Triviality and Vacuum Stability Bounds

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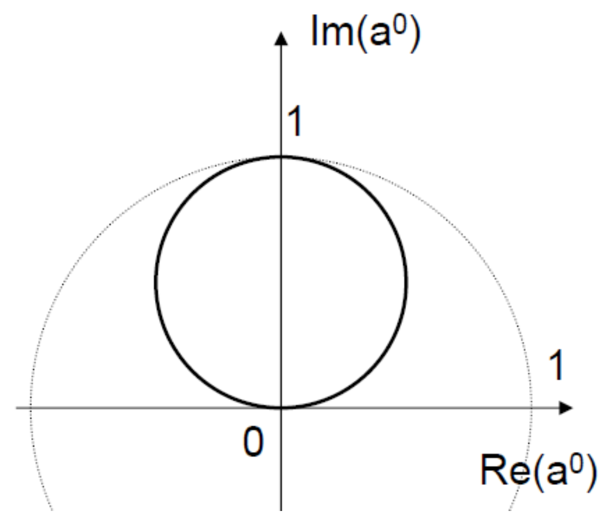
(postdoc at MSU, 2000-2001)

Unitarity Bounds

Elastic scattering $\boxed{W_L^+ W_L^- \to W_L^+ W_L^-}$

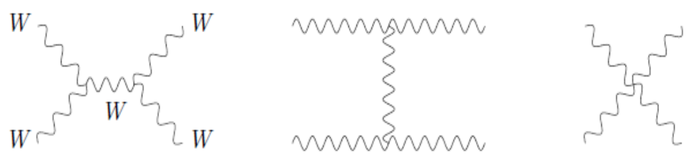
S-wave amplitude $(a^0)_{fi}$

$$|a^0|^2 = \mathrm{Im}(a^0)$$



Tree level unitarity $\boxed{|a^0| < 1}$

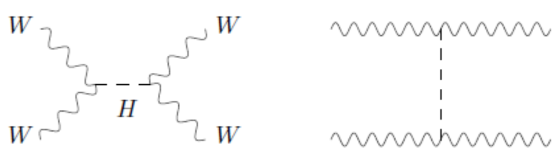
(1) No Higgs boson



$$a^0 \simeq \frac{G_F\, s}{8\pi\sqrt{2}}$$

$$\boxed{s < \frac{8\pi\sqrt{2}}{G_F} \sim (1.6\mathrm{TeV})^2}$$

(2) Including Higgs mediation



$$a^0 \simeq G_F m_H^2$$

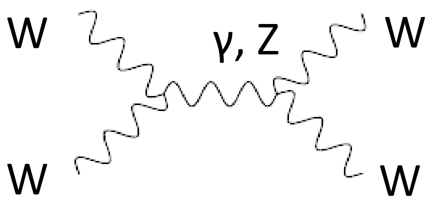
$$\boxed{m_H < 1\ \mathrm{TeV}}$$

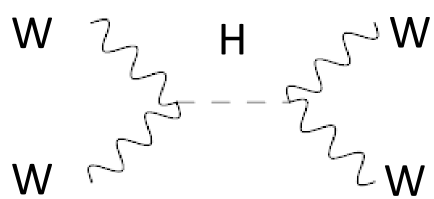
Lee, Quigg, Thacker, 1977

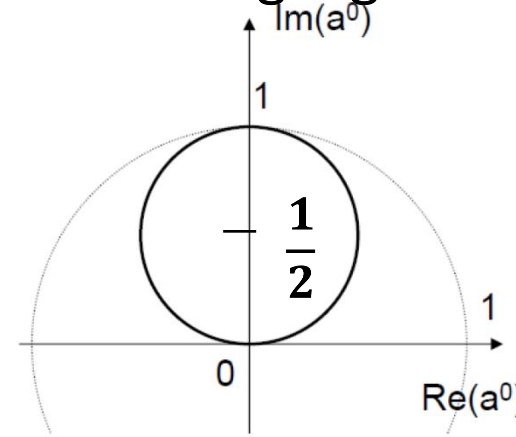
There must be anything at the TeV scale !

Tree-level unitarity

Consider scattering amplitude for the longitudinal weak gauge boson in the SM. For example, $W_L^+ W_L^- \to W_L^+ W_L^-$







The 0th partial wave amplitude is written as

$$|a^0|^2 \leq \mathrm{Im}(\mathrm{a}^0).$$

The tree-level S wave amplitude is real. The allowed region of a^0 is expressed by

$$\left|a^0_{tree}\right| < 1.$$

In high energy limit, the coupling constant λ is constrained because

$$a^0_{tree} = -\frac{G_F m_h^2}{4\sqrt{2}\pi} \quad \text{and } m_h^2 = 2\lambda v^2.$$

Tree-level unitarity

In the SM, 4x4 scattering matrix is made by neutral two body states.

$$\{W_L^+ W_L^-, Z_L Z_L, HH, Z_L H\}$$

$$a^0 = -\frac{\lambda}{4\pi}\begin{pmatrix} 1 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 \\ \frac{\sqrt{2}}{4} & \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{\sqrt{2}}{4} & \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \xrightarrow{\text{Diagonalize}} |a^0| = \begin{pmatrix} |a| & 0 & 0 & 0 \\ 0 & |b| & 0 & 0 \\ 0 & 0 & |c| & 0 \\ 0 & 0 & 0 & |d| \end{pmatrix} < \frac{1}{2}$$

Diagonalizing this matrix, the constraint is stronger

Tree-level unitarity

In the high energy limit,

$$\{W_L^+W_L^-, Z_LZ_L, HH, Z_LH\} \approx \{w^+w^-, zz, hh, zh\}$$

Equivalence theorem

There are 14 neutral, 8 singly and a doubly charged two body states in the THDM.

$$\{w_1^+w_2^-, w_2^+w_1^-, h_1z_2, h_2z_1, z_1z_2, h_1h_2\},$$

$$\left\{w_1^+w_1^-, w_2^+w_2^-, \frac{1}{\sqrt{2}}z_1z_1, \frac{1}{\sqrt{2}}z_2z_2, \frac{1}{\sqrt{2}}h_1h_1, \frac{1}{\sqrt{2}}h_2h_2\right\}, \{h_1z_1, h_2z_2\},$$

$$\{h_1w_1^+, h_2w_1^+, z_1w_1^+, z_2w_1^+, h_1w_2^+, h_2w_2^+, z_1w_2^+, z_2w_2^+\}, \{w_1^+, w_2^+\}.$$

Tree-level unitarity

The scattering matrix is made by two body states.
example, neutral two body states
$\{w_1^+ w_2^-, w_2^+ w_1^-, h_1 z_2, h_2 z_1, z_1 z_2, h_1 h_2\}$ in the THDM

