FRONTIERS IN PHYSICS

COLLIDER PHYSICS

UPDATED EDITION

ABP

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QCD and Collider Physics

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CAMBRIDGE MONOGRAPHS
ON PARTICLE PHYSICS, NUCLEAR PHYSICS
AND COSMOLOGY

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Introduction to Collider Physics (II)

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Living in fewer/more dimensions

Experimentalists | **Phenomenologists** | **Theorists**

d=1+2 | d=1+3 | d=1+9?

Our notation:

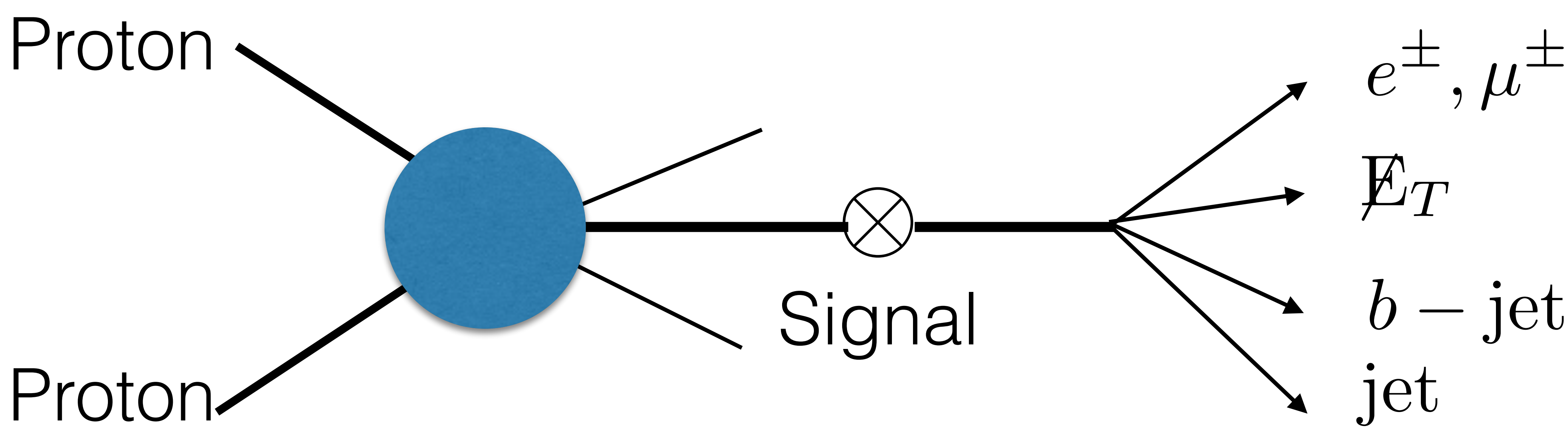
p^α P^μ P^ω

- How do we go from 1+3 to 1+2 dimensions?

The KEY of Collider Physics

Goal: seeing the signal on top of huge backgrounds

$$\frac{\text{Signal}}{\text{Background}}$$



1) Choose signal channels which have large cross-sections
2) Choose the decay mode with a clean collider signature

A Simple Demonstration of Collider Simulation

PHYSICAL REVIEW D **69**, 075008 (2004)

Associated production of *CP*-odd and charged Higgs bosons at hadron colliders

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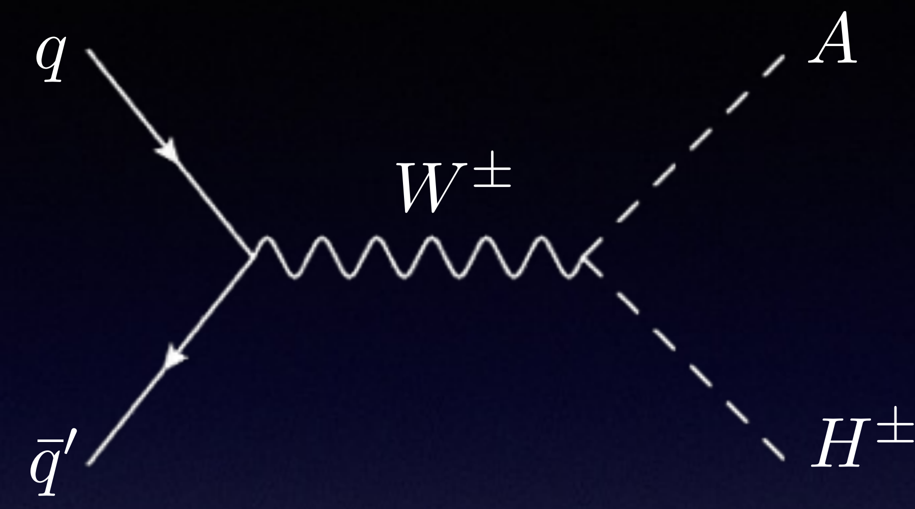
I want to show you how phenomenologists think

The Goal

- *AH*⁺ associate production
  - ➡ A long ignored channel in "*Higgs Hunter's Guide*"
  - ➡ A demo for collider simulations



Collider Phenomenology ≠ Mad-suite (MadGraph/Event/Analysis)

You **MUST** fully understand your results.
You cannot simply say "*I got them from Mad-...*"

Motivation

Physics predictions depend strongly on the details of SUSY parameters.

A typical SUSY phenomenology study depends on at least two SUSY parameters, e.g. $\tan\beta$ and m_A, and various physics reach depends on other SUSY parameters as well.

Very often, the physics reach of a process is expressed in terms of bounds on

$$\sigma(\text{production}) \times \text{Br}(\text{decay branching ratio})$$

where both σ and Br depend on SUSY parameters

In general detection efficiency also depends on SUSY parameters.

Our task is to find a SUSY process

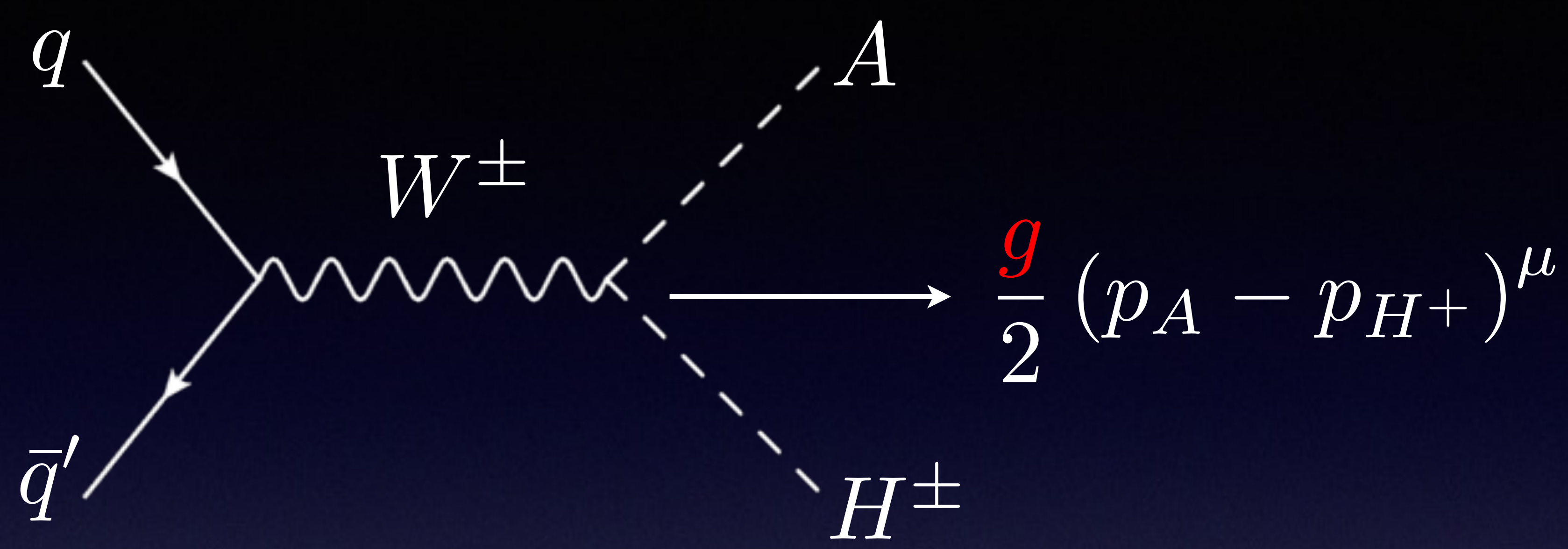
- whose tree level $\sigma_{\rm prod}$ depends on only ONE SUSY parameter that can be determined by kinematic variable (e.g. invariant mass).
- that is not sensitive to the detailed SUSY parameters via radiative corrections.
- that can bound the SUSY models by (product of)

  Br (decay branching ratio)

  without convoluting with $\sigma_{\rm prod}$.
- that can be used to distinguish MSSM from its alternatives, e.g. 2HDM.
- whose final state particle kinematics can be properly modeled without specifying any SUSY parameters.

  → The detection efficiency can be accurately determined.

The promising process $pp \to W^\pm \to AH^\pm$



The production cross section in general depends on two masses: m_A and m_{H^+}, e.g. in 2HDM.

But in MSSM,

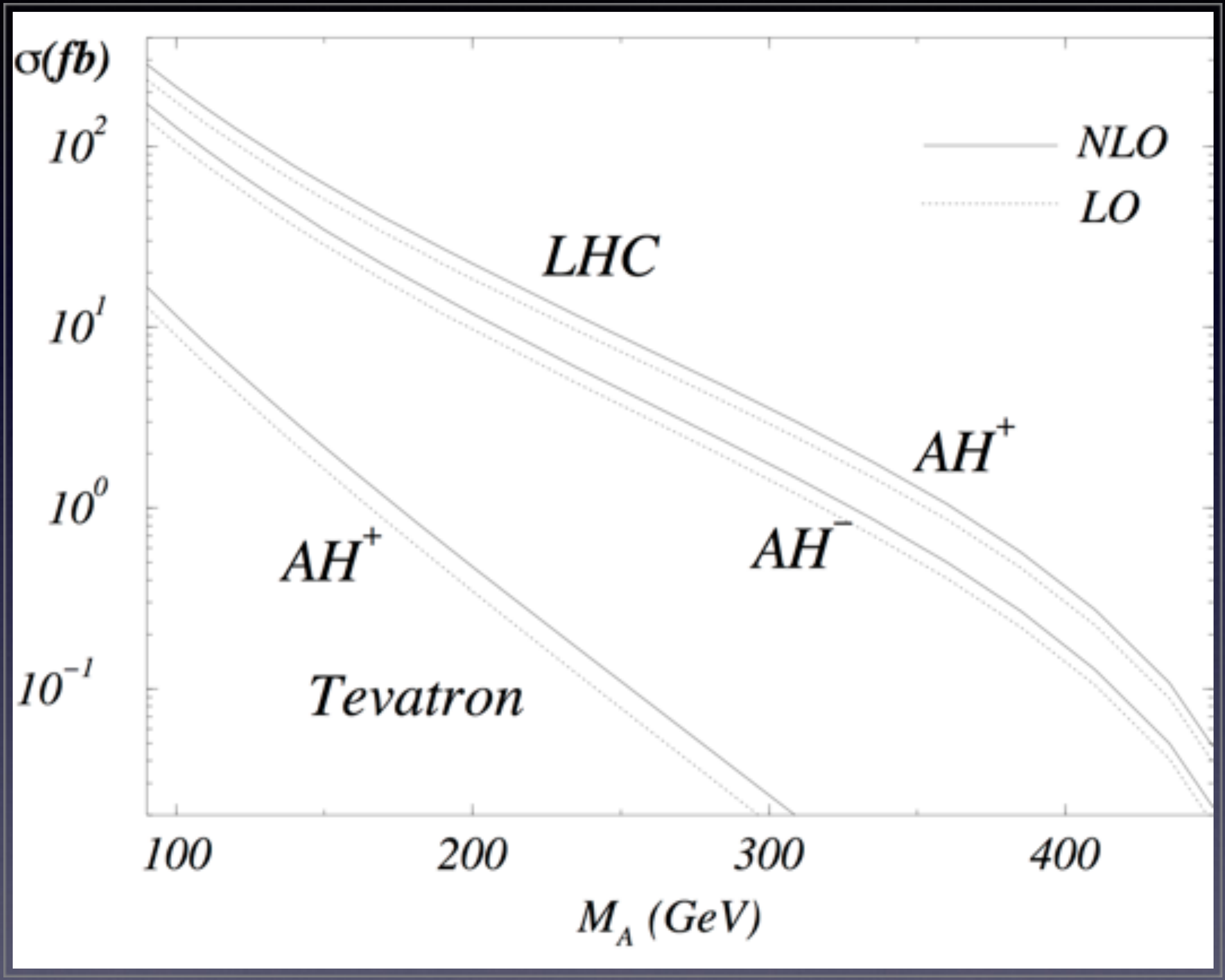
$$m_{H^+}^2 = m_A^2 + m_W^2$$

→ $\sigma_{\rm prod}$ only depends on g and m_A .

(m_A can be determined from its decay kinematics, e.g. the invariant mass $m_{b\bar{b}}$ in $A \to b\bar{b}$.)

Production rates

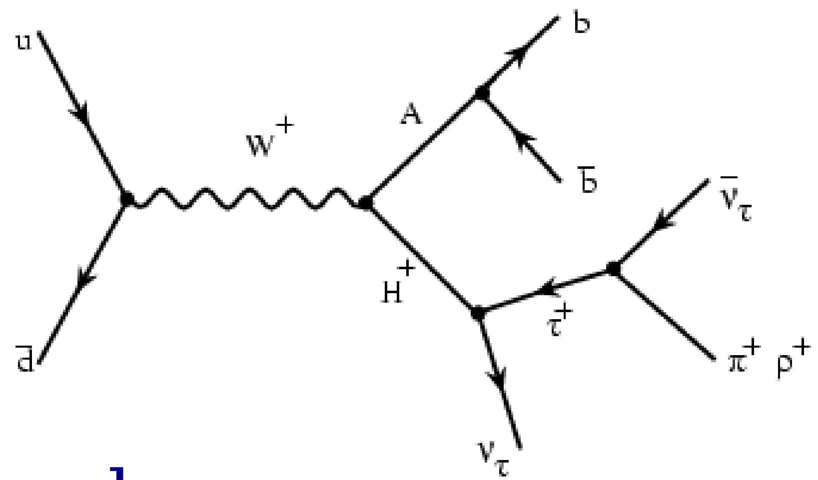
- NLO QCD correction is about 20%
- Uncertainty due to PDF is about 5% at LHC for m_A =120GeV.
- The one-loop electroweak correction to the production rate is smaller than the PDF uncertainty.



- The MSSM mass relation between A and H^+ holds well beyond tree level.

Signal and BKGD

Signal

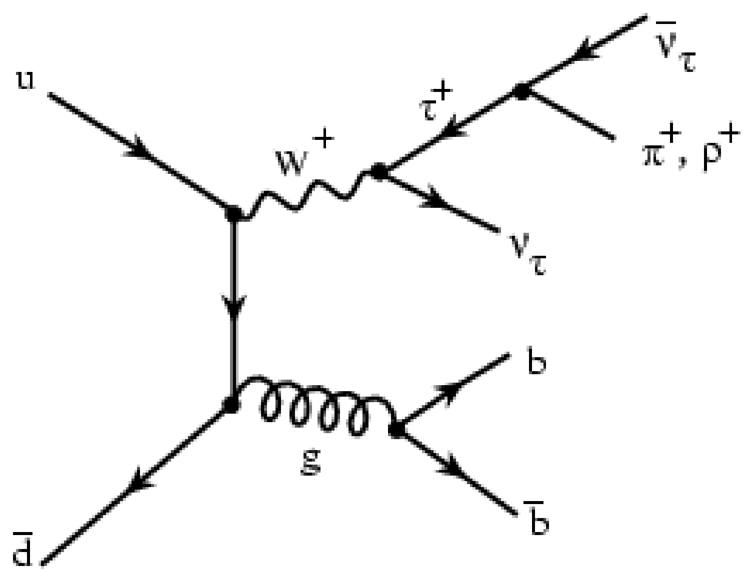


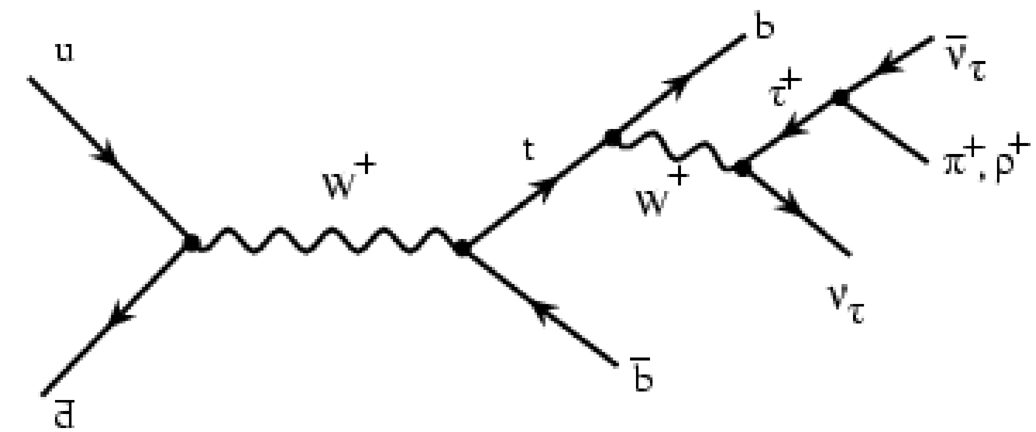
Veto additional lepton and jet from the parton level background events that satisfy

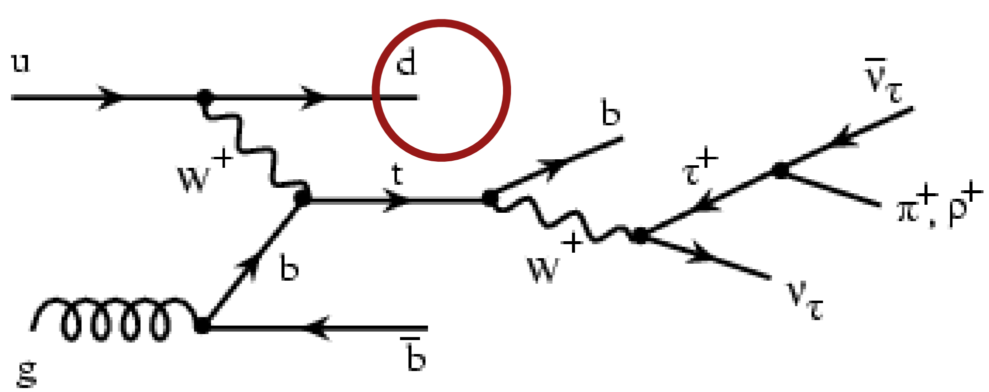
$$p_T(\text{lepton}) > 10\,\text{GeV}, \text{and } |\eta(\text{lepton})| < 3$$

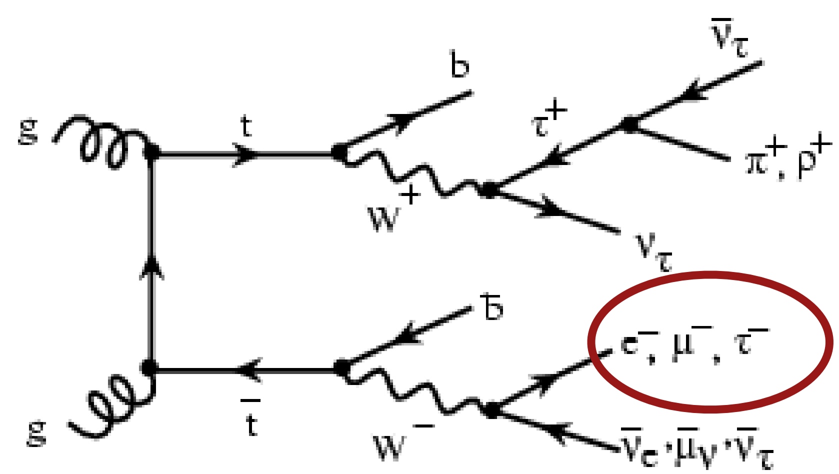
$$p_T(\text{jet}) > 10\,\text{GeV}, \text{and } |\eta(\text{jet})| < 3.5$$

Backgrounds









Model parameters and basic cuts

- The model parameters, production rates and decay BRs

Sets	A	B	C
m_A/Γ_A	101 /3.7	165.7 /5.6	250 /7.9
m_h/Γ_h	96.8 / 3.3	112 /0.04	112 /0.01
m_H/Γ_H	113 /0.38	163 /5.5	247.8 /7.8
m_{H+}/Γ_{H+}	126 / 0.43	182 / 0.68	261.4 / 4.2
$\sigma(AH^+)$ $[fb]$	164	36	5.4
$\sigma(HH^+)$ $[fb]$	137.4	37.4	5.4
$Br(A \to b\bar{b})$	0.91	0.90	0.89
$Br(H \to b\bar{b})$	0.90	0.90	0.89
$Br(H^+ \to \tau^+\nu)$	0.98	0.90	0.00
$Br(H^+ \to t\bar{b})$	0.00	0.09	0.79
$Br(\tau^+ \to \pi^+\nu)$	0.11	0.11	0.11

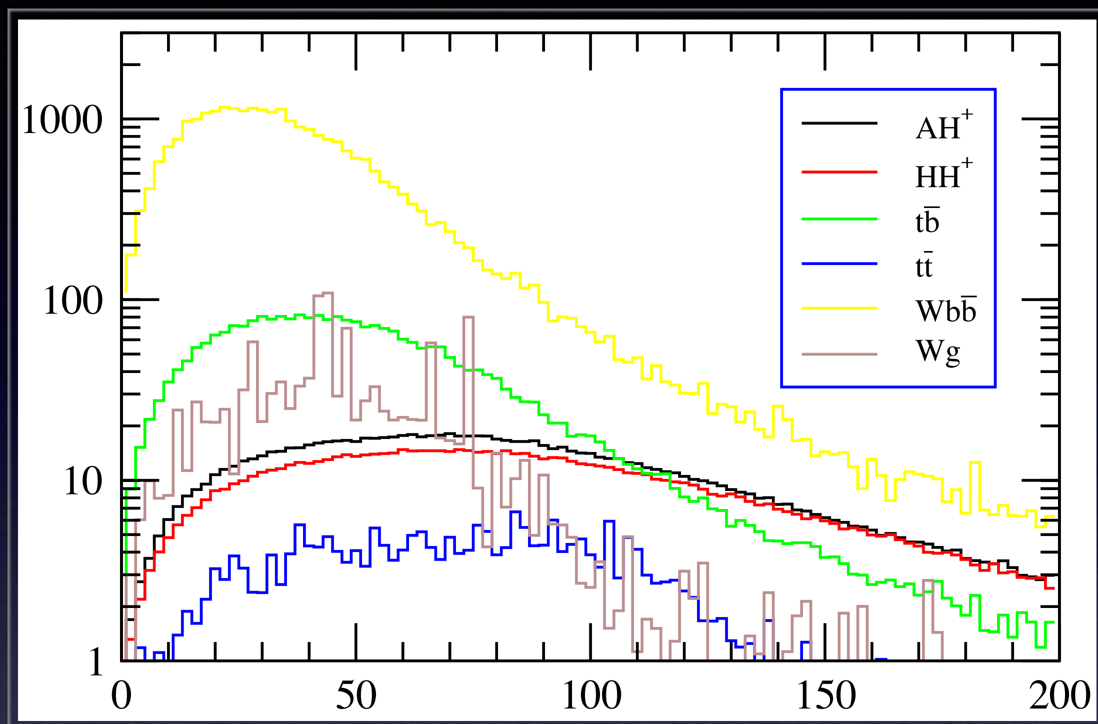
where tan $\beta = 40, \mu = M = 500$GeV.

