# Lepton Colliders

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# Lepton Colliders

Consider $e^-e^+$ colliders:

| Name | $\sqrt{s}$(GeV) | $L(10^{30} cm^{-2} s^{-1})$ | Years | Detectors | Location |
|---|---|---|---|---|---|
| LEP | 110<br>161-209 | 24<br>100 | 89-95<br>96-00 | ALEPH, L3,<br>DELPHI, OPAL | CERN |
| SLC | 100 | 3 | 92-98 | SLD | SLAC |
| BEPC | 4.4<br>4.2 | 12.6<br>1000 | 89-05<br>07-now | BES | China |
| Tristan | 64 | 37 | 89-95 | TOPAZ<br>AMY, VENUS | Japan |

Refer to PDG (High-energy collider parameters: $e^+e^-$ colliders) for details.

- Event number $N$ is given by $N = \sigma\mathcal{L}$, where $\sigma$ is cross section and $\mathcal{L}$ is integrated luminosity.

- 1 barn is $10^{-24}\text{cm}^2$.

## Inclusive Rates

Total cross section

$$R=\frac{\sigma\left(e^{-}e^{+}\rightarrow\mathrm{hadrons}\right)}{\sigma\left(e^{-}e^{+}\rightarrow\mu^{-}\mu^{+}\right)}. \tag{1}$$

At leading order (LO),

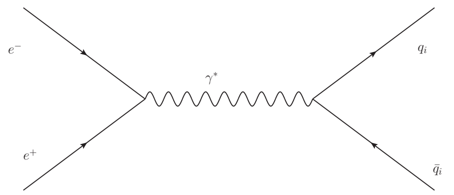
$$\sigma\left(e^{-}e^{+}\rightarrow\mathrm{hadrons}\right)=\sum_{q_i=u,d,s,\cdots}\sigma\left(e^{-}e^{+}\rightarrow q_i\bar{q}_i\right), \tag{2}$$

where the mass of the quark $(q_i)$ has to be small than $\sqrt{s}/2$, and $\sqrt{s}$ is the center-of-mass energy of $e^{-}e^{+}$.

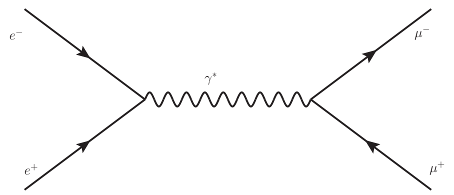
For example, at $\sqrt{s} = 8$ GeV,

$$\begin{aligned}
& \sigma\left(e^- e^+ \to \text{hadrons}\right) \\
= & \sum_{q_i = u,d,s,c} \sigma\left(e^- e^+ \to q_i \bar{q}_i\right)
\end{aligned}$$