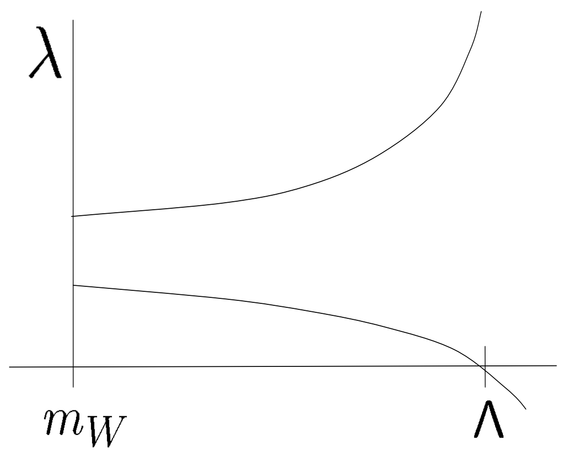
$$a_0^{\mathrm{I}} = -\frac{1}{32\pi}\begin{pmatrix} 2(\lambda_3+\lambda_4) & 4\lambda_5^* & -i\lambda_{45}^{-\,*} & i\lambda_{45}^{-\,*} & \lambda_{45}^{+\,*} & \lambda_{45}^{+\,*} \\ 4\lambda_5 & 2(\lambda_3+\lambda_4) & i\lambda_{45}^{-} & -i\lambda_{45}^{-} & \lambda_{45}^{+} & \lambda_{45}^{+} \\ i\lambda_{45}^{-} & -i\lambda_{45}^{-\,*} & 2\lambda_{345}^{-} & 2\mathrm{Re}(\lambda_5) & 2\mathrm{Im}(\lambda_5) & 2\mathrm{Im}(\lambda_5) \\ -i\lambda_{45}^{-} & i\lambda_{45}^{-\,*} & 2\mathrm{Re}(\lambda_5) & 2\lambda_{345}^{-} & -2\mathrm{Im}(\lambda_5) & -2\mathrm{Im}(\lambda_5) \\ \lambda_{45}^{+} & \lambda_{45}^{+\,*} & 2\mathrm{Im}(\lambda_5) & -2\mathrm{Im}(\lambda_5) & 2\lambda_{345}^{+} & 2\mathrm{Re}(\lambda_5) \\ \lambda_{45}^{+} & \lambda_{45}^{+\,*} & 2\mathrm{Im}(\lambda_5) & -2\mathrm{Im}(\lambda_5) & 2\mathrm{Re}(\lambda_5) & 2\lambda_{345}^{+} \end{pmatrix}$$

$$|a_0^{\mathrm{I}}| = \begin{pmatrix} |e_1| & 0 & 0 & 0 & 0 & 0 \\ 0 & |e_2| & 0 & 0 & 0 & 0 \\ 0 & 0 & |f_+| & 0 & 0 & 0 \\ 0 & 0 & 0 & |f_-| & 0 & 0 \\ 0 & 0 & 0 & 0 & |f_1| & 0 \\ 0 & 0 & 0 & 0 & 0 & |f_2| \end{pmatrix} < 1$$

Theoretical Constraint on m_H in SM

Requirement for the stability of the theory below a cutoff scale Λ

$$m_H^2 = 2\lambda v^2$$

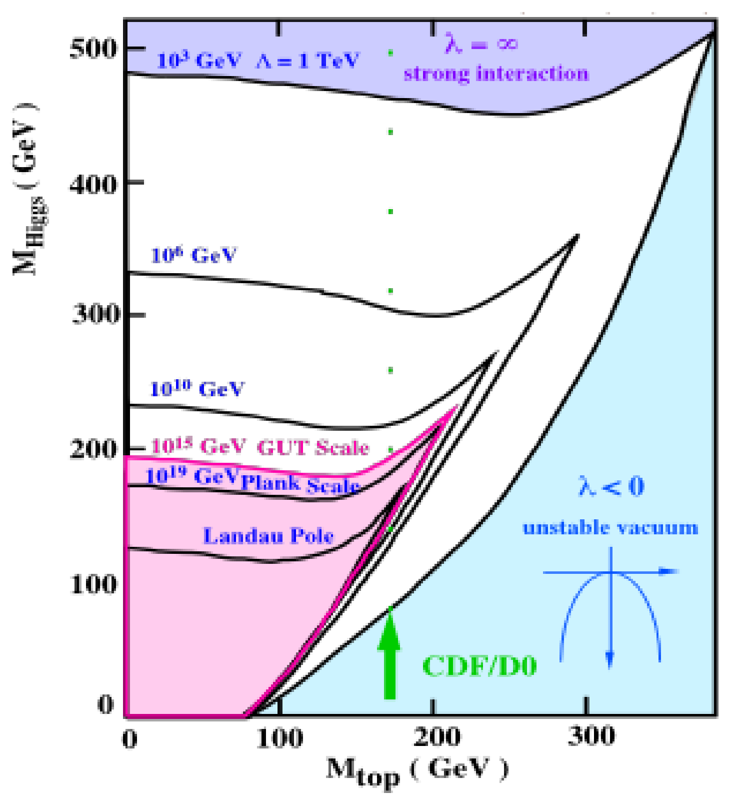


RGE

$$16\pi^2 \mu \frac{d}{d\mu}\lambda = 24\lambda^2 - 6y_t^4 + \ldots.$$

$$140 < m_H < 175\text{GeV}\ (\Lambda = 10^{19}\text{GeV})$$

$$70 < m_H < 480\text{GeV}\ (\Lambda = 1\text{TeV})$$



Two Higgs Doulet Model (2HDM)

• 2HDM is a simplest extension of the MSM Higgs sector for various theoretical motivations (extra CP phase, EW baryogenesis, SUSY, Little Higgs, etc)

Higgs potential

$$
\begin{aligned}
V_{\text{2HDM}}(\Phi_1, \Phi_2) &= m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - (m_3^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) \\
&+ \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 \\
&+ \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \left\{\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right\}(\Phi_1^\dagger\Phi_2) + \text{h.c.}\right],
\end{aligned}
$$

$m_3^2,\ \lambda_{5-7} \in \boldsymbol{C}$ (sources of explicit CP violation)

In the MSSM: $\lambda_1 = \lambda_2 = (g_2^2 + g_1^2)/4, \quad \lambda_3 = (g_2^2 - g_1^2)/4, \quad \lambda_4 = g_2^2/2,$
$\lambda_5 = \lambda_6 = \lambda_7 = 0.$

Yukawa interaction

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= \bar{q}_L(f_1^{(d)}\Phi_1 + f_2^{(d)}\Phi_2)d_R + \bar{q}_L(f_1^{(u)}\tilde{\Phi}_1 + f_2^{(u)}\tilde{\Phi}_2)u_R \\
&+ \bar{l}_L(f_1^{(e)}\Phi_1 + f_2^{(e)}\Phi_2)e_R + \text{h.c.}, \quad (\tilde{\Phi}_i = i\tau^2\Phi_i^*)
\end{aligned}
$$

