

1           **Uncertainties and Error Growth in Forecasting the Record-Breaking Rainfall in**  
2                                   **Zhengzhou, Henan on 19–20 July 2021**

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## Abstract

This study explores the controlling factors of the uncertainties and error growth at different spatial and temporal scales in forecasting the high-impact extremely heavy rainfall event that occurred in Zhengzhou, Henan Province China on 19–20 July 2021 with a record-breaking hourly rainfall exceeding 200 mm and a 24-h rainfall exceeding 600 mm. Results show that the strengths of the mid-level low-pressure system, the upper-level divergence, and the low-level jet determine both the amount of the extreme 24-h accumulated and hourly rainfall at 0800 UTC. The forecast uncertainties of the accumulated rainfall are insensitive to the magnitude and the spatial structure of the tiny, unobservable errors in the initial conditions of the ensemble forecasts generated with Global Ensemble Forecast System (GEFS) or sub-grid-scale perturbations, suggesting that the predictability of this event is intrinsically limited. The dominance of upscale rather than upamplitude error growth is demonstrated under the regime of  $k^{-5/3}$  power spectra by revealing the inability of large-scale errors to grow until the amplitude of small-scale errors has increased to an adequate amplitude, and an apparent transfer of the fastest growing scale from smaller to larger scales with a slower growth rate at larger scales. Moist convective activities play a critical role in enhancing the overall error growth rate with a larger error growth rate at smaller scales. In addition, initial perturbations with different structures have different error growth features at larger scales in different variables in a regime transitioning from the  $k^{-5/3}$  to  $k^{-3}$  power law. Error growth with Conditional Nonlinear Optimal Perturbation (CNOP) tends to be more upamplitude relative to the GEFS or sub-grid-scale perturbations possibly owing to the inherited error growth feature of CNOP perturbation, the inability of convective parameterization scheme to rebuild the  $k^{-5/3}$  power spectra at the mesoscales, and different error growth characteristics in the  $k^{-5/3}$  and  $k^{-3}$  regimes.

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47 **1. Introduction**

48 Wide-spread torrential rainfall hit Henan province on 17–22 July 2021. The most intense  
49 episode of rainfall occurred on 19–20 July 2021 in Zhengzhou, the capital city of Henan  
50 Province, and the surrounding area. The 24-h accumulated rainfall from 0000 UTC 20 July to  
51 0000 UTC 21 July (LST = UTC + 8) exceeded 600 mm and several stations recorded their  
52 respective historically highest daily accumulated rainfall (Ran et al. 2021; Shi et al. 2021). The  
53 highest hourly rainfall of 201.9 mm in metropolitan Zhengzhou city, occurred from 0800 UTC to  
54 0900 UTC 20 July, marks a new record of hourly rain rate in mainland China (Shi et al. 2021;  
55 Sun et al. 2021; Zhuang and Xing 2021). Associated hazards on that day, especially inland flash  
56 flooding, led to 380 casualties.

57 The geographical locations and topographical features of Henan Province make it prone to  
58 many different types of heavy-rain-producing weather systems (e.g., Liang et al. 2020). The  
59 devastating rainfall event on 19–20 July 2021 has been found to be contributed by various  
60 environmental forcing such as liftings associated with upper-level troughs and mid-level vortices,  
61 moisture transportations associated with the subtropical high to the east, Typhoon In-Fa to the  
62 southeast, and Typhoon Cempaka to the south, as well as low-level convergence associated with  
63 low-level jet and local topography (Ran et al. 2021; Sun et al. 2021). Many dynamical and  
64 thermodynamical parameters during this event deviated significantly from the climatology of  
65 major torrential rainfall events in this area, especially low-level vorticity and column-integrated  
66 precipitable water (Zhang et al. 2021).

67 In spite of the extremeness of this event, operational weather forecast offices of Henan  
68 Province and Zhengzhou predicted the occurrence of this extreme rainfall event a few days  
69 before and issued several warnings for a vast region in Henan Province in the following days

70 prior to this event. However, the most intense rainfall centers in the operational forecasts are  
71 several hundred kilometers away from the actual epicenter. Similarly, forecasts from several  
72 global and regional numerical weather prediction (NWP) models show large variations in terms  
73 of the location and the intensity of the highest accumulated rainfall (Shi et al. 2021). This work  
74 aims to understand these discrepancies between forecasted and observed rainfall in this record-  
75 breaking rainfall event by examining the forecast uncertainties and the associated error growth  
76 mechanism.

77 The extent of accuracy in numerical weather forecasting is often referred to as “atmospheric  
78 predictability” and was first proposed by Lorenz (1963). Lorenz (1996) categorized this problem  
79 into practical predictability (Lorenz 1982), or the forecast capability given currently available  
80 knowledge and techniques, and intrinsic predictability (Lorenz 1969), or the longest possible  
81 forecast extent given nearly perfect knowledge and techniques. One important aspect of practical  
82 predictability originates from uncertainties in representing key environmental forcing. Key  
83 environment forcing for a heavy rainfall event is generally identified using ensemble-based  
84 sensitivity analysis (ESA), which measures linear relationships between a scalar forecast metric  
85 and atmospheric state variables through ensemble statistics (Hakim and Torn 2008), or the  
86 conditional nonlinear optimal perturbation (CNOP; Mu and Duan 2003) method, which is an  
87 adjoint-based method that takes nonlinear processes into account. By diagnosing key  
88 environmental factors in extreme rainfall events, understanding on the rainfall forecast  
89 uncertainties can be improved (e.g., Hawblitzel et al. 2007; Lynch and Schumacher 2014; Yu  
90 and Meng 2016; Zhang and Meng 2018). Based on ESA, Zhang and Meng (2018) revealed the  
91 importance of well-forecast low-level jet locations in determining the performance of ensemble  
92 rainfall forecast during a persistent heavy rainfall event in Guangdong, China, in early spring of

93 2014. Based on both ESA and CNOP, Yu and Meng (2016) consistently demonstrated the  
94 essential role of the mid-level trough in the westerly flow and the associated low-level low in the  
95 high-impact rainfall event in Beijing, China, on 21 July 2012. Yu and Meng (2022) found that the  
96 CNOP with moist physics identified the sensitive areas at both the lower levels and upper levels  
97 for four typical heavy rainfall events in north China. The upper-level sensitive area, which  
98 corresponds to the upper-level weather systems, is associated with high baroclinicity, while the  
99 lower-level sensitive area, which corresponds to the lower-level weather systems, is associated  
100 with the moist physics. Although there have been studies revealing weather systems that may  
101 have affected this record-breaking extreme heavy rainfall in Henan (e.g., Ran et al. 2021; Sun et  
102 al. 2021), the key environmental factors and their associated forecast uncertainties remain  
103 unknown.

104 Unlike practical predictability that is primarily controlled by uncertainties in NWP models  
105 and initial conditions, intrinsic predictability is primarily limited by error growth mechanisms  
106 that are inherently embedded in the dynamical and thermodynamical processes of the weather  
107 (e.g., Melhauser and Zhang 2012; Sun and Zhang 2016). Zhang et al. (2007) presented the  
108 conceptual model of how tiny, unobservable errors will limit the predictability at the mesoscales:  
109 those small-amplitude small-scale errors will grow upscales and rapidly spread with the help of  
110 moist convective processes, saturate at smaller scales and transfer to progressively larger scales  
111 through geostrophic adjustments, and eventually limit the predictability of mesoscale and  
112 synoptic scales. This conceptual model has been proved by many following studies (e.g., Judt  
113 2018; Selz 2019; Selz and Craig 2015; Sun and Zhang 2016, 2020; Sun et al. 2017; Zhang et al.  
114 2016; Zhang et al. 2019).

115 Several studies argue that large-scale errors are just as important as, if not more than, small-  
116 scale errors (Durrán and Gingrich 2014; Durrán and Weyn 2016; Nielson and Schumacher 2016;  
117 Zhang 2021), and errors grow upamplitude at all model-resolved scales simultaneously rather  
118 than transfer upscales (Weyn and Durrán 2017; Judt 2018, 2020). It should be noted that these  
119 different disagreements are essentially equivalent: small-scale errors are more important if the  
120 errors are governed by upscale growth, because the upscale growth of small-scale errors will  
121 dominate the existing large-scale errors (Zhang et al. 2007); while large-scale errors are more  
122 important if the errors are governed by upamplitude growth, because large-scale errors can grow  
123 to greater amplitudes owing to the greater base energy at these scales (Durrán and Weyn 2016).  
124 Understanding the relative importance of errors at different spatial scales will facilitate a better  
125 understanding of the error growth mechanisms. Therefore, many of the previous studies have  
126 used high-resolution, convection-permitting ensemble forecasts that incorporate initial condition  
127 uncertainties of different amplitude and/or spatial scales to examine the error growth  
128 mechanisms (e.g., Melhauser and Zhang 2012; Nielsen and Schumacher 2016; Zhang et al. 2016;  
129 Weyn and Durrán 2019). However, previous studies either examined the sensitivity of the  
130 forecast error growth to different amplitude and horizontal scales of homogeneous initial  
131 uncertainties or did not examine this sensitivity when flow-dependent initial uncertainties were  
132 imposed, while how sensitive the forecast error growth is to different amplitude and horizontal  
133 scales of flow-dependent unobservable initial uncertainties in a high-resolution convection-  
134 permitting ensemble forecast on a real-world high-impact rainfall event remains unknown.

135 In addition to the scale and amplitude, the structure of initial perturbations may influence the  
136 forecast uncertainty and error growth features as well. Initial perturbations with different  
137 structures are mainly generated through breeding vectors, singular vector, random sampling from

138 a climatologically based background error covariance such as CV3 from the WRFDA package,  
139 and the CNOP method. Mu et al. (2007) found that CNOP-type error tends to have a seasonal  
140 dependent evolution and produces the most considerable negative effect on the forecast results.  
141 Adding CNOP to the initial condition yields a spring predictability barrier phenomenon, while  
142 adding perturbations with the same magnitude but a different structure from the CNOP does not.  
143 How sensitive the forecast error growth is to different structures of initial uncertainties in a real-  
144 world extremely heavy rain event is also one interesting question to answer.

145 Therefore, to explore the uncertainties and error growth in forecasting this high-impact  
146 torrential rainfall event at different spatial and temporal scales, we present a suite of analyses  
147 using forecasts from numerical models ranging from global models to regional, convection-  
148 permitting models in this study. This includes ESA using the Observing System Research and  
149 Predictability Experiment (THORPEX) Interactive Grand Global Ensemble (TIGGE; Bougeault  
150 et al. 2010), the CNOP method using coarse-resolution simulations from the Pennsylvania State  
151 University–National Center for Atmospheric Research (PSU–NCAR) fifth-generation Mesoscale  
152 Model (MM5; Grell et al. 1995), and high-resolution convection-permitting ensemble  
153 simulations from the Weather Research and Forecasting (WRF) model with initial perturbations  
154 of different amplitudes and spatial scales. Section 2 summarizes the data, methods, models, and  
155 experiment designs that are used in this study. Key environmental factors related to the forecast  
156 uncertainties of the rainfall are revealed in section 3. Section 4 examines the error growth  
157 features and their sensitivities to different scales, amplitudes and structures. Section 5 gives the  
158 summary.

159

## 160 **2. Data and methodology**

161 *2.1 Observed 24-h accumulated rainfall*

162 Hourly rain gauge data provided by the China Meteorological Administration with an  
163 average site spacing of ~5–10 km was interpolated to a 0.1°×0.1° grid using a Cressman  
164 interpolation method (Cressman 1959). Fig. 1a shows the 24-h accumulated rainfall from 1200  
165 UTC 19 July to 1200 UTC 20 July 2021. The accumulated rainfall intensely concentrated over  
166 northern Henan Province, with a maximum of 505.54 mm, a considerable area that exceeds 400  
167 mm, and an area-average of 74.49 mm over the inner box of Fig. 1a.

168

169 *2.2 The TIGGE ensemble and the ensemble sensitivity analysis*

170 Forecasts on the 24-h accumulated rainfall from 1200 UTC 19 July to 1200 UTC 20 July  
171 over the focused region given in Fig. 1a were quantitatively evaluated using the TIGGE  
172 ensembles, with a forecast initialization of 0000 UTC 19 July. Twelve global models from  
173 TIGGE evaluated in the present study are listed in Table 1. The TIGGE-derived fields were  
174 interpolated into a 0.1° × 0.1° grid to facilitate the comparison with observations.

175 Four global models with better forecast performances (see Section 3 for details) were then  
176 selected to identify the key factors for the extreme heavy rainfall using ESA. We calculated the  
177 area-averaged 24-h accumulated rainfall over the focused regions (32.5–36.5° N, 111–115° E,  
178 inner box in Fig. 1a) from 1200 UTC 19 July to 1200 UTC 20 July as the forecast metric ( $P$ ).  
179 The Pearson correlation coefficient ( $R$ ) was used to measure the correlation between the forecast  
180 metrics and the variables of interest ( $X$ ) at different forecast times and pressure levels, and was  
181 calculated as follows (Hakim and Torn 2008):

182

$$R = \frac{\sum_{i=1}^n (X_i - \bar{X})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2 \sum_{i=1}^n (P_i - \bar{P})^2}} \quad (1)$$

183 where the overbar represents the ensemble mean and  $n$  is the ensemble size (73 herein  
184 combining the 4 selected models).

185

### 186 *2.3 Configurations and experiment design of the regional convection-permitting ensemble*

187 High-resolution convection-permitting ensemble simulations using the Advanced Research  
188 WRF (ARW/WRF; Skamarock et al. 2021) dynamical core were used to examine the intrinsic  
189 predictability and error growth in forecasting this event. Three one-way nested domains using the  
190 ARW/WRF model, version 4.2, are configured with horizontal grid spacings of 27, 9, and 3 km,  
191 and  $210 \times 130$ ,  $340 \times 280$ , and  $301 \times 301$  horizontal grids, respectively. There are 51 hybrid terrain-  
192 pressure levels, and the upper-most level is located at 50 hPa. Physical parameterization schemes  
193 are selected after trial-and-error tests, including the aerosol-aware Thompson and Eidhammer  
194 (2014) microphysics scheme, modified Tiedke cumulus scheme (Zhang and Wang 2017; only  
195 applied in the 27-km domain), revised MM5 scheme for surface layer processes (Jimenez et al.  
196 2012), thermal diffusion scheme for land surface processes, Yonsei University PBL scheme  
197 (Hong et al. 2006), and RRTMG schemes for longwave and shortwave radiation (Iacono et al.  
198 2008).