- Imposing basic cuts

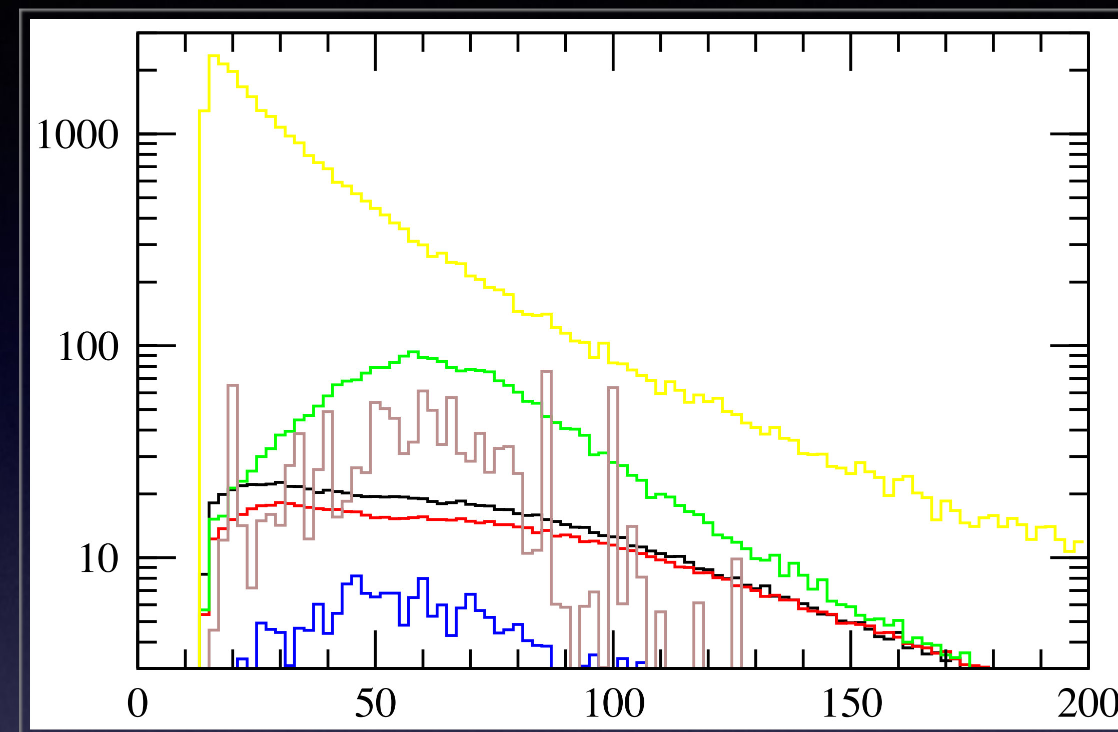
$$p_T(b,\bar{b},\pi^+) > 15 \text{ GeV}, \left|\eta(b,\bar{b},\pi^+)\right| < 3.5, \Delta R(b,\bar{b},\pi^+) > 0.4$$

Set A (m_A=101GeV)

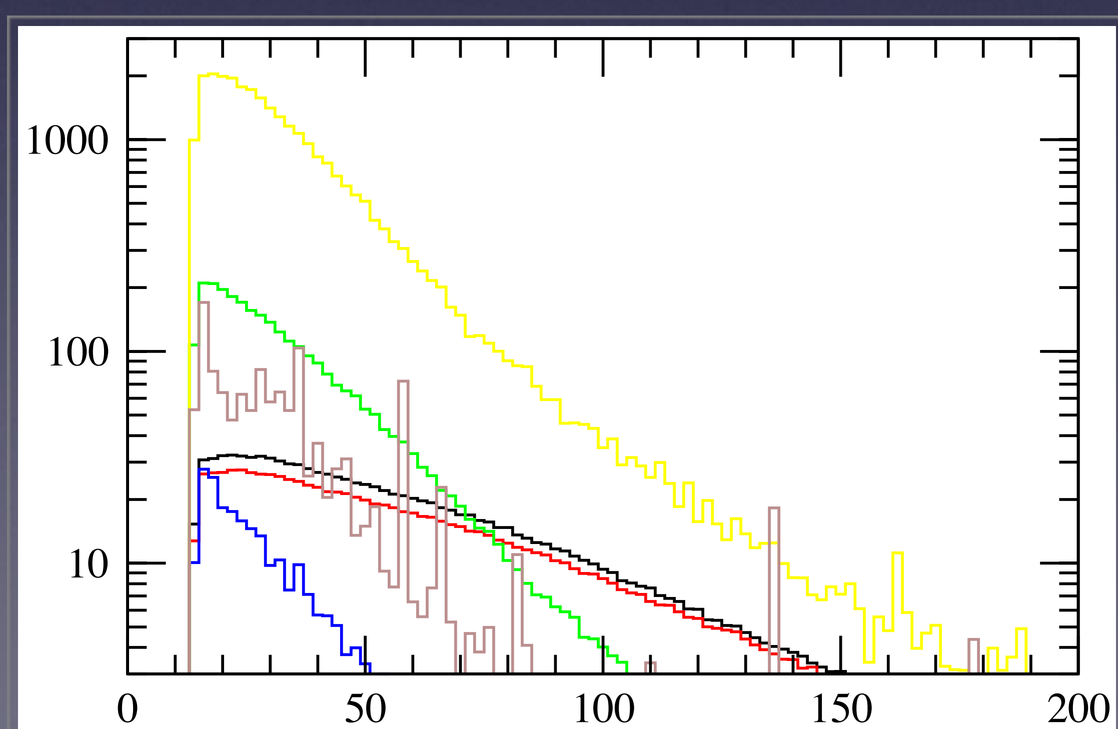
Kinematics Distributions



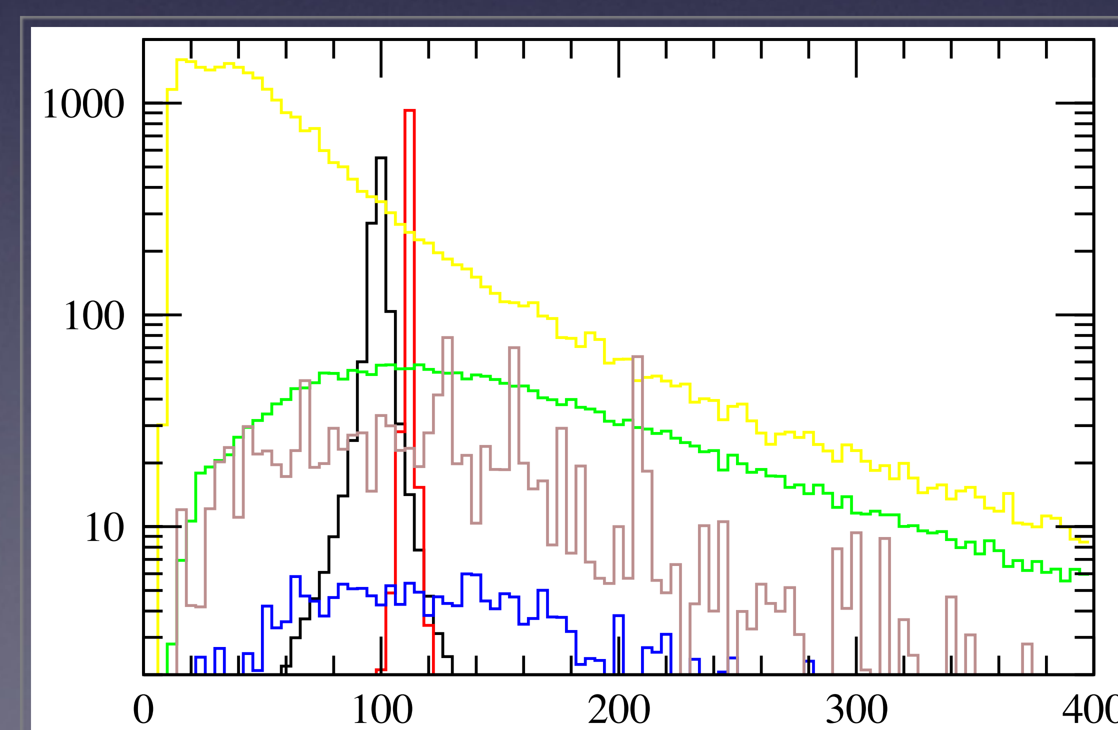
$\not{E}_T$(GeV)



p_T^b(GeV)



$p_T^{\pi^+}$(GeV)



$m_{b\bar{b}}$(GeV)

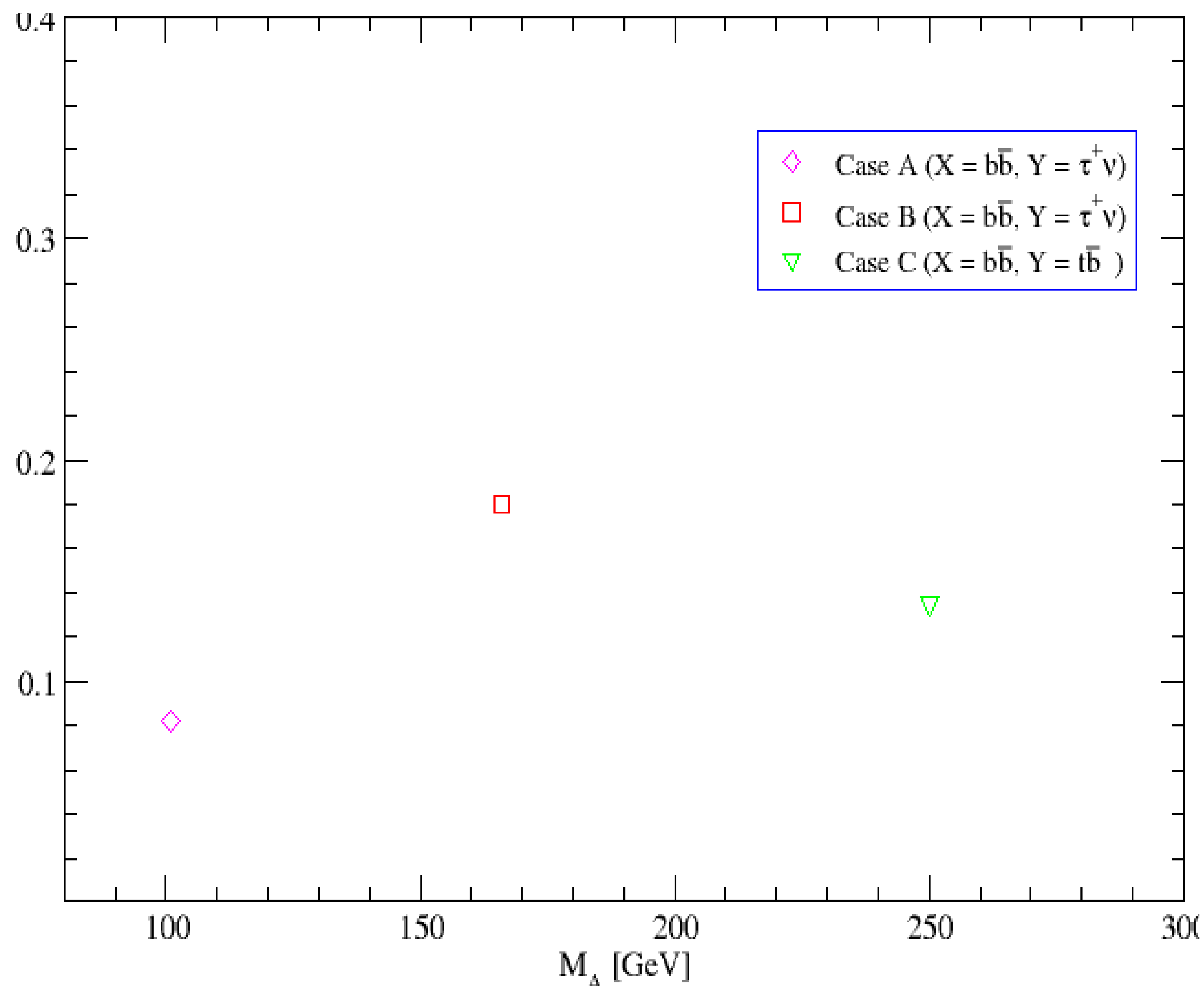
Significance

- Numbers of signal and background events at LHC with $100fb^{-1}$.
  The b-tagging efficiency (50%, for tagging both b and $\bar{b}$ jets) is included, and the kinematic cuts listed in each column are applied sequentially.

- Signal: AH^+

	Basic Cuts	$\not{E}_T > 50$	$P_T^{\pi} > 40$	$90 < M_{b\bar{b}} < 110$ [GeV]
AH^+	507	391	241	216
HH^+	48	38	24	0
$Wb\bar{b}$	11555	3111	864	67
$t\bar{b}$	1228	614	163	12
Wg	567	236	68	11
$t\bar{t}$	110	80	17	2
Signal (S)	507	391	241	216
Bckg (B)	13507	4078	1135	92
S/B	0.038	0.095	0.212	2.35
$S/\sqrt{B}$	4.36	6.12	7.14	22.5
$\sqrt{S+B}/S$	0.23	0.17	0.15	0.08

Constraint on MSSM



Constraints on the product of branching ratios

$$B(A \to b\bar{b}) \times B(H^+ \to \tau^+ \nu_\tau)$$

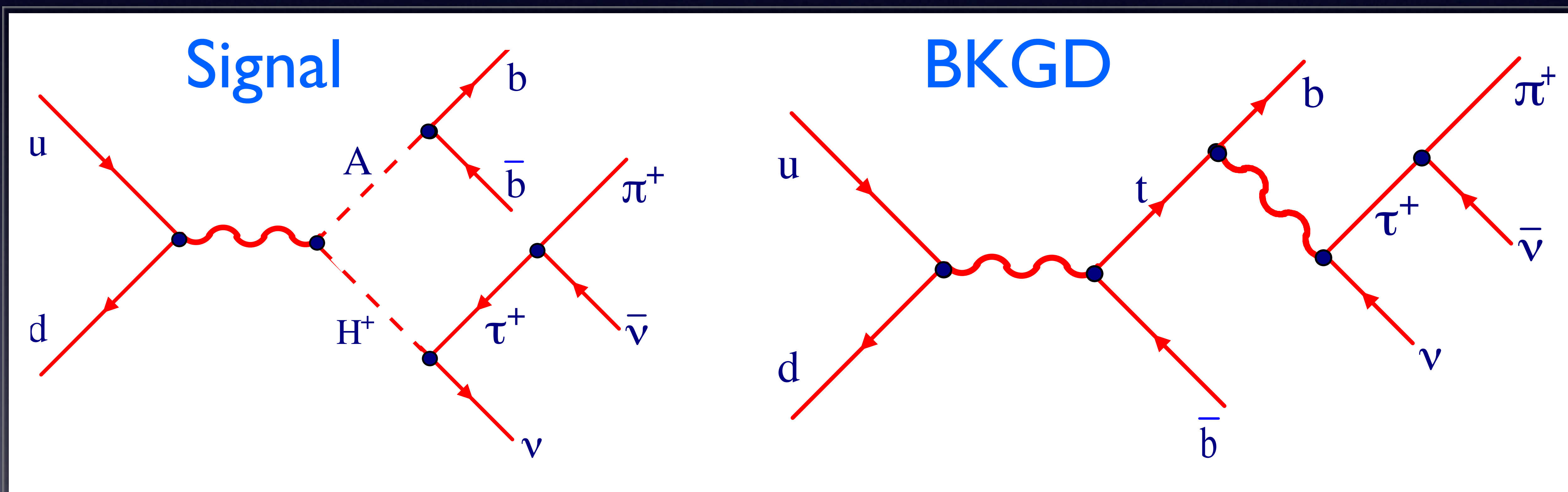
as a function of M_A for Case A and Case B, and

$$B(A \to b\bar{b}) \times B(H^+ \to t\bar{b})$$

for Case C, at the LHC, where τ^+ decays into $\pi^+ \bar{\nu}_\tau$ channel.

So far so good, *but*

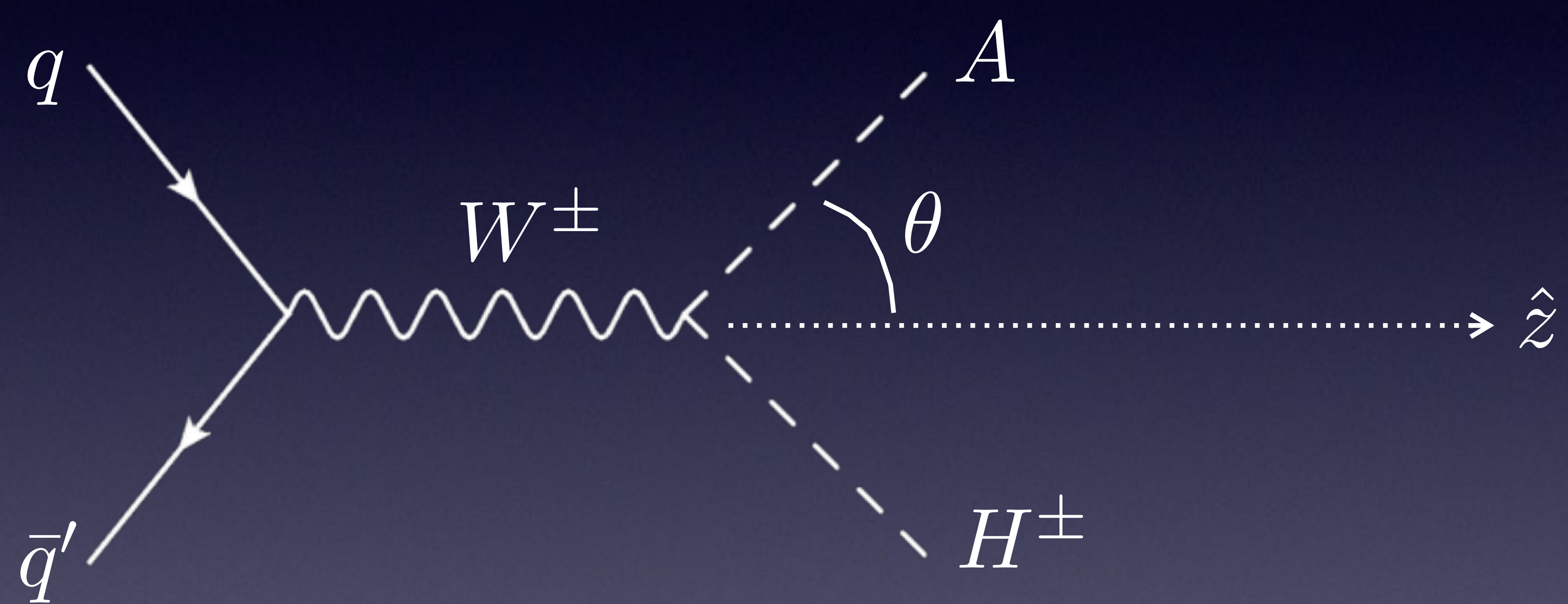
- Why are all the P_T distributions of the signal events much harder than those of the SM backgrounds?



Answer: Matrix element
(spin correlations)

A quick question

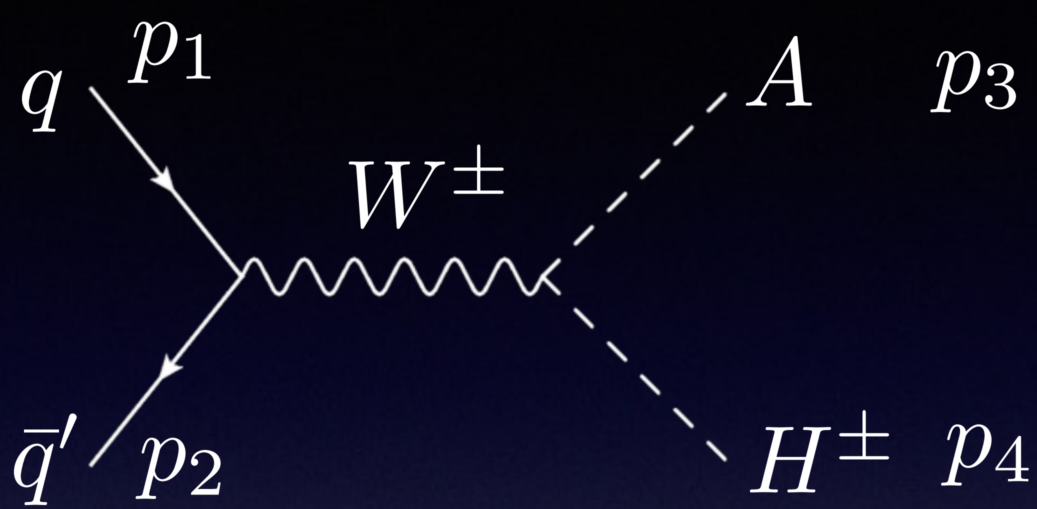
- What does the matrix element square look like in the c.m. frame?



(a) $(1+\cos\theta)^2$ (b) $(1-\cos\theta)^2$ (c) $\sin^2\theta$

(d) $\sin^2\frac{\theta}{2}$ (e) $\cos^2\frac{\theta}{2}$ (f) $\sin^2\theta\cos^2\theta$

Matrix element square



In the c.m. frame

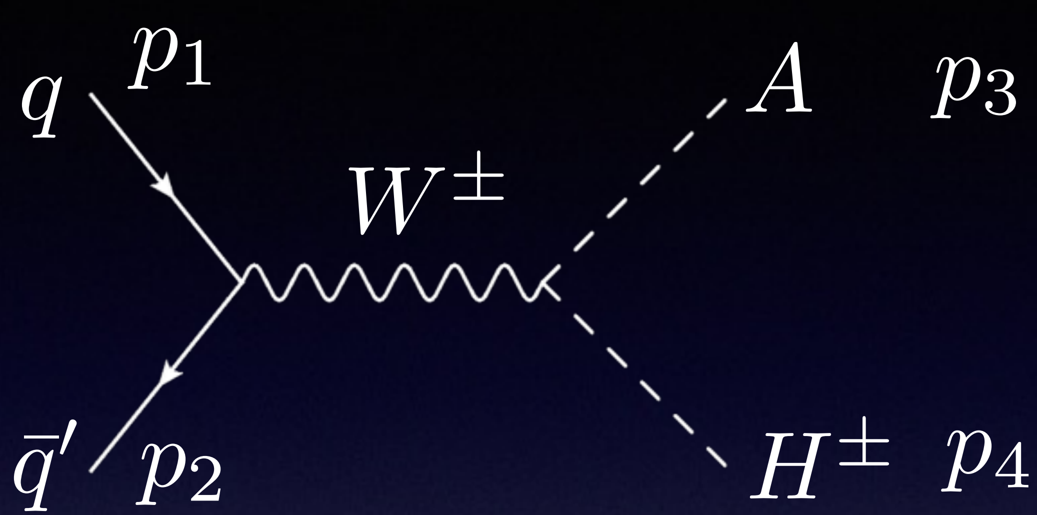
$$p_1 = (E, 0, 0, E)$$
$$p_2 = (E, 0, 0, -E)$$
$$p_3 = (E_3, Ps_\theta, 0, Pc_\theta)$$
$$p_4 = (E_4, -Ps_\theta, 0, -Pc_\theta)$$

$$|\mathcal{M}|^2 = \left(\frac{g}{\sqrt{2}}\right)^2 \left(\frac{g}{2}\right)^2 \frac{1}{(\hat{s} - m_W^2)^2 + M_W^2 \Gamma_W^2} \times \mathrm{Tr}\left[(\not{p}_3 - \not{p}_4)\not{p}_1(\not{p}_3 - \not{p}_4)\not{p}_2 P_R\right]$$

$$= \left(\frac{g}{\sqrt{2}}\right)^2 \left(\frac{g}{2}\right)^2 \frac{1}{(\hat{s} - m_W^2)^2 + M_W^2 \Gamma_W^2} \times \left[4\hat{t}\hat{u} - 4m_A^2 m_{H^+}^2\right]$$

$$= \left(\frac{g}{\sqrt{2}}\right)^2 \left(\frac{g}{2}\right)^2 \frac{1}{(\hat{s} - m_W^2)^2 + M_W^2 \Gamma_W^2} (4E^2) \times \left\{4P^2 \sin^2\theta\right\}$$

Matrix element square



In the c.m. frame

$$p_1 = (E, 0, 0, E)$$
$$p_2 = (E, 0, 0, -E)$$
$$p_3 = (E_3, Ps_\theta, 0, Pc_\theta)$$
$$p_4 = (E_4, -Ps_\theta, 0, -Pc_\theta)$$

$$|\mathcal{M}|^2 = \left(\frac{g}{\sqrt{2}}\right)^2 \left(\frac{g}{2}\right)^2 \frac{1}{(\hat{s} - m_W^2)^2 + M_W^2 \Gamma_W^2} \times \mathrm{Tr}\left[(\not{p}_3 - \not{p}_4)\not{p}_1(\not{p}_3 - \not{p}_4)\not{p}_2 P_R\right] = \left(\frac{g}{\sqrt{2}}\right)^2 \left(\frac{g}{2}\right)^2 \frac{1}{(\hat{s} - m_W^2)^2 + M_W^2 \Gamma_W^2} \times \left[4\hat{t}\hat{u} - 4m_A^2 m_{H^+}^2\right]$$

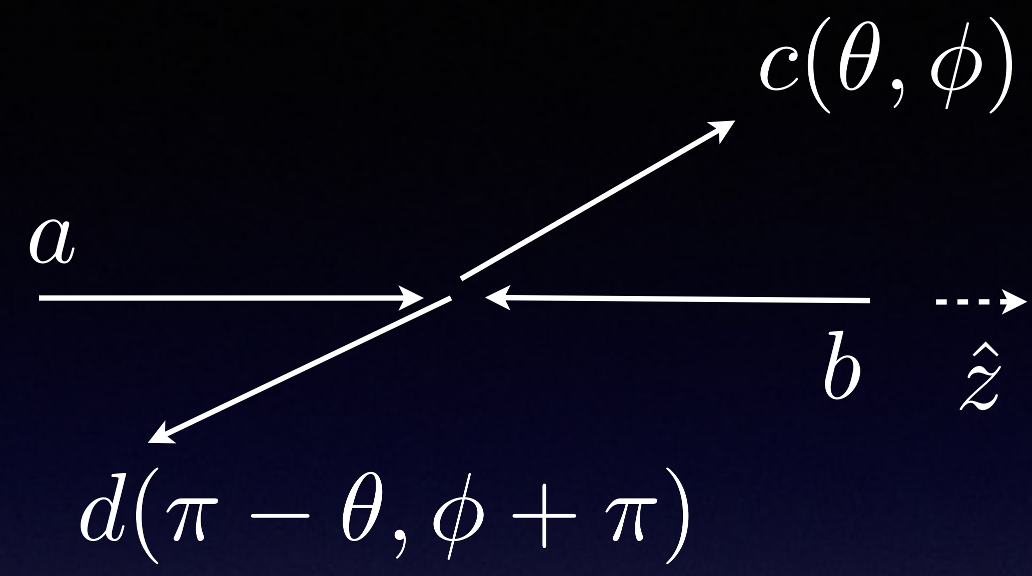
$$= \left(\frac{g}{\sqrt{2}}\right)^2 \left(\frac{g}{2}\right)^2 \frac{1}{(\hat{s} - m_W^2)^2 + M_W^2 \Gamma_W^2} (4E^2) \times \left\{4P^2 \sin^2\theta\right\}$$

Can we get the angular dependence directly without any lengthy calculation?