The two-Higgs-doublet model with a softly broken Z_2 symmetry

Simplest extended Higgs sector with $\rho_{\text{tree}} = 1$ FCNC suppress

- Higgs potential ($\supset$ MSSM Higgs sector)

$$V_{\text{THDM}} = +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \underline{m_3^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right)}$$
$$+\frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2$$
$$+\lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 + \frac{\lambda_5}{2} \left[\left(\Phi_1^\dagger \Phi_2\right)^2 + (\text{h.c.})\right]$$

- THDM with a softly-broken discrete symmetry:

$(\Phi_1 \to \Phi_1,\ \Phi_2 \to -\Phi_2)$: Natural flaovr conservation

- Yukawa interaction (Model I, II):

$$\mathcal{L}_{\text{I}} = -y_D \overline{Q}_L \Phi_1 b_R - y_U \bar{t}_R \Phi_1^\dagger Q_L + (h.c.)$$
$$\mathcal{L}_{\text{II}} = -y_D \overline{Q}_L \Phi_1 b_R - y_U \bar{t}_R \Phi_2^\dagger Q_L + (h.c.)$$

2HDM

The simplest model with ρ=1 at the tree level

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + i a_i) \end{bmatrix} \quad (i=1,2) \qquad \langle \Phi_i \rangle = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} v_i \end{bmatrix} \qquad \frac{v_2}{v_1} \equiv \tan\beta$$

Mass eigenstates and angles

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix} \qquad \begin{bmatrix} z_1^0 \\ z_2^0 \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} z^0 \\ A^0 \end{bmatrix}$$

$$\begin{bmatrix} w_1^\pm \\ w_2^\pm \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} w^\pm \\ H^\pm \end{bmatrix}$$

Mass eigenstates

$$\Phi_1 \text{ and } \Phi_2 \Rightarrow \underline{h, \quad H,} \quad A^0, \ H^\pm \oplus \text{Goldstone bosons}$$

↑ ↑ ↑charged

CPeven CPodd

FCNC Suppression

- THDM with a softly-broken discrete symmetry:

  $(\Phi_1 \to \Phi_1,\ \Phi_2 \to -\Phi_2)$: Natural flaovr conservation

  - Yukawa interaction (Model I, II):

$$\mathcal{L}_{\mathsf{I}} = -y_D \overline{Q}_L \Phi_1 b_R - y_U \bar{t}_R \Phi_1^\dagger Q_L + (h.c.)$$
$$\mathcal{L}_{\mathsf{II}} = -y_D \overline{Q}_L \Phi_1 b_R - y_U \bar{t}_R \Phi_2^\dagger Q_L + (h.c.)$$

- Higgs potential ($\supset$ MSSM Higgs sector)

$$\begin{aligned} V_{\mathsf{THDM}} = & +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \underline{m_3^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right)} \\ & + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 \\ & + \lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 + \frac{\lambda_5}{2} \left[\left(\Phi_1^\dagger \Phi_2\right)^2 + (\mathrm{h.c.})\right] \end{aligned}$$

Yukawa interaction

There are four types of the Yukawa interactions under the discrete symmetry (to avoid FCNC)

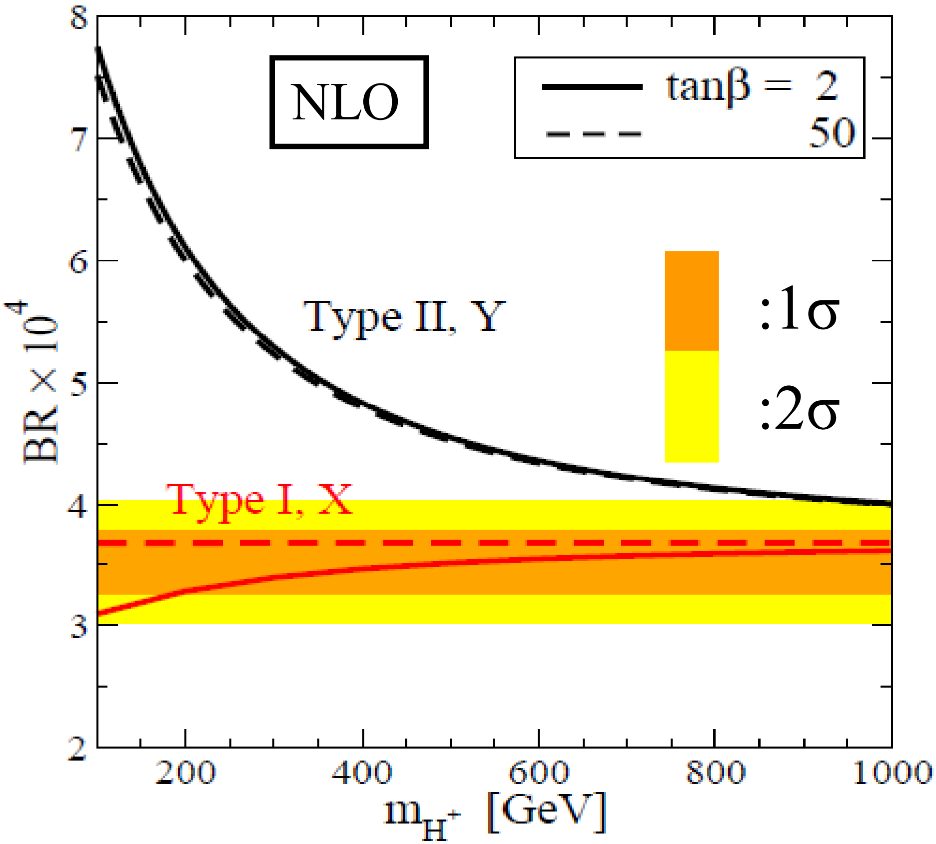
- Type-II THDM: MSSM
  - Φ_1 couples to down-type quarks and leptons.
  - Φ_2 couples to up-type quarks.
  - $b \to s\gamma$ bound is very strong
  - ➡ **Light H^+ is forbidden**
- Type-X THDM : AKS model
  - Φ_1 couples to leptons
  - Φ_2 couples to quarks
  - $b \to s\gamma$ bound is mild

	Φ_1	Φ_2	Q^i	L^i	u_R^i	d_R^i	e_R^i
Type-I	+	−	+	+	−	−	−
Type-II	+	−	+	+	−	+	+
Type-X	+	−	+	+	−	−	+
Type-Y	+	−	+	+	−	+	−

➡ **Light H^+ is possible !**

In this talk, I mainly discuss the Type-X THDM and the MSSM

Aoki, SK, Tsumura, Yagyu, PRD80, 015017 (2009)



NLO calculation

Ciuchini et al.Nucl. Phys. B 527, 21 (1998).

NNLO calculation

Misiak, Steinhauser, Nucl. Phys. B 764, 62 (2007).

Misiak et al., PRL. 98, 022002 (2007).