$$= \sum_{q_i = u,d,s,c} \left( \quad \right)$$



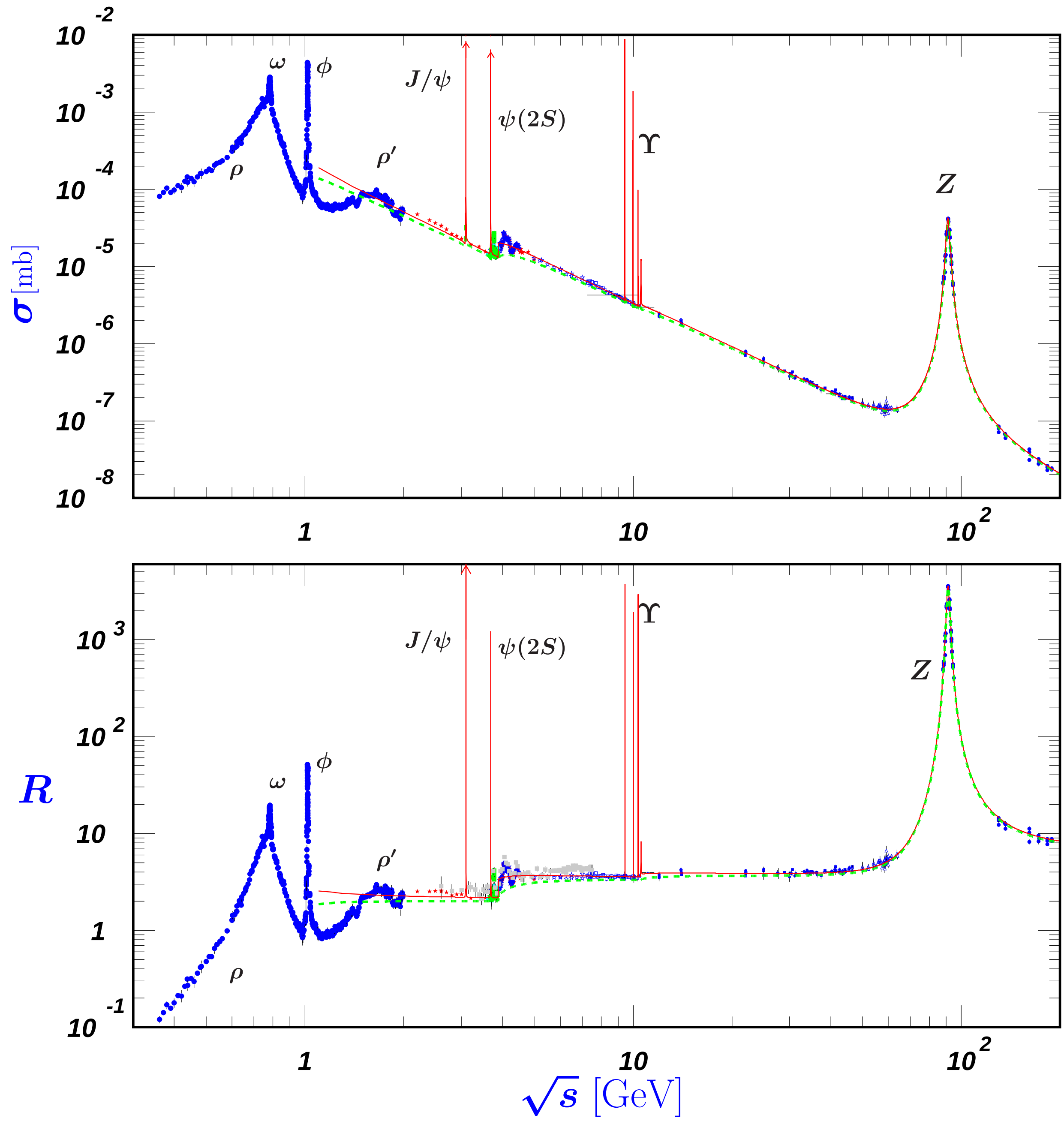

$$\simeq \sum_{q_i = u,d,s,c} {\color{red}N_C} \cdot \left(Q_{q_i}\right)^2 \cdot \left( \quad \right)$$




Note: Quark has three colors ($N_C = 3$). ($SU(3)_C$)

Hence,

$$\begin{aligned}
R &= ({\color{red}3}) \cdot \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right] \\
&= {\color{red}3} \cdot \frac{10}{9} = \frac{10}{3}. \qquad (3)
\end{aligned}$$


σ [mb]
R
√s [GeV]
ρ
ω
φ
ρ′
J/ψ
ψ(2S)
Υ
Z
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^3
10^2
10
1
10^-1
1
10
10^2