Helicity amplitude

- 2 to 2 scattering

$$\underset{\lambda_a}{a} + \underset{\lambda_b}{b} \to \underset{\lambda_c}{c} + \underset{\lambda_d}{d}$$



- Jacob-Wick formalism (partial wave decomposition)

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(p_f - p_i)\mathcal{M}_{fi}$$

$$\mathcal{M}_{fi} = \frac{8\pi}{\sqrt{\beta_i \beta_f}} \sum_{J=0}^{\infty} (2J+1) T^J_{\lambda_a \lambda_b; \lambda_c \lambda_d}(E_{cm}) d^J_{\lambda_i \lambda_f}(\theta) e^{i(\lambda_i - \lambda_f)\phi}$$

$$\lambda_i = \lambda_a - \lambda_b$$
$$\lambda_f = \lambda_c - \lambda_d$$

$d^J_{\lambda_i \lambda_f}(\theta)$ *Wigner d-function*

ϕ angle is trivial in general.

PDG Book: d-Functions

$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$

$d^{\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$

$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$

$d^{j}_{m',m} = (-1)^{m-m'} d^{j}_{m,m'} = d^{j}_{-m,-m'}$

(Partial tables at top edge)

m_1	m_2			J / M
0	−1	1/2	1/2	2
−1	0	1/2	−1/2	−2
−1	−1	1		

m_1	m_2			J / M
−1	−1	2/3	1/3	3
−2	0	1/3	−2/3	−3
−2	−1	1		

3/2×3/2

m_1	m_2	J=3, M=+3
+3/2	+3/2	1

m_1	m_2	J=3, M=+2	J=2, M=+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	J=3, M=−2	J=2, M=−2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	J=3, M=−3
−3/2	−3/2	1

$d^{1}_{0,0} = \cos\theta$ $\quad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2}$ $\quad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$

$d^{1}_{1,1} = \frac{1+\cos\theta}{2}$ $\quad d^{1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$ $\quad d^{1}_{1,-1} = \frac{1-\cos\theta}{2}$

2×3/2

m_1	m_2	J=7/2, M=+7/2
+2	+3/2	1

m_1	m_2	J=7/2, M=+5/2	J=5/2, M=+5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	J=7/2, M=+3/2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	J=7/2, M=+1/2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	J=7/2, M=−1/2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	J=7/2, M=−3/2	J=5/2, M=−3/2	J=3/2, M=−3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	J=7/2, M=−5/2	J=5/2, M=−5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	J=7/2, M=−7/2
−2	−3/2	1

2×2

m_1	m_2	J=4, M=+4
+2	+2	1

m_1	m_2	J=4, M=+3	J=3, M=+3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	J=4, M=+2	J=3, M=+2	J=2, M=+2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	J=4, M=+1	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	J=4, M=0	J=3, M=0	J=2, M=0	J=1, M=0	J=0, M=0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	J=4, M=−1	J=3, M=−1	J=2, M=−1	J=1, M=−1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	J=4, M=−2	J=3, M=−2	J=2, M=−2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	J=4, M=−3	J=3, M=−3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	J=4, M=−4
−2	−2	1

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^{2}_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^{2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^{2}_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^{2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

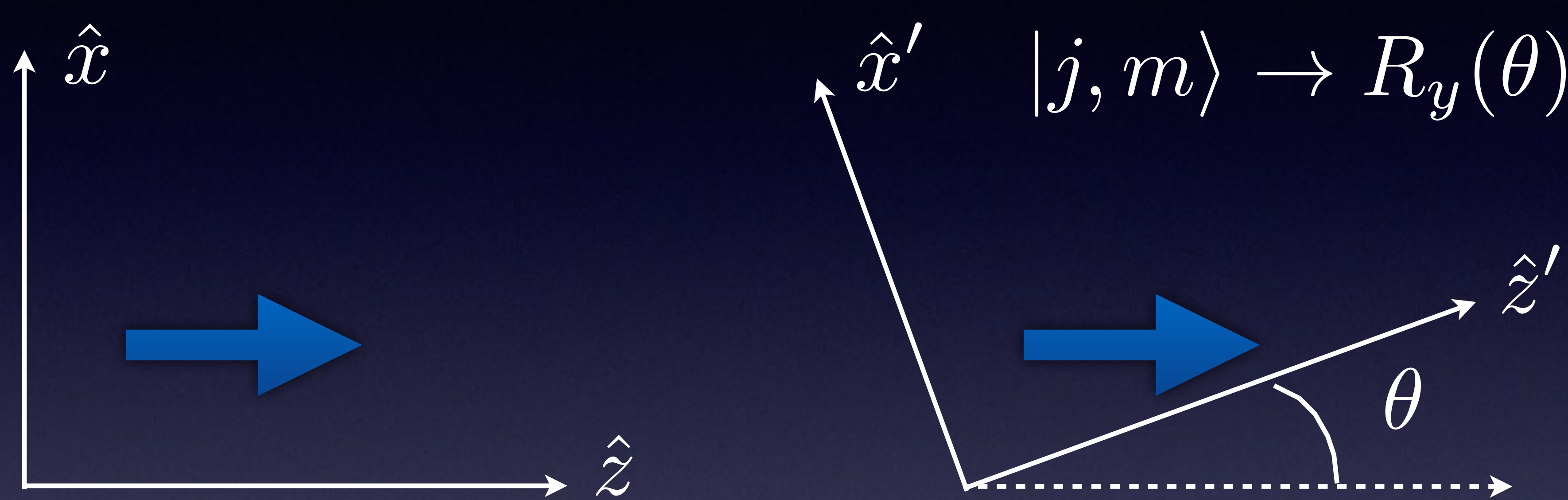
$$d^{2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^{2}_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Angular momentum in QM

- Consider a vector $|j,m\rangle$ $\qquad J^2 \quad J_3$

$$|j,m\rangle \rightarrow R_y(\theta)\,|j,m\rangle$$



$$R_y(\theta) = e^{i\theta J_y}$$

Rotation matrices

$$d^j_{m\rightarrow m'}(\theta) \equiv d^j_{m,m'}(\theta) = \langle jm'|R_y(\theta)|jm\rangle$$

The modulus squared is the probability that a particle $J_3 = m$ will have $J_3 = m'$ after the rotation to the new frame.

AH^+ Production

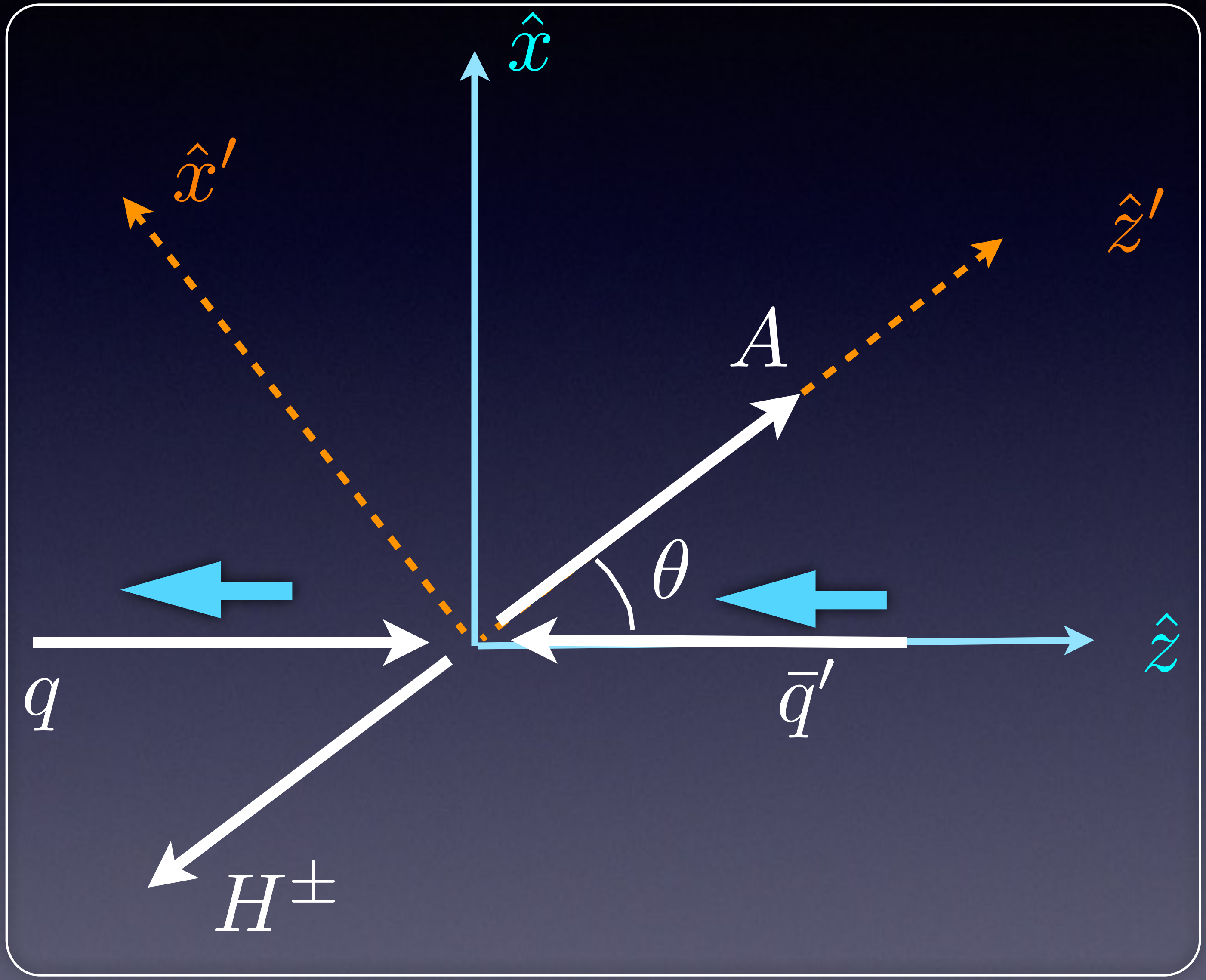
- Rotation matrices of spin-1

$$J_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$d^1_{-1,0} = -\frac{1}{\sqrt{2}}\sin\theta$$

$$\lambda_i = \lambda_q - \lambda_{\bar{q}'}$$
$$= -1/2 - 1/2 = -1$$

$$\lambda_f = \lambda_A - \lambda_{H^\pm} = 0$$



(1) Only longitudinal W-boson contributes.

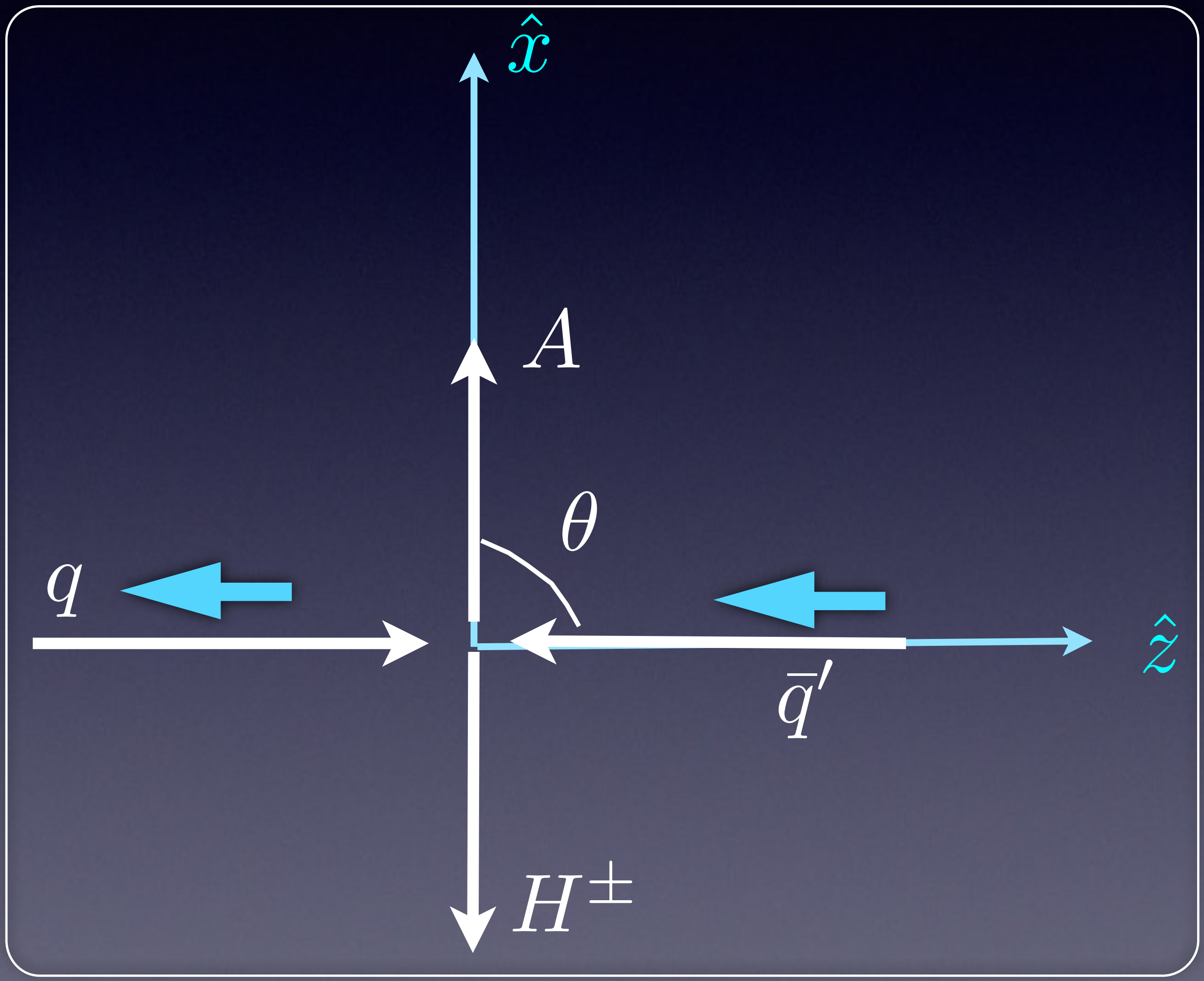
(2) A and H^+ stay in p-wave. What does that mean?

AH^+ Production

- Rotation matrices of spin-1

$$J_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$d^1_{-1,0} = -\frac{1}{\sqrt{2}}\sin\theta$$



(1) Only longitudinal W-boson contributes.

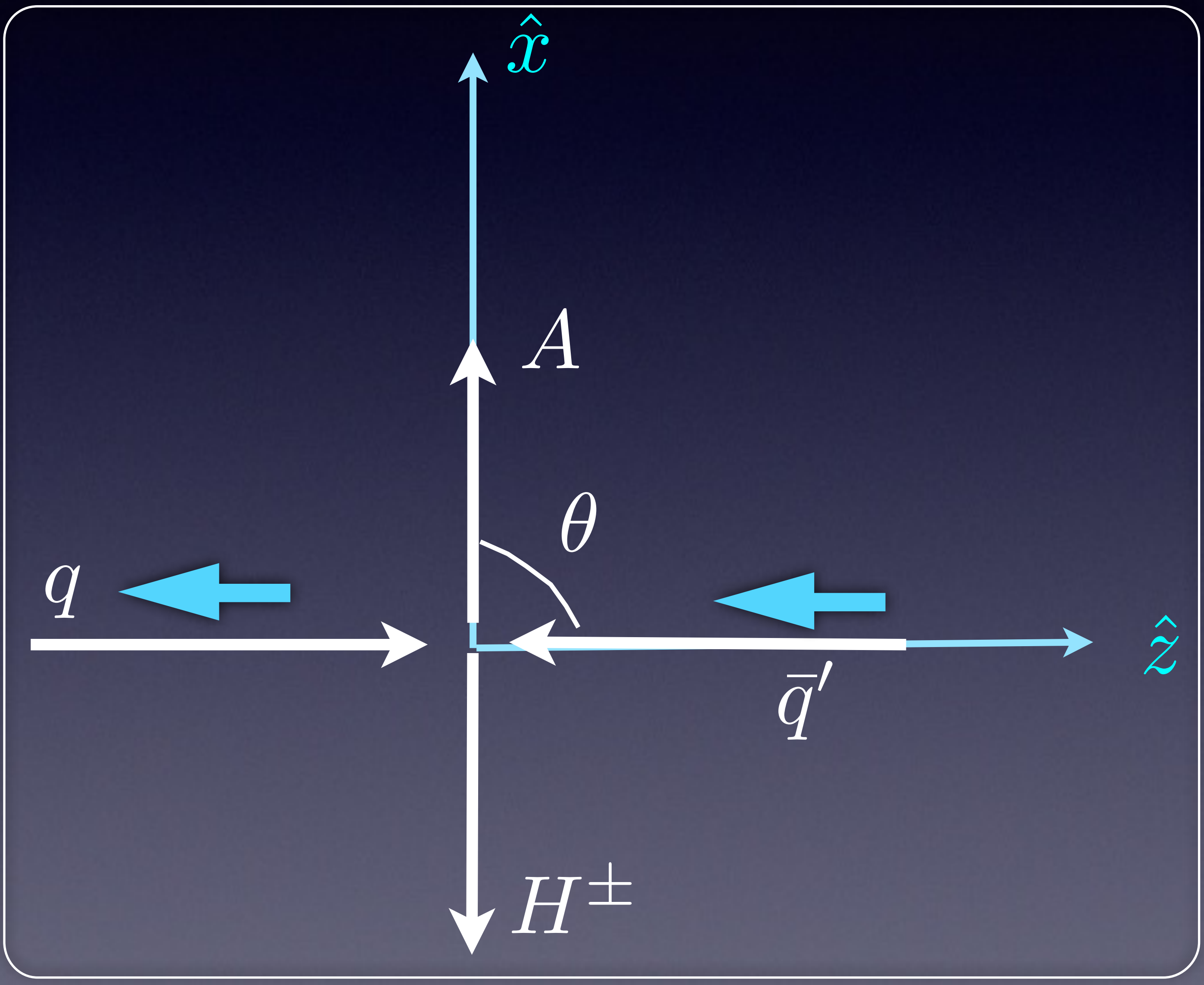
(2) *A* and H^+ stay in *p*-wave

AH^+ Production

- Rotation matrices of spin-1

$$J_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$d^1_{-1,0} = -\frac{1}{\sqrt{2}}\sin\theta$$

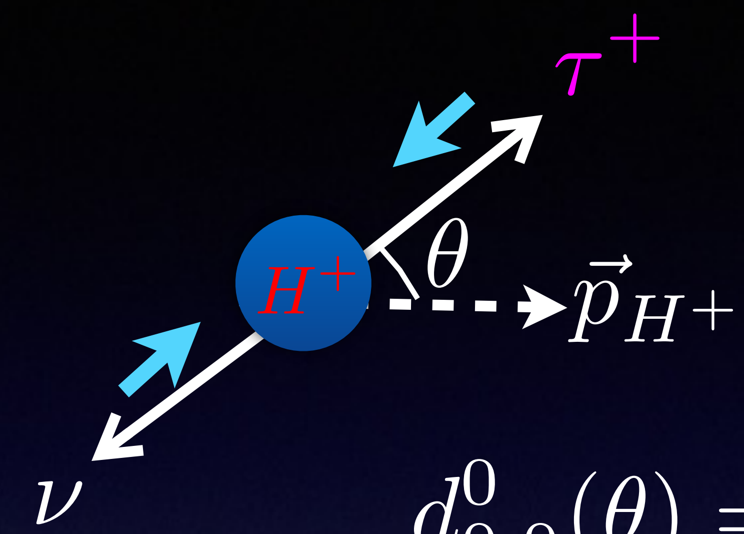


(1) Only longitudinal W-boson contributes.

(2) *A* and H^+ stay in *p*-wave → Large P_T's of *A* and H^+ (and their decay products)

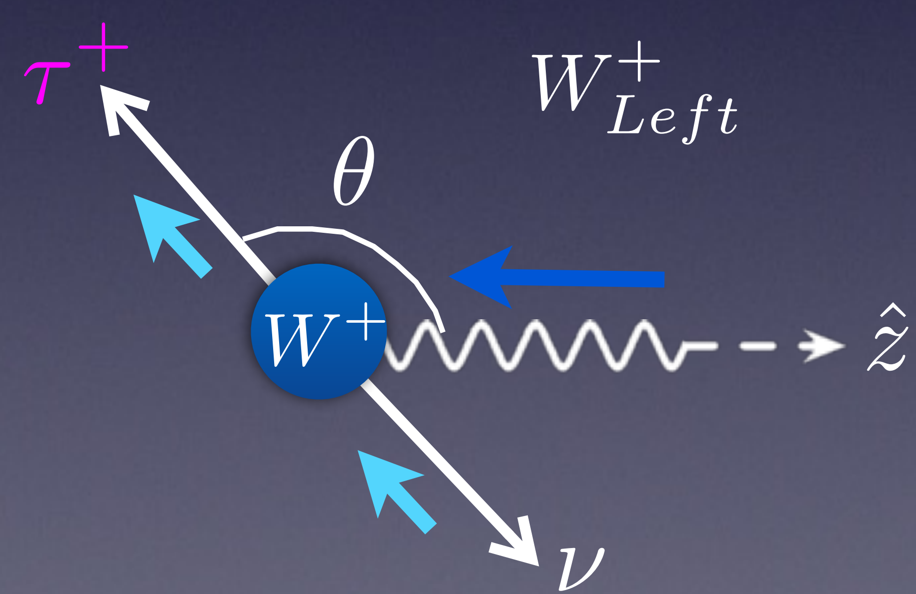
Tau is polarized

- Tau-lepton from H^+ decay is left-handedly polarized

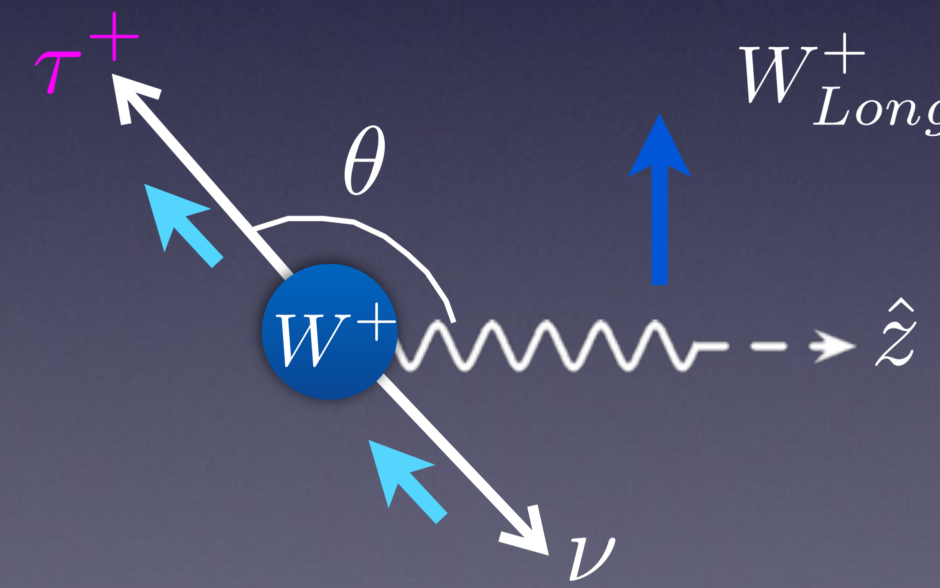


$$d^0_{0,0}(\theta) = 1$$

- Tau-lepton from W^+ decay is right-handedly polarized



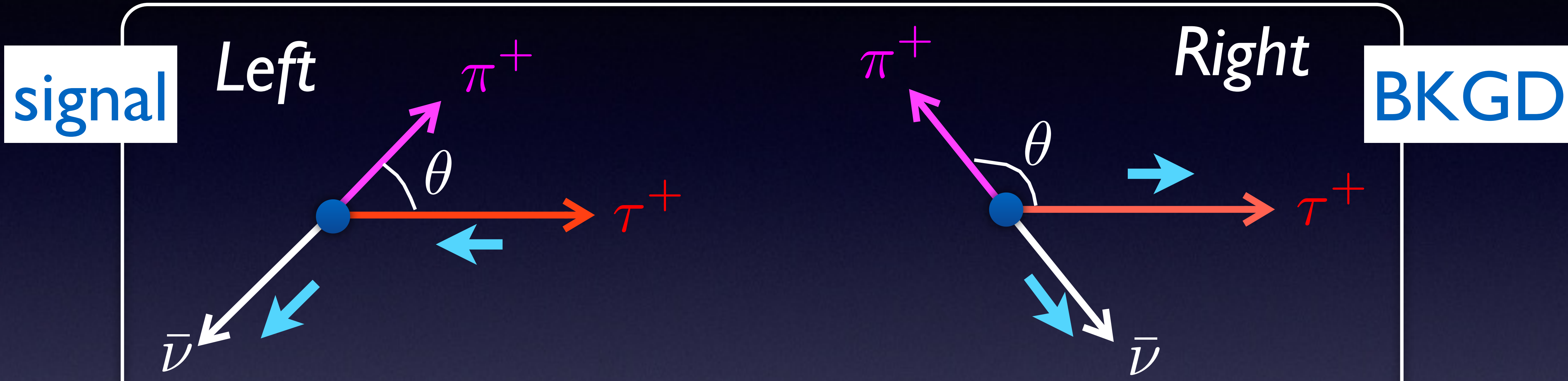
$$d^1_{-1,1}(\theta) = \frac{1 - \cos\theta}{2}$$



$$d^1_{0,1} = -\frac{\sin\theta}{\sqrt{2}}$$

p_{π^+} depends on τ^+ polarization

- A left-handed τ^+ produces a harder π^+

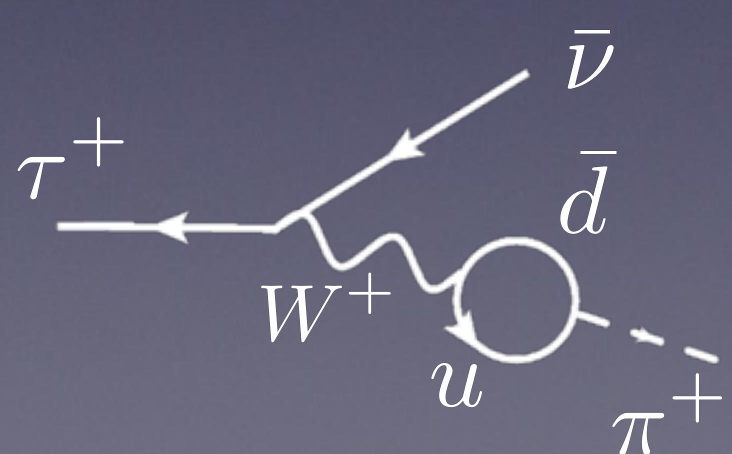


$$d^{\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}} = \cos\frac{\theta}{2} \qquad d^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}} = -\sin\frac{\theta}{2}$$