Light charged Higgs bosons are possible in the **Type-X** (and **MSSM)**

8 parameters : $\Rightarrow \{m_h,\ m_H,\ m_A,\ m_{H^\pm},\ \alpha,\ \beta,\ v,\ M_{\text{soft}}\ \}$

v (VEV) $\simeq 246$ GeV, $\tan\beta\ (= \langle\Phi_2\rangle/\langle\Phi_1\rangle)$

α: mixing angle between h and H

$M_{\text{soft}}\ \ (= \frac{m_3}{\sqrt{\cos\beta\sin\beta}})$: soft-breaking scale of the discrete symm.

$$m_h^2 = v^2\left(\lambda_1\cos^4\beta + \lambda_2\sin^4\beta + \frac{\lambda}{2}\sin^2 2\beta\right) + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_H^2 = M_{\text{soft}}^2 + v^2\,(\lambda_1 + \lambda_2 - 2\lambda)\sin^2\beta\cos^2\beta + \mathcal{O}\left(\frac{v^2}{M_{\text{soft}}^2}\right),$$

$$m_{H^\pm}^2 = M_{\text{soft}}^2 - \frac{\lambda_4+\lambda_5}{2}v^2,$$

$$m_A^2 = M_{\text{soft}}^2 - \lambda_5 v^2.$$

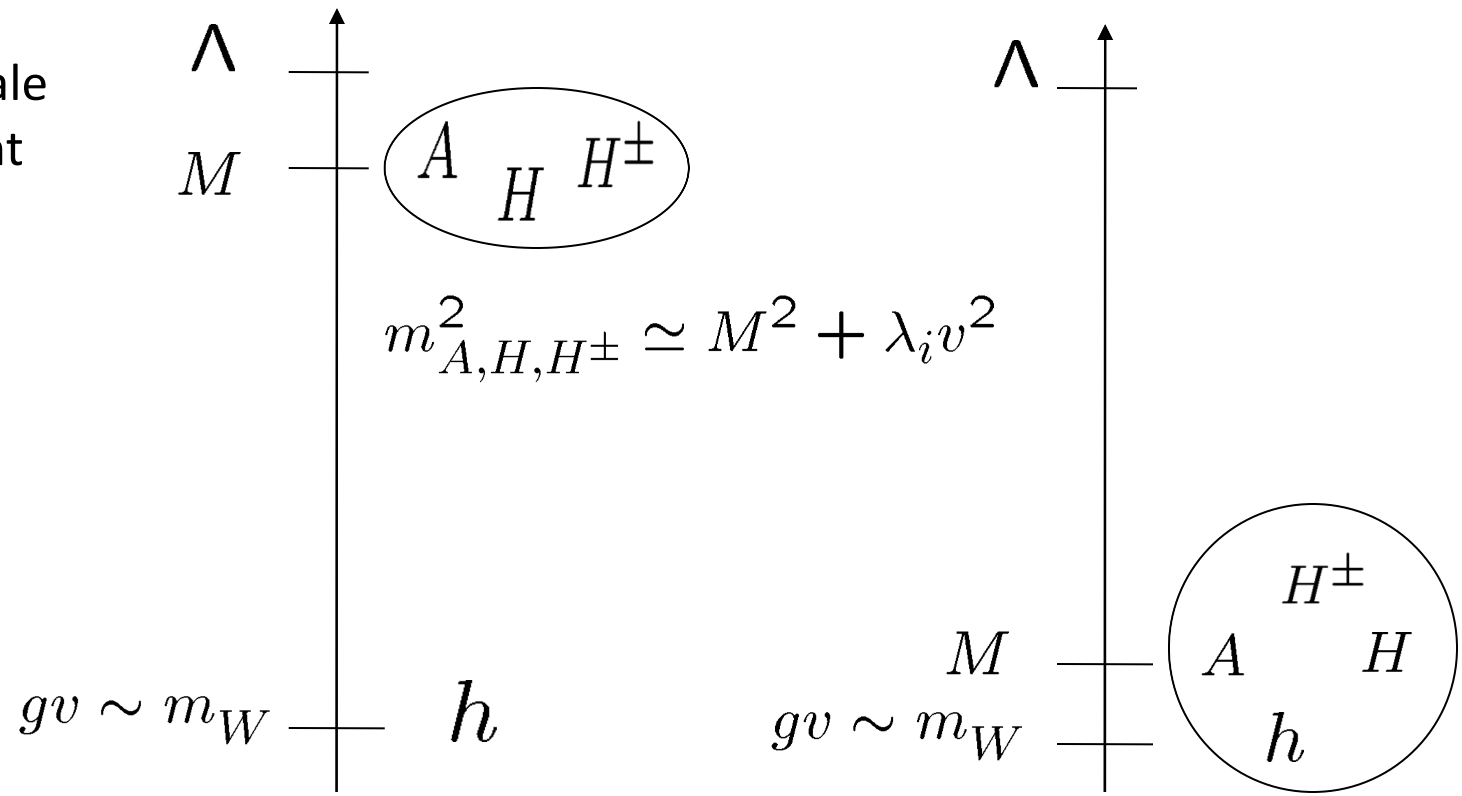
M_{soft}: soft breaking scale

of the discrete symmetry

Decoupling/Non-decoupling

Λ: Cutoff

M: Mass scale irrelevant to VEV



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{v^2}{\Lambda^2}\mathcal{O}^{(6)}$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{nonSM}} + \frac{v^2}{\Lambda^2}\mathcal{O}^{(6)}$$

Non-decoupling effect

Type2-2HDM (MSSM) Higgs couplings

Higgs mixing

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

VEV's: $v_1^2 + v_2^2 = v^2 \simeq (246\ \text{GeV})^2$

$$\tan\beta = \frac{v_2}{v_1}$$

SM

2HDM type2 (MSSM)

Gauge coupling:

$\phi VV \quad (V = Z, W)$ $\Rightarrow$ hVV: $\sin(\beta-\alpha)$, HVV: $\cos(\beta-\alpha)$

Yukawa coupling:

$\phi b\bar{b}$ $\Rightarrow$ $hb\bar{b}$: $\dfrac{\sin\alpha}{\cos\beta}$, $Hb\bar{b}$: $\dfrac{\cos\alpha}{\cos\beta}$

$\phi t\bar{t}$ $\Rightarrow$ $Ht\bar{t}$: $\dfrac{\sin\alpha}{\sin\beta}$, $ht\bar{t}$: $\dfrac{\cos\alpha}{\sin\beta}$