199 In order to examine the influence of the amplitude and scale of the initial uncertainties on the  
200 intrinsic predictability of this event, a total of four ensemble forecasts, each containing 40  
201 ensemble members that run from 0600 UTC 19 July to 1200 UTC 20 July, are designed. Two of  
202 them incorporate initial uncertainties from relatively large scales. We first derive the  
203 perturbations from the 20-member  $0.5^\circ \times 0.5^\circ$  Global Ensemble Forecast System (GEFS) analyses  
204 valid at 0600 UTC 19 July and the 20-member GEFS 6-hour forecasts from 0000 UTC 19 July  
205 (also valid at 0600 UTC 19 July) by subtracting their respective ensemble mean from each of the

206 respective 20 members. Temperature, water vapor mixing ratio, and the two components of the  
207 horizontal wind are processed. Then, the 40 perturbations are scaled by a factor of 0.1 and added  
208 to the GFS analysis valid at 0600 UTC 19 July to generate 40 initial conditions (ICs) with  
209 uncertainties that are an order of magnitude smaller than current global model analysis  
210 uncertainties, which is necessary because intrinsic predictability examines error growth  
211 mechanisms resulted from tiny, unobservable initial uncertainties. These 40 ICs are used to  
212 initialize the “LARGE” ensemble forecast. The initial perturbations of the LARGE ensemble are  
213 further scaled by a factor of 0.1 (therefore a 0.01 factor from their original values) to form the  
214 ICs that initialize the “LARGE0.1” ensemble forecast.

215 The other two ensembles contain initial uncertainties that are concentrated at smaller scales.  
216 To facilitate this purpose, we first run a short-term deterministic forecast from the GFS analysis  
217 valid at 0600 UTC 19 July 2021 using a configuration of model domains that cover exactly the  
218 same region but using horizontal grid spacings that are 1/3 of their original values (i.e., 9, 3, and  
219 1 km for the three domains). Then, values of temperature, water vapor mixing ratio, and the two  
220 horizontal components of the wind at each grid point of the original model domains are replaced  
221 by randomly, nonrepetitively selected values from the adjacent 3×3 grid points in the higher-  
222 resolution 9-3-1-km simulation, similar to the generation of initial perturbations of Zhang et al.  
223 (2016). Since each grid points in the original model domain corresponds to 8 surrounding grid  
224 points in the 9-3-1-km domain (excluding the grid points that are collocated), each 9-3-1-km  
225 simulation output can be used to generate 8 different perturbations. Five model outputs from  
226 0655 to 0700 UTC 19 July 2021 (when small-scale structures are sufficiently developed while no  
227 significant precipitation occur) from the 9-3-1-km simulation, each 72 seconds apart, are used to  
228 generate 40 ICs (8 for each output) that contain uncertainties that represent flow-dependent

229 features that the original model resolutions are not able to resolve, and these ICs are used to  
230 initialize the “SMALL” ensemble forecast. Similar to the LARGE0.1 ensemble, the initial  
231 perturbations of the SMALL ensemble are also multiplied by 0.1 to initialize the “SMALL0.1”  
232 ensemble forecast. Although the perturbations in SMALL and SMALL0.1 are drawn from  
233 simulation outputs close to 0700 UTC, they are nonetheless added to the GFS analysis valid at  
234 0600 UTC 19 July, consistent with LARGE and LARGE0.1.

235

#### 236 *2.4 Description of CNOP and its experiment design*

237 CNOP is the initial perturbations that maximize the cost function under certain initial  
238 constraint conditions (Mu and Duan, 2003). The cost function is defined as  $J(\delta x_0) = \|M(x_0 +$   
239  $\delta x_0) - M(x_0)\|$ , and the initial perturbations  $\delta x_0^*$  is called CNOP, if and only if  $J(\delta x_0^*) =$   
240  $\max_{\|\delta x_0\| \leq \beta} J(\delta x_0)$ .  $M$  is the nonlinear operator.  $x_0$  is the state vector  $x$  at the initial time and  
241 the  $M(x_0)$  represents the value of  $x$  at forecast time  $t$ .  $\beta$  is used to constrain the values of the  
242 initial perturbations.

243 The norm used to constrain the cost function and the initial perturbations is the total moist  
244 energy (TME) norm (Ehrendorfer et al. 1999), which is calculated as follows:

$$245 \quad \text{TME} = \frac{1}{2} \left( u^2 + v^2 + \frac{C_p}{T_r} T^2 + \frac{L^2}{C_p T_r} q^2 + R_a T_r \left( \frac{P_s}{P_r} \right)^2 \right), \quad (2)$$

246 where  $C_p$  ( $1005.7 \text{ J kg}^{-1} \text{ K}^{-1}$ ) is the specific heat at constant pressure,  $T_r$  (270 K) is the reference  
247 temperature,  $R_a$  ( $287.04 \text{ J kg}^{-1} \text{ K}^{-1}$ ) is the gas constant of dry air,  $L$  ( $2.5104 \times 10^6 \text{ J kg}^{-1}$ ) is the  
248 latent heat of condensation per unit mass,  $P_r$  (1000 hPa) is the reference pressure, and  $u$ ,  $v$ ,  $T$ ,  $q$   
249 and  $P_s$  are the two horizontal wind components, temperature, water vapor mixing ratio, and  
250 surface pressure, respectively. The sensitive area, in which area the weather systems can be

251 regarded as the key weather systems to the heavy rainfall (Yu and Meng 2016, 2022), is defined  
252 as the location of the top 1% vertically integrated TME of the entire model simulation domain.

253 In this study, the CNOP is calculated based on the MM5 model (Grell et al. 1995) and its  
254 tangent linear and adjoint models (Zou et al. 1997) using the spectral projected gradient 2 (SPG2;  
255 Birgin et al. 2001) optimization algorithm. The model domain has 90×65 horizontal grids with a  
256 horizontal resolution of 60 km and 21 terrain-following levels in the vertical from the surface to  
257 50 hPa. The initial and boundary conditions are provided by the National Centers for  
258 Environmental Prediction (NCEP) final analysis (FNL) of  $1^\circ \times 1^\circ$  at a 6-h interval. The large-  
259 scale precipitation scheme, the Anthes-Kuo cumulus parameterization scheme, and the bulk  
260 planetary boundary layer scheme are used. In order to reveal the sensitive areas of the extreme  
261 hourly rainfall at 0800 UTC 20 July, the starting and ending times are 0600 UTC 19 July and  
262 0800 UTC 20 July 2021, respectively. The verification area covers the location of the heavy  
263 rainfall (the inner green box in Fig. 2a).

264 In order to examine the evolution of initial perturbations with different structures and their  
265 impact on the rainfall forecast, the starting time of CNOP is the same as that in the convection-  
266 permitting ensemble forecast experiments. In details, the perturbations of CNOP and three  
267 random members from the LARGE ensemble are added to the GFS analysis at 0600 UTC 19  
268 July 2020 as initial conditions to calculate the perturbation development using the WRF model  
269 with the same physical parameterization schemes as those used in the LARGE ensemble, except  
270 for using a domain coverage and horizontal grid spacing the same as those for the MM5 model  
271 used to calculate the CNOP. The perturbations of the three random members from the outer-most  
272 domain of the high-resolution LARGE ensemble (see Section 2.3) were interpolated to the  
273 CNOP model grid, and the magnitude of the CNOP perturbations are scaled down to be the same

274 as the LARGE perturbations in terms of the mean TME in the area of interests (the black inner  
275 box in Fig. 2a).

276

### 277 **3. Key environmental factors related with the forecast uncertainties of the rainfall**

#### 278 *3.1 Evaluation of rainfall forecast in TIGGE ensembles*

279 Due to the relatively coarse horizontal resolutions of the global models (Table 1), the TIGGE  
280 ensemble forecast generally underestimates the rainfall amount (Fig. 3a). Nevertheless, BoM,  
281 NCMRWF, UKMO, and KMA stand out from the 12 TIGGE models with higher  $P$  (Fig. 3a)  
282 and are utilized for further ESA. Ensemble mean rainfall of the 4 models generally reproduces  
283 the rainfall distribution at the threshold of 50 mm (Fig. 1b), with a much smaller maximum of  
284 186.88 mm located more to the south compared with observation (Fig. 1a). In terms of forecast  
285 skills, KMA has the highest equitable threat score (ETS; Wilks 1995) at thresholds of 100 mm  
286 and 150 mm, while NCMRWF still retains some skills at higher thresholds such as 250 mm and  
287 300 mm (Figs. 3b–e).

288 Typical members with good and poor rainfall forecasts are selected based on ETSs of  
289 individual members (Fig. S1) and subjective comparison with the observed rainfall pattern.  
290 Members 01, 17, 49, and 71 are eventually chosen as good members, while members 35, 42, 72,  
291 and 40 are chosen as poor members (Fig. S2). Comparisons are then performed between good  
292 and poor members to obtain more physical insights into the correlation patterns of the ESA.

293

#### 294 *3.2 Results from ensemble-based sensitivity analysis*

295 Synoptic circulation systems in mid-troposphere are highly correlated with the extreme  
296 heavy rainfall. There are prominent negative correlations between 500-hPa  $Z$  and  $P$  in central  
297 China, especially in northern Henan Province from 0600 UTC 19 July to 1200 UTC 20 July,

298 with the strongest negative correlation of  $\sim -0.8$  occurring at 1800 UTC 19 July (Figs. 4a, d).  
299 Consistently, the composite of good members is characterized by a deeper mid-level low  
300 compared with that in poor members (Figs. 4e, f). This result suggests that the mid-level low  
301 significantly contributes to a large rainfall accumulation. We thus choose 1800 UTC 19 July to  
302 examine the key environmental forcing for the 24-h accumulated rainfall. Fig. 4a also shows  
303 positive correlation over the subtropical high as well as the ridge region near the southwest  
304 periphery of the subtropical high, but with a much smaller area with a confidence level of 95%  
305 and above. Together with the comparison on 500 hPa between typical members (Figs. 4b, c),  
306 these results consistently suggest that a stronger subtropical high with a deeper ridge on its  
307 southwestern flank is favorable for a larger rainfall accumulation, possibly through preventing  
308 the low pressure vortex from rapidly moving to the east.

309 The upper-level jet stream also plays an essential role in the extreme heavy rainfall process.  
310 In the correlation map between 200-hPa  $Z$  and  $P$  (Fig. 5a), there are significant positive  
311 correlations of  $\sim 0.4$  to the north of Henan Province and negative correlations of  $\sim -0.3$   
312 downstream. This result indicates that a deeper ridge and trough on 200 hPa, namely, a wavier  
313 upper-level circulation is beneficial for the rainfall accumulation. The deeper ridge is associated  
314 with a stronger upper-level northwesterly jet stream to the north of the focused region (Fig. 5b),  
315 which may provide more sufficient upper-level divergence that favors the heavy rainfall process.

316 The correlation map between 850-hPa  $Z$  and  $P$  (Figs. 5c, e) is generally similar to that on  
317 500 hPa (Figs. 4a, d), reinforcing that lower geopotential height (herein a deeper low-level  
318 trough) over the focused region and a stronger subtropical high to the east are favorable for the  
319 rainfall accumulation. In the correlation map between 850-hPa horizontal wind speed (Figs. 5d,  
320 f), there are positive correlations of  $\sim 0.6$  to the south and east of the focused region, as well as

321 over the northern periphery of Typhoon In-Fa. This result implies that the southerly and  
322 southeasterly low-level jets upstream of the focused region, which could be strengthened by the  
323 warm ridge extending southwestward from the subtropical high (Ran et al. 2021), are essential  
324 during the extreme heavy rainfall process by providing abundant moisture from the south.

325 Locations of the two tropical cyclones are remotely relevant with the heavy rainfall  
326 accumulation. The difference in the composite 850-hPa relative vertical vorticity between good  
327 members at 1800 UTC 19 July are given in Fig. 6a. Compared with poor members, the good  
328 members are characterized by a stronger vorticity over the focused region, which is corroborated  
329 with the deeper low-level vortex suggested by Fig. 5c. Moreover, there are dipoles of vorticity  
330 differences over two tropical cyclones (Fig. 6a), and the dipoles become stronger ( $\sim 6 \times 10^{-5} \text{ s}^{-1}$ )  
331 later on at 0000 UTC 20 July (Fig. 6b), indicating the locations of tropical cyclones are  
332 associated with the rainfall accumulation over the focused region. Consistently, the composite of  
333 good members is featured with a Cempaka located more to the southeast and an In-Fa located  
334 slightly more to the south, which is especially true on 0000 UTC 20 July in Fig. 6b. These  
335 variations of tropical cyclone locations may be closely related to the southwestward intrusion of  
336 the subtropical high. The southwestward intrusion of subtropical high may increase the  
337 geopotential height gradient over the northern area of Typhoon In-Fa, which enhances the low-  
338 level easterlies as revealed in Fig. 5d and thus facilitates more moisture transportation to the  
339 focused region.

340

### 341 *3.3 Results from CNOP*

342 Key environmental forcings for the 24-h accumulated rainfall identified using ESA are  
343 generally consistent with those for the hourly extreme rainfall at 0800 UTC 20 July identified

344 using CNOP. Three sensitive areas are identified by the CNOP (Fig. 2a). The vertical  
345 distribution of the horizontally integrated TME of CNOP (hereafter referred to as hTME) over  
346 the sensitive area B, which is located in the middle of the verification area, peaks at the middle  
347 ( $\sim 500$  hPa) and upper level ( $\sim 300$  hPa, Fig. 2b). This sensitive area is corresponding to the low  
348 pressure vortex and its associated shear line at 500 hPa (Fig. 2d) and the ridge at 300 hPa (Fig.  
349 2c). The hTME over the sensitive area A, which is located to the south of the verification area  
350 (Fig. 2a), peaks at lower ( $\sim 850$  hPa) and middle levels ( $\sim 500$  hPa, Fig. 2b). This sensitive area  
351 is corresponding to the southeasterly flow to the south of the shear line extending from the low  
352 vortex at 850 hPa (Fig. 2e) and 500 hPa (Fig. 2d). The environmental systems at sensitive areas  
353 A and B identified by CNOP are consistent with those identified by ESA. The CNOP also  
354 identifies the sensitive area C in the northwest of the verification area (Fig. 2a) at  $\sim 300$  hPa (Fig.  
355 2b), which is corresponding to the westerly trough (Fig. 2c) neighboring the ridge.