Homework:

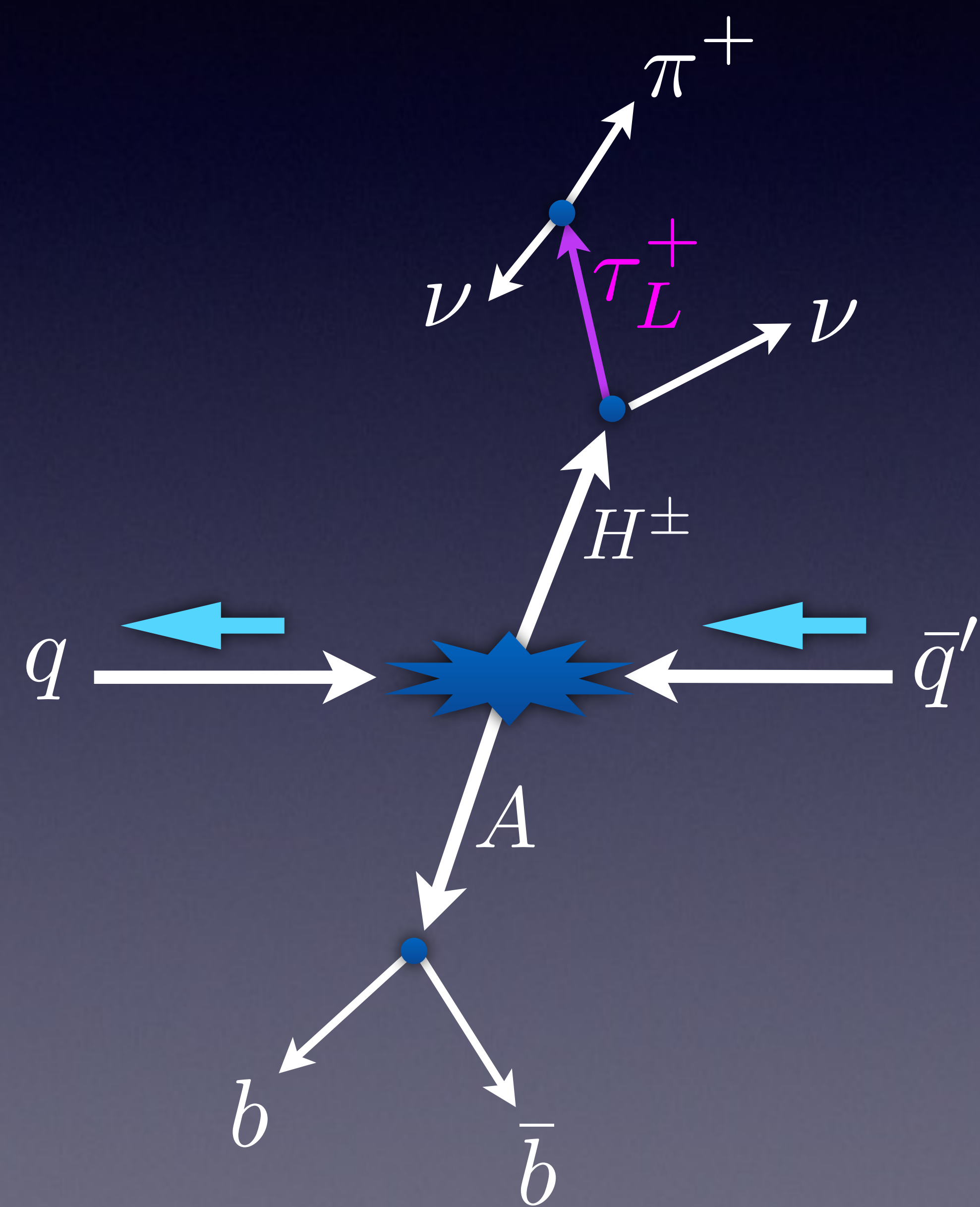
Verify the above angular dependence with the following effective interaction

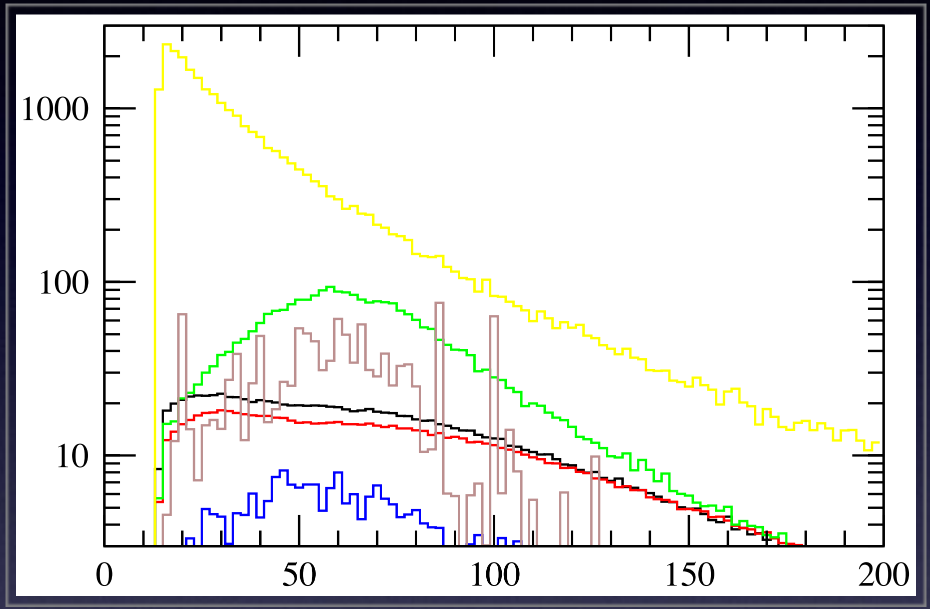
$$\left(\partial^{\mu}\pi^{-}\right)\overline{\tau^{+}}\gamma_{\mu}P_{L}\nu$$



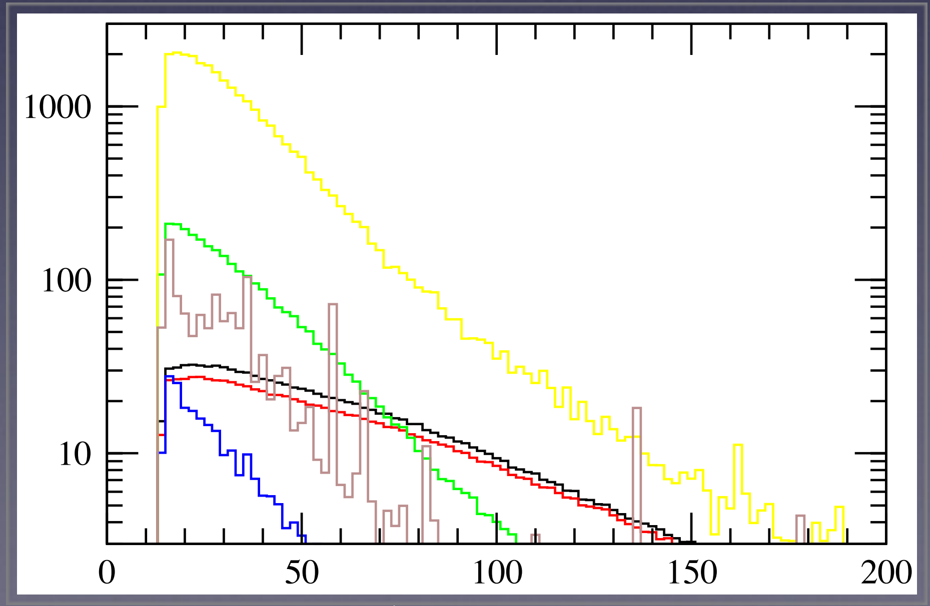
Interim summary

- Spin correlations force the scalars and its decay products in the signal events being highly boosted while those of the backgrounds are anti-boosted.





$p_T^b\,(\mathrm{GeV})$

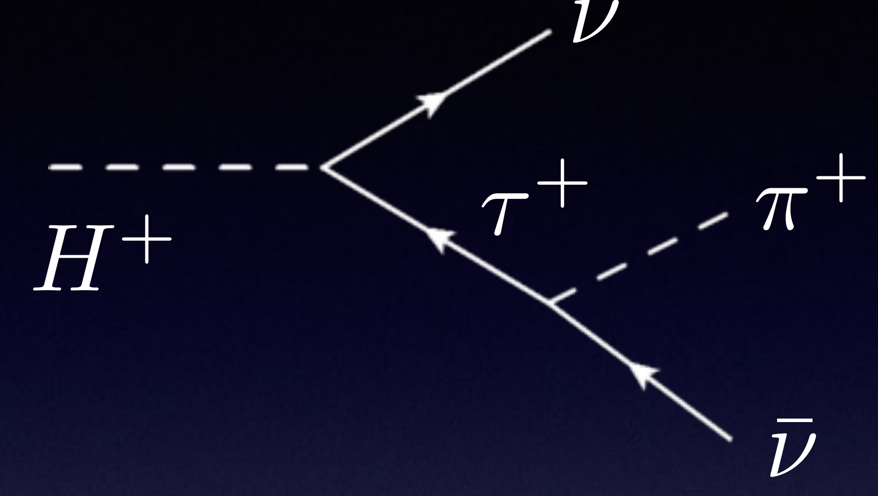


$p_T^{\pi^+}\,(\mathrm{GeV})$

Mass measurement of H^+

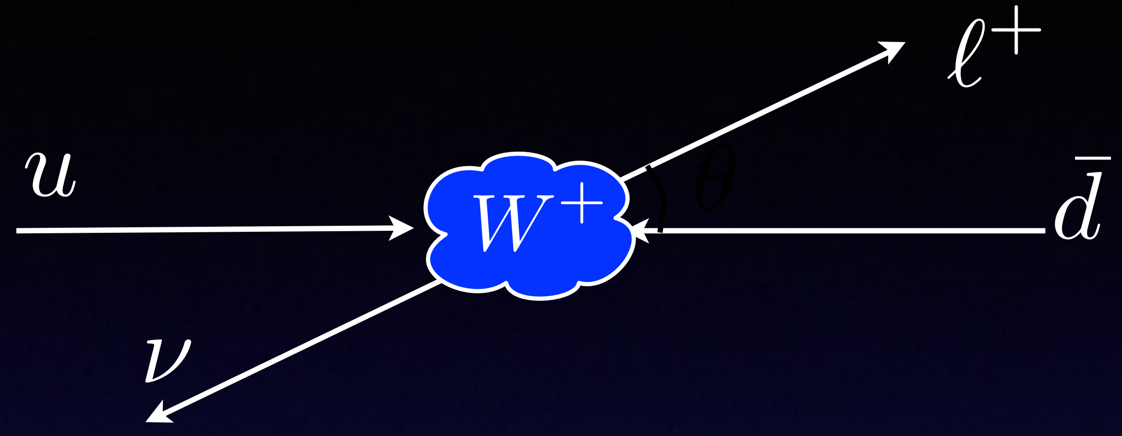
- Experimental difficulty:

Given a measured MET, can we tell it is one or two neutrinos?



- Four exceptions:
  - H^+: $H^+ \to \tau^+\nu \to \pi^+\nu\bar{\nu} \to \pi^+ \not{E}_T$ spin corr.
  - $t\bar{t}$: $t\bar{t} \to b\bar{b}\ell^+\ell'^-\nu\bar{\nu} \to b\bar{b}\ell^+\ell'^- \not{E}_T$ on mass shell conditions
  - h : $h \to W^+W^- \to \ell^+\ell'^-\nu\bar{\nu} \to \ell^+\ell'^- \not{E}_T$ spin corr.
  - h : $h \to \tau^+\tau^- \to \pi^+\pi^-\nu\bar{\nu} \to \pi^+\pi^- \not{E}_T$ kinematics

Transverse Mass

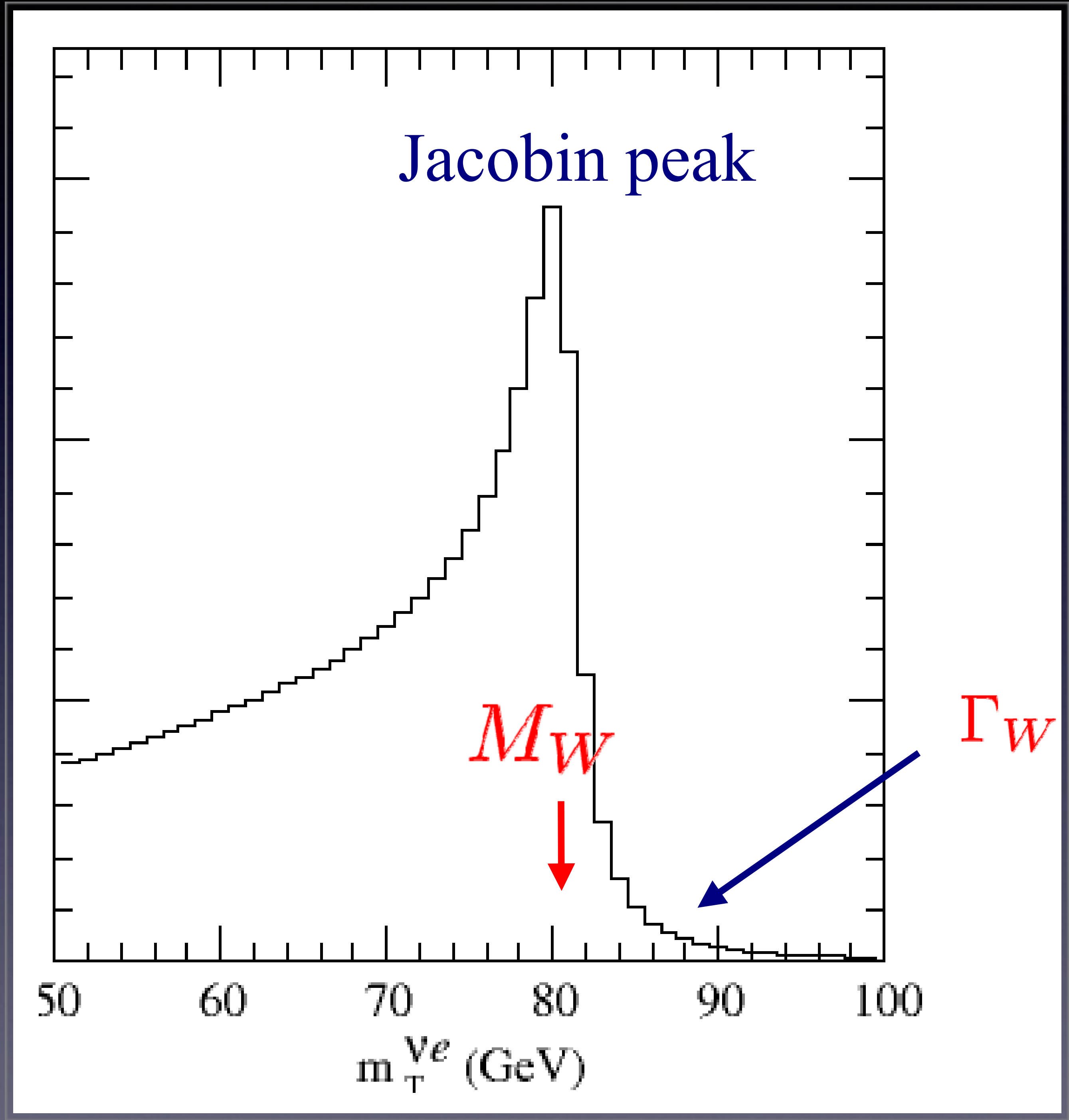


★ TM: measuring the mass of the W-boson in the leptonic decay channel

$$m_T^2 = 2\left(E_T^\ell \not{E}_T - p_T^\ell \not{p}_T\right) = 2p_T^\ell \not{E}_T(1-\cos\phi)$$

★ The true mass of the W boson satisfies
$$m_T^2 \le m_W^2$$

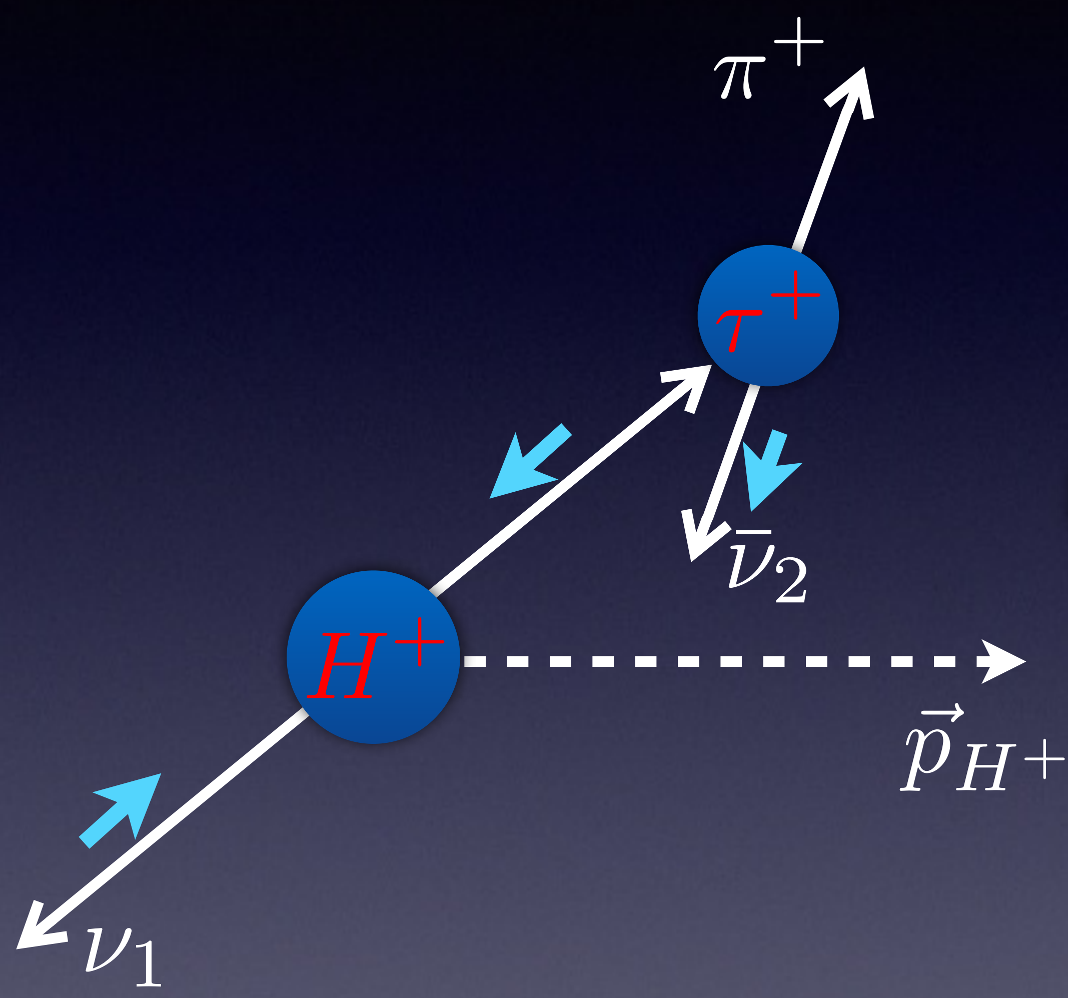
★ The end point of the transverse mass distribution is the W boson mass.



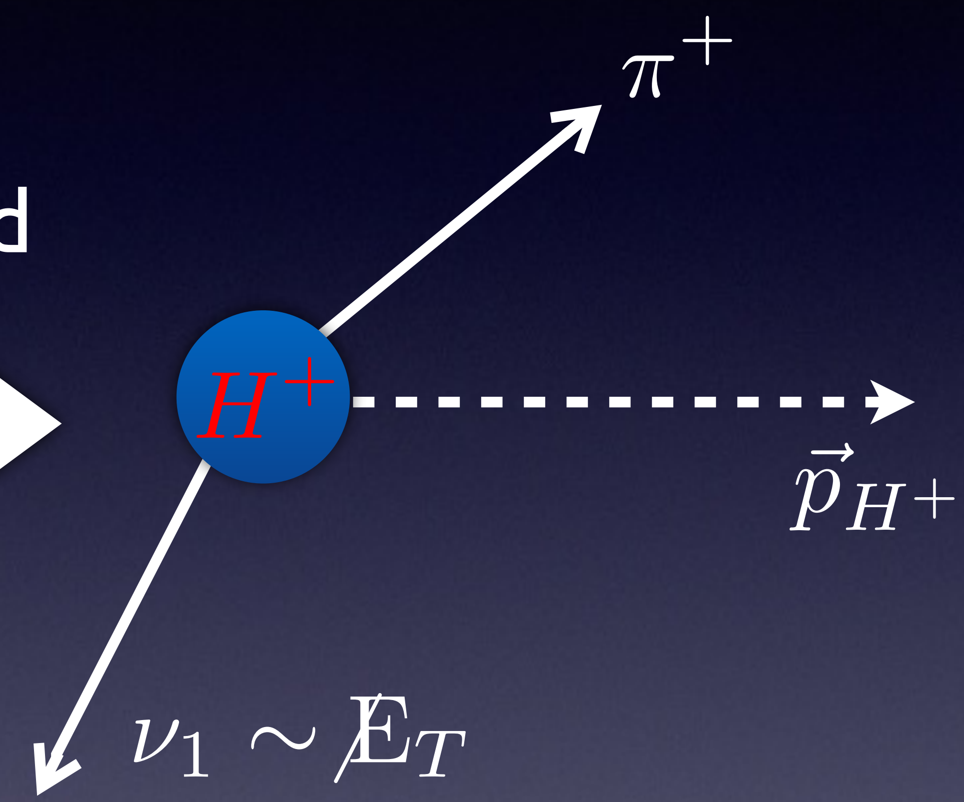
$$\frac{d\sigma}{dm_T^2} \sim \frac{1}{\sqrt{1-m_T^2/\hat{s}}}$$

H^+ reconstruction

- Spin correlation dominates



simplified as



Neutrino from tau decay is anti-boosted such that it tends to be very soft.