SM-like regime

hVV	HVV
$\sin(\beta-\alpha)$	$\cos(\beta-\alpha)$

$$\sin(\beta-\alpha) \simeq 1$$

Only the lightest Higgs h couples to weak gauge bosons

h behaves like the SM Higgs boson

$$g_{hVV} \simeq g_{hVV}(SM)$$

Heavy Higgs boson couplings with gauge bosons vanish

$$g_{HVV} \simeq 0$$

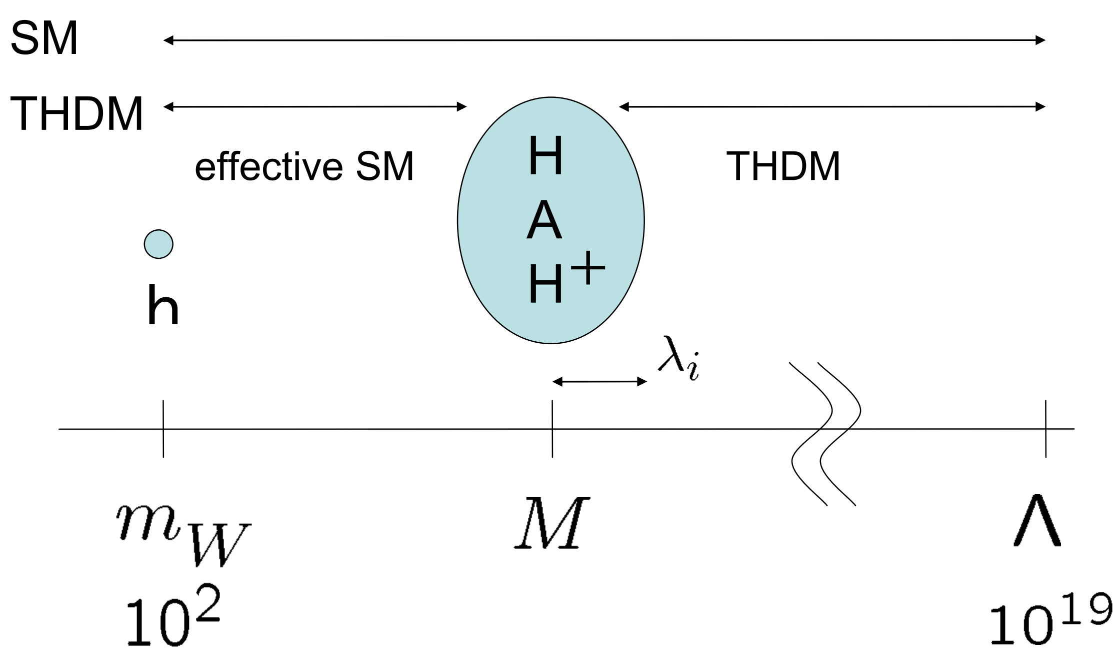
Many λcouplings → mass prediction changed

Lightest Higgs mass

$$m_h^2 = \lambda v^2$$

Additional Higgs masses

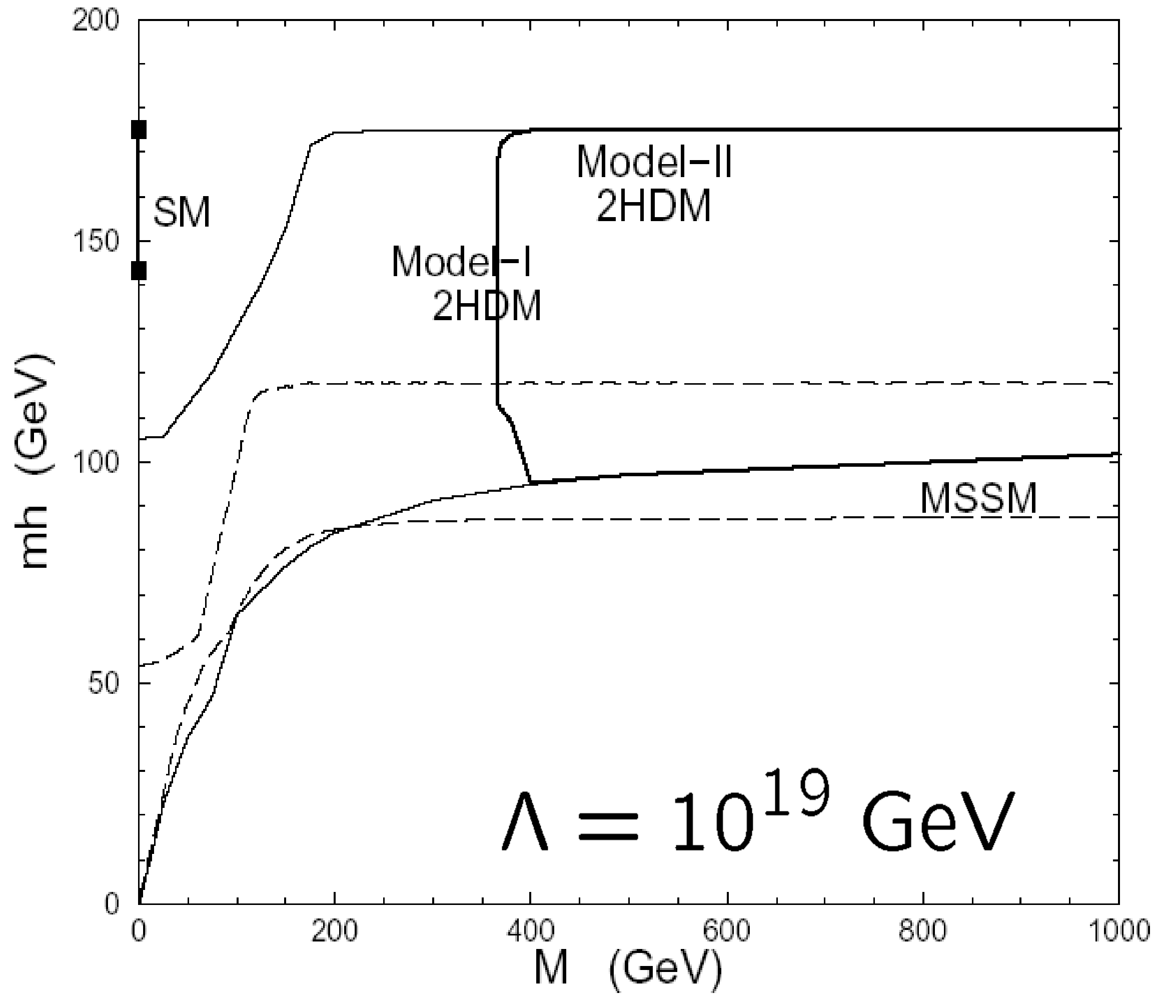
$$m_\phi^2 \simeq M^2 + \lambda' v^2$$



RGE

$$16\pi^2 \mu \frac{d}{d\mu} \lambda = 24\lambda^2 - 6 y_t^2 + A(\lambda', \lambda'', \dots)$$

Mass of the lightest Higgs boson constrained by triviality and vacuum stability



SM
2HDM type1
2HDM type 2
(MSSM)

The predicted region of mass can be differ even if all the other phenomena behave like the SM in the low energy.