356

#### 357 **4. Error growth features and their sensitivities to different scales, amplitudes, and** 358 **structures**

359 All four convection-permitting ensemble forecasts show very similar distribution, structure,  
360 and values of the accumulated rainfall as well as the uncertainties across their ensemble members,  
361 and higher accumulated rainfall amounts are generally collocated with greater uncertainties  
362 (Fig.7). On the one hand, the similarity of rainfall region across all ensemble forecasts suggests  
363 that the general location of where rainfall will occur is quite predictable. However, the large  
364 uncertainties of the 24-hour accumulated precipitation in these forecasts with minute initial  
365 perturbations, as well as the insensitivity of these forecast uncertainties to the spatial scale or

366 amplitude of the initial perturbations, suggest that the predictability of the extreme rainfall during  
367 this event is intrinsically limited and highly unpredictable in a deterministic forecasting system.

368

369 *4.1 Overall power spectrum and error growth features with respect to different scales and*  
370 *amplitudes*

371 Although all ensemble forecasts show very similar uncertainties for their 24-hour  
372 accumulated precipitation forecasts as well as the power spectra of various state variables at the  
373 end of the ensemble forecast at 1200 UTC 20 July (figure not shown), the growth of the  
374 ensemble spread at the first several hours show different characteristics associated with the  
375 spatial scale and amplitude of the initial perturbations, which is shown in Fig. 8 for the energy  
376 spectra of the ensemble spread (simply “error energy” hereafter) of the U-wind component (the  
377 power spectra of temperature and water vapor mixing ratio are qualitatively similar and therefore  
378 omitted).

379 For LARGE and LARGE0.1, because of the  $0.5^\circ \times 0.5^\circ$  horizontal grid spacing of the GEFS  
380 analyses that are used to generate their initial perturbations, most of the error energy is  
381 concentrated in relatively large scales and the energy decreases rapidly for wavelengths below  
382  $\sim 200$  km (Figs. 8a, b), consistent with the statement that the smallest resolvable features of a  
383 numerical model are roughly 4 to 6 times of its horizontal grid spacing (e.g., Skamarock 2004).  
384 The missing error energy at shorter wavelengths is quickly filled as the simulation goes on.  
385 However, the error energy “plateau” at wavelengths longer than  $\sim 200$  km does not increase for  
386 the first 3 to 4 hours (Figs. 8a, b). The error energy at relatively large scales only starts to  
387 increase when the error energy at relatively small scales has grown to an amplitude that is  
388 comparable to the large-scale errors, and LARGE0.1 starts to increase slightly earlier than

389 LARGE due to its smaller initial error energy (Fig. 8b). If we look at how much error at each  
390 scale grows every hour by examining the ratios of the error energy spectra of two consecutive  
391 hours, it is clear that error growth is greater in smaller scales at earlier times, and shifts to larger  
392 scales at later times, for both LARGE and LARGE0.1 ensembles (Figs. 8e, f). Furthermore,  
393 accompanying this shift from smaller to larger scales of error growth peaks, the amplitude of the  
394 peaks also gradually decreases as they move toward larger scales, suggesting a slower growth at  
395 larger scales (Figs. 8e, f). The behavior of error growth of the LARGE and LARGE0.1  
396 ensembles suggest that the most prominent growth of errors – even when only large-scale  
397 uncertainties are imposed at the initial conditions – occurs at smaller scales first, then gradually  
398 transitions to larger scales (“upscale growth”), and the speed of error growth at smaller scales is  
399 faster than later at larger scales, consistent with the three-stage error growth model of Zhang et al.  
400 (2007).

401 On the other hand, the SMALL and SMALL0.1 ensembles first show apparent adjustment  
402 from 0 h to 1 h resulting from the unbalances in the sub-grid-scale initial perturbations. Unlike  
403 the error energy spectra of LARGE and LARGE0.1 that drastically decreases for wavelengths  
404 smaller than ~200 km, the error energy spectra of SMALL and SMALL0.1 at 0 h and 1 h are  
405 almost flat across the entire range of the wavelengths (Figs. 8c, d); however, since larger scales  
406 has greater base energy than smaller scales, therefore the overall “flat” error energy spectra of  
407 SMALL and SMALL0.1 actually indicates that errors are more concentrated at smaller scales  
408 than larger scales, opposite to the power spectra of LARGE and LARGE0.1 that errors are more  
409 concentrated at larger scales. Compared with the growth of error energy of LARGE and  
410 LARGE0.1 (Figs. 8a, b), the errors seem to be growing at all scales simultaneously for SMALL  
411 and SMALL0.1 (Figs. 8c, d), similar to the error growth after 4 h in LARGE0.1 (Fig. 8b, cyan

412 color). However, if we look at the error growth ratios, there is also a shift of error growth peaks  
413 from the smaller scale with larger amplitude at earlier times to the larger scale with smaller  
414 amplitude at later times (Figs. 8g, h), similar to what we have already observed for the LARGE  
415 and LARGE0.1 ensembles (Figs. 8e, f), especially for SMALL0.1 which has smaller initial  
416 errors (Fig. 8h).

417 In general, although the error grows up at all scales in some of the circumstances, the  
418 wavelengths at which the fastest error growth occurs shift upscales for initial condition  
419 perturbations from both small and large scales with different initial amplitudes. This remains true  
420 in sensitivity experiments that completely remove initial perturbations at scales smaller than 200  
421 km of LARGE and SMALL ensembles, while another sensitivity experiment that only keeps  
422 SMALL's initial perturbations at scales smaller than 200 km shows that error at smaller scales  
423 can grow without larger-scale errors (see supplement Fig. S3).

424

#### 425 *4.2 Impact of moist process on the power spectrum and error growth features with respect to* 426 *different scales and amplitudes*

427 Error growth rates at regions with and without precipitation are distinct due to the dominant  
428 role of moist convective processes in error growth at mesoscales (Zhang et al. 2007). Fig. 9a  
429 shows the root-mean difference kinetic energy (RMDKE; e.g., Zhang et al. 2002) averaged over  
430 regions with (“moist” in Fig. 9a) and without (“dry” in Fig. 9a) precipitation, defined as the  
431 ensemble mean precipitation rate exceeding or lower than  $10^{-6}$  mm h<sup>-1</sup>, in the ensembles. The  
432 characteristics of temperature and moisture are generally the same as RMDKE. RMDKE is  
433 defined as

434 
$$RMDKE = \sqrt{\frac{\sum_i \frac{1}{2}(u_i'^2 + v_i'^2)}{n}}, \quad (3)$$

435 where  $u'$  and  $v'$  are the differences between an ensemble member and the ensemble mean for  
 436 the two components of the horizontal wind,  $i$  is all the grid points within the moist or the dry  
 437 region from all the 40 ensemble members, and  $n$  is the quantity of all  $i$  grid points. It is  
 438 apparent from Fig. 9a that the error growth rate for the first 6 to 8 hours is much faster in the  
 439 moist region than in the dry region for all the ensembles, proving the critical role of moist  
 440 convective processes in boosting the error growth at mesoscales. Furthermore, while the four  
 441 ensembles contain initial perturbations from different scales with different amplitudes, the curves  
 442 of their respective RMDKE, both in the moist and the dry region, are generally parallel with each  
 443 other. This suggests that, at least for the four ensembles that we have examined for this event, the  
 444 error growth mechanisms at the first 6 to 8 hours are likely independent of the scales and  
 445 amplitudes (when they are already very small) of the initial perturbations.

446 We further decompose the horizontal winds into three different scales with partitions at 30  
 447 and 200 km, then categorize the decomposed wind components into moist and dry regions and  
 448 examine how RMDKE grows with and without precipitation at different scales (Fig. 9b). Similar  
 449 to what we have observed in the temporal evolution of the error energy spectra (Fig. 8), the  
 450 distinctions of error growth rates in moist and dry regions are greater at smaller scales than at  
 451 larger scales: the 0–6-h error growth rate in the moist region is 3.76 times of the dry region  
 452 growth rate at the small scales (< 30 km), while this ratio of moist-region versus dry-region error  
 453 growth rate is 2.77 at the medium scale (30–200 km), and it becomes almost comparable in these  
 454 two regions at the large scale (> 200 km). The “stall” of error growth at the first several hours at  
 455 the largest scales (Fig. 8) is also apparent in Fig. 9b that the large-scale RMDKE almost does not

456 grow at the beginning of the forecasts, unlike the other two scales. The small-scale RMDKE at 6-  
457 hour lead-time is smaller than the medium-scale RMDKE (Fig. 9b), because the fastest error  
458 growth scale moves beyond 30 km after about 2 h (Figs. 8e–h).

459 In short, ensemble forecasts show that the predictability of this rainfall event is intrinsically  
460 limited. Reducing initial error amplitudes will not lead to improved forecasts. No matter what the  
461 spatial scales and amplitudes the initial perturbations are, the error energy spectra have no  
462 difference after 6–8 h. This time scale is consistent with many other studies (e.g., Durran and  
463 Gingrich 2014, Durran and Weyn 2016). There is also an apparent upscale growth of errors with  
464 errors at smaller scales growing faster, and errors grow faster in regions where precipitation  
465 occurs than that in the no-precipitation region due to the dominant role of moist convective  
466 processes in mesoscale error growth, both consistent with the three-stage error growth  
467 conceptual model of Zhang et al. (2007). Additionally, it is shown for the first time in peer-  
468 reviewed literatures that the error growth at larger scales depends on the smaller-scale errors that  
469 larger-scale errors will not grow until smaller-scale errors have grown to an amplitude that is  
470 comparable to larger-scale errors, while error growth at smaller scales is independent of larger-  
471 scale errors. This suggests that the mechanism governing error growth of this event in our  
472 ensembles is primarily the upscale growth rather than the upamplitude growth.

473

#### 474 *4.3 Error growth features with respect to different structures*

475 Larger error growth is observed in the CNOP than the LARGE (it should be noted that in this  
476 section, “LARGE” refers to the simulation with perturbations derived from the LARGE  
477 ensemble in Section 4.1 and 4.2, rather than the LARGE ensemble itself) in the whole integration  
478 time from 0600 UTC 19 July to 1200 UTC 20 July (Fig.10a). The CNOP grows faster than the

479 LARGE at the first several hours, and the growth rate of the CNOP and the LARGE become  
480 similar after that (Fig.10b).

481 Similar vertical distributions are observed in CNOP and the 3 members of LARGE in the  
482 first several hours (Fig. 11c), while the horizontal distributions are greatly different from each  
483 other. The sensitive areas identified by CNOP are corresponding to the key synoptic weather  
484 systems, while those of LARGE aren't (Figs. 11a, b; Figs. 2c–e). Large vertically-integrated  
485 TME of CNOP is generally collocated with the hourly precipitation simulation at both the initial  
486 hours (Fig. 11d) and times after that (Fig. 11g). The large perturbation developments at different  
487 vertical levels are corresponding to the key synoptic weather systems that are associated with the  
488 rainfall at the whole integration time, which are the low-level water vapor convergence area, the  
489 mid-level low and the upper-level divergence area (Figs. 12a, c, e, g, i, k). However, at the first  
490 several hours in LARGE (~10 h), the large vertically-integrated TME is not quite consistent with  
491 the simulated hourly precipitation (Fig. 11e) and the large perturbation developments at different  
492 vertical levels are not associated with the key synoptic weather systems as good as in CNOP  
493 (Figs. 12b, f, j). After the first several hours, the LARGE development patterns become similar  
494 to the CNOP (Fig. 11h) and are better corresponding to the key synoptic weather systems  
495 mentioned above (Figs. 12d, h, l).

496 Faster error growth at smaller scales than that at larger scales is also observed in CNOP and  
497 LARGE in the first several hours (Fig. 13). However, different error growth features are found at  
498 larger scales in different variables for these two types of perturbations with different structures.  
499 While the characteristics of error growth of temperature in both CNOP and LARGE forecasts are  
500 similar to those observed in the previous subsection that larger scale errors stall when smaller  
501 scale errors grow for the first few hours (“upscale”; Figs. 13e–h), error growth of  $Q_v$  in both

502 CNOP and LARGE forecasts are more uniform (“upamplitude”; Figs. 13i, j), although smaller  
503 scale errors grow slightly faster than larger scale errors at the beginning (Figs. 13k, l). On the  
504 other hand, error growth of the U-wind component shows different behavior in the two forecasts:  
505 it is more upamplitude with the CNOP initial perturbations (Fig. 13a), while more upscale with  
506 the LARGE initial perturbations (Fig. 13b). This result suggests that the error growth of U is  
507 more sensitive to the structure of initial perturbation than those of temperature and Qv.

508 The reasons for the more upamplitude features are two folds related to the structure of the  
509 initial perturbation and the wavelength regime used for forecasts. On the one hand, the more  
510 upamplitude features in U and Qv of the CNOP may be contributed partly by the inherent faster  
511 error growth associated with the large-scale flow patterns that are well collocated with rainfall.  
512 On the other hand, the distribution of atmospheric kinetic energy with respect to wavelengths has  
513 already transitioned from a  $k^{-5/3}$  power law at the smallest scales of the CNOP and LARGE  
514 forecasts to a  $k^{-3}$  power law at the largest scales of these forecasts (e.g., Skamarock 2004), and  
515 Rotunno and Snyder (2008) and Durran and Gingrich (2014) show that error grows more upscale  
516 in the  $k^{-5/3}$  regime while more upamplitude in the  $k^{-3}$  regime. Additionally, Skamarock (2004)  
517 shows that forecasts with parameterized convection (like the CNOP and LARGE forecasts) are  
518 not able to build the  $k^{-5/3}$  energy spectrum and hinders error growth at smaller scales compared  
519 with forecasts with explicit convection, which may enhance the upamplitude tendency for the  
520 forecasts. Therefore, mixed behavior of both upscale and upamplitude error are observed in the  
521 three variables, with Qv showing the strongest upamplitude characteristics as the convective  
522 parameterization scheme directly impacts it while it is more of an indirect impact on the  
523 temperature and U-wind component through modified convective activities.

524 In short, CNOP has larger error growth at the whole integration time and a much faster  
525 growth rate at the first several hours than the LARGE. In addition, error growth tends to be more  
526 upamplitude in these coarse resolution forecast especially with the CNOP. The error grows at  
527 larger scales may be related to both the inherited feature of CNOP perturbation, the inability of  
528 the convective parameterization scheme to rebuild the  $k^{-5/3}$  atmospheric power spectra at the  
529 mesoscales, and different error growth characteristics in the  $k^{-5/3}$  and  $k^{-3}$  regimes.