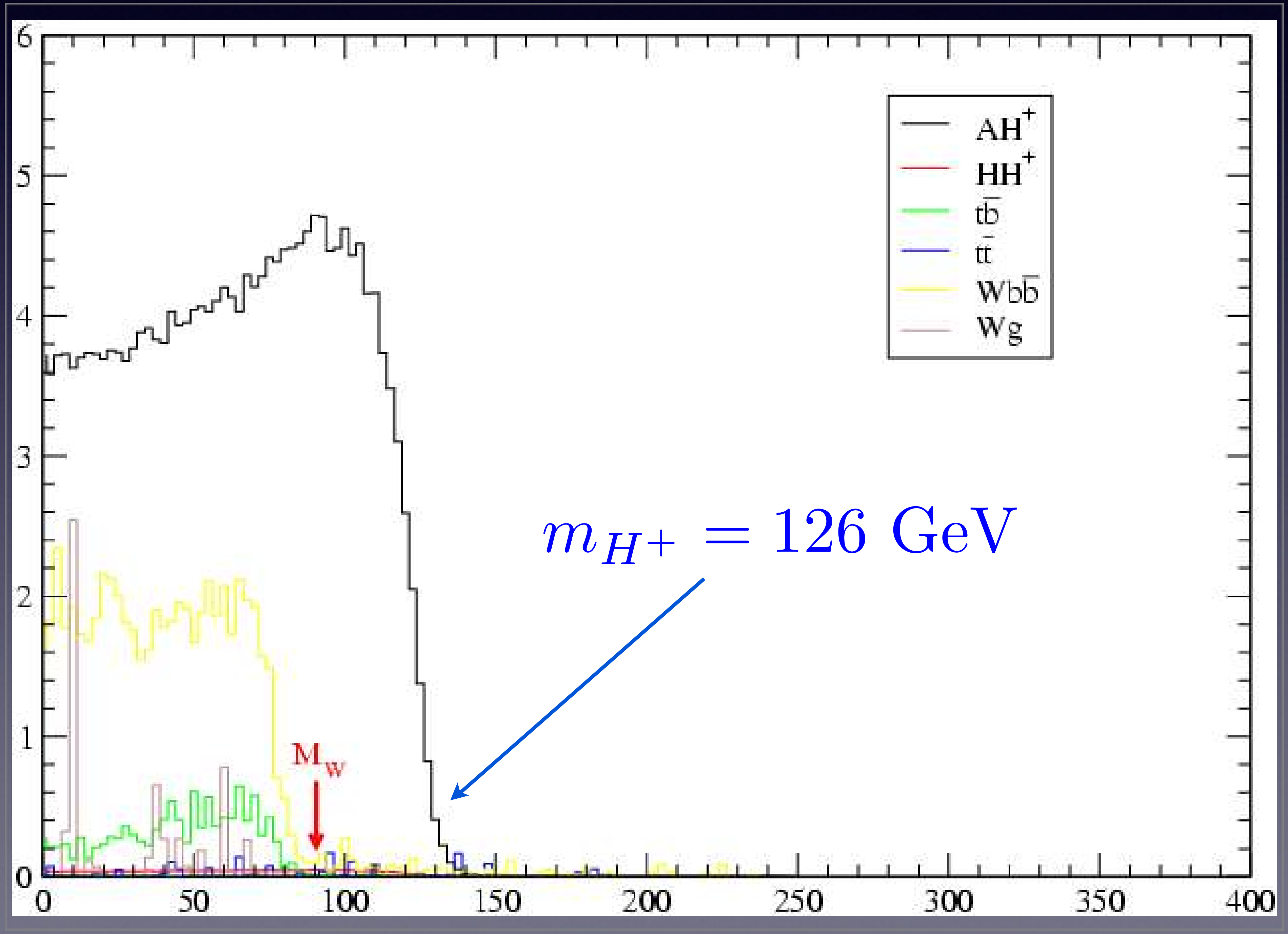
$$m_T = \sqrt{2 p_T(\pi^+)\ \not{E}_T (1 - \cos\Delta\phi)}$$

$\Delta\phi$ is the azimuthal angle between π^+ and $\not{E}_T$

Transverse mass of π^+ and $\not{E}_T$

- Transverse mass of H+ after imposing the mass window cut on the two b-jets

$$\left|M(b\bar{b}) - 100\right| < 10 \text{ GeV}$$

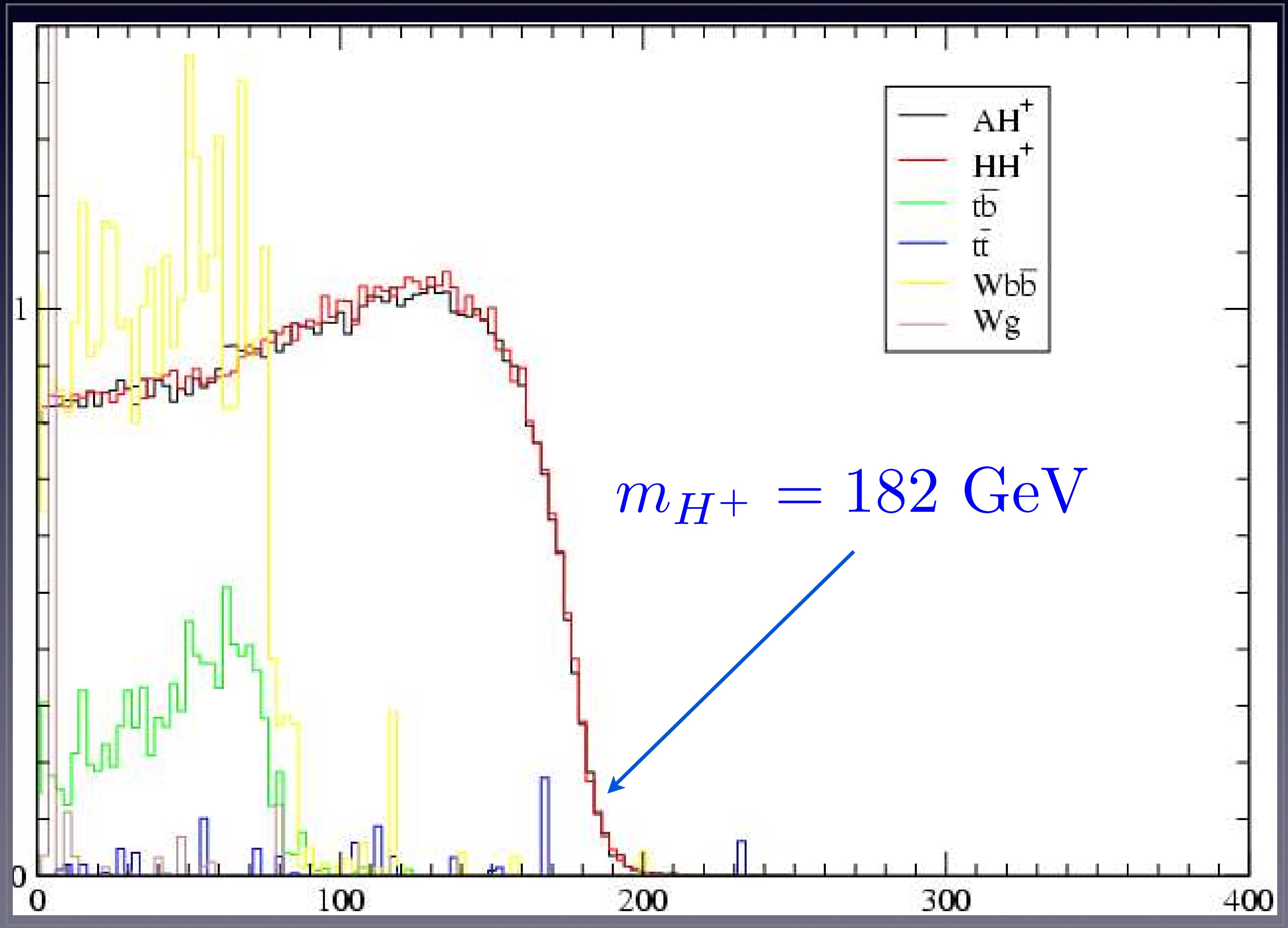


Set-A

Transverse mass of π^+ and $\not{E}_T$

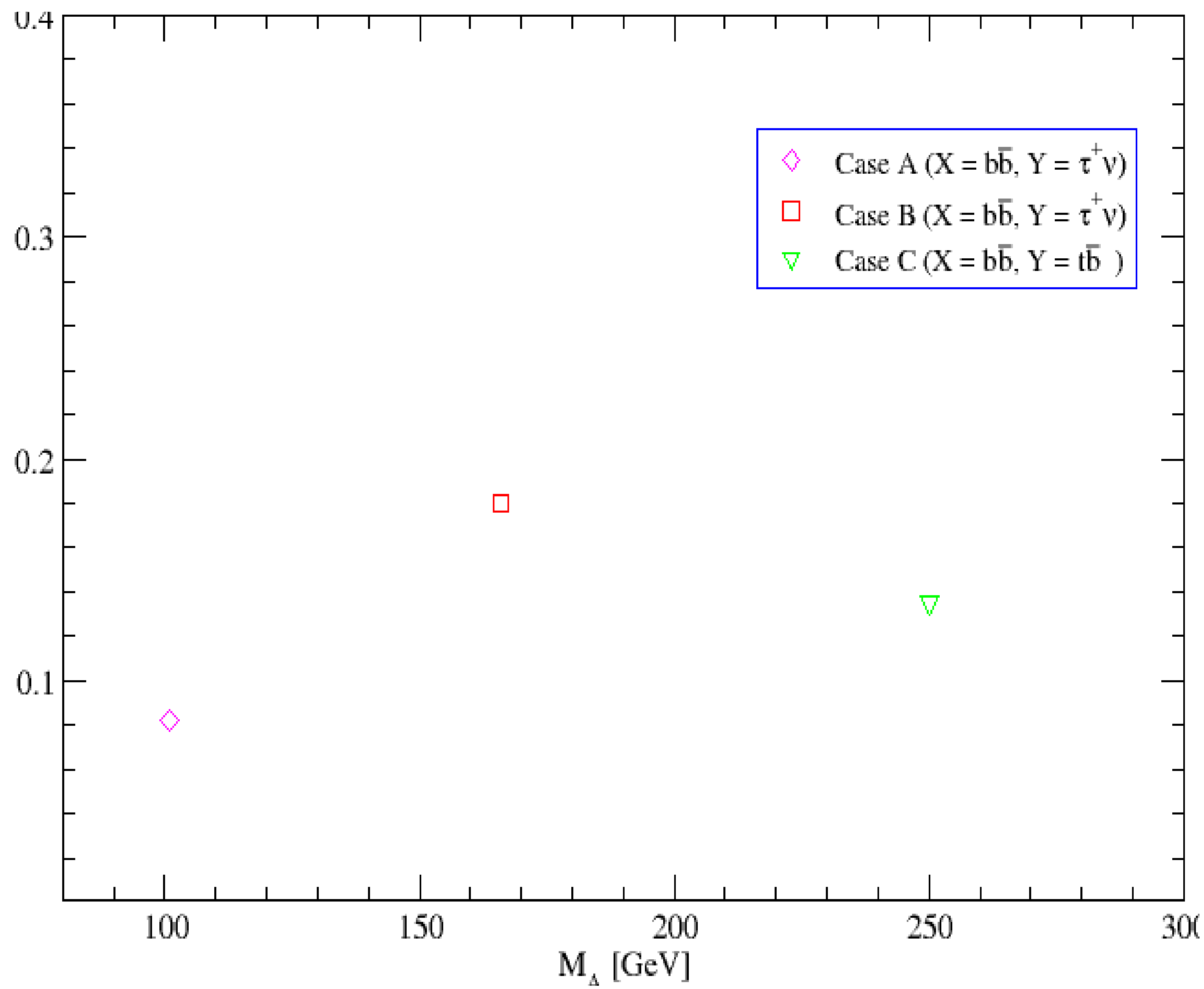
- Transverse mass of H+ after imposing the mass window cut on the two b-jets

$$\left|M(b\bar{b}) - 165\right| < 10 \text{ GeV}$$



Set-B

Constraint on MSSM



Constraints on the product of branching ratios

$$B(A \to b\bar{b}) \times B(H^+ \to \tau^+ \nu_\tau)$$

as a function of M_A for Case A and Case B, and

$$B(A \to b\bar{b}) \times B(H^+ \to t\bar{b})$$

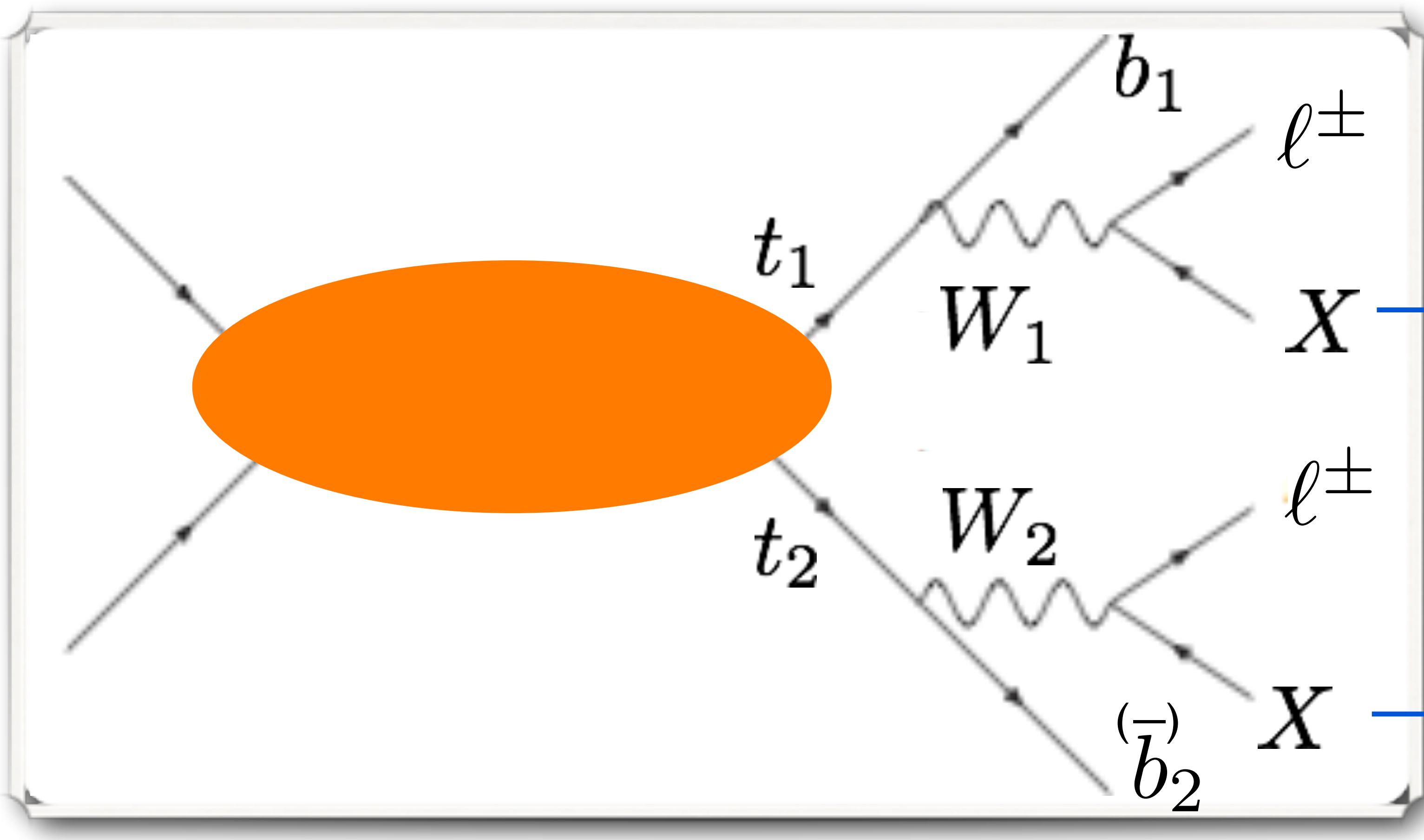
for Case C, at the LHC, where τ^+ decays into $\pi^+ \bar{\nu}_\tau$ channel.

Auxiliary material (I)
Two neutrinos

- *H*⁺: $H^+ \to \tau^+\nu \to \pi^+\nu\bar{\nu} \to \pi^+ \not{E}_T$ spin correlation
- $t\bar{t}$: $t\bar{t} \to b\bar{b}\ell^+\ell'^-\nu\bar{\nu} \to b\bar{b}\ell^+\ell'^- \not{E}_T$ on-shell conditions
- *h* : $h \to W^+W^- \to \ell^+\ell'^-\nu\bar{\nu} \to \ell^+\ell'^- \not{E}_T$ spin correlation
- *h* : $h \to \tau^+\tau^- \to \pi^+\pi^-\nu\bar{\nu} \to \pi^+\pi^- \not{E}_T$ kinematics

t-tbar in dilepton mode

★ Four unknowns and four on-shell conditions



6 unknowns
-2 from MET

$$m_{W_1}^2 = (p_{\mu_1} + p_{\nu_1})^2$$
$$m_{W_2}^2 = (p_{\mu_2} + p_{\nu_2})^2$$
$$m_{t_1}^2 = (p_{W_1} + p_{b_1})^2$$
$$m_{t_2}^2 = (p_{W_2} + p_{b_2})^2$$

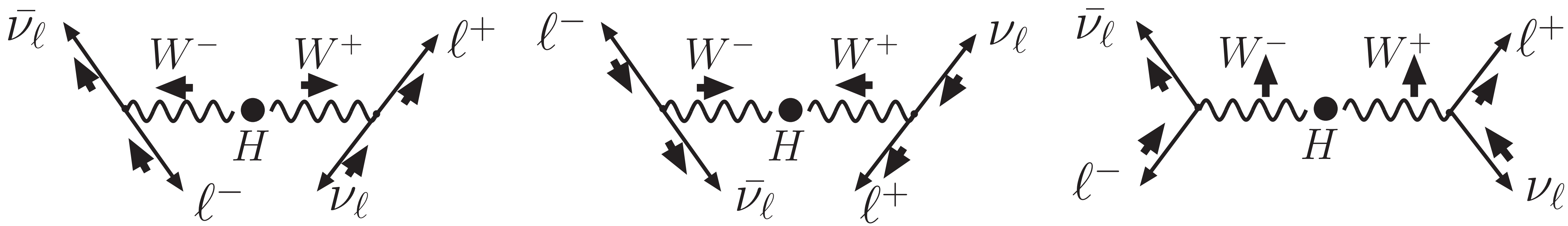
Quartic equation
(correct paring is necessary)

$$p_x^4(\nu_1) + a\ p_x^3(\nu_1) + b\ p_x^2(\nu_1) + c\ p_x(\nu_1) + d = 0$$

~~Two complex~~, two real solutions

Higgs search in WW dilepton mode

- Spin correlation demands both leptons moving in parallel

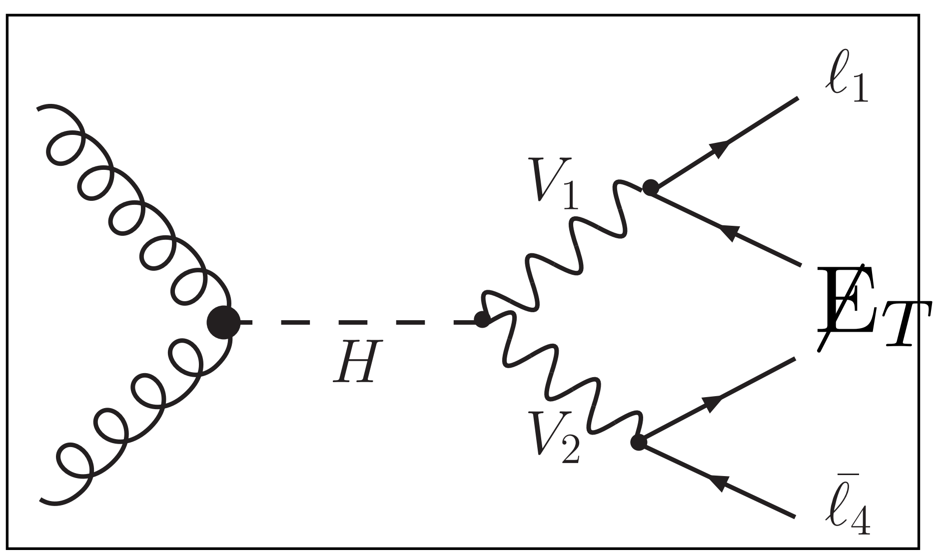


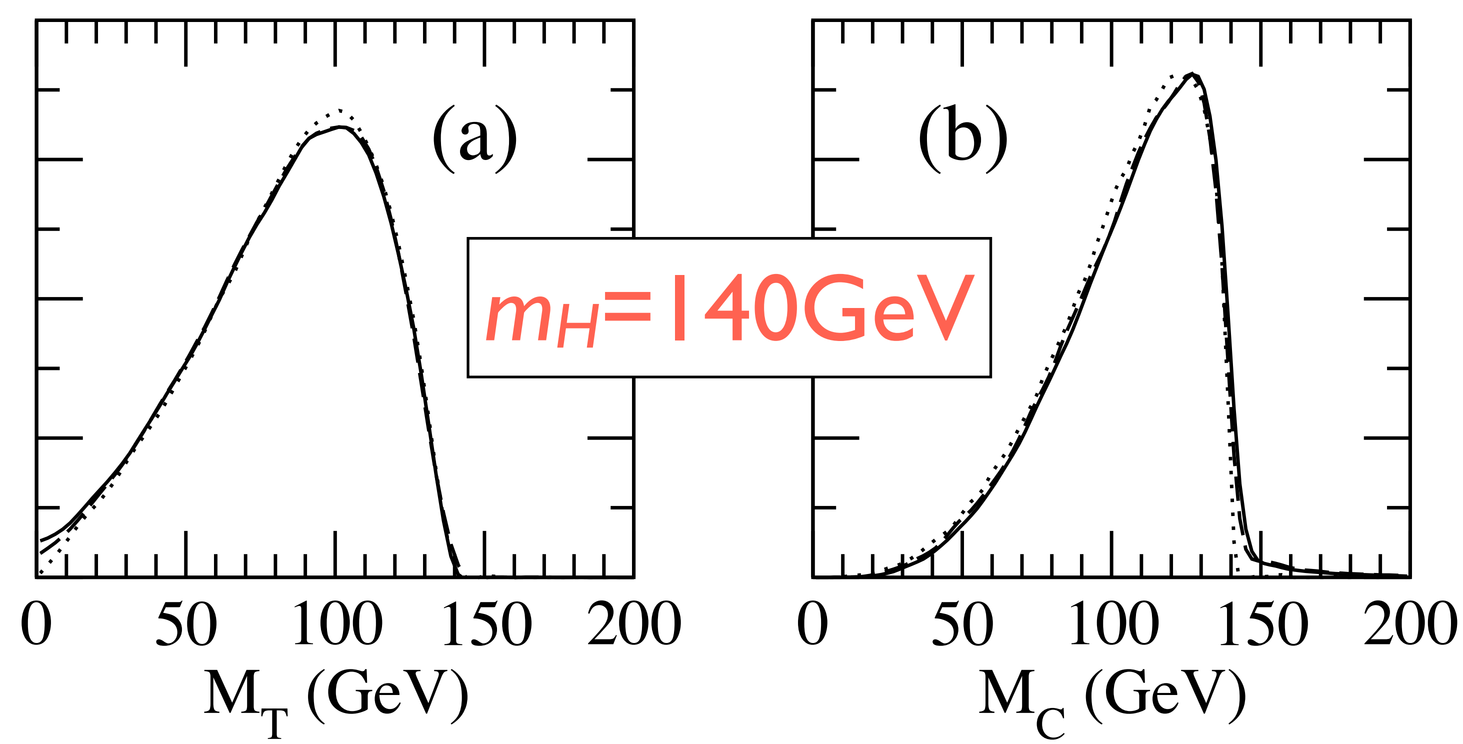
Rainwater, Zeppenfeld,
Phys. Rev. D61 (2000) 093005
Q.-H. Cao and C.-R. Chen,
Phys. Rev. D76 (2007) 075007

- Transverse cluster mass

$$M_T = \sqrt{2 p_T^{LL} \not{E}_T (1 - \cos\Delta\phi(p_T^{LL}, \not{E}_T))},$$

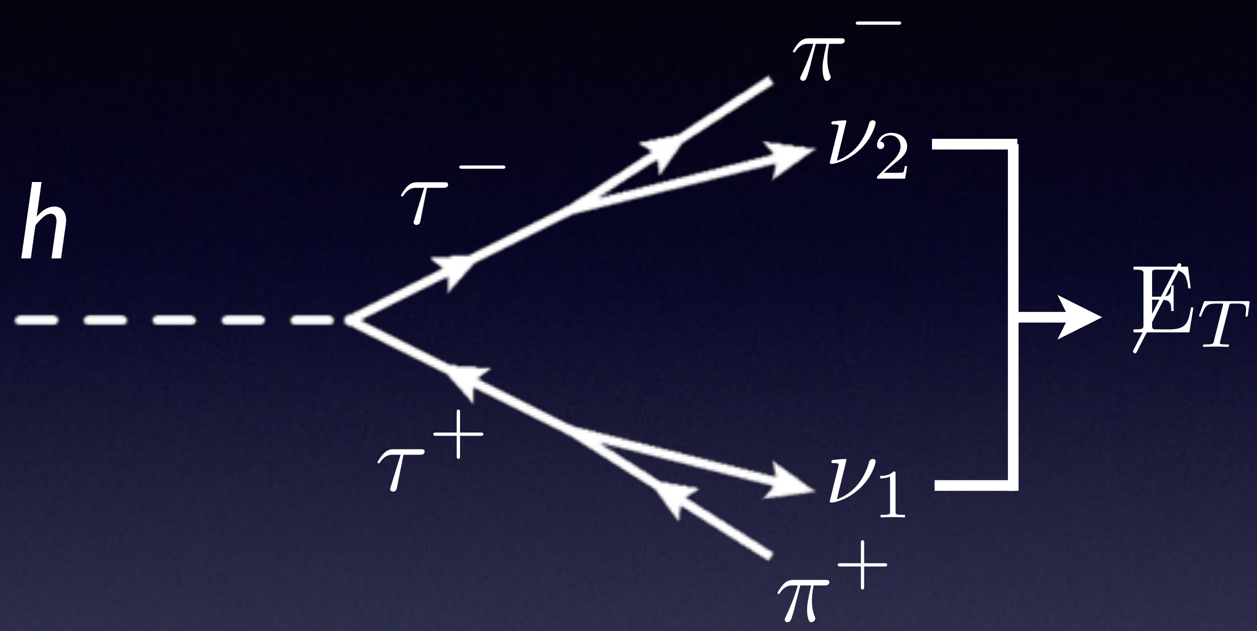
$$M_C = \sqrt{p_T^{LL2} + m_{LL}^2} + \not{E}_T,$$