Kanemura, Kasai, Okada 1999

Aoki, Kanemura, Seto, PRL 102 (2009)
Aoki, Kanemura, Seto, PRD 80 (2009)

The Aoki-Kanemura-Seto model

SM + **Extended Higgs sector** + **TeV-RH ν** — Z_2-odd

Symmetry: $SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2 \times \tilde{Z}_2$

Particle: $\Phi_1, \Phi_2 \rightarrow$ h, H, A, $H^\pm$ — Z_2-even

h, H: CP-even; A: CP-odd; $H^\pm$: Charged

SM-like Higgs

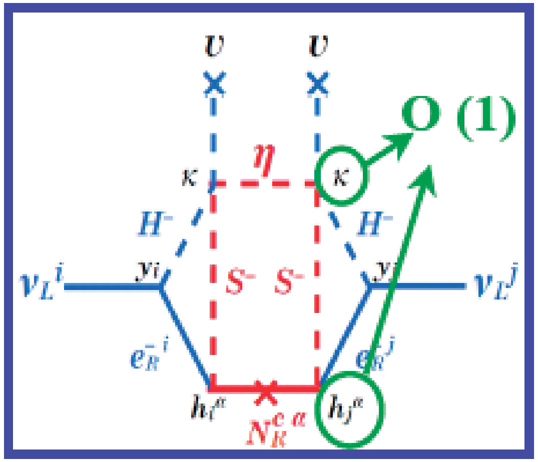
η, $S^\pm$, N_R — Z_2-odd

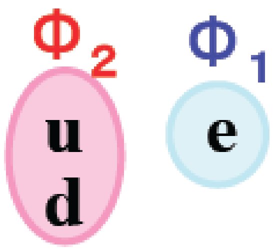
Z_2 (exact) : To forbid m_ν under the 2-loop level

Guarantee of DM stability

$\tilde{Z}_2$ (softly broken) : To forbid FCNC at the tree-level

- **Tiny neutrino mass**
  →It is naturally generated at the 3-loop level.
- **Dark matter**
  →The lightest Z_2-odd scalar is the candidate : η
- **Baryogenesis** *Cohen, Kaplan, Nelson, hep-ph/9302210* *Fromme, Huber, Seniuch, JHEP11 (2006)*
  →Electroweak baryogenesis
- A light H^+ [O(100)GeV]
  → Type-X two Higgs doublet model (2HDMX)
  Aoki, Kanemura, Tsumura, Yagyu, PRD 80 (2009)





Theoretical consistency of the model

The AKS model is an effective theory whose cutoff scale should be from **multi-TeV** to **10TeV**, since we do not prefer to unnatural fine-tuning of correction of scalar boson masses.

In particular in this model

- Some of the coupling constants are O(1), since neutrino masses are generated at the 3-loop level.
- There are a lot of additional scalar bosons (H, A, $H^{\pm}$, $S^{\pm}$, η).

It is non-trivial about the consistency of the model.

We have to examine whether the model has allowed parameter regions which satisfy the conditions of **vacuum stability** and **triviality** even when the cutoff scale is 10TeV.

Vacuum stability and triviality bounds

Vacuum stability bound

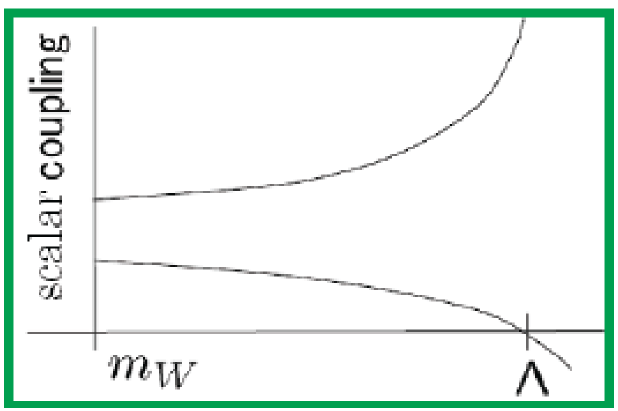
We require that scalar potentials do not have a negative coefficient in any direction even when order parameters take large value:

$$\lim_{r\to\infty} V\,(rv_1, rv_2, \ldots, rv_n) > 0$$

Triviality bound

The condition of the reliable perturbative calculation.

We require that all of the coupling constants which vary according to the RGEs do not become strong coupling up to the cutoff scale.



RGE

$$\frac{d}{d\mu}\lambda_i = \frac{1}{16\pi^2}\left[\lambda_1^2 + \lambda_2^2 + \cdots - y_{f_1}^4 - y_{f_2}^4 - \cdots\right]$$

Bosonic loop contributes positive,
fermionic loop contributes negative for scalar couplings.

Vacuum stability in the AKS model

Deshpande, Ma, PRD 18 (1978) for 2HDM

Kanemura, Kasai, Lin, Okada, Tseng, Yuan PRD 64 (2000) for Zee model

$$\boxed{\lambda_1(\mu) > 0, \;\; \lambda_2(\mu) > 0, \;\; \lambda_S(\mu) > 0, \;\; \sigma_1(\mu) > 0, \;\; \sigma_2(\mu) > 0, \;\; \lambda_\eta(\mu) > 0,}$$

$$\boxed{\begin{aligned} &\rho_1(\mu) + \sqrt{\lambda_1(\mu)\lambda_S(\mu)/2} > 0, \\ &\rho_2(\mu) + \sqrt{\lambda_2(\mu)\lambda_S(\mu)/2} > 0, \\ &\bar{\lambda}(\mu) + \sqrt{\lambda_1(\mu)\lambda_2(\mu)/2} > 0, \end{aligned}}$$

$$\boxed{\bar{\lambda}(\mu) = \lambda_3(\mu) + \min[0, \lambda_4(\mu) + \lambda_5(\mu), \lambda_4(\mu) - \lambda_5(\mu)]}$$

$$\boxed{\begin{aligned} &2\lambda_1(\mu) + 2\lambda_2(\mu) + 4\lambda_3(\mu) + 4\rho_1(\mu) + 4\rho_2(\mu) + \lambda_S(\mu) \\ &+ 4\sigma_1(\mu) + 4\sigma_2(\mu) + \frac{2\lambda_\eta(\mu)}{3} + 4\xi(\mu) - 16\sqrt{2}\kappa(\mu) > 0 \end{aligned}}$$

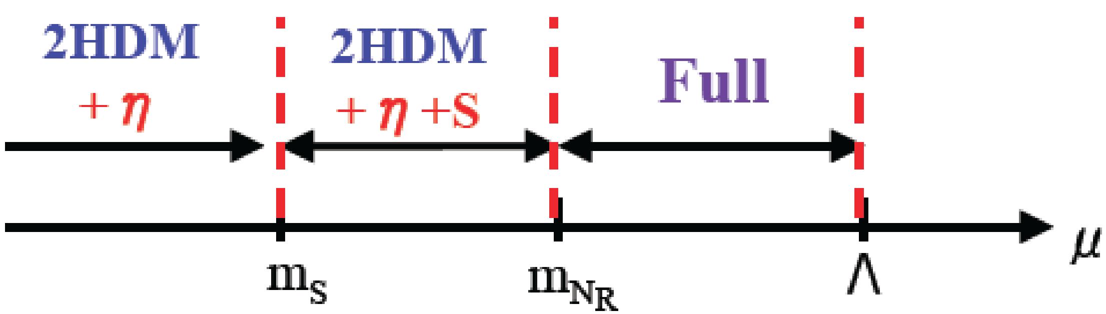
Triviality bound in the AKS model

Scalar couplings: $|\lambda(\mu)|, |\sigma(\mu)|, |\rho(\mu)|, |\kappa(\mu)|, |\xi(\mu)| \,|_{\mu<\Lambda} < 8\pi$