530

## 531 **5. Concluding remarks**

532 This study explores the controlling factors of the uncertainties and error growth features with  
533 various initial scales, amplitudes, and structures in forecasting the high-impact extremely heavy  
534 rainfall event that occurred in Henan Province China on 17–22 July 2021. The most intense  
535 events happened during 19–20 July 2021, when the metropolitan area of Zhengzhou, the capital  
536 city of Henan Province, and the surrounding area received record-breaking hourly rainfall of  
537 201.9 mm and 24-hour accumulated rainfall of over 600 mm. In spite of warnings that were  
538 issued for several days prior to this event, large uncertainties exist in operational forecasts in the  
539 location and intensity of the highest accumulated rainfall of this event. A suite of analyses,  
540 including ensemble sensitivity analysis (ESA) using an ensemble of global models, conditional  
541 nonlinear optimal perturbation (CNOP) method using a coarse-resolution regional model, and  
542 ensemble simulations using a high-resolution convection-permitting regional model, is designed  
543 in this study.

544 Using the four models that most accurately predicted the rainfall amount and location of this  
545 event from the TIGGE ensemble, ESA reveals several dominating synoptic features that  
546 determine the forecast uncertainties of this event. The most significant contributor is found to be

547 the mid-to-lower low-pressure system directly over Henan Province. The upper-level deeper  
548 ridge and trough that are associated with a stronger jet stream are found to provide stronger  
549 upper-level divergence and hence stronger lifting and more favorable for heavy rainfall. In  
550 addition, the positions of the two tropical cyclones and the associated low-level jets are also  
551 important for the rainfall. Likely being associated with the southwestward extending of a ridge  
552 from the subtropical high, when Typhoon Cempaka is more located to the southeast or Typhoon  
553 In-Fa is more located to the south, the low-level jets are enhanced and so is the total amount of  
554 precipitation in Henan Province. Similar to the 24-h accumulated rainfall, the hourly extreme  
555 rainfall at 0800 UTC 20 July is also sensitive to the upper-level ridge, mid-level low, and low-  
556 level trough extending from the low pressure vortex as revealed by our CNOP analysis.

557 High-resolution convection-permitting ensemble forecasts with flow-dependent,  
558 unobservably small initial perturbations show that rainfall area is quite predictable but the  
559 predictability of this event is intrinsically limited in terms of the maximum values of 24-hour  
560 accumulated precipitation. Reducing initial perturbations by an order of magnitude will not lead  
561 to reduced forecast uncertainties, no matter the spatial scales of the initial perturbations are  
562 relatively large (from a global model) or small (from sub-grid-scale unresolved uncertainties).

563 The evolution of the energy spectra of the forecast errors is insensitive to the amplitudes or  
564 spatial scales or structures of the initial perturbations after 6 to 8 hours. The intrinsically limited  
565 predictability of convectively driven extreme rainfall events is widely recognized, and this  
566 insensitivity of forecast errors to the amplitudes and spatial scales in initial perturbations is  
567 aligned with previous studies of other extreme rainfall events under different synoptic regimes,  
568 including Mei-yu rainfall (Bei and Zhang 2007), warm-sector rainfall (Wu et al. 2020), frontal  
569 and pre-frontal rainfall (Weyn and Durran 2019), and organized convective systems (Nielsen and

570 Schumacher 2016, Weyn and Durran 2019). However, for the initial perturbation generated with  
571 GEFS or sub-grid-scale uncertainties, one outstanding discovery is the behavior of large-scale  
572 flow-dependent errors in the absence of the small-scale errors: to the knowledge of the authors,  
573 this is the first study that shows the inability of large-scale errors to grow until the amplitude of  
574 small-scale errors have increased to an adequate amplitude, confirming that errors of smaller  
575 scale grow faster than those of larger scale.

576 In addition, the error growth rate with respect to different spatial scales and time – despite  
577 whether large-scale or small-scale initial uncertainties are imposed – also shows an apparent  
578 transfer of the fastest growing scale from smaller to larger scales with a slower growth rate at  
579 larger scales. This result suggests that although upamplitude growth and upscale growth coexist,  
580 the dominant mechanism controlling the error growth is their upscale transfer, at least for the  
581 ensemble forecasts of this high-impact event examined in this study. Faster error growth is also  
582 observed in regions where precipitation occurs, suggesting the importance of moist convective  
583 processes in controlling the error growth of this event. Whether this behavior of large-scale flow-  
584 dependent errors holds true for other events and how sensitive this behavior is to different  
585 strengths of synoptic forcing remain unknown and deserve further studies.

586 The sensitivity of the error growth to different structures of initial perturbations was also  
587 examined with the distribution of atmospheric kinetic energy transitioning from the  $k^{-5/3}$  to  $k^{-3}$   
588 regimes. Results show that CNOP has larger error growth at the whole integration time and a  
589 much faster growth rate at the first several hours than the GEFS or sub-grid-scale perturbations.  
590 Different error growth features at larger scales are observed in different variables for the  
591 perturbations with different structures. CNOP pattern initial perturbations, whose error growth  
592 well corresponds to the rainfall associated key synoptic weather systems at the whole integration

593 time, show more upamplitude feature with an error growth at the initial hours at both smaller and  
594 larger scales for U-component and water vapor mixing ratio. However, the error growth feature  
595 of temperature is not quite sensitive to the structure of initial perturbations. The error growth at  
596 larger scales may be owing to the inherited feature of CNOP perturbation, the inability of the  
597 convective parameterization scheme to rebuild the  $k^{-5/3}$  power spectra at the mesoscales, and  
598 different error growth characteristics in the  $k^{-5/3}$  and  $k^{-3}$  regimes.

599 To conclude, this study suggests that the forecast uncertainties of the record-breaking  
600 extreme rainfall event that occurred in Henan Province China on 19–20 July 2021 are associated  
601 with many different factors across different spatial scales. Practically, because of incomplete  
602 knowledge of the atmosphere, model deficiencies, and imperfect data assimilation techniques,  
603 initial conditions of different models disagree in terms of their representations of the upper-level  
604 ridge and trough, the mid-level low-pressure system directly over Henan Province, as well as the  
605 low-level jet associated with the warm ridge of the subtropical high and the two distant typhoons  
606 to the southeast, which leads to diverse forecasts of the total accumulated rainfall. However,  
607 even we have a nearly perfect model with nearly perfect estimations of the atmospheric  
608 conditions, tiny, unobservable errors will grow upscale and, to a slightly lesser extent,  
609 upamplitude with the help of moist convective processes, and intrinsically prevents the accurate  
610 predictions of the location and strength of the accumulated rainfall in a deterministic sense.  
611 Although the universality of some of these conclusions needs to be further examined under  
612 different scenarios, they nonetheless highlight the importance of further developing advanced  
613 data assimilation techniques that can make better use of existing but underutilized observations,  
614 as well as the benefits of ensemble forecasts that consider uncertainties in initial conditions over

615 deterministic forecasts, in improving practical predictability of extreme weather events and  
616 providing more useful numerical weather predictions as forecast guidance in the future.

617

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628

## 629 **References**

630 Bei N, Zhang F. 2007. Impacts of initial condition errors on mesoscale predictability of heavy  
631 precipitation along the Mei-yu front of China. *Q. J. R. Meteor. Soc.*, 133: 83–99.

632 Birgin E G, Martínez J E, Raydan M. 2001. Algorithm 813: SPG—software for convex-  
633 constrained optimization. *ACM Transactions on Mathematical Software*, 27: 340–349.

634 Bougeault P., and Coauthors. 2010: The THORPEX Interactive Grand Global Ensemble. *Bull.*  
635 *Am. Meteorol. Soc.*, 91: 1059–1072.

636 Cressman G P. 1959. An operational objective analysis system. *Mon. Wea. Rev.*, 87: 367–374.

- 637 Durran D R, Gingrich M. 2014. Atmospheric predictability: Why butterflies are not of practical  
638 importance. *J. Atmos. Sci.*, 71: 2476–2488.
- 639 Durran D R, Weyn J A, 2016: Thunderstorms do not get butterflies. *Bull. Am. Meteorol. Soc.*, 97:  
640 237–243.
- 641 Ehrendorfer M, Errico R M, Raeder K D. 1999. Singular-vector perturbation growth in a  
642 primitive equation model with moist physics. *J. Atmos. Sci.*, 56: 1627–1648.
- 643 Grell G A, Dudhia J, Stauffer D R. 1995. A description of the fifth-generation Penn State/NCAR  
644 Mesoscale Model (MM5). NCAR Tech. Note NCAR/TN-398+STR, 121 pp.
- 645 Hakim G J, Torn R D. 2008. Ensemble synoptic analysis. *Synoptic–dynamic meteorology and  
646 weather analysis and forecasting: A tribute to Fred Sanders*, Meteor. Monogr., 55.
- 647 Hawblitzel D P, Zhang F, Meng Z, Davis C A. 2007. Probabilistic evaluation of the dynamics  
648 and predictability of the mesoscale convective vortex of 10–13 June 2003. *Mon. Wea. Rev.*,  
649 135: 1544–1563.
- 650 Hong S-Y, Noh Y, Dudhia J. 2006. A new vertical diffusion package with an explicit treatment  
651 of entrainment processes. *Mon. Wea. Rev.*, 134: 2318–2341.
- 652 Iacono M J, Delamere J S, Mlawer E J, Shephard M W, Clough S A, Collins W D. 2008.  
653 Radiative forcing by long-lived greenhouse gases: Calculations with the AER radiative  
654 transfer models. *J. Geophys. Res.*, 113: D13103.
- 655 Jimenez P, Dudhia F, Gonzalez-Ruoco J F, Navarro J, Montavez J P, Garcia-Bustamente E. 2012.  
656 A revised scheme for the WRF surface layer formulation. *Mon. Wea. Rev.*, 140: 898–918.
- 657 Judt F. 2018. Insights into atmospheric predictability through global convection-permitting  
658 model simulations. *J. Atmos. Sci.*, 75: 1477–1497.

- 659 Judt F. 2020. Atmospheric predictability of the tropics, middle latitudes, and polar regions  
660 explored through global storm-resolving simulations. *J. Atmos. Sci.*, 77: 257–276.
- 661 Liang Y, Qiao C, Dong J. 2020. Spatial-temporal distribution and impact analysis of the first  
662 rainstorm in Henan Province of the recent 34 years. *Meteorol. Environ. Sci. (in Chinese)*, 43:  
663 26–32.
- 664 Lorenz E N. 1963. Deterministic nonperiodic flow. *J. Atmos. Sci.*, 20: 130–141.
- 665 Lorenz E N, 1969. The predictability of a flow which possesses many scales of motion. *Tellus*,  
666 21A: 289–307.
- 667 Lorenz E N, 1982. Atmospheric predictability experiments with a large numerical model. *Tellus*,  
668 34A: 505–513.
- 669 Lorenz E N, 1996. Predictability – A problem partly solved. In: *Proceedings of Seminar on*  
670 *Predictability*. Reading, United Kingdom, ECMWF, 1–18.
- 671 Lynch S L, Schumacher R S. 2014. Ensemble-based analysis of the May 2010 extreme rainfall in  
672 Tennessee and Kentucky. *Mon. Wea. Rev.*, 142: 222–239.
- 673 Melhauser C, Zhang F. 2012. Practical and intrinsic predictability of severe and convective  
674 weather at the mesoscales. *J. Atmos. Sci.*, 69: 3350–3371.
- 675 Mu M, Duan W. 2003: A new approach to studying ENSO predictability: Conditional nonlinear  
676 optimal perturbation. *Chinese Sci. Bull.*, 48: 1045–1047.
- 677 Mu M, Xu H, Duan W. 2007. A kind of initial errors related to “spring predictability barrier” for  
678 El Nino events in Zebiak-Cane model. *Geophys. Res. Lett.*, 34: L03709.
- 679 Mu M, Wang H, Zhou F. 2007. A preliminary application of conditional nonlinear optimal  
680 perturbation to adaptive observation. *Chinese J. Atmos. Sci. (in Chinese)*, 31: 1102–1112.

- 681 Nielsen E R, Schumacher R S. 2016. Using convection-allowing ensembles to understand the  
682 predictability of an extreme rainfall event. *Mon. Wea. Rev.*, 144: 3651–3676.
- 683 Ran L, Li S, Zhou Y, Yang S, Ma S, Zhou K, Shen D, Jiao B, Li N. 2021. Observational analysis  
684 of the dynamic, thermal, and water vapor characteristics of the “7.20” extreme rainstorm  
685 event in Henan Province. *Chin. J. Atmos. Sci. (in Chinese)*, 45: 1366–1383.
- 686 Rotunno R, Snyder C. 2008. A generalization of Lorenz’s model for the predictability of flows  
687 with many scales of motion. *J. Atmos. Sci.*, 65: 1063–1076.
- 688 Selz T. 2019. Estimating the intrinsic limit of predictability using a stochastic convection scheme.  
689 *J. Atmos. Sci.*, 76: 754–765.
- 690 Selz T, Craig G. 2015. Upscale error growth in a high-resolution simulation of a summertime  
691 weather event over Europe. *Mon. Wea. Rev.*, 143: 813–827.
- 692 Shi W, Li X, Zeng M, Zhang B, Wang H, Zhu K, Zhuge X. 2021. Multi-model comparison and  
693 high-resolution regional model forecast analysis for the “7·20” Zhengzhou severe heavy rain.  
694 *Trans. Atmos. Sci. (in Chinese)*, 44: 688–702.
- 695 Skamarock W C. 2004. Evaluating mesoscale NWP models using kinetic energy spectra. *Mon.*  
696 *Wea. Rev.*, 132: 3019–3032.
- 697 Skamarock W C, and Coauthors. 2021. A Description of the Advanced Research WRF Model  
698 Version 4.3. NCAR Technical Note NCAR/TN-556+STR.
- 699 Sun Y Q, Zhang F. 2016. Intrinsic versus practical limits of atmospheric predictability and the  
700 significance of the butterfly effect. *J. Atmos. Sci.*, 73: 1419–1438.
- 701 Sun Y Q, Zhang F. 2020. A new theoretical framework for understanding multiscale atmospheric  
702 predictability. *J. Atmos. Sci.*, 77: 2297–2309.