Higgs search in tau-tau mode

- Collinear approximation

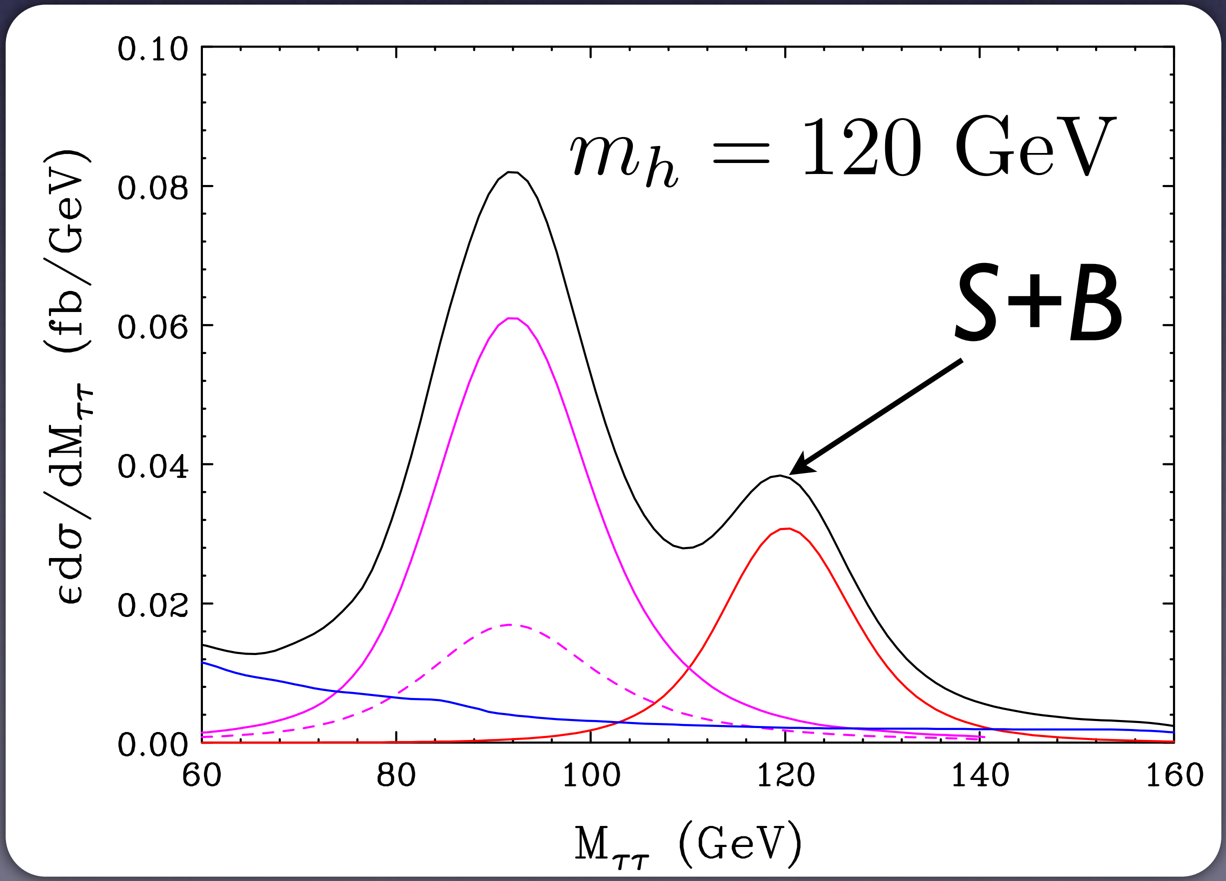


$$p_{\tau^+} = x p_{\pi^+} + (1-x) p_{\nu_1}$$

$$p_{\tau^-} = y p_{\pi^-} + (1-y) p_{\nu_2}$$

$$\not{E}_x = \left(\frac{1}{x} - 1\right) p^x_{\pi^+} + \left(\frac{1}{y} - 1\right) p^x_{\pi^-}$$

$$\not{E}_y = \left(\frac{1}{x} - 1\right) p^y_{\pi^+} + \left(\frac{1}{y} - 1\right) p^y_{\pi^-}$$



further demands

$x > 0, y > 0$

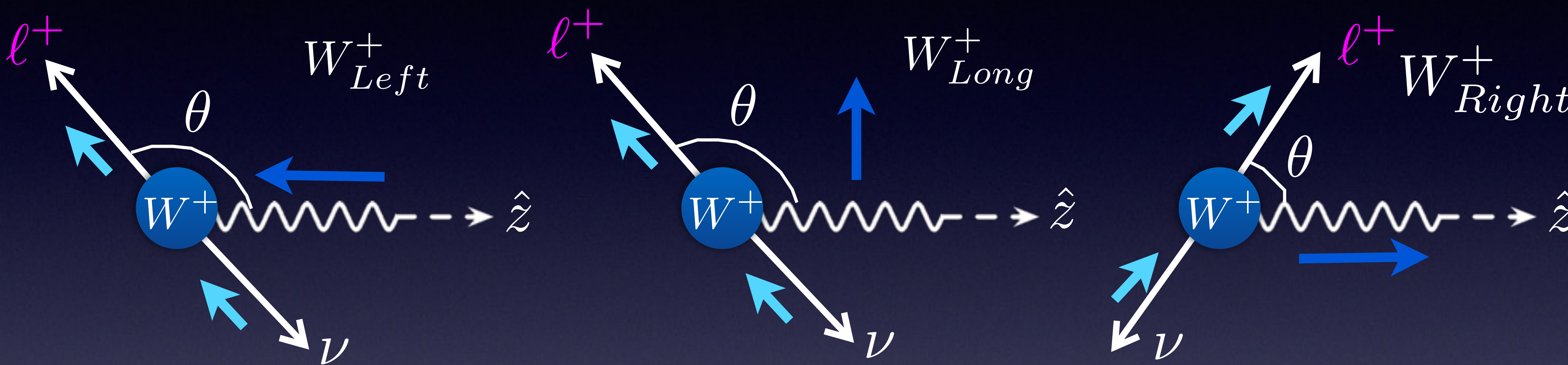
Plehn, Rainwater, Zeppenfeld,
Phys. Rev. D61 (2000) 093005

Auxiliary material (II)

W-boson Helicity as a measure of the chirality structure of the *W-t-b* coupling

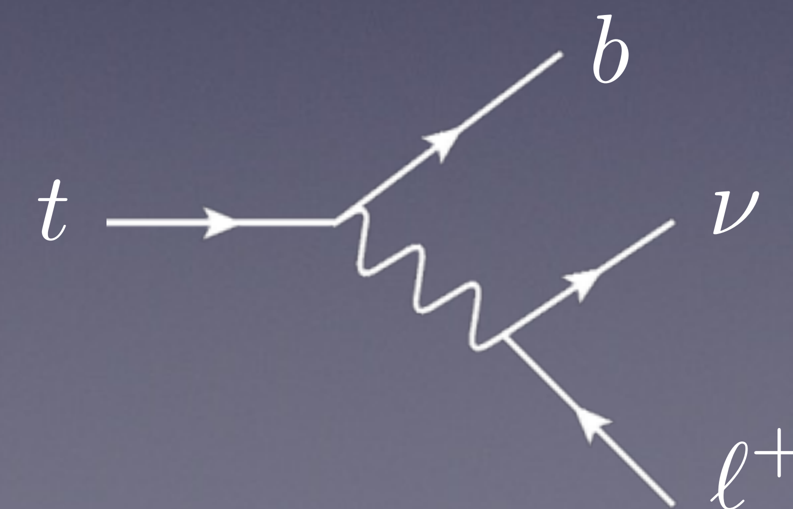
W-boson helicity

- can be measured from the charged-lepton angular distribution



$$d^1_{-1,1}(\theta) = \frac{1-\cos\theta}{2} \qquad d^1_{0,1} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,1}(\theta) = \frac{1+\cos\theta}{2}$$

"V-A" coupling

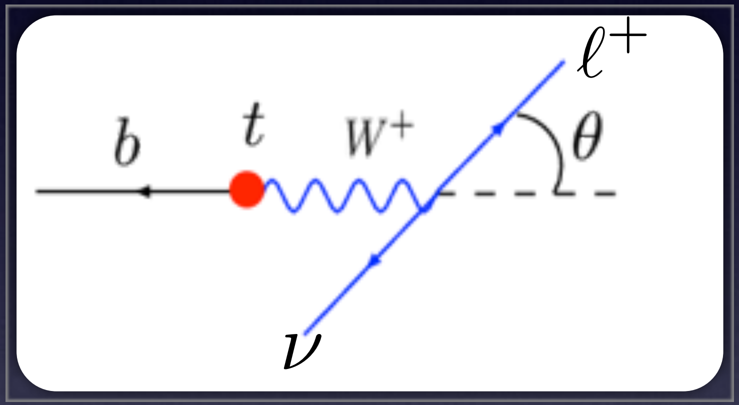


W Helicity from Top Decay

- A good probe of the handness of *W-t-b* coupling

$$\frac{1}{\Gamma_t}\frac{d\Gamma_t}{d\cos\theta} = f_0\frac{3}{4}\sin^2\theta + f_-\frac{3}{8}(1-\cos\theta)^2 + f_+\frac{3}{8}(1+\cos\theta)^2$$

$$\cos\theta \simeq \frac{2m_{be}^2}{m_t^2 - m_W^2} - 1$$

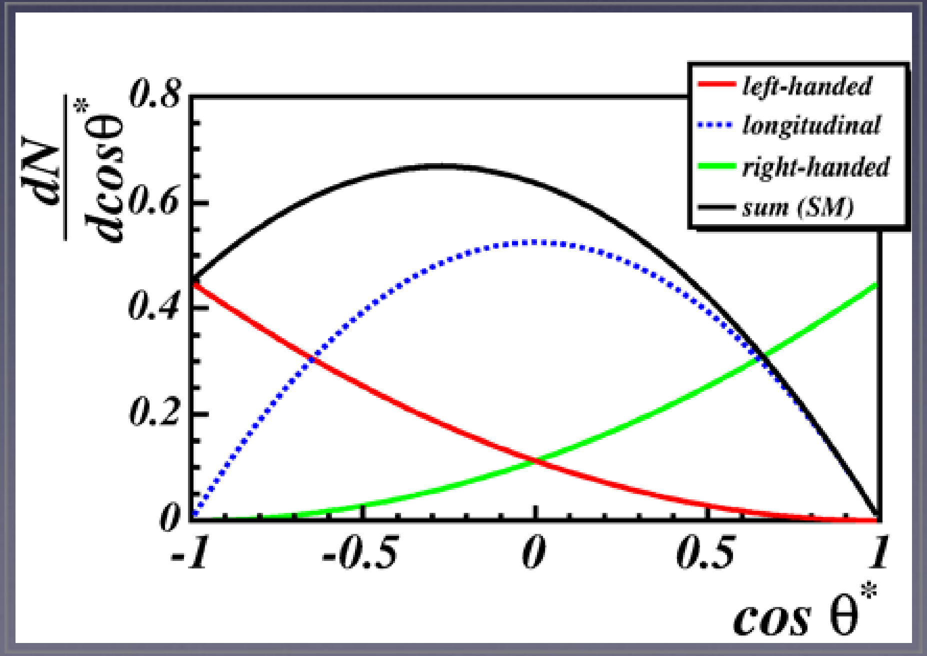


In the SM at the tree level:

$$f_0 = 0.7,\ f_- = 0.3,\ f_+ = 0$$

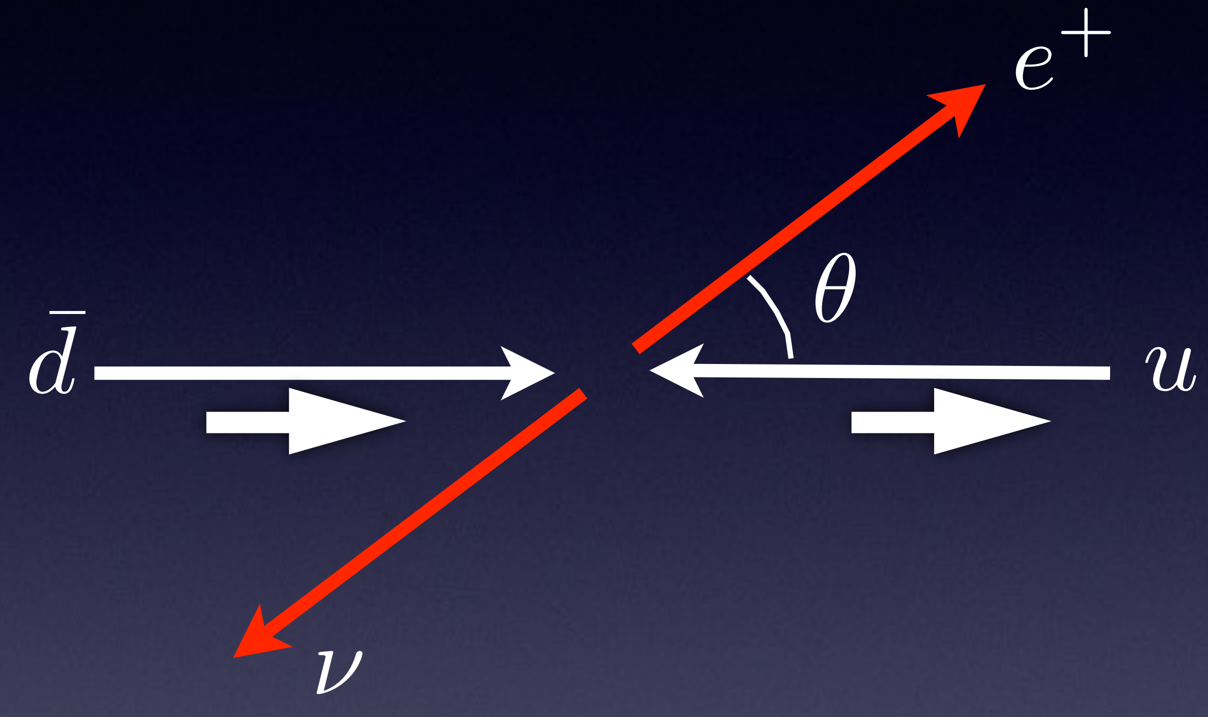
$$f_0 = \frac{\Gamma(t \to bW_{Long})}{\Gamma(t \to bW_{Long}) + \Gamma(t \to bW_T)}$$

$$\simeq \frac{m_t^2}{m_t^2 + 2m_W^2}$$



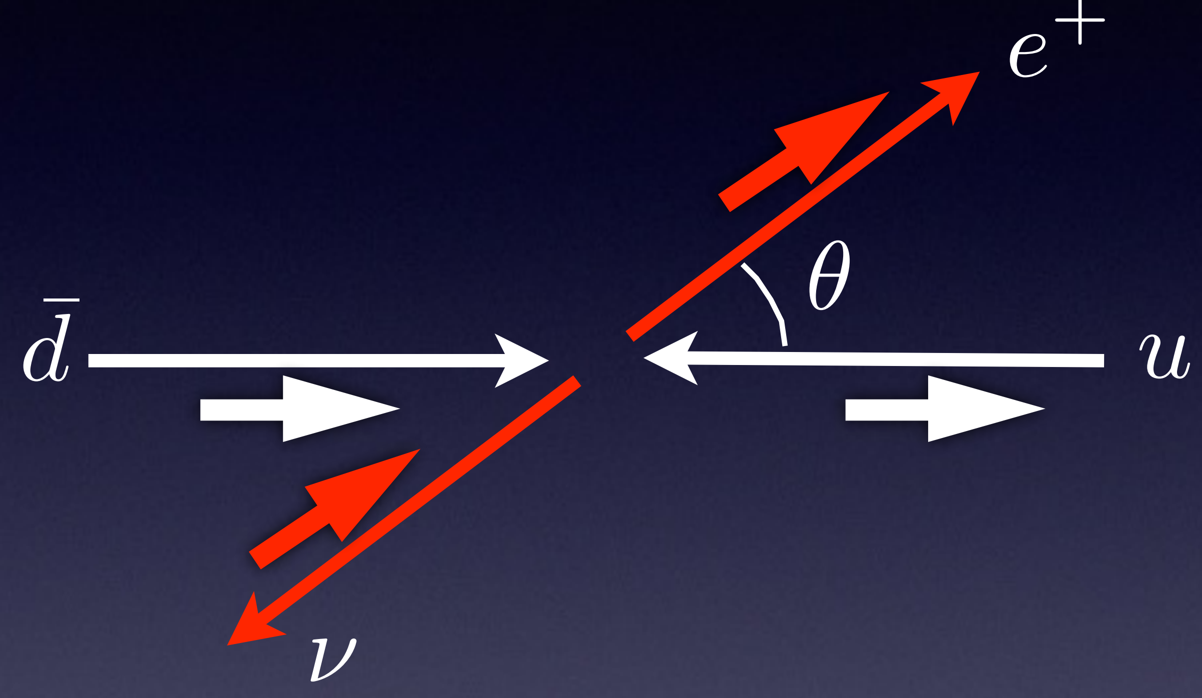
Quiz

- Angular distribution of the Drell-Yan process $u\bar{d} \to e^+\nu$



Quiz

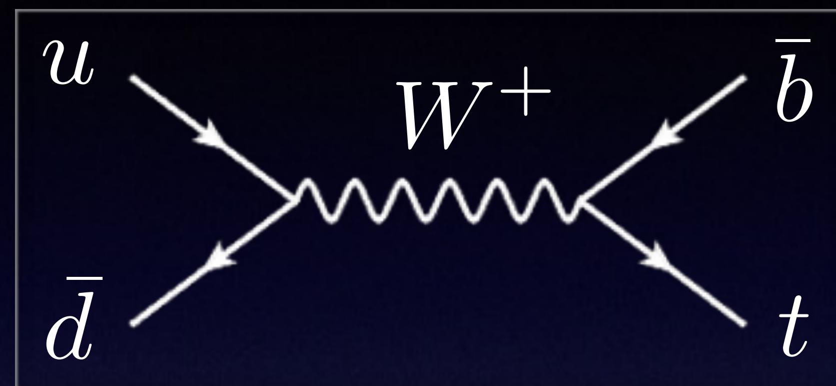
- Angular distribution of the Drell-Yan process $u\bar{d} \to e^+\nu$