**$R$ in Light-Flavor, Charm, and Beauty Threshold Regions**

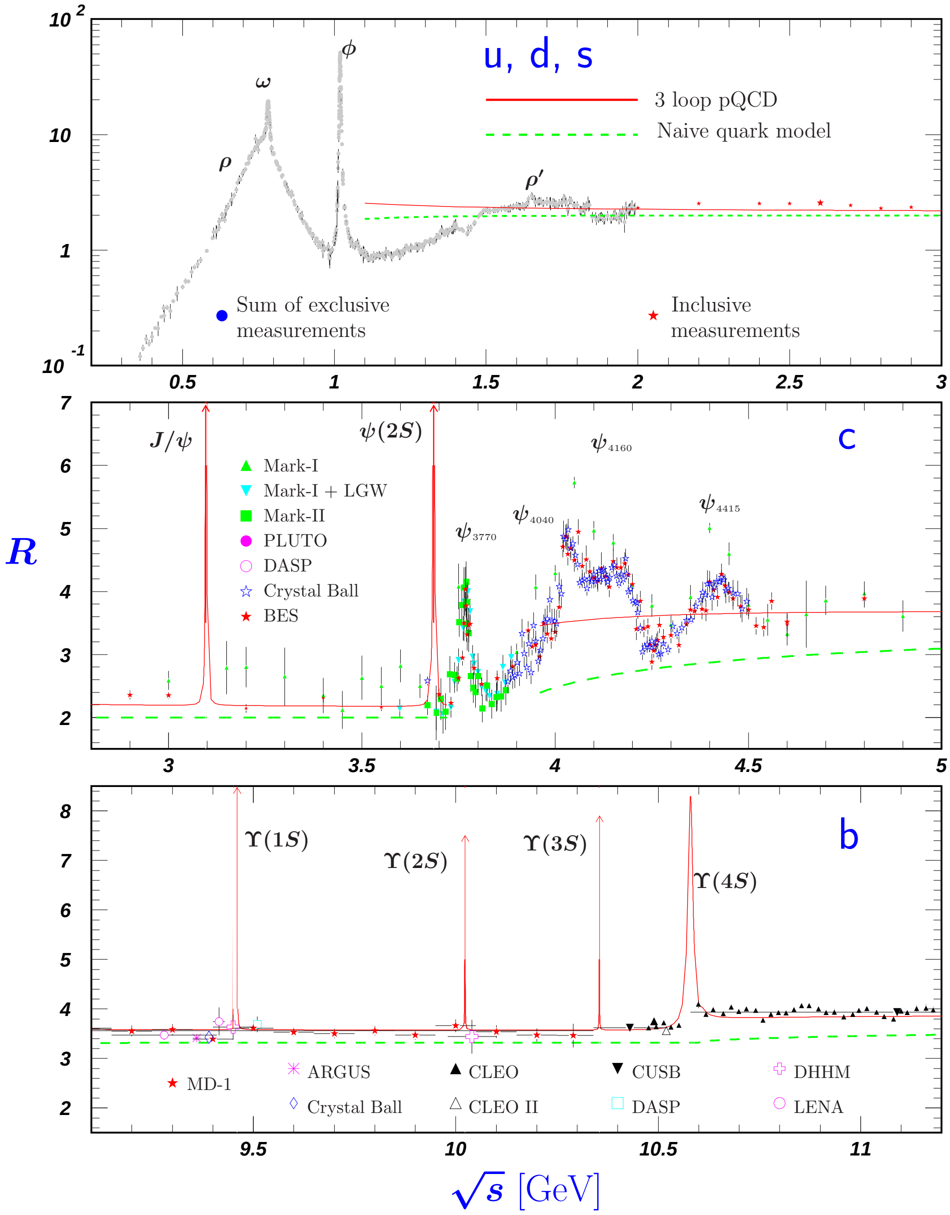



**Figure 40.7:** $R$ in the light-flavor, charm, and beauty threshold regions. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are the same as in Fig. 40.6. **Note:** CLEO data above $\Upsilon(4S)$ were not fully corrected for radiative effects, and we retain them on the plot only for illustrative purposes with a normalization factor of 0.8. The full list of references to the original data and the details of the $R$ ratio extraction from them can be found in `[arXiv:hep-ph/0312114]`. The computer-readable data are available at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, August 2007) See full-color version on color pages at end of book.