- 703 Sun Y Q, Rotunno R, Zhang F. 2017. Contribution of moist convection and internal gravity  
704 waves to building the atmospheric  $-5/3$  kinetic energy spectra. *J. Atmos. Sci.*, 74: 185–201.
- 705 Sun Y, Xiao H, Yang H, Ding J, Fu D, Guo X, Feng L. 2021. Analysis of dynamic conditions  
706 and hydrometeor transport of Zhengzhou super heavy rainfall event on 20 July 2021 based on  
707 optical flow field of remote sensing data. *Chin. J. Atmos. Sci. (in Chinese)*, 45: 1384–1399.
- 708 Thompson G, Eidhammer T. 2014. A study of aerosol impacts on clouds and precipitation  
709 development in a large winter cyclone. *J. Atmos. Sci.*, 71: 3636–3658.
- 710 Weyn J A, Durran D R. 2017. The dependence of the predictability of mesoscale convective  
711 systems on the horizontal scale and amplitude of initial errors in idealized simulations. *J.*  
712 *Atmos. Sci.*, 74: 2191–2210.
- 713 Weyn J A, Durran D R. 2019. The scale dependence of initial-condition sensitivities in  
714 simulations of convective systems over the southeastern United States. *Q. J. R. Meteorol.*  
715 *Soc.*, 145: 57–74.
- 716 Wilks D S. 1995. *Statistical methods in the atmospheric sciences: An introduction* (p. 467). San  
717 Diego, CA: Academic Press.
- 718 Wu N, Zhuang X, Min J, Meng Z. 2020. Practical and intrinsic predictability of a warm-sector  
719 torrential rainfall event in the south China monsoon region. *J. Geophys. Res.*, 125:  
720 e2019JD031313.
- 721 Yu H, Meng Z. 2016. Key synoptic-scale features influencing the high-impact heavy rainfall in  
722 Beijing, China on 21 July 2012. *Tellus*, 68A: 31045.
- 723 Yu H, Meng Z. 2022: The impact of moist physics on the sensitive area identification for heavy  
724 rainfall associated weather systems. *Adv. Atmos. Sci.*, 39(5),684-696.

- 725 Zhang C, Wang Y. 2017. Projected future changes of tropical cyclone activity over the western  
726 North and South Pacific in a 20-km-mesh regional climate model. *J. Climate*, 30: 5923–5941.
- 727 Zhang F, Snyder C, Rotunno R. 2002. Mesoscale predictability of the “surprise” snowstorm of  
728 24–25 January 2000. *Mon. Wea. Rev.*, 130: 1617–1632.
- 729 Zhang F, Bei N, Rotunno R, Snyder C, Epifanio C. 2007. Mesoscale predictability of moist  
730 baroclinic waves: Convection-permitting experiments and multistage error growth dynamics.  
731 *J. Atmos. Sci.*, 64: 3579–3594.
- 732 Zhang F, Sun Y Q, Magnusson L, Buizza R, Lin S-J, Chen J-H, Emanuel K. 2019. What is the  
733 predictability limit of midlatitude weather? *J. Atmos. Sci.*, 76: 1177–1091.
- 734 Zhang M, Meng Z. 2018. Impact of synoptic-scale factors on rainfall forecast in different stages  
735 of a persistent heavy rainfall event in south China. *J. Geophys. Res.*, 123: 3574–3593.
- 736 Zhang X. 2021. Impacts of different perturbation methods on multiscale interactions between  
737 multisource perturbations for convection-permitting ensemble forecasting during SCMREX.  
738 *Quart. J. Roy. Meteor. Soc.*, 147: 3899–3921.
- 739 Zhang X, Yang H, Wang X, Shen L, Wang D, Li H. 2021. Analysis on characteristic and  
740 abnormality of atmospheric circulations of the July 2021 extreme precipitation in Henan.  
741 *Trans. Atmos. Sci. (in Chinese)*, 44: 672–687.
- 742 Zhang Y, Zhang F, Stensrud D J, Meng Z. 2016. Intrinsic predictability of the 20 May 2013  
743 tornadic thunderstorm event in Oklahoma at storm scales. *Mon. Wea. Rev.*, 144: 1273–1298.
- 744 Zhuang Y, Xing A. 2021. History must not repeat itself—urban geological safety assessment is  
745 essential. *Nat. Hazards*, <https://doi.org/10.1007/s11069-021-05111-y>.
- 746 Zou, X L, Vandenberghe F, Ponca M, Kuo Y-H. 1997. Introduction to adjoint techniques and  
747 the MM5 adjoint modeling system. NCAR Tech. Note NCAR/TN2435+STR, 110 pp.

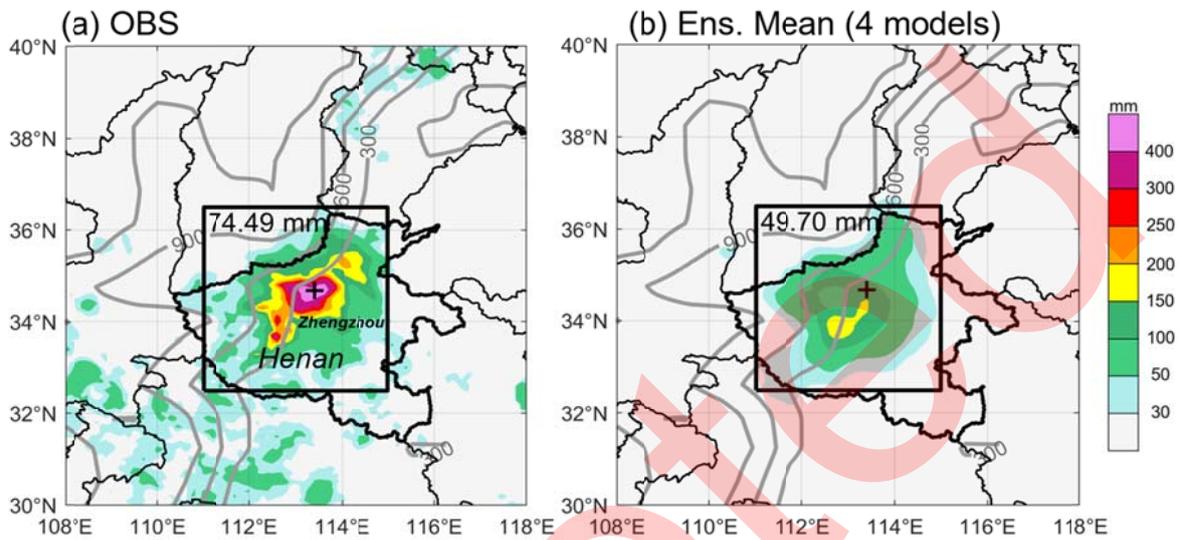
748 **Tables**

749 **Table 1.** Descriptions of TIGGE models used in the study. More details about the TIGGE models can be  
750 found at <https://confluence.ecmwf.int/display/TIGGE/Models>. The models in italics are used by ESA.  
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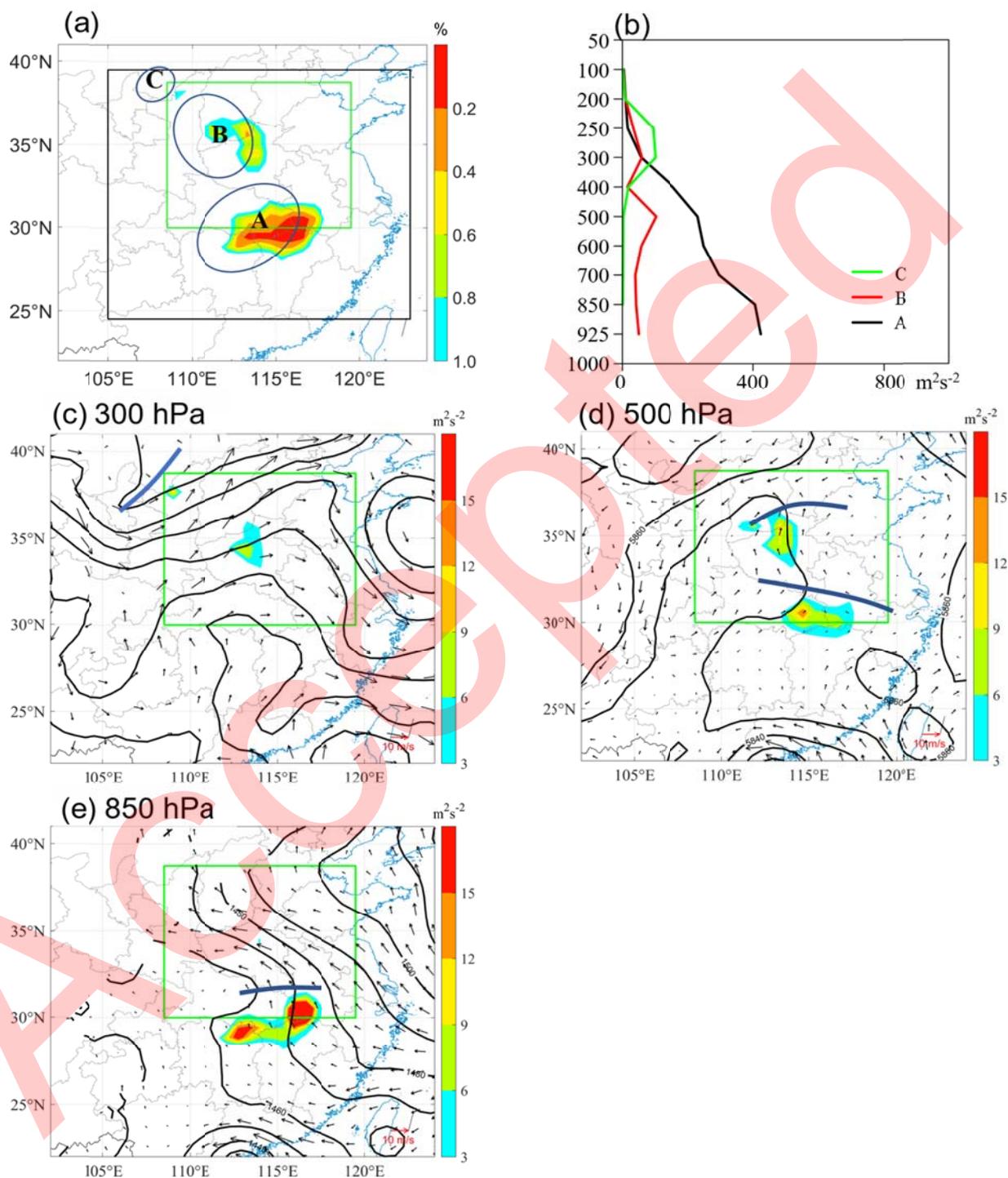
	<b>Model</b>	<b>Original Resolution (km)</b>	<b>Ensemble Size</b>
1	<i>BoM</i>	<i>30-45</i>	<i>17</i>
2	CMA	50	30
3	DWD	40	40
4	ECCC	39	21
5	ECMWF	16/32 (after day 10)	51
6	IMD	12	21
7	JMA	139	51
8	<i>KMA</i>	<i>33</i>	<i>25</i>
9	Météo France	7.5-37	35
10	NCEP	25	31
11	<i>NCMRWF</i>	<i>13</i>	<i>12</i>
12	<i>UKMO</i>	<i>21</i>	<i>18</i>

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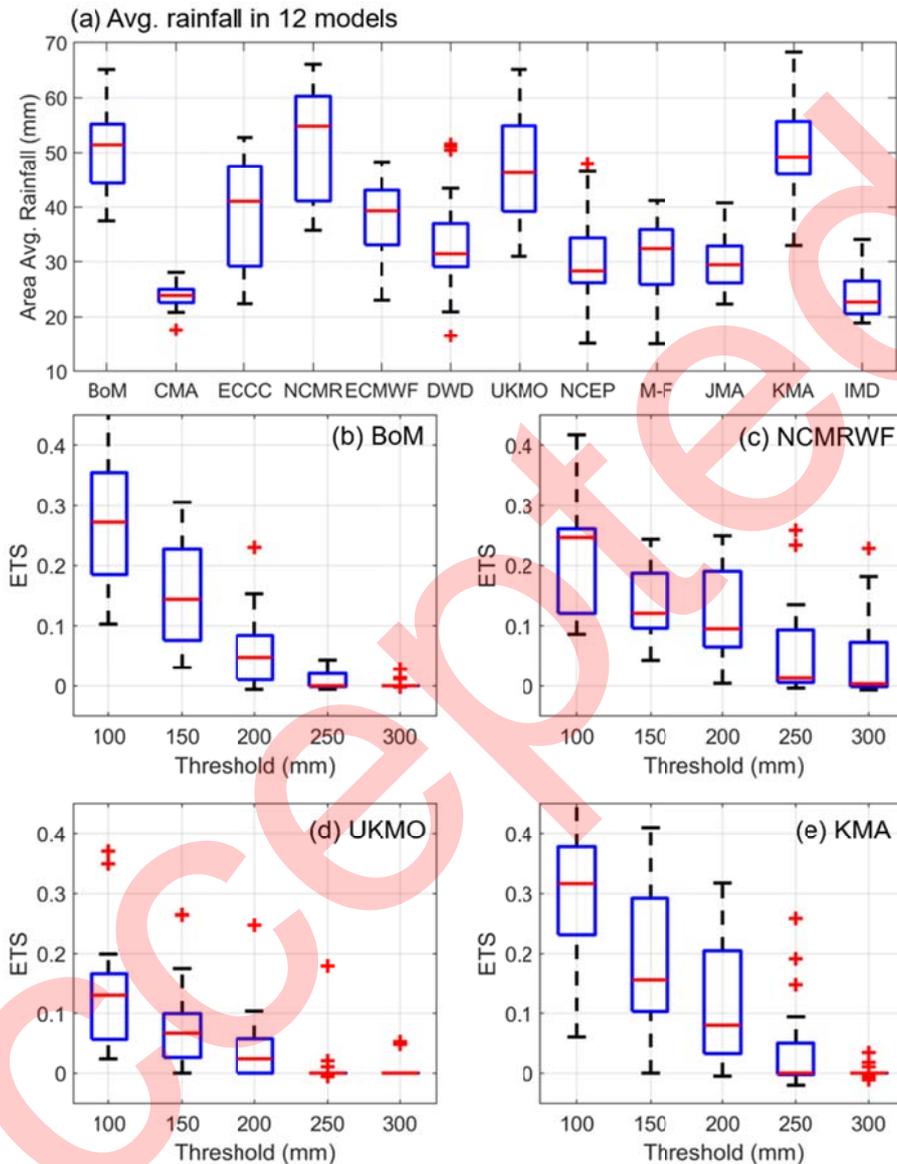
754 **Figures**