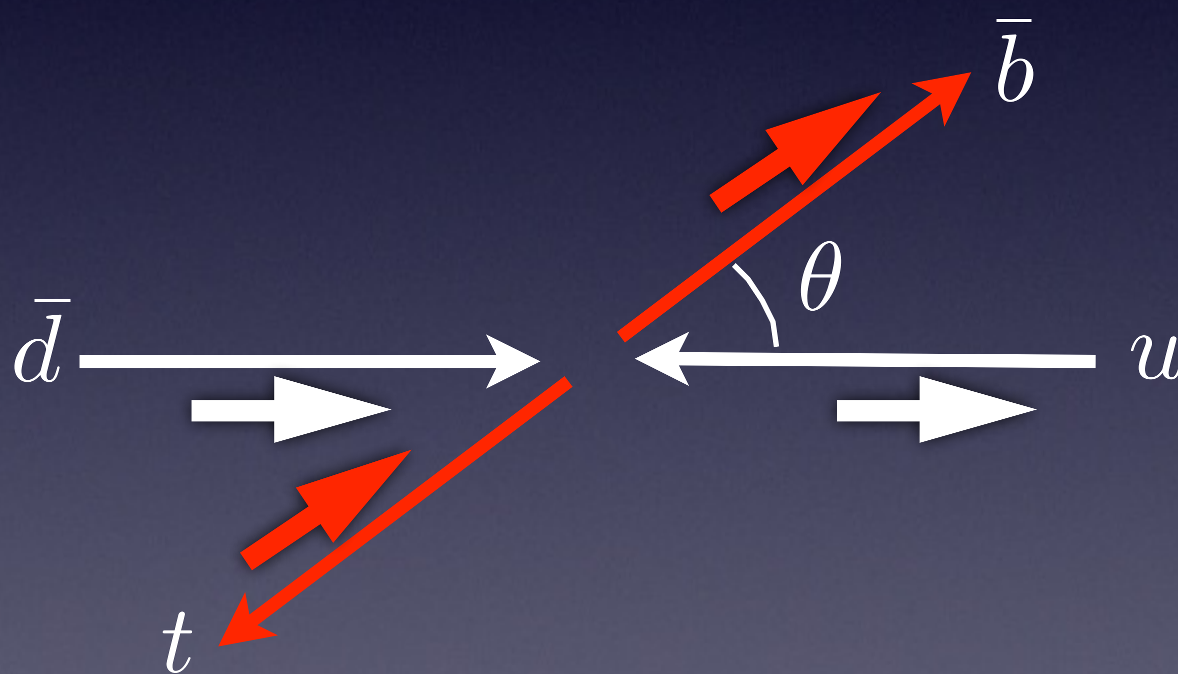


$$d^1_{1,1} = 1 + \cos\theta$$

Quiz

- Angular distribution of the single-top process $u\bar{d} \to t\bar{b}$

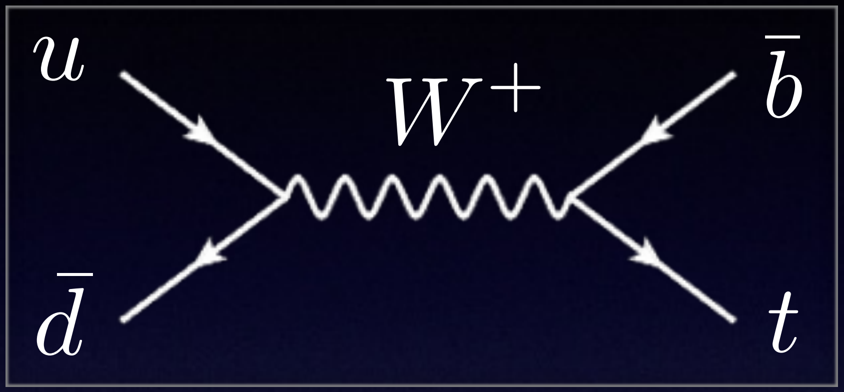


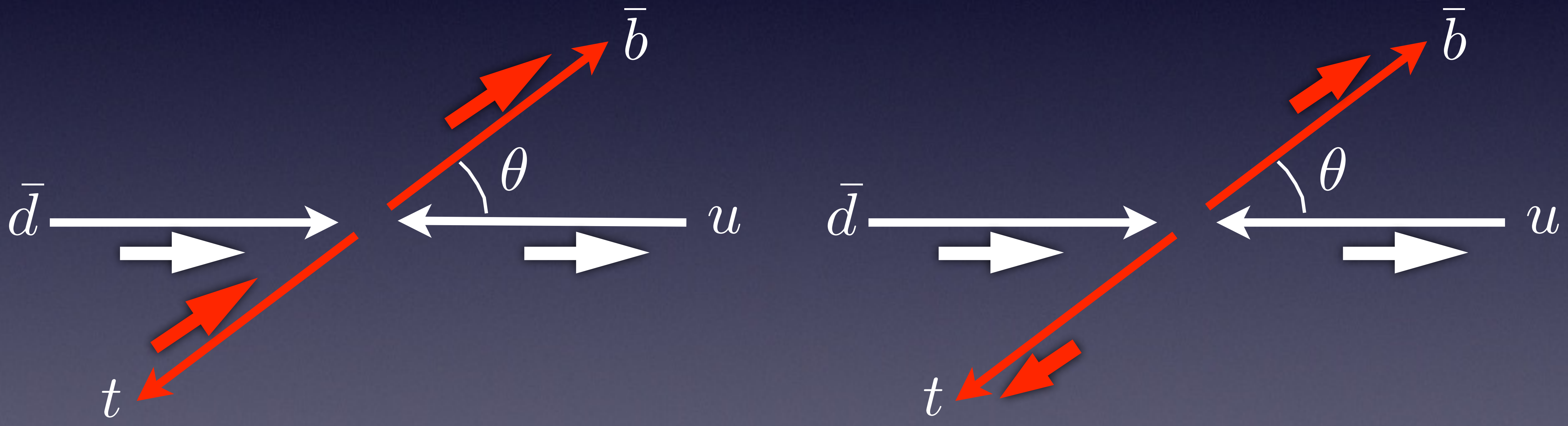


$$d^1_{1,1} = 1 + \cos\theta$$

Quiz

- Angular distribution of the single-top process $u\bar{d} \to t\bar{b}$

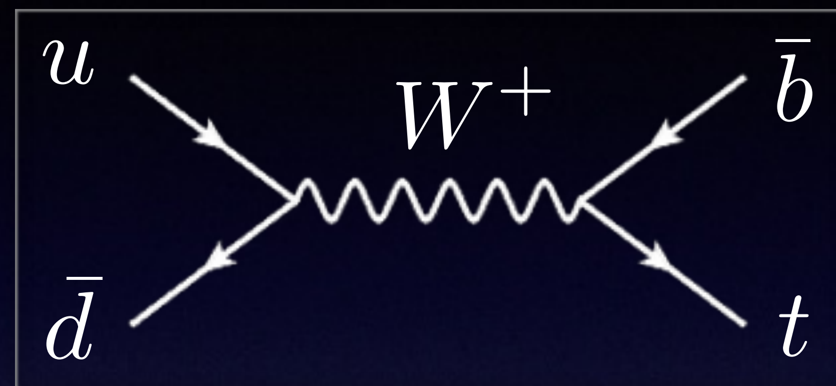


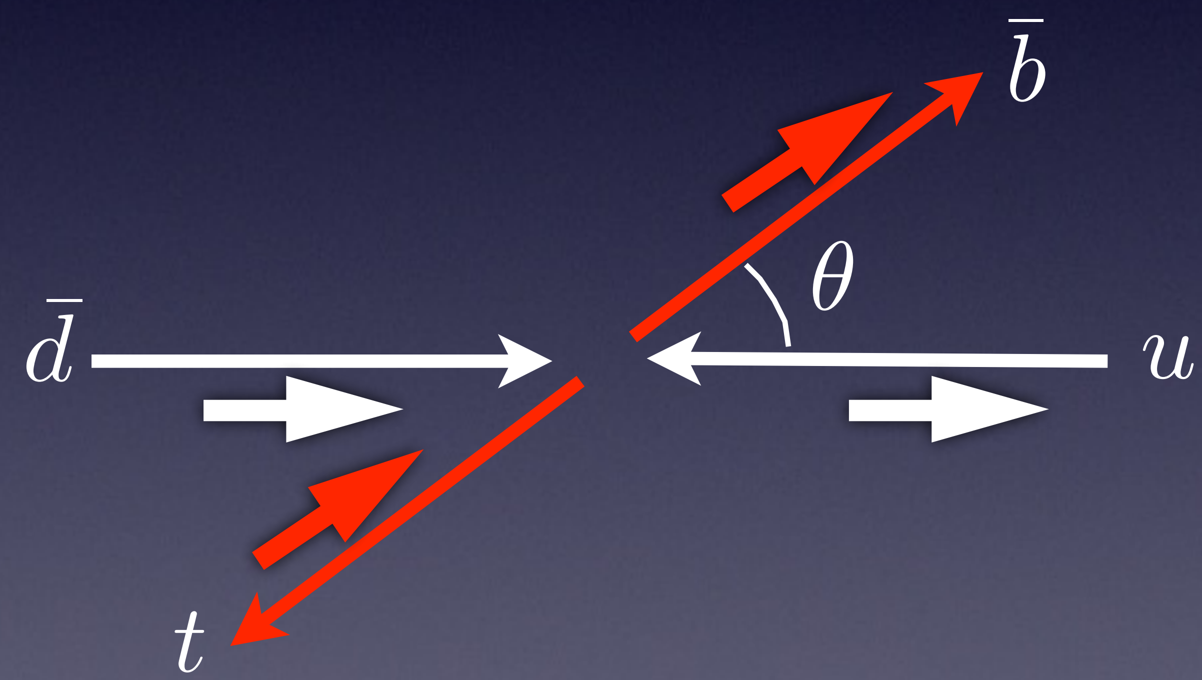


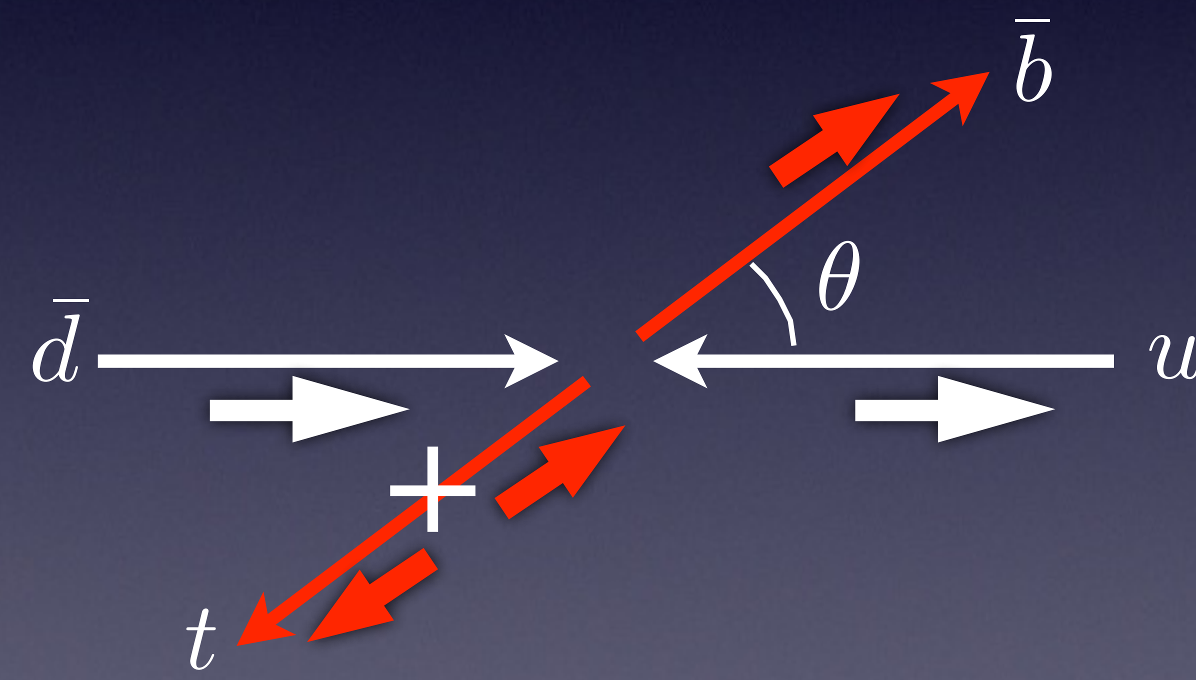
$$d^1_{1,1} = 1 + \cos\theta \qquad d^1_{1,0} = \sin\theta$$

Quiz

- Angular distribution of the single-top process $u\bar{d} \to t\bar{b}$







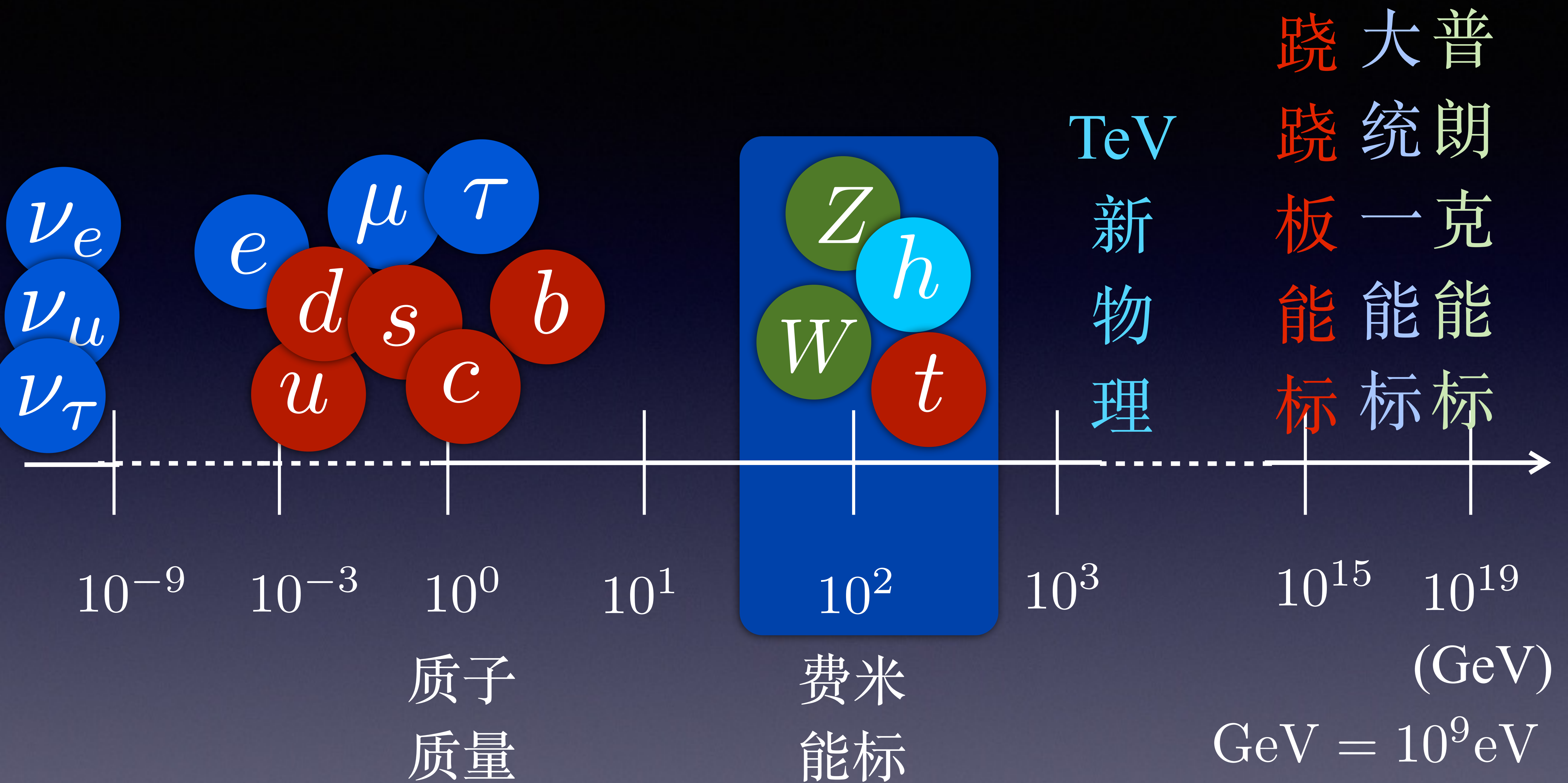
$$d^1_{1,1} = 1 + \cos\theta$$

$$d^1_{1,0} = \sin\theta$$

$$\mathcal{M} \propto m_t$$

Top-Quark Physics

标准模型



Top Quark and Higgs Boson Phenomenology

Why top-quark?

$$\mathcal{L} = \left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) - \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

$$+ y_f \bar{F}_L \Phi f_r + \cdots$$

1995

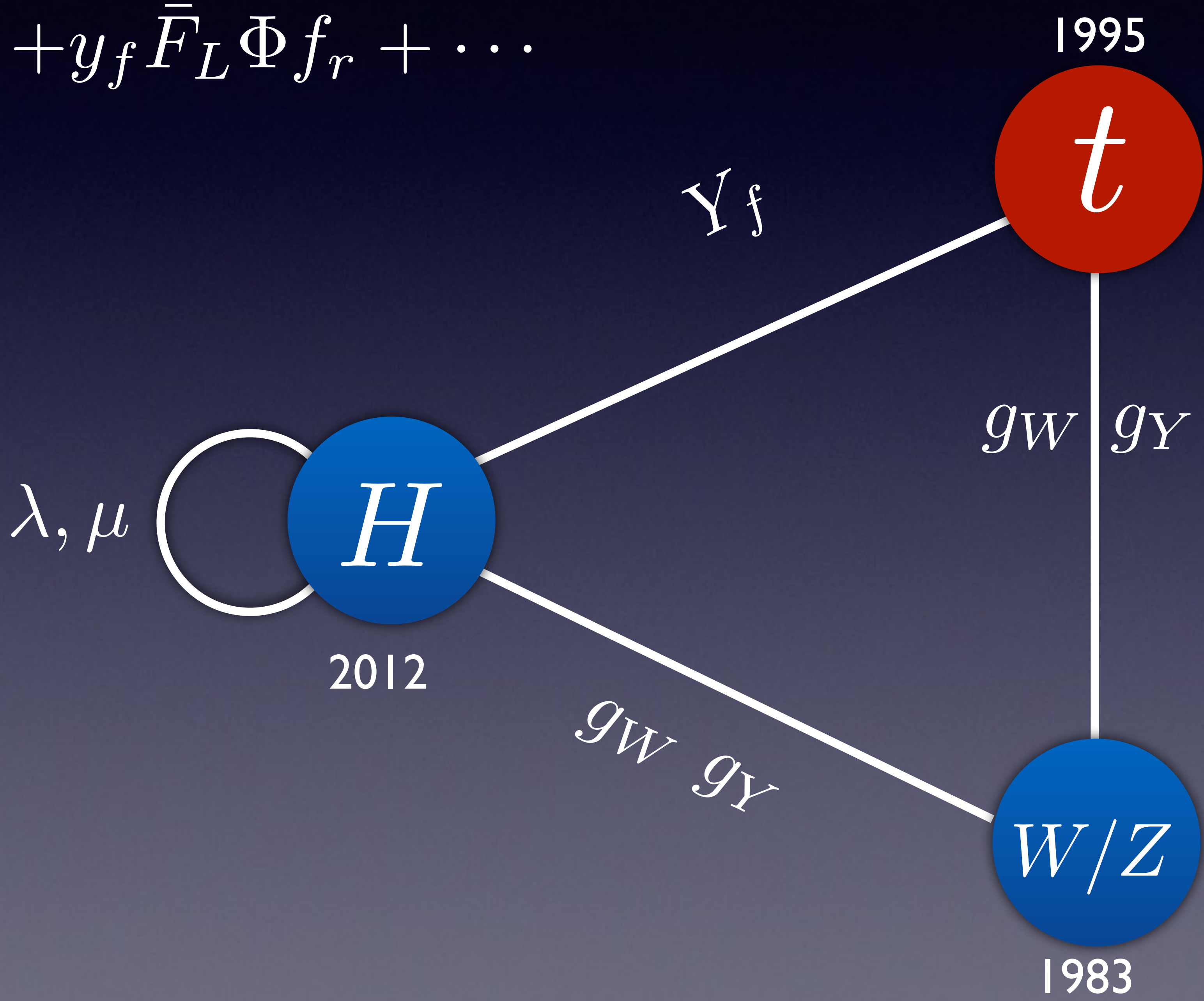
2012

1983

Why top-quark?

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

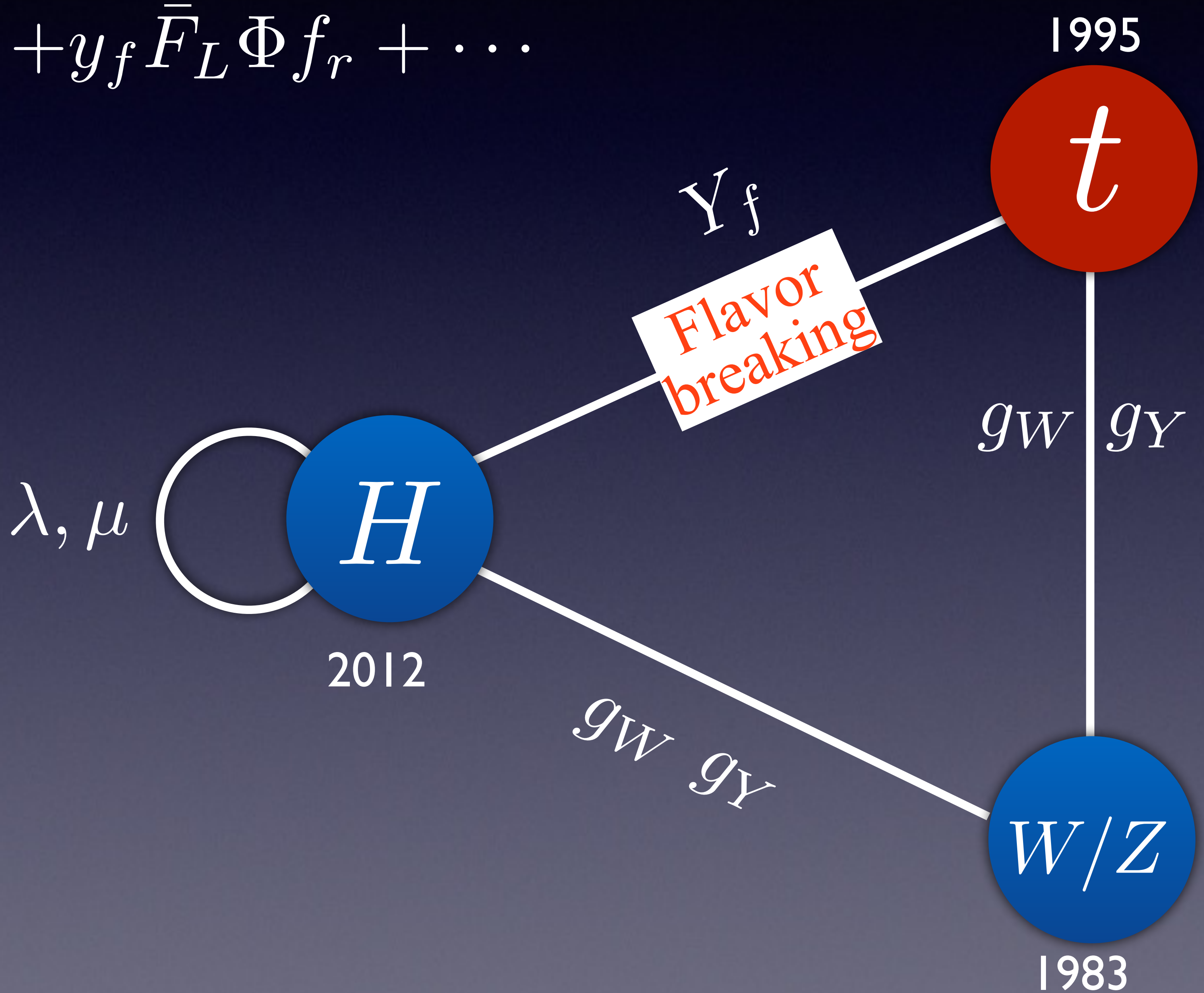
$$+ y_f \bar{F}_L \Phi f_r + \cdots$$

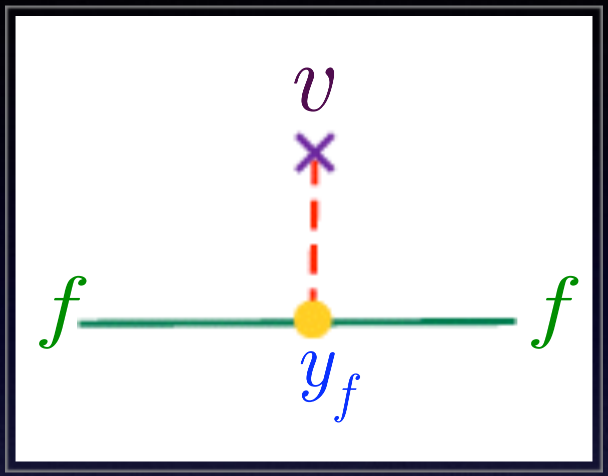


Why top-quark?

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

$$+y_f \bar{F}_L \Phi f_r + \cdots$$

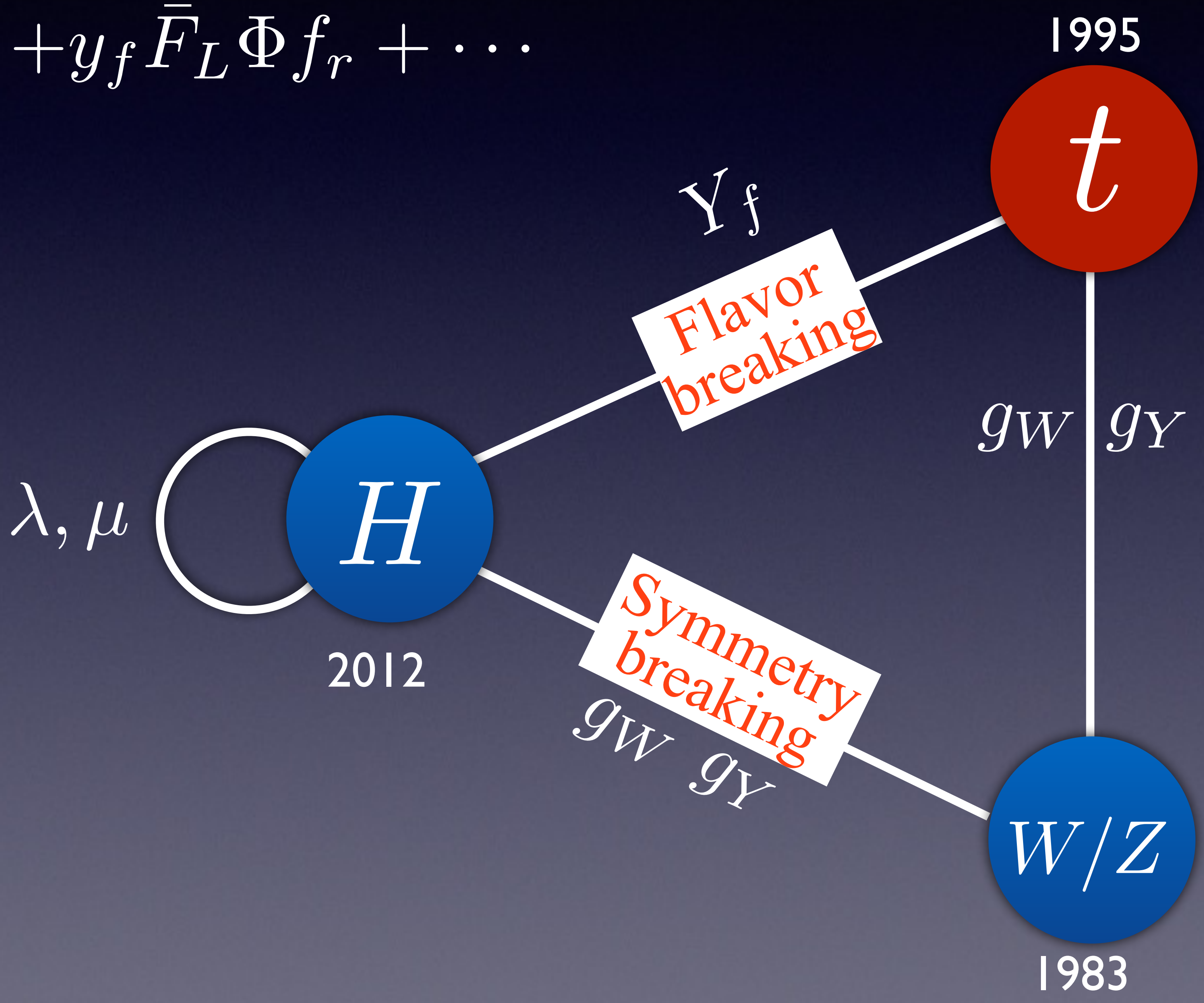


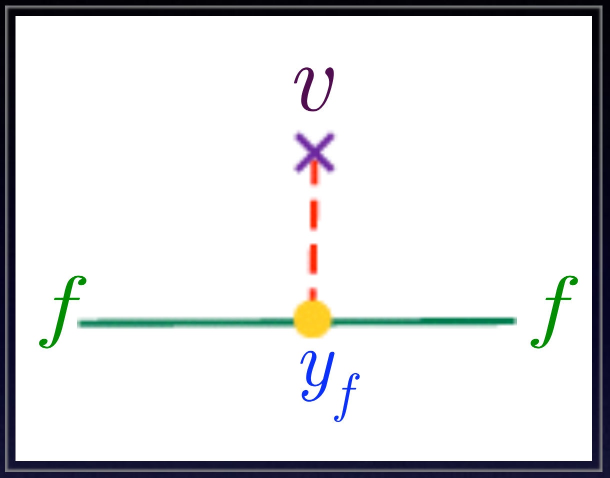


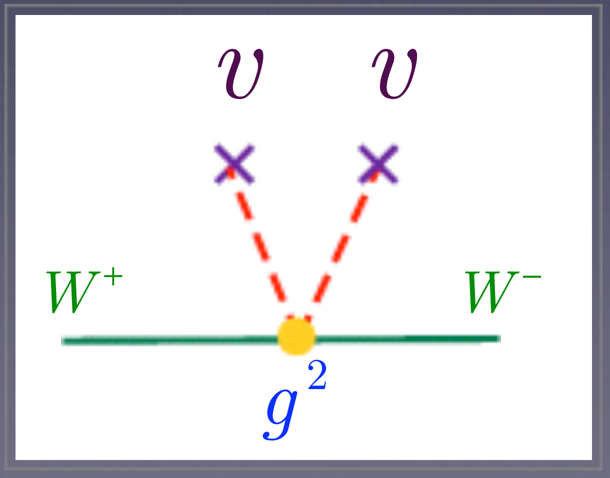
Why top-quark?