# Exclusive Observables–Jets

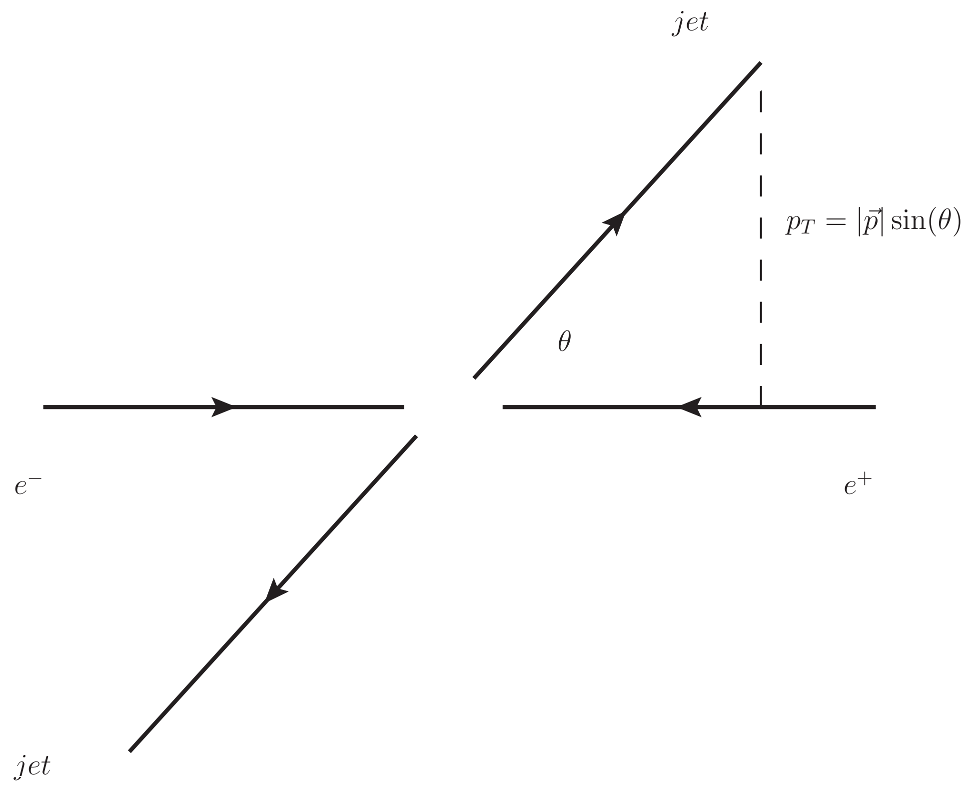
- There is no free quark
  - QCD confinement
- Quarks have to hadronize into hadrons
  - Final state fragmentation
- For large $\sqrt{s}$, final state hadrons like to move together and form two jets.
- The following figure is an example of real data collected from the DELPHI detector on the Large Electron-Positron (LEP) collider at CERN. Here a quark pair is seen as a pair of hadron jets in the detector.

