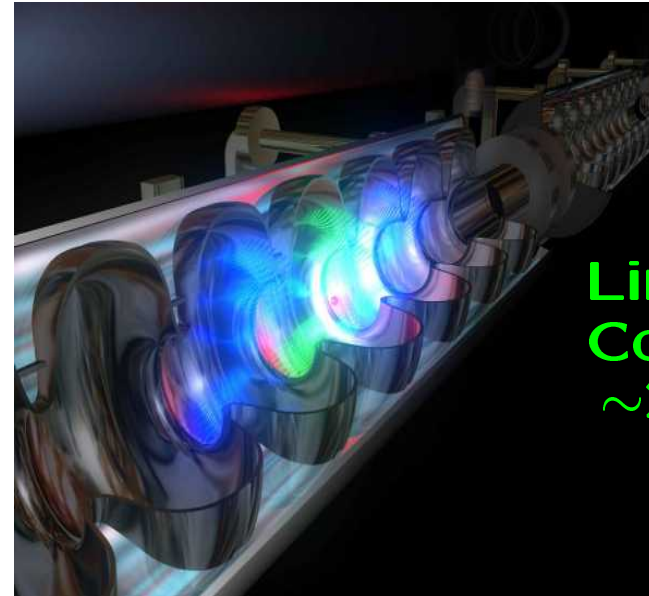
(3) COURAGE !!!



## Outlook ...The Future ...

- ▶ HEP does have vigorous EXPs upcoming...The Future is bright...

CERN  
LHC  
2007



Linear  
Colliders  
~2012

- ▶ CERN Large Hadron Collider (LHC) will turn on by 2007. Particle physicists are looking forward to upcoming Original Discoveries.....
- ▶ Next International Linear Collider (LC) will make Complementary Precision Probes and establish New Physics Beyond the Standard Model!
- ★ Many more in addition to LHC & LC ..... cf. my Friday Seminar:

Structure of Cosmological CP Violation via Neutrino Seesaw