Yukawa couplings: $y_t^{\,2}(\mu), y_b^2(\mu), y_\tau^{\,2}(\mu), \; h^2(\mu) \,|_{\mu<\Lambda} < 4\pi$

Kanemura, Kasai, Lin, Okada, Tseng, Yuan PRD 64 (2000)

- We analyze scale dependence of coupling constants by using RGEs at the 1-loop level then search the allowed parameter regions as a function of cutoff scale.
- We take into account the threshold effect as follows:



RGEs in the AKS model

Scalar couplings

$$\beta(\lambda_1) \sim \frac{1}{16\pi^2}\Big[12\lambda_1^2 + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 + 4\lambda_3\lambda_4 + 2\rho_1^2 + \sigma_1^2 - 4y_\tau^4 + 4y_\tau^2\lambda_1\Big]$$

$$\beta(\lambda_2) \sim \frac{1}{16\pi^2}\Big[12\lambda_2^2 + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 + 4\lambda_3\lambda_4 + 2\rho_2^2 + \sigma_2^2 - 12y_t^4 - 12y_b^4 + (12y_t^2 + 12y_b^2)\lambda_2\Big]$$

$$\beta(\lambda_3) \sim \frac{1}{16\pi^2}\Big[6\lambda_1\lambda_3 + 2\lambda_1\lambda_4 + 6\lambda_2\lambda_3 + 2\lambda_2\lambda_4 + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 + 2\rho_1\rho_2 + \sigma_1\sigma_2 + 4\kappa^2 + (6y_t^2 + 6y_b^2 + 2y_\tau^2)\lambda_3\Big]$$

$$\beta(\lambda_4) \sim \frac{1}{16\pi^2}\Big[2(\lambda_1 + \lambda_2 + 4\lambda_3 + 2\lambda_4)\lambda_4 + 8\lambda_5^2 - 8\kappa^2 + (6y_t^2 + 6y_b^2 + 2y_\tau^2)\lambda_4\Big]$$

$$\beta(\lambda_5) \sim \frac{1}{16\pi^2}\Big[2(\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4)\lambda_5 + (6y_t^2 + 6y_b^2 + 2y_\tau^2)\lambda_5\Big]$$

$$\beta(\lambda_S) \sim \beta(\lambda_S)\frac{1}{16\pi^2}\Big[8\rho_1^2 + 8\rho_2^2 + 5\lambda_S^2 + 2\xi^2 + 4\sum_{i,\alpha}(h_i^\alpha)^2\lambda_S - 8\sum_{i,j}\sum_{\alpha,\beta} h_i^\alpha h_i^\beta h_j^\beta h_j^\alpha\Big]$$

$$\beta(\lambda_\eta) \sim \frac{1}{16\pi^2}\Big[12(\sigma_1^2 + \sigma_2^2) + 3\lambda_\eta^2 + 6\xi^2\Big]$$

$$\beta(\xi) \sim \frac{1}{16\pi^2}\Big[4\rho_1\sigma_1 + 4\rho_2\sigma_2 + 2\lambda_S\xi + \lambda_\eta\xi + 4\xi^2 + 2\sum_{\alpha,i}(h_i^\alpha)^2\xi\Big]$$

$$\beta(\rho_1) \sim \frac{1}{16\pi^2}\Big[6\lambda_1\rho_1 + 4\lambda_3\rho_2 + 2\lambda_4\rho_2 + 2\rho_1\lambda_S + 4\rho_1^2 + \sigma_1\xi + 8\kappa^2 + (+2\sum_{i,\alpha}(h_i^\alpha)^2 + 2y_\tau^2)\rho_1\Big]$$

$$\beta(\rho_2) \sim \frac{1}{16\pi^2}\Big[6\lambda_2\rho_2 + 4\lambda_3\rho_1 + 2\lambda_4\rho_1 + 2\rho_2\lambda_S + 4\rho_2^2 + \sigma_2\xi + 8\kappa^2 + (+2\sum_{i,\alpha}(h_i^\alpha)^2 + 6y_t^2 + 6y_b^2)\rho_2\Big]$$

$$\beta(\sigma_1) \sim \frac{1}{16\pi^2}\Big[6\lambda_1\sigma_1 + (4\lambda_3 + 2\lambda_4)\sigma_2 + \sigma_1\lambda_\eta + 2\rho_1\xi + 16\kappa^2 + 2y_\tau^2\sigma_1\Big]$$

$$\beta(\sigma_2) \sim \frac{1}{16\pi^2}\Big[6\lambda_2\sigma_2 + (4\lambda_3 + 2\lambda_4)\sigma_1 + \sigma_2\lambda_\eta + 2\rho_2\xi + 16\kappa^2 + (+6y_t^2 + 6y_b^2)\sigma_2\Big]$$

$$\beta(\kappa) \sim \frac{1}{16\pi^2}\kappa\Big[2\lambda_3 - 2\lambda_4 + 2\xi + 2\sigma_1 + 2\sigma_2 + 2\rho_1 + 2\rho_2 + \sum(h_i^\alpha)^2 + 3y_t^2 + 3y_b^2 + y_\tau^2\Big]$$