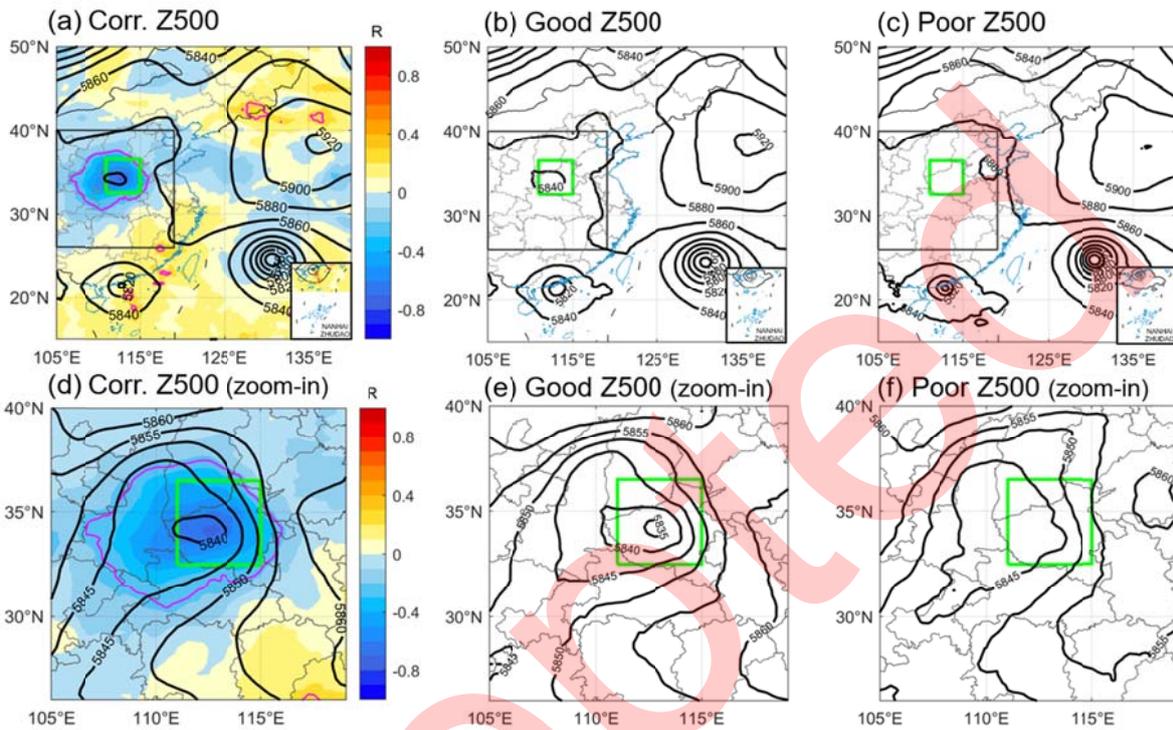
755  
756 **Figure 1.** Rainfall distribution in observation and the mean of ensemble forecasts. (a) Observed 24-h  
757 accumulated rainfall from 1200 UTC 19 July to 1200 UTC 20 July 2021 (shading; units: mm) and terrain  
758 height (grey contour; units: m). (b) Same as (a) but for ensemble mean rainfall forecast initialized at 0000  
759 UTC 19 July based on 4 best-performed models (BoM, NCMR, UKMO, and KMA; see text for details).  
760 The black box denotes the focused region, and the area-averaged 24-h accumulated rainfall is given on  
761 the top left of the box. The location of Zhengzhou City is marked as black cross and Henan Province is  
762 outlined in solid black.



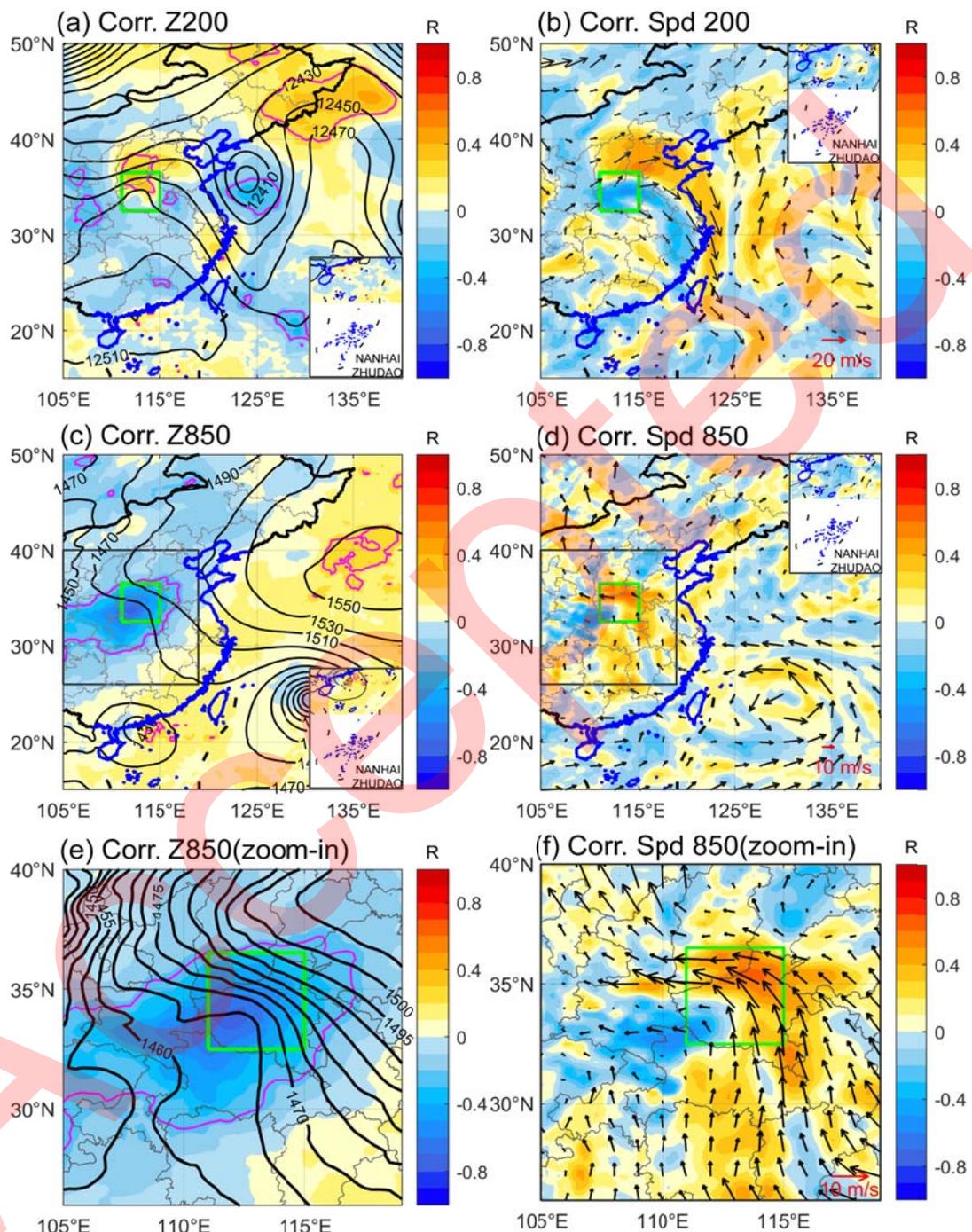
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 764 **Figure 2.** (a) Sensitive areas (shading) identified by the CNOP and (b) the vertical distribution of hTME  
 765 (units:  $m^2 s^{-2}$ ) over sensitive area A (black line), B (red line) and C (green line) in panel (a). Wind (vector;  
 766 units:  $m s^{-1}$ ), Z (contour; units: gpm), and TME (shading; units:  $m^2 s^{-2}$ ) of CNOP at (c) 300 hPa, (d) 500  
 767 hPa, and (e) 850 hPa. The blue line in (c) denotes the trough extending from the cold vortex, and the blue  
 768 line in (d) and (e) denotes the shear line from the low vortex. The inner green square box in (a, c, d, e)  
 769 denotes the verification area, and the inner black box in (a) denotes the area for the scale analysis.



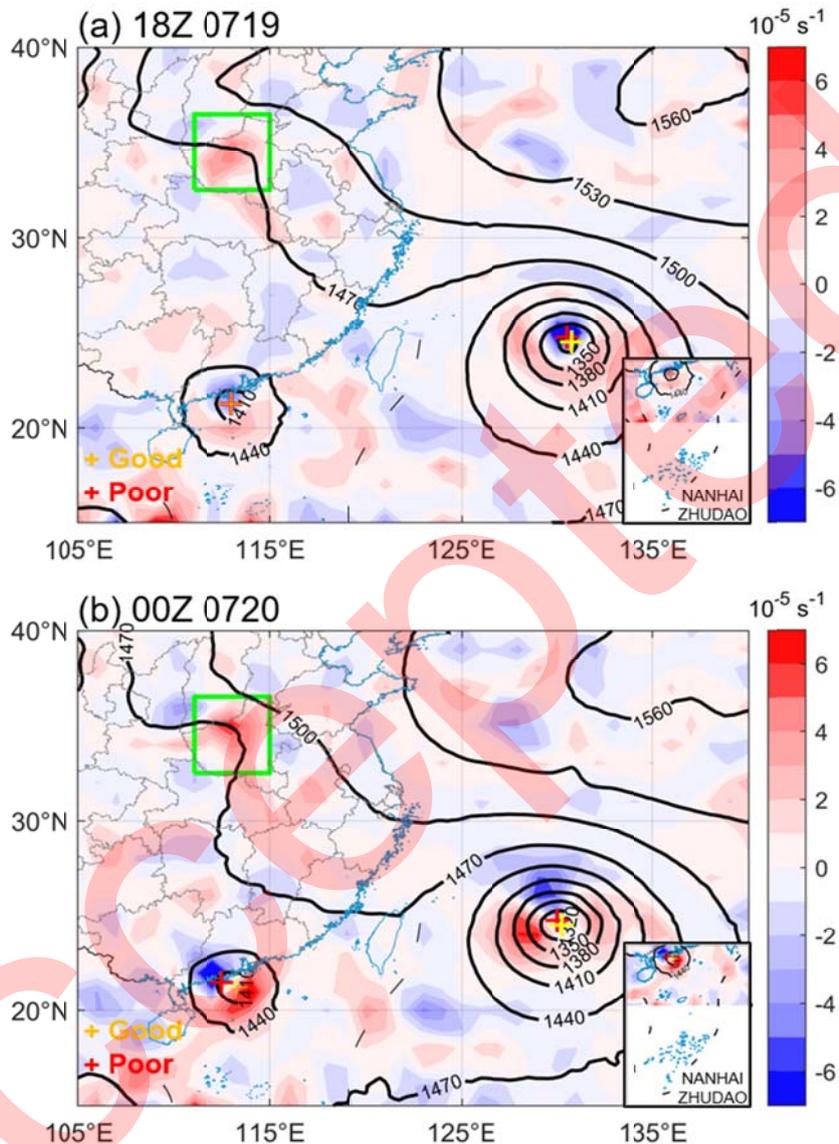
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 771 **Figure 3.** Quantitative precipitation evaluation of ensemble forecasts. (a) Boxplot of area-averaged 24-h  
 772 accumulated rainfall of ensemble members from the 12 TIGGE models. Boxplot of ETSs of ensemble  
 773 members from (b) BoM, (c) NCMRWF, (d) UKMO, and (e) KMA models at different thresholds from  
 774 100 mm to 300 mm over the focused region. The whiskers extend to the most extreme data points that are  
 775 not considered outliers. Points are identified as outliers if they are larger than  $q_3 + 1.5(q_3 - q_1)$  or smaller  
 776 than  $q_1 - 1.5(q_3 - q_1)$ , where  $q_1$  and  $q_3$  are the 25th and 75th percentiles.



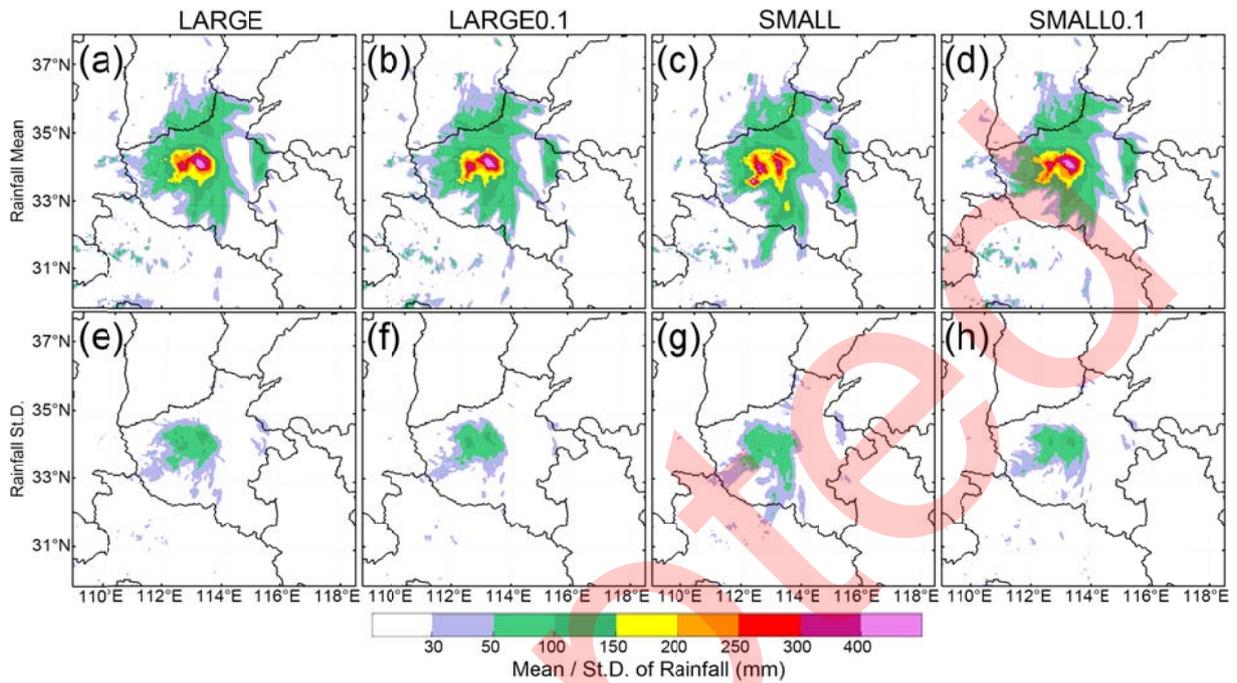
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 778 **Figure 4.** (a) Correlation coefficients (shading; magenta contours for 95% confidence) between the 500-  
 779 hPa Z at 1800 UTC 19 July and the area-averaged 24-h accumulated rainfall, with ensemble mean 500-  
 780 hPa Z at respective time contoured in bold black (in gpm); the green box denotes the focused region same  
 781 as the black one in Fig. 1; the grey box denotes the spatial range of (d)–(f). (b) Composite of good  
 782 members on 500-hPa Z at 1800 UTC 19 July contoured in black (in gpm). (c) is the same as (b) but for  
 783 composite of poor members. (d)–(f) are the same as (a)–(c) but zoomed in over the grey box given in (a)–  
 784 (c).