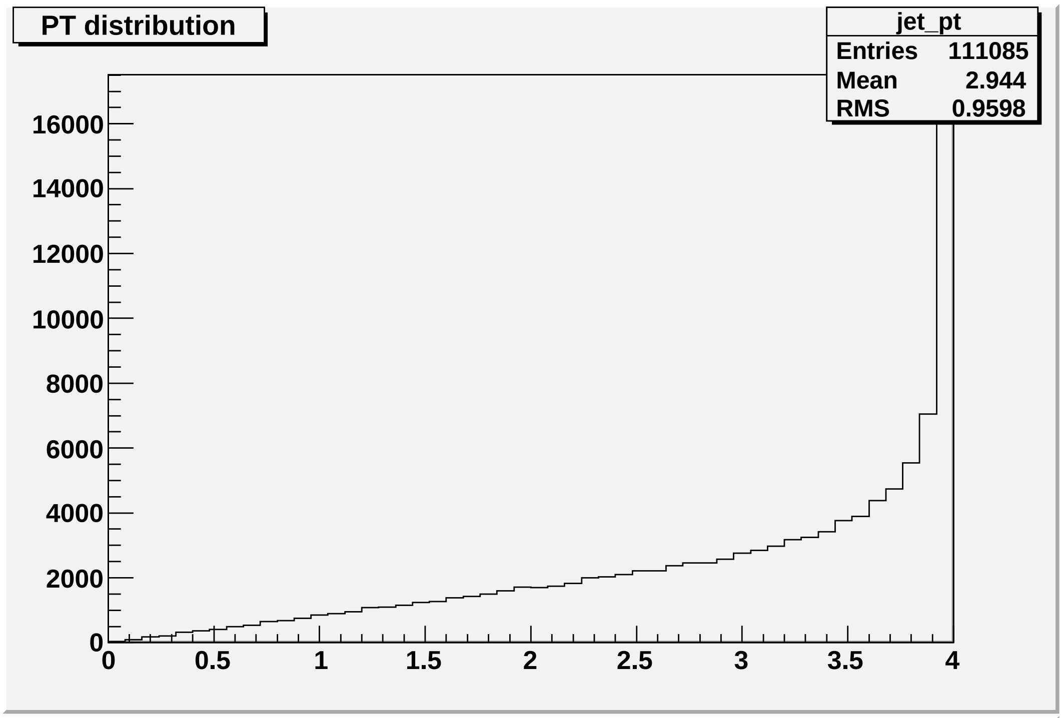

DELPHI Interactive Analysis

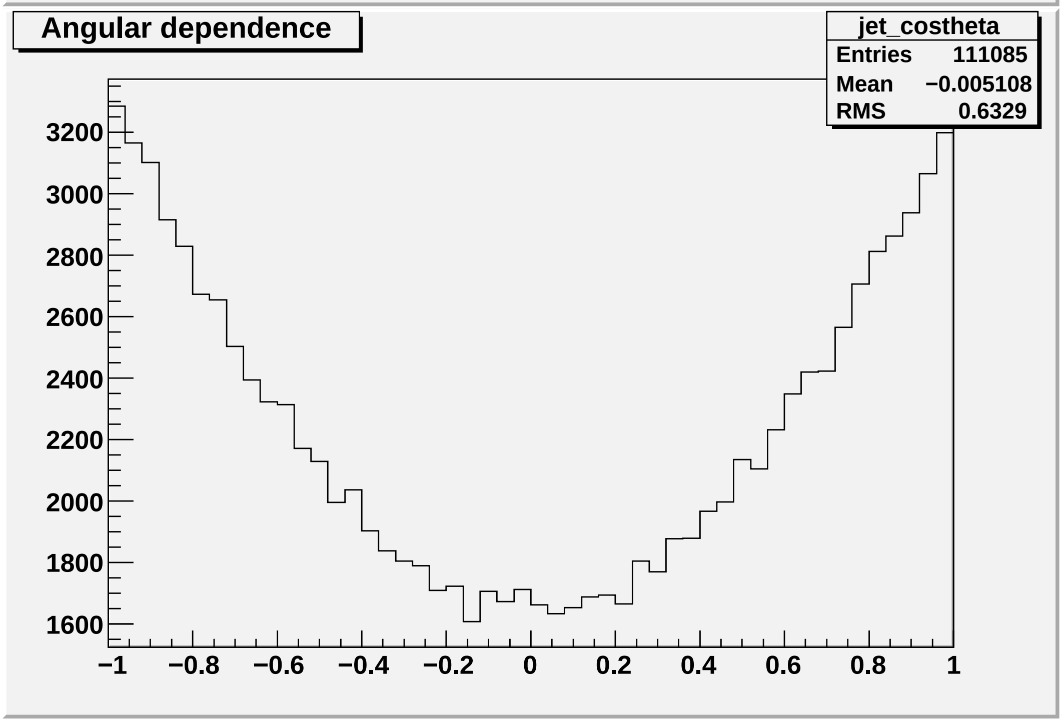
# Jets

- The characteristic feature of the two jets can be described by the $q_i$ and $\bar{q}_i$ partons in the final state
  - Parton-hadron duality

- For example: for $\sqrt{s} = 8$ GeV, some of the kinematic distributions of the quark jet are given below.




PT distribution
jet_pt
Entries 111085
Mean 2.944
RMS 0.9598
16000
14000
12000
10000
8000
6000
4000
2000
0
0 0.5 1 1.5 2 2.5 3 3.5 4


Angular dependence
jet_costheta
Entries 111085
Mean −0.005108
RMS 0.6329
3200
3000
2800
2600
2400
2200
2000
1800
1600
−1 −0.8 −0.6 −0.4 −0.2 0 0.2 0.4 0.6 0.8 1

# Project-2

- Use CalcHEP or MadGraph to calculate the leading order $P_T$ and $\cos\theta$ distributions of the quark ($q_i$) jet for $e^-e^+ \to q_i\bar{q}_i$ at $\sqrt{s} = 8$GeV.

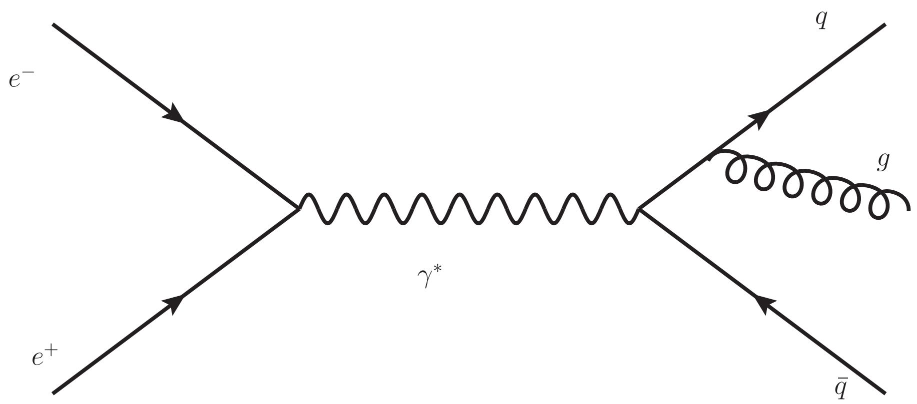
- Repeat the above task for $q_i = u, d, s, c$ and compare their results.