Yukawa couplings

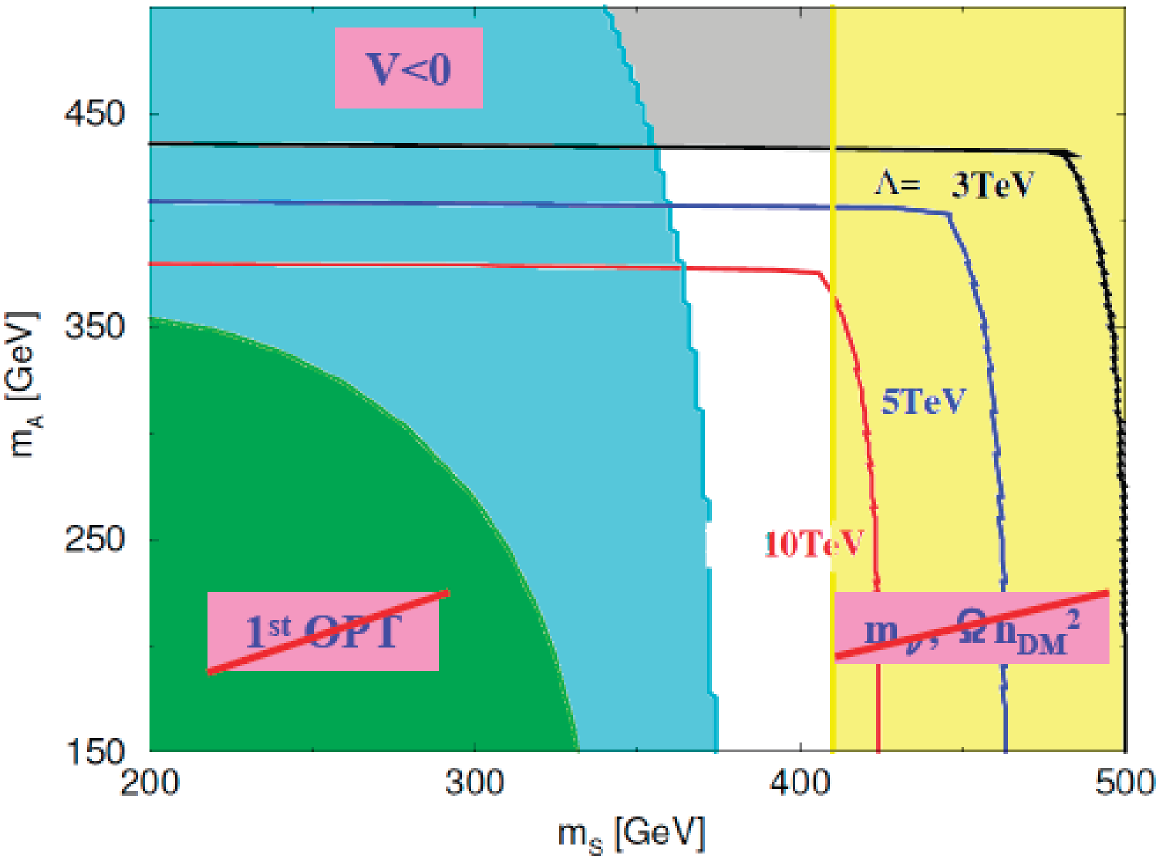
$$\beta(y_t) \sim \frac{1}{16\pi^2}\Big[\frac{9}{2}y_t^3 + \frac{3}{2}y_t y_b^2\Big]$$

$$\beta(y_b) \sim \frac{1}{16\pi^2}\Big[\frac{9}{2}y_b^3 + \frac{3}{2}y_t^2 y_b\Big]$$

$$\beta(y_\tau) \sim \frac{1}{16\pi^2}\Big[\frac{5}{2}y_\tau^3\Big]$$

$$\beta(h_i^\alpha) \sim \frac{1}{16\pi^2}\Big[\frac{1}{2}h_i^\alpha\sum_j(h_j^\alpha)^2 + \frac{1}{2}h_i^\alpha\sum_\beta(h_i^\beta)^2 + h_i^\alpha\sum_{j,\beta}(h_j^\beta)^2$$

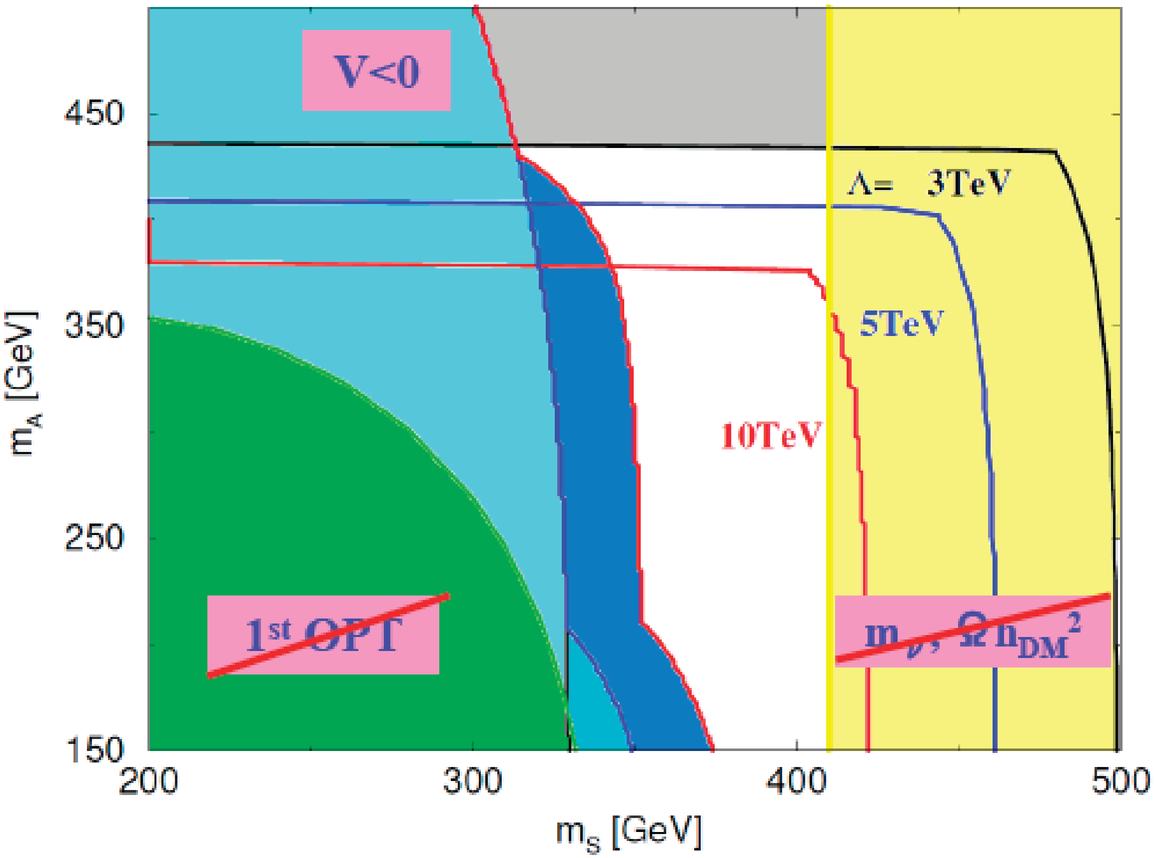
Allowed regions from vacuum stability, 1^{st}-OPT, DM abundance and neutrino mass



$\sin(\beta - \alpha)=1$: SM-like limit
$\tan\beta=25$
$\kappa=1.2$
$\xi=3.0$ $[\xi|S|^2\eta^2]$
M_R=3TeV
m_h=120GeV
$M=m_{H+}=m_H$=100GeV
μ_S=200GeV
μ_η=30GeV

There are allowed regions even when Λ is around 10TeV which satisfy the condition from vacuum stability and triviality not be inconsistent with experimental bounds.

Allowed regions from vacuum stability, 1^{st}-OPT, DM abundance and neutrino mass



$\sin(\beta - \alpha)=1$: SM-like limit

$\tan\beta=25$

$\kappa=1.2$

$\xi=4.0$ $[\xi |S|^2 \eta^2]$

$M_R=3\text{TeV}$

$m_h=120\text{GeV}$

$M=m_{H+}=m_H=100\text{GeV}$

$\mu_S=200\text{GeV}$

$\mu_\eta=30\text{GeV}$

There are allowed regions even when Λ is around 10TeV which satisfy the condition from vacuum stability and triviality not be inconsistent with experimental bounds.