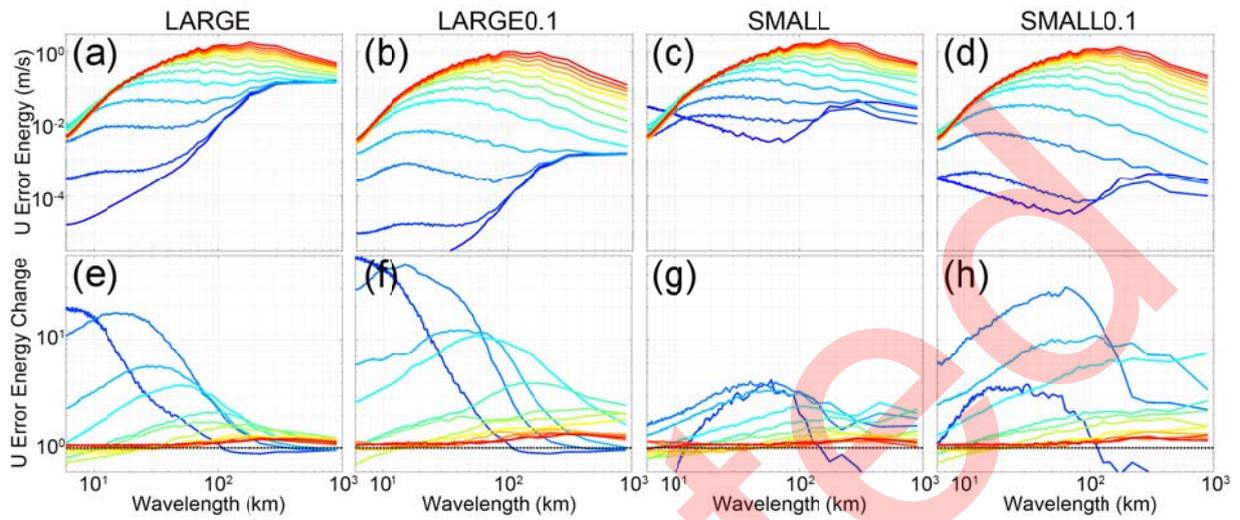
785  
 786 **Figure 5.** (a) Correlation coefficients (shading; magenta contours for 95% confidence) between the 200-  
 787 hPa Z at 1800 UTC 19 July and the area-averaged 24-h accumulated rainfall, with ensemble mean 200-  
 788 hPa Z at respective time contoured in bold black (in gpm). (b) Correlation coefficients (shading) between  
 789 the 200-hPa horizontal wind speed at 1800 UTC 19 July and the area-averaged 24-h accumulated rainfall,  
 790 with ensemble mean 200-hPa horizontal wind vector at respective time (reference vector given in bottom  
 791 right). (c) and (d) are the same as (a) and (b) but for 850 hPa, where the grey box denotes the spatial range  
 792 of (e) and (f). (e) and (f) are the same as (c) and (d) but zoomed in over the grey box given in (c) and (d).



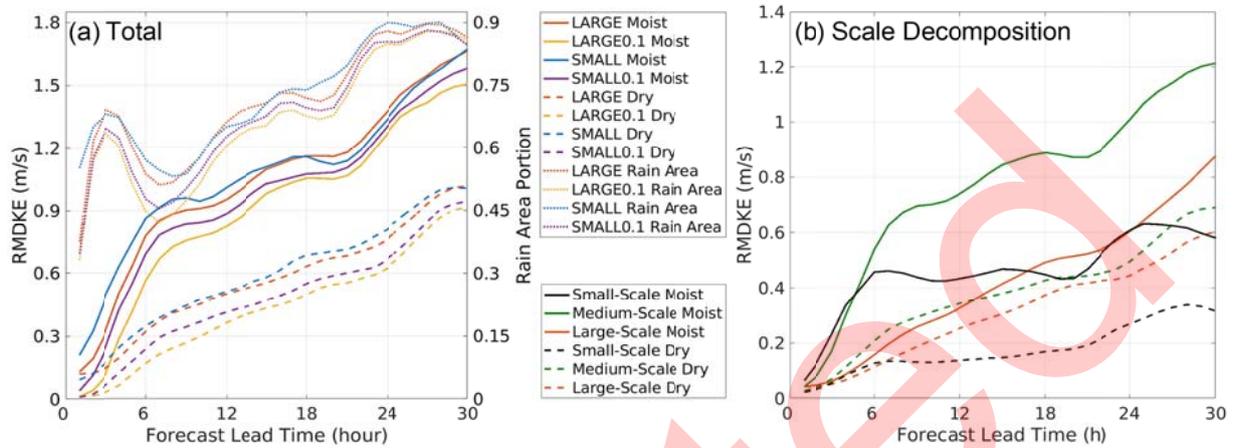
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 794 **Figure 6.** (a) Differences of 850-hPa vertical relative vorticity between composite of good members and  
 795 poor members (Good – Poor) on 1800 UTC 19 July; ensemble mean 850-hPa Z is contoured in black (in  
 796 gpm); locations of tropical cyclones (identified based on the minimum of 850-hPa Z) in the composite of  
 797 good (poor) members were denoted by yellow (red) crosses. (b) is the same as (a) but for 0000 UTC 20  
 798 July. The green box indicates the focused region same as the black one in Fig. 1.



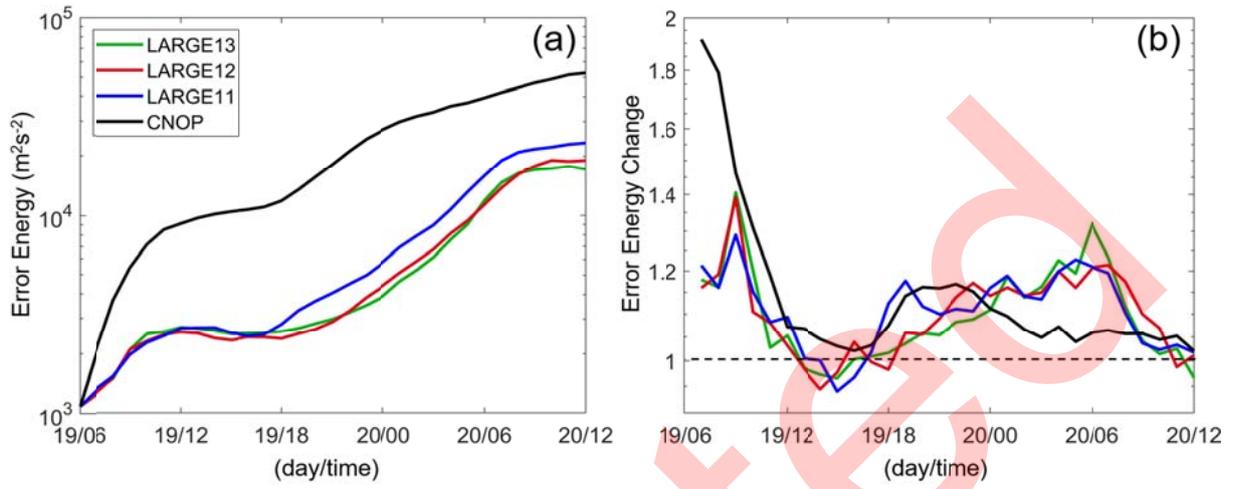
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 800 **Figure 7.** (upper panels) ensemble mean and (lower panels) ensemble standard deviation of 24-hour  
 801 accumulated rainfall from 1200 UTC 19 July 2021 to 1200 UTC 20 July 2021 for the (first column)  
 802 LARGE, (second column) LARGE0.1, (third column) SMALL, and (fourth column) SMALL0.1  
 803 ensemble forecasts.



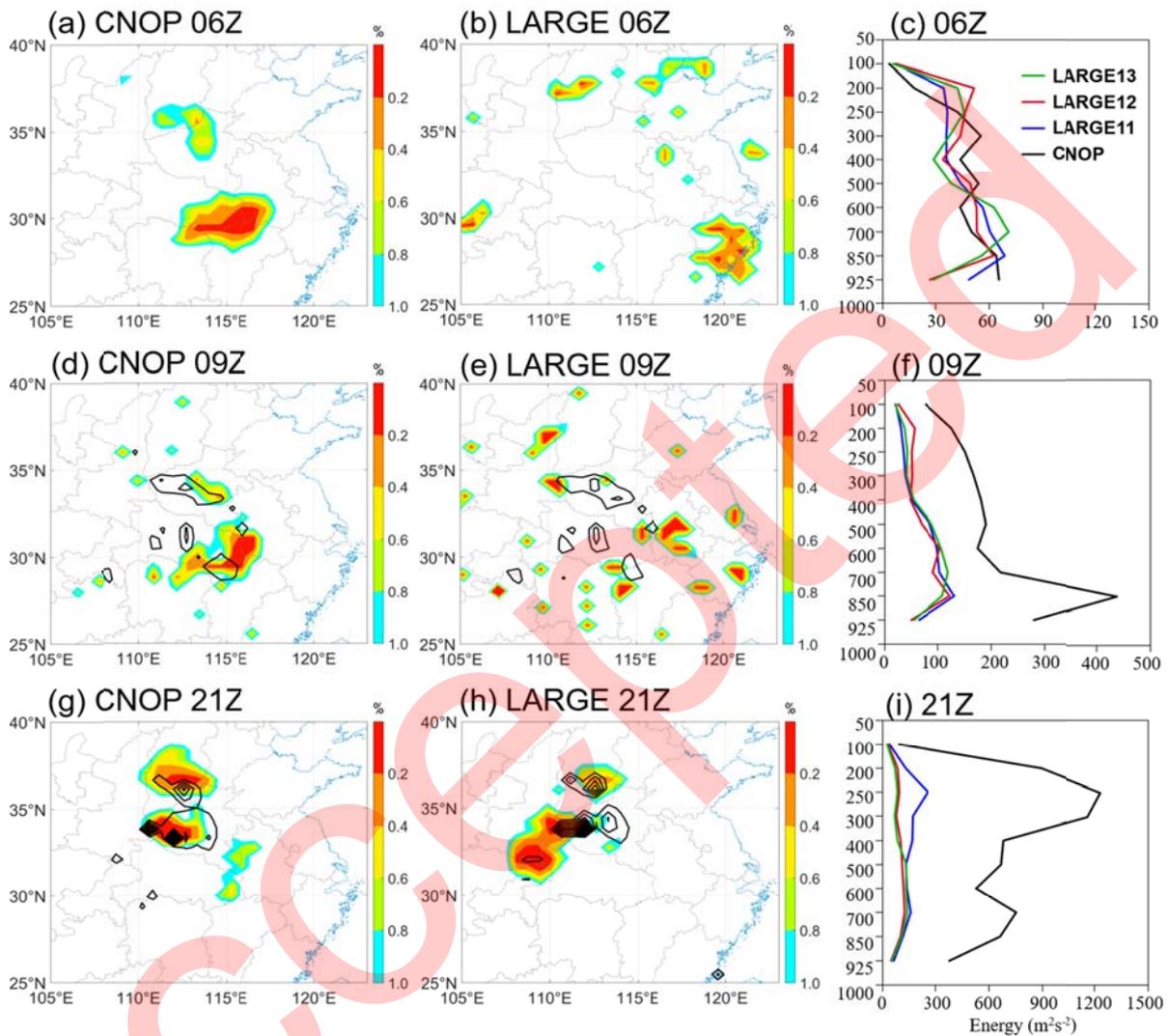
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 805 **Figure 8.** (upper panels) Hourly ensemble-mean power spectra and (lower panels) hourly growth of the  
 806 ensemble-mean power spectra of the U-wind perturbations from 0600 UTC to 1800 UTC of 19 July 2021  
 807 (0 to 12 hours of the ensemble simulation) for the (first column) LARGE, (second column) LARGE0.1,  
 808 (third column) SMALL, and (fourth column) SMALL0.1 ensemble forecasts. Blue colors denote earlier  
 809 times (shorter simulation lengths) and red colors denote later times (longer simulation lengths).