- Calculate their total cross sections.

# Jets

- Jet has not only momentum, energy, but also mass and distinct profile.

- At NLO,




Is this a two-jet or three-jet events?

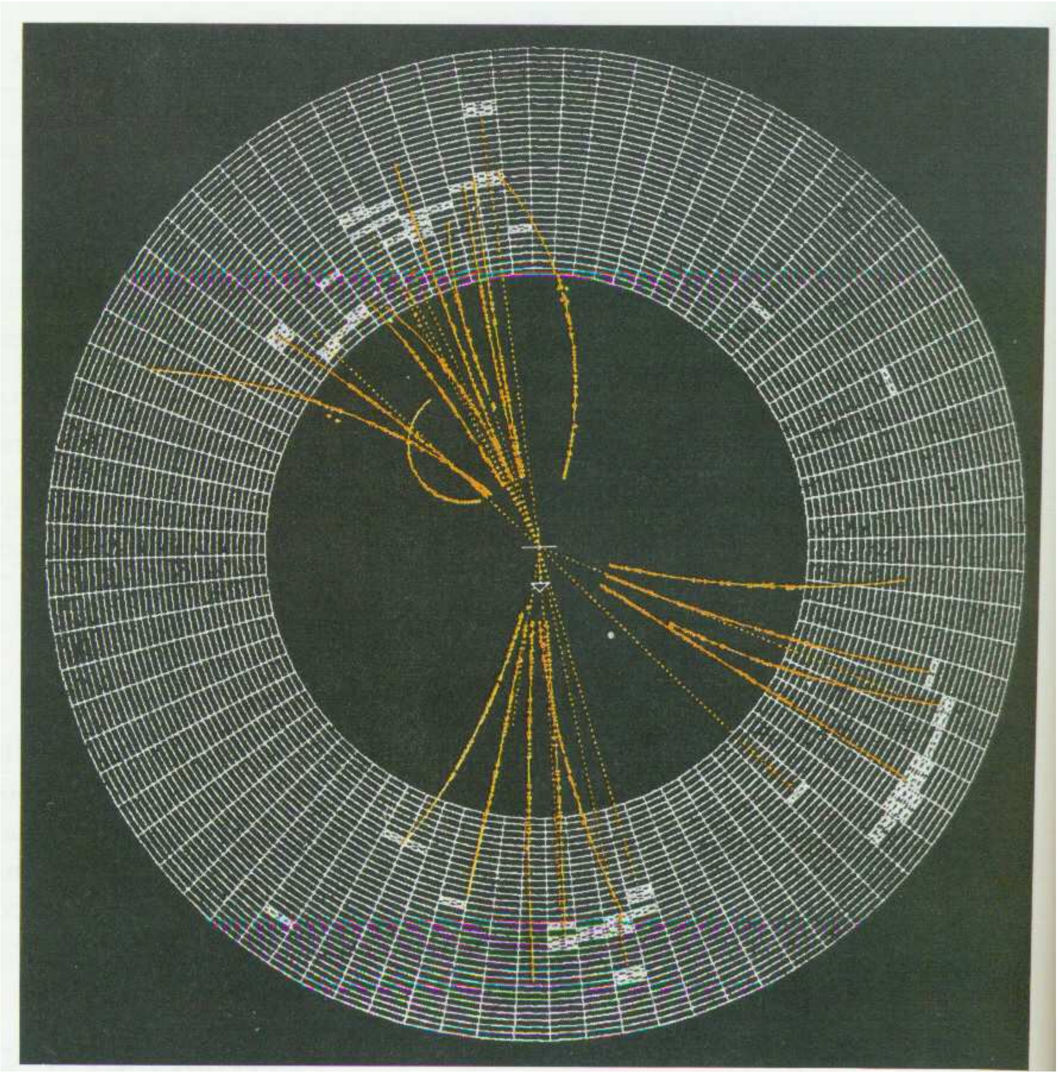
– Jet algorithm is needed to compare theory to data.

- The particle multiplicity of gluon jet (hadronization) is different from quark jet (hadronization).