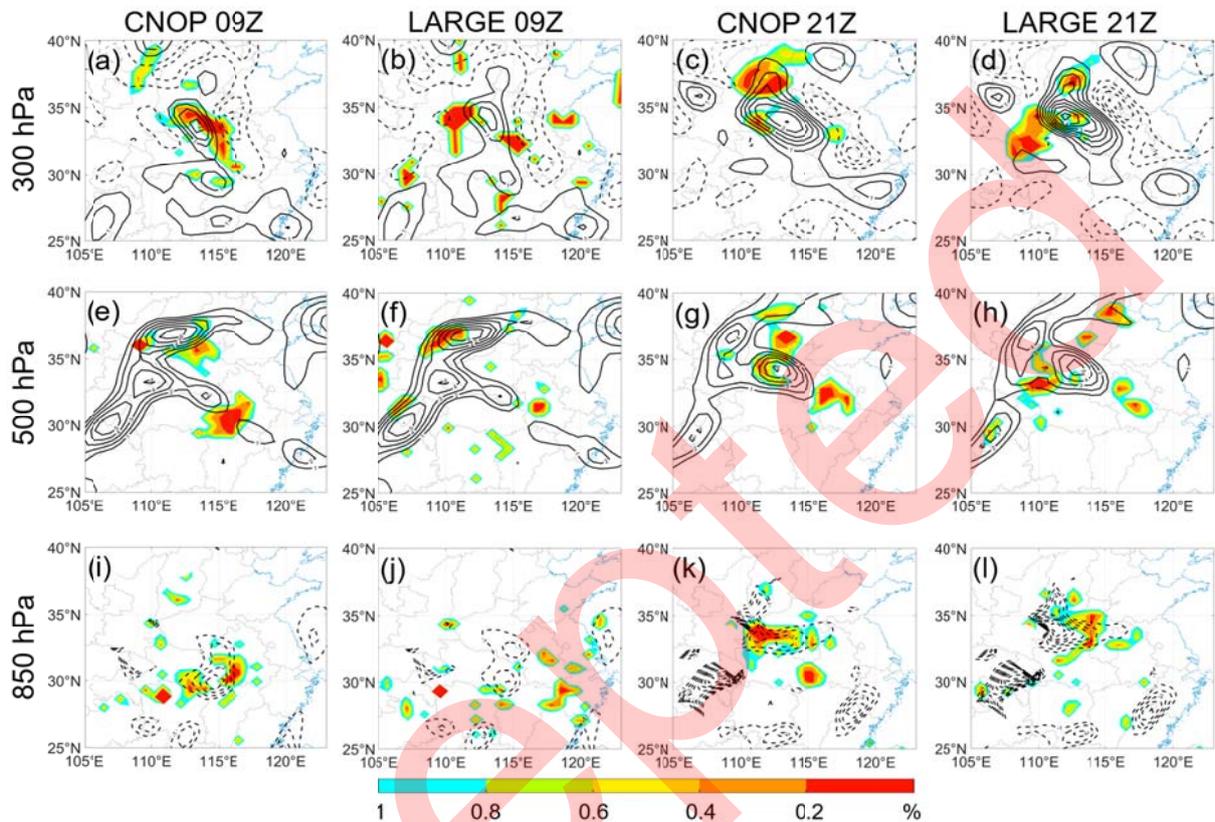
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 811 **Figure 9.** RMDKE of (a) the original model forecast fields, and (b) the model forecast fields decomposed  
 812 into small scales (< 30 km), medium scales (30–200 km), and large scales (> 200 km) for regions with  
 813 (solid lines) and without (dashed lines) precipitation. The portion of the model domain that is covered by  
 814 precipitation is also plotted in (a) as the dotted lines. The lines in (b) are the averages of respective, scale-  
 815 decomposed RMDKE values of the four ensembles (i.e., LARGE, LARGE0.1, SMALL, and SMALL0.1).



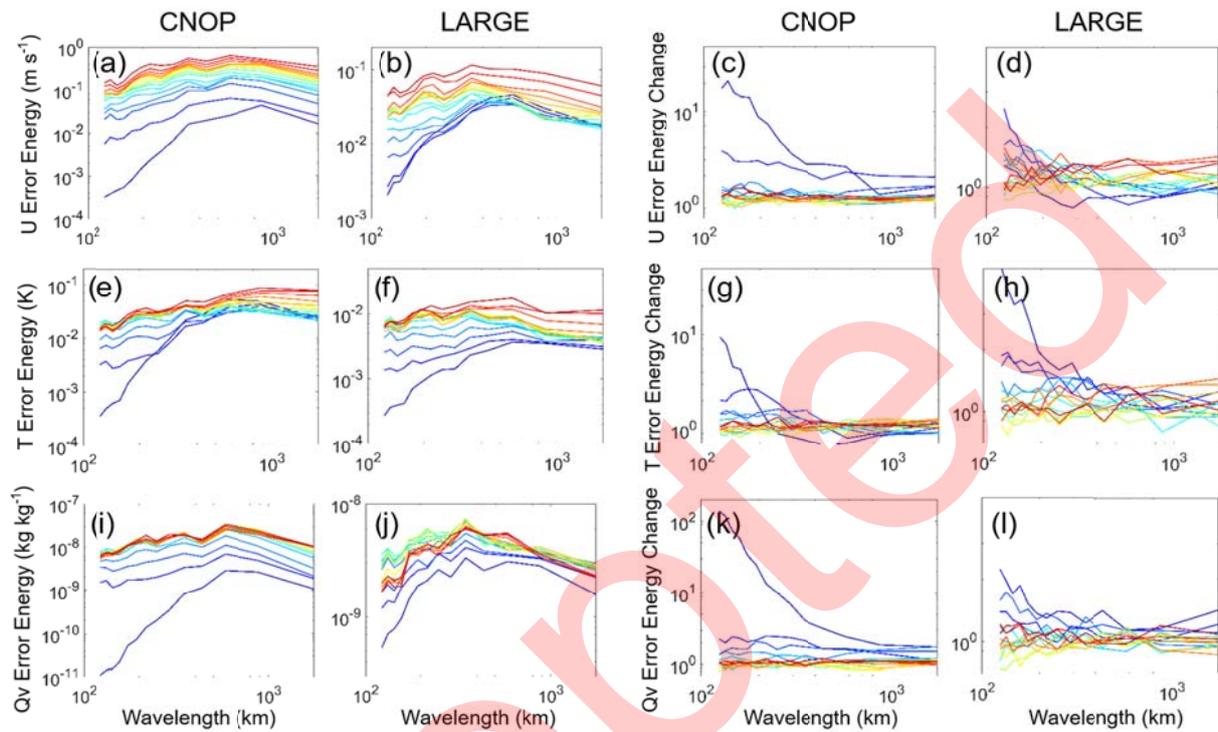
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 817 **Figure 10.** The (a) TME (units:  $\text{m}^2 \text{s}^{-2}$ ) and (b) hourly growth of the TME over the area interested (inner  
 818 black box in Fig.2a) for the CNOP (black line) and LARGE (red, blue and green lines).



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 820 **Figure 11.** The development of the CNOP and LARGE at (upper panels) 0600 UTC, (middle panels)  
 821 0900 UTC, and (bottom panels) 2100 UTC 19 July 2021. The vertical integrated TME (shading, the top 1%  
 822 energy) for (first column) CNOP and (second column) LARGE\_11, and (third column) the vertical  
 823 distribution of the hTME (units:  $\text{m}^2 \text{s}^{-2}$ ) for CNOP and LARGE are shown. The simulated 1-h  
 824 precipitation (contour, every 2 mm) at the corresponding time are also shown in (d, e, g, h).



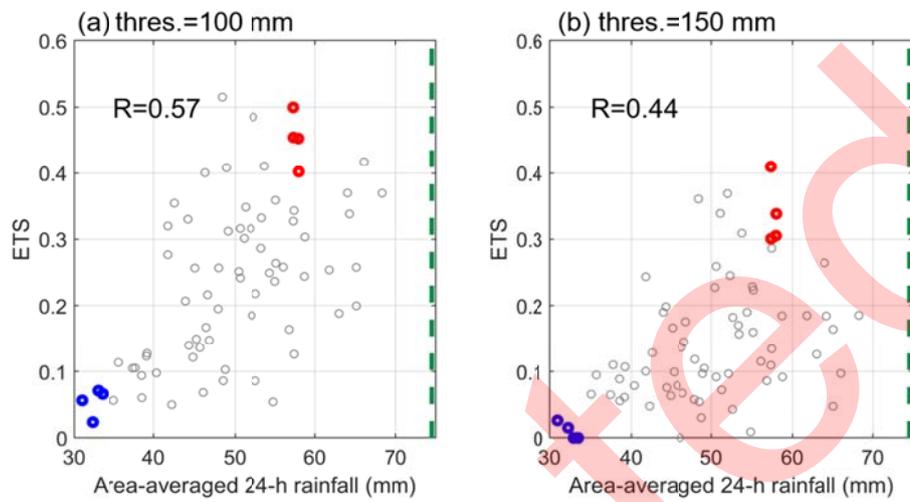
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 826 **Figure 12.** The development of the (first and third columns) CNOP and (second and fourth columns)  
 827 LARGE\_11 at (first and second columns) 0900 UTC and (third and fourth columns) 2100 UTC 19 July  
 828 2021. The TME (shading, the top 1% energy) at (upper panels) 300 hPa, (middle panels) 500 hPa, and  
 829 (bottom panels) 850 hPa. Also shown are (upper panels) the horizontal wind divergence (contour, units:  
 830  $10^{-5} \text{ s}^{-1}$ , solid line denotes divergence and dotted line denotes convergence) at 300 hPa, (middle panels)  
 831 the vertical relative vorticity (contour, units:  $10^{-5} \text{ s}^{-1}$ ) at 500 hPa, and (bottom panels) the water vapor  
 832 flux divergence (contour, units:  $10^{-4} \text{ kg}/(\text{kg}\cdot\text{s})$ ) at 850 hPa for the (first and third columns) CNOP and  
 833 (second and fourth columns) LARGE\_11. The solid contour line denotes the divergence area at 300 hPa  
 834 and positive vorticity at 500 hPa, and the dotted line denotes the convergence area at 850 hPa.



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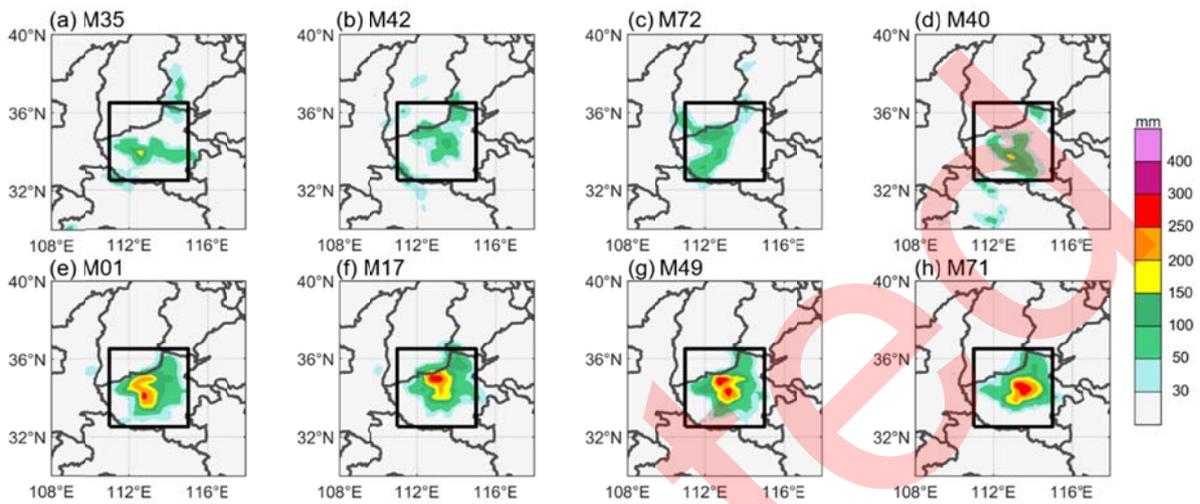
**Figure 13.** (first and second columns) Hourly power spectra and (third and fourth columns) hourly growth of the power spectra of (upper panels) the U-wind perturbations, (middle panels) the T perturbations, and (bottom panels) the Qv perturbations from 0600 UTC to 2100 UTC 19 July 2021 (0 to 15 hours of the simulation) for the (first and third column) CNOP, (second and fourth column) LARGE\_11. Blue colors denote earlier times (shorter simulation lengths) and red colors denote later times (longer simulation lengths).

843 **Supplements**



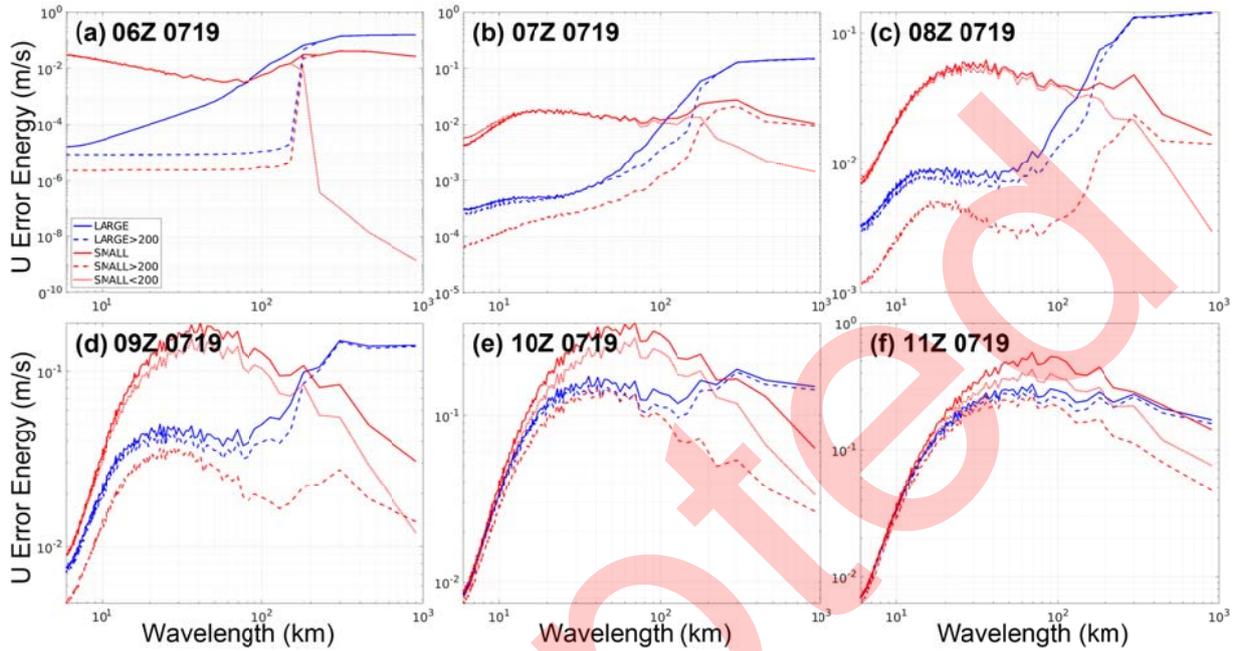
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845 **Figure S1.** Scatter plots of 24-h area-averaged rainfall versus ETSs for the thresholds of (a) 100  
846 mm and (b) 150 mm. Good (poor) members are plotted in red (blue) circles, and other members  
847 are plotted in grey circles. The dashed green line represents the observed 24-h area-averaged  
848 rainfall. Correlation coefficient between ETSs and area-averaged rainfall are given in the upper  
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**Figure S2.** 24-h accumulated rainfall distribution of poor members (a–d) and good members (e–h) (shading; mm) with member number given in the panel label. The black box denotes the focused region in Fig. 1a.



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**Figure S3.** Hourly evolutions of power spectra of the U-wind perturbations from 0600 UTC to 1100 UTC 19 July 2021 for the original LARGE and SMALL ensembles and three sensitivity experiments: “LARGE>200” that removes all initial perturbations of horizontal scales smaller than 200 km, “SMALL>200” that removes all initial perturbations of horizontal scales smaller than 200 km, and “SMALL<200” that removes all initial perturbations of horizontal scales larger than 200 km.