Approximately, their ratio (for gluon and light quark jets with the same energy) is

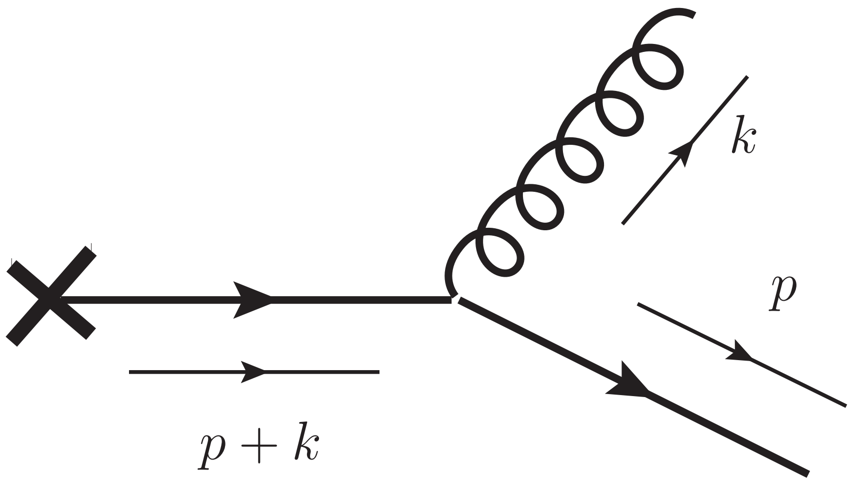
$$\simeq \frac{C_A}{C_F} = \frac{N_C}{(N_C^2 - 1)/2N_C} = \frac{3}{4/3} = \frac{9}{4}. \qquad (4)$$

This was checked in the $3$-jet event. The following figure shows the event signature at OPAL at LEP.



# Soft and Collinear Gluons

In perturbative QCD, the process involved in an outgoing quark with gluon radiation,



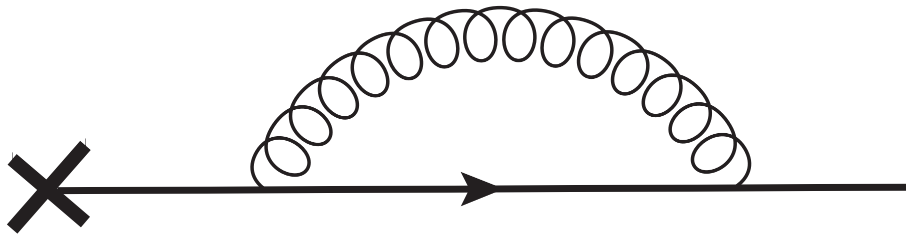

where the propogator takes the form

$$\frac{1}{(p+k)^2} \simeq \frac{1}{2p \cdot k}, \tag{5}$$

for $k^2 = 0$ and $p^2 \simeq 0$.

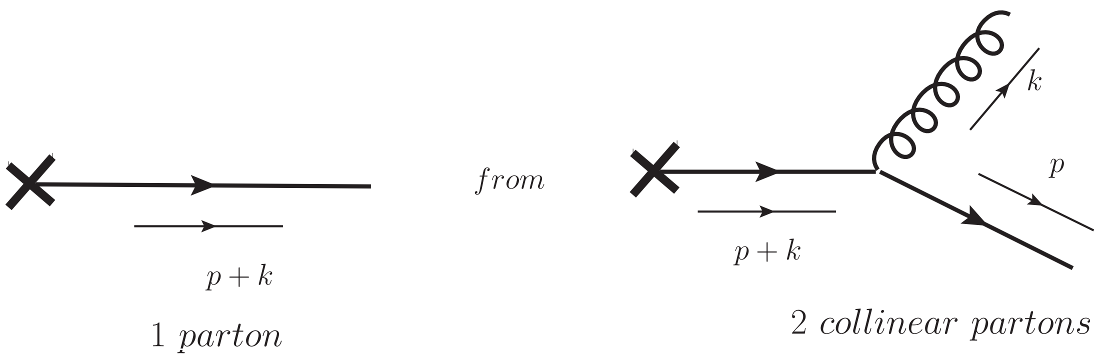
The calculation blows up when

- $k \to 0$, (soft gluon), which requires the inclusion of virtual corrections



to cancel all the soft singularities.

- $k \parallel p$, (collinear gluon). We could define an infrared-safe observables (such as a "cone jet") to compare to data (which is always finite). Namely, we do not distinguish




  – Jet functions