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

$$+ y_f \bar{F}_L \Phi f_r + \cdots$$



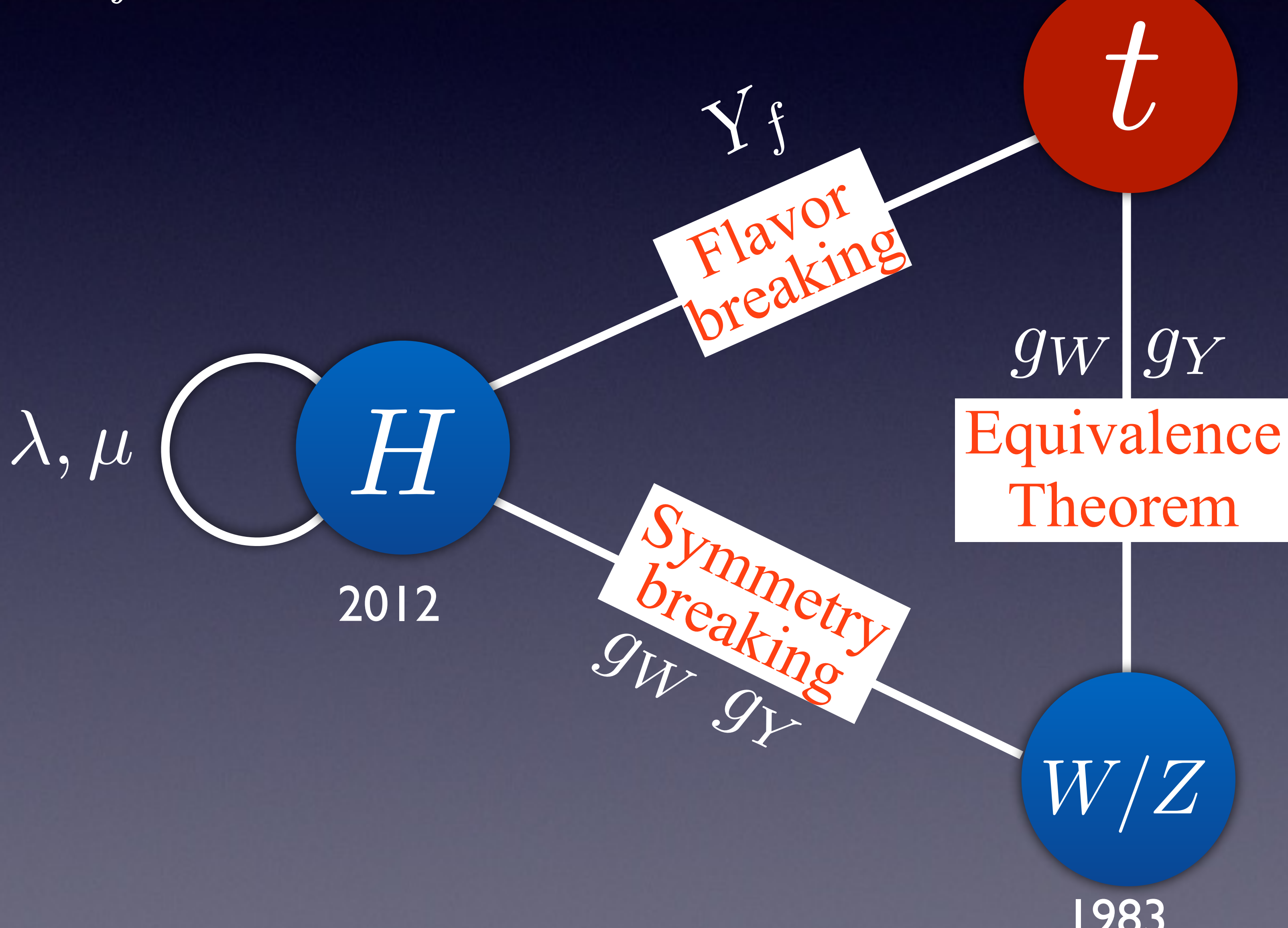


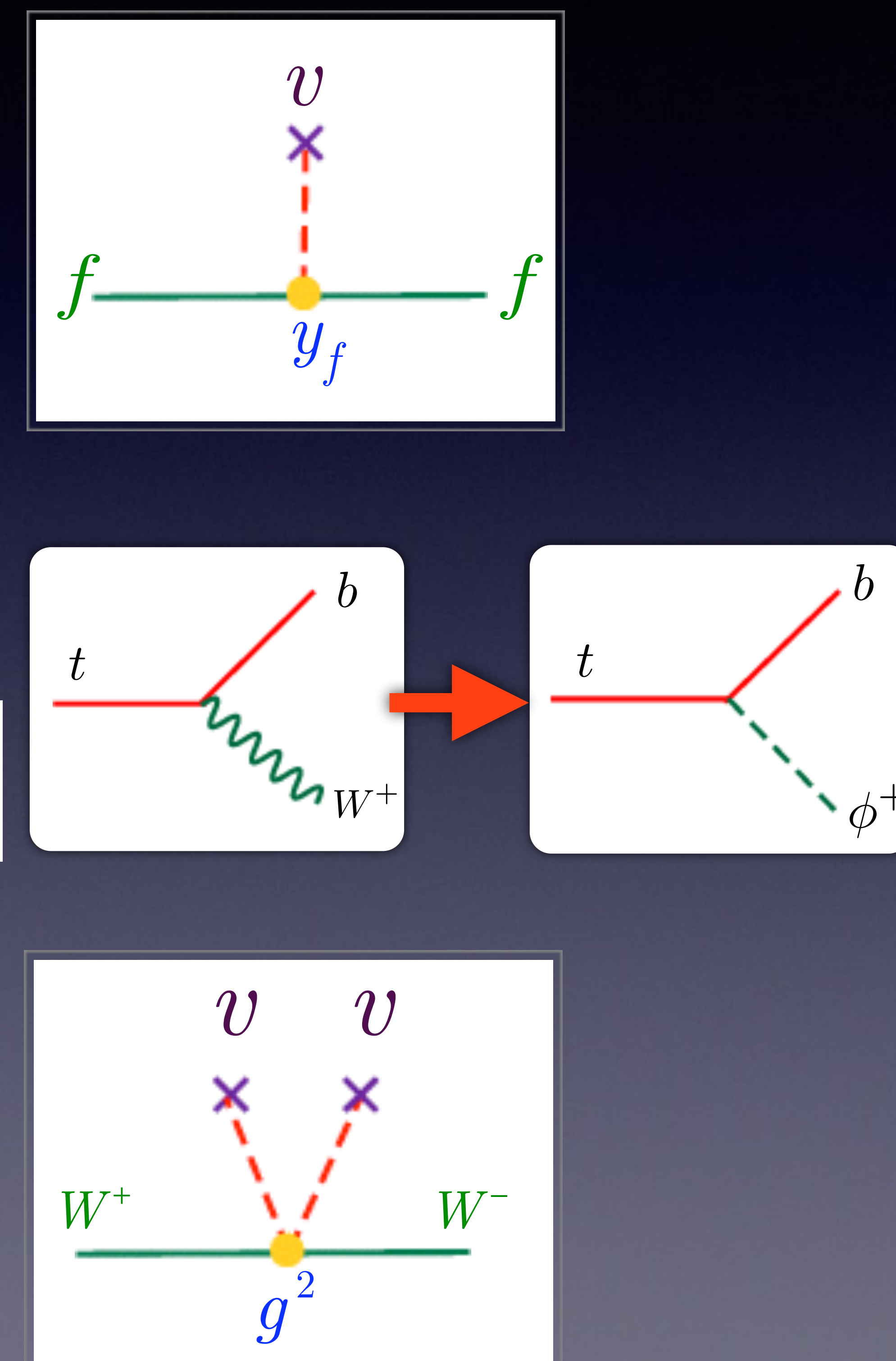


Why top-quark?

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

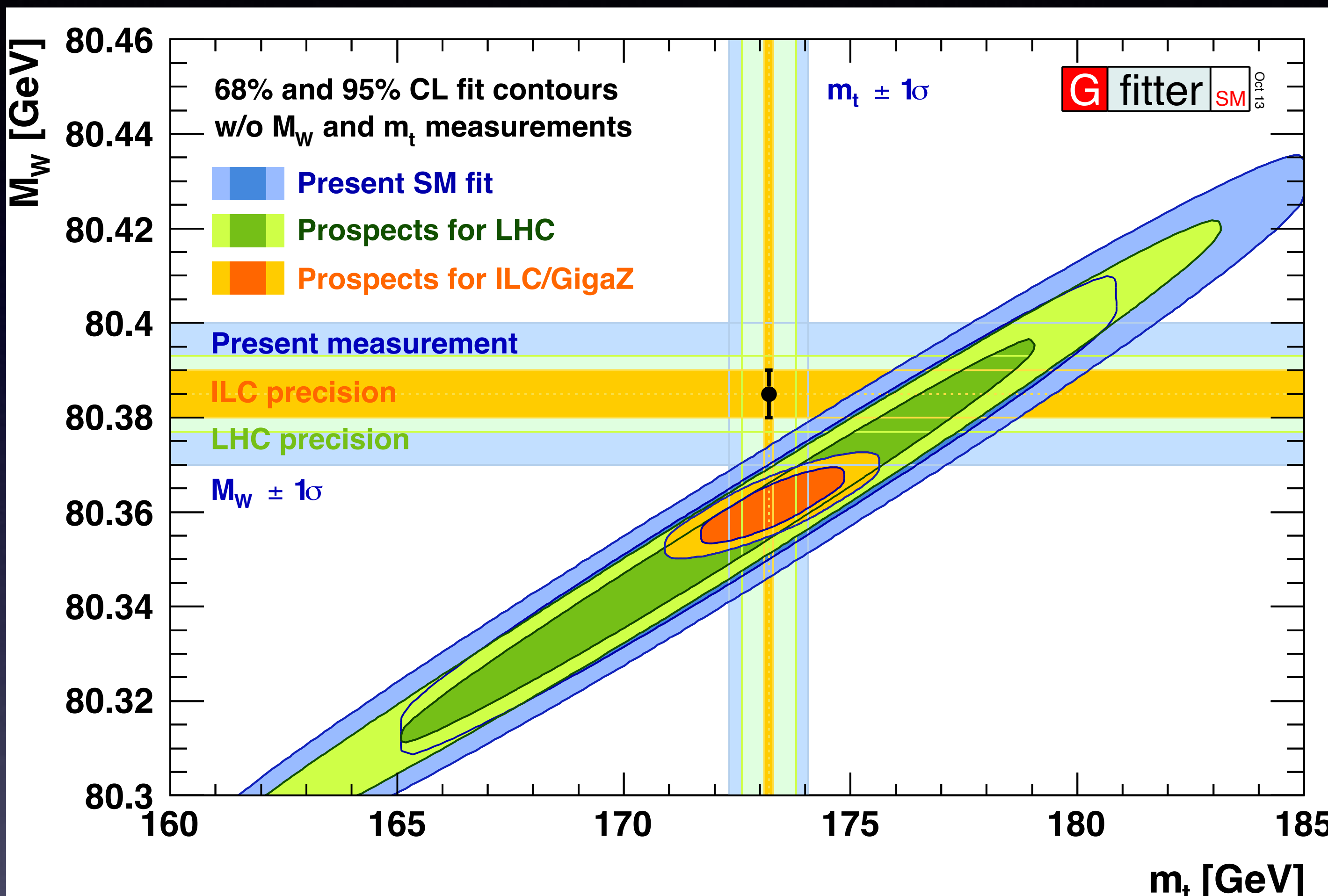
$$+ y_f \bar{F}_L \Phi f_r + \cdots$$

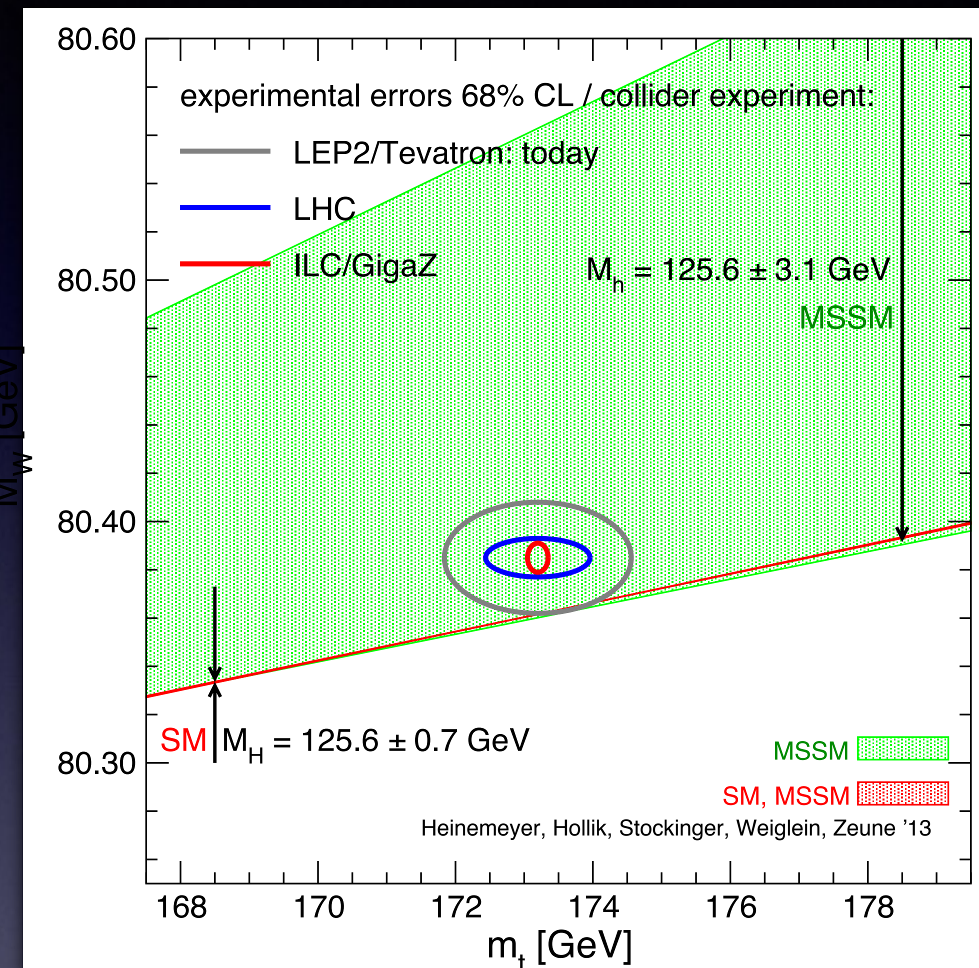




Mass Precision

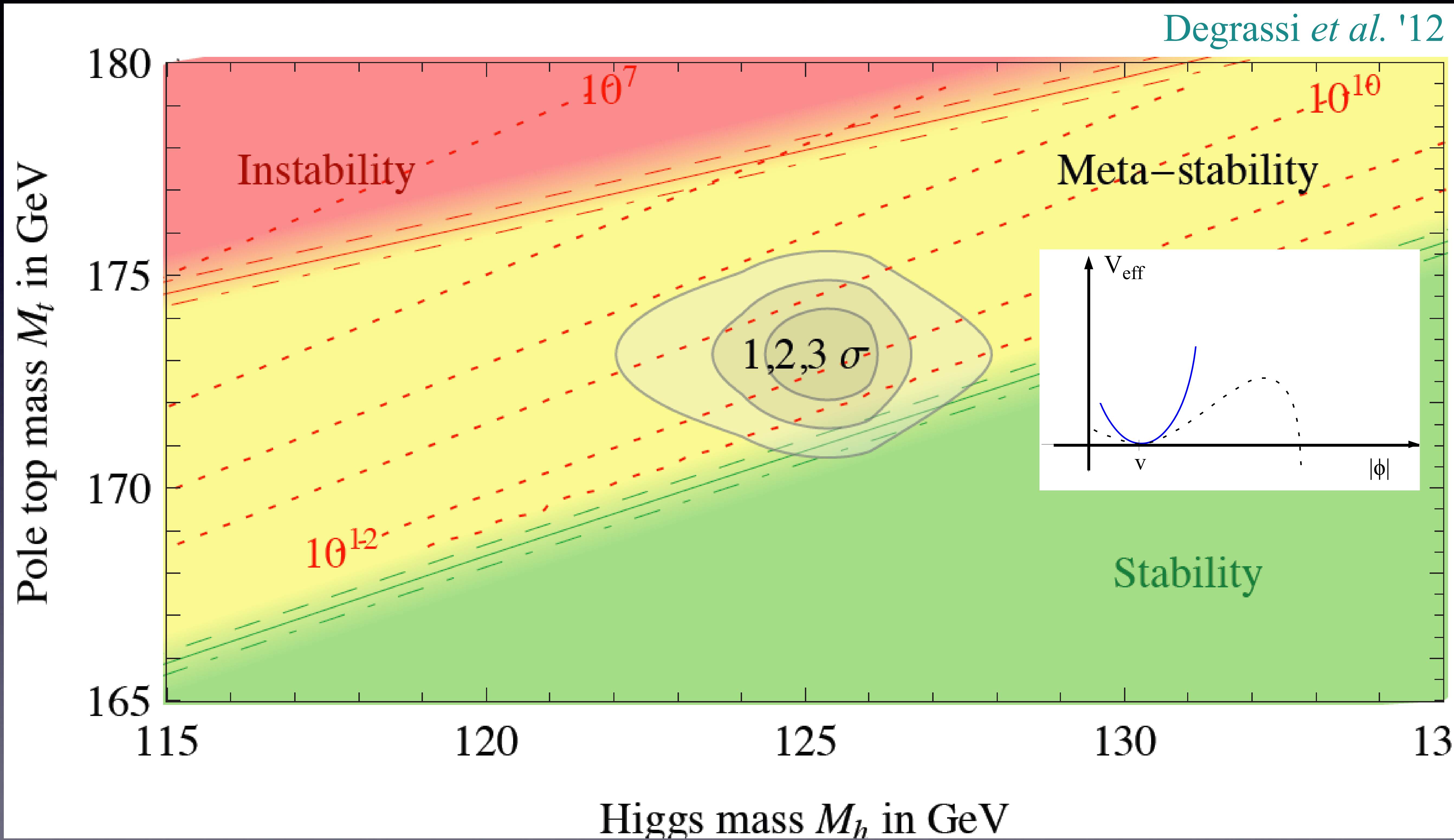
Top-quark, W-boson and Higgs boson





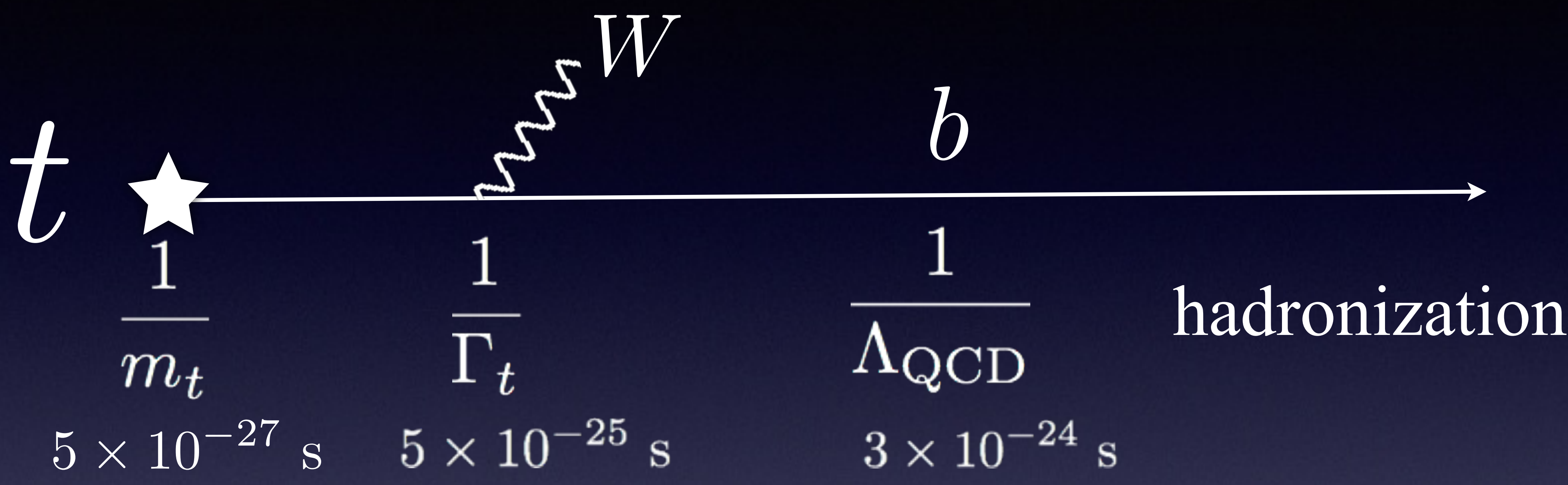
	LHC	LHC	ILC/GigaZ	ILC	ILC	ILC	TLEP	SM prediction
$\sqrt{s}$ [TeV]	14	14	0.091	0.161	0.161	0.250	0.161	-
$\mathcal{L}[\text{fb}^{-1}]$	300	3000		100	480	500	3000×4	-
ΔM_W [MeV]	8	5	-	4.1-4.5	2.3-2.9	2.8	< 1.2	4.2(3.0)
$\Delta \sin^2\theta^{\ell}_{\text{eff}}$ $[10^{-5}]$	36	21	1.3	-	-	-	0.3	3.0(2.6)

Top-Quark Mass vs Higgs-Boson Mass



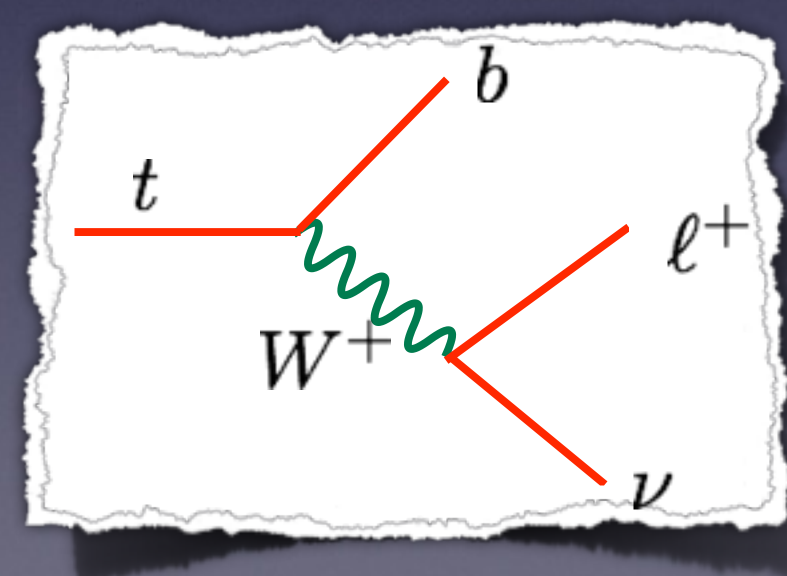
Top-quark: the only bare quark in SM

- Short lifetime:



- "bare" quark :

  spin info well kept
  among its decay products



Charged lepton: the top-spin analyzer

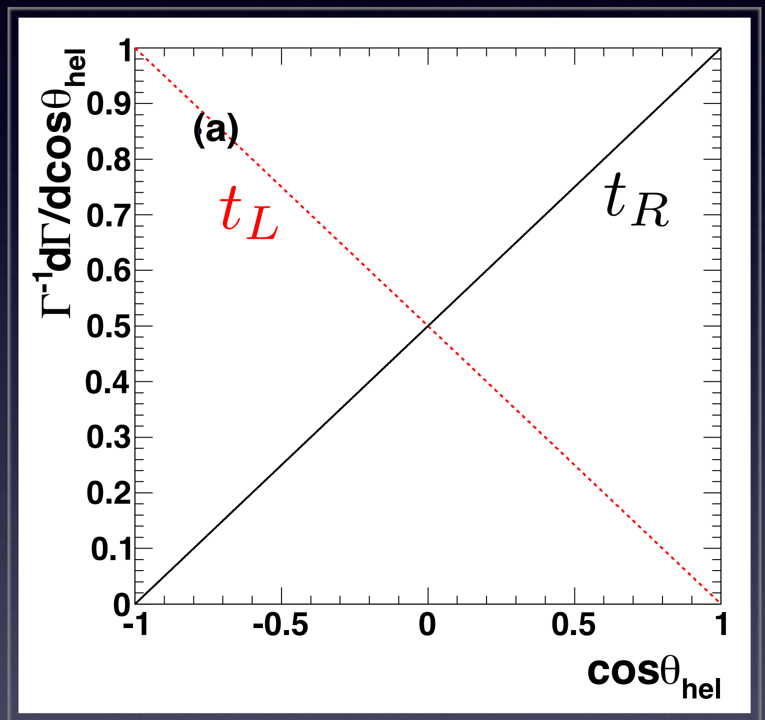
- The charged-lepton tends to *follow* the top-quark spin direction.

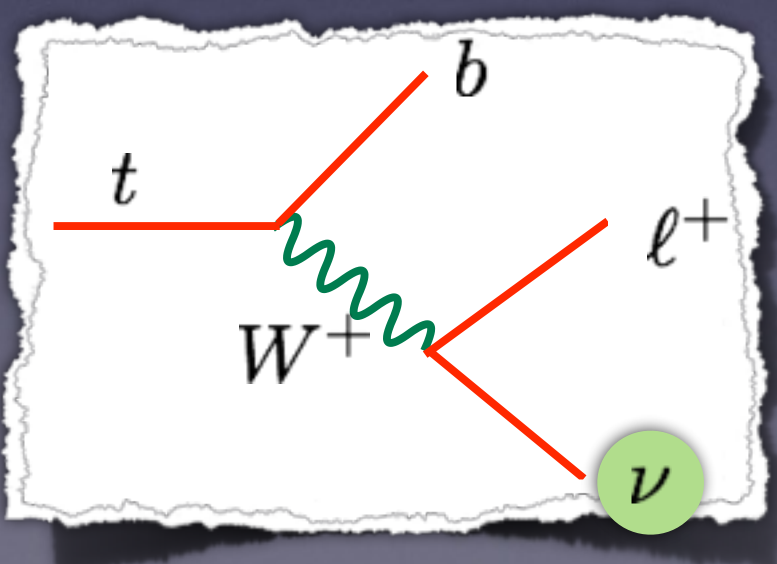
- In top-quark rest frame

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_{\rm hel}} = \frac{1+\lambda_t\cos\theta_{\rm hel}}{2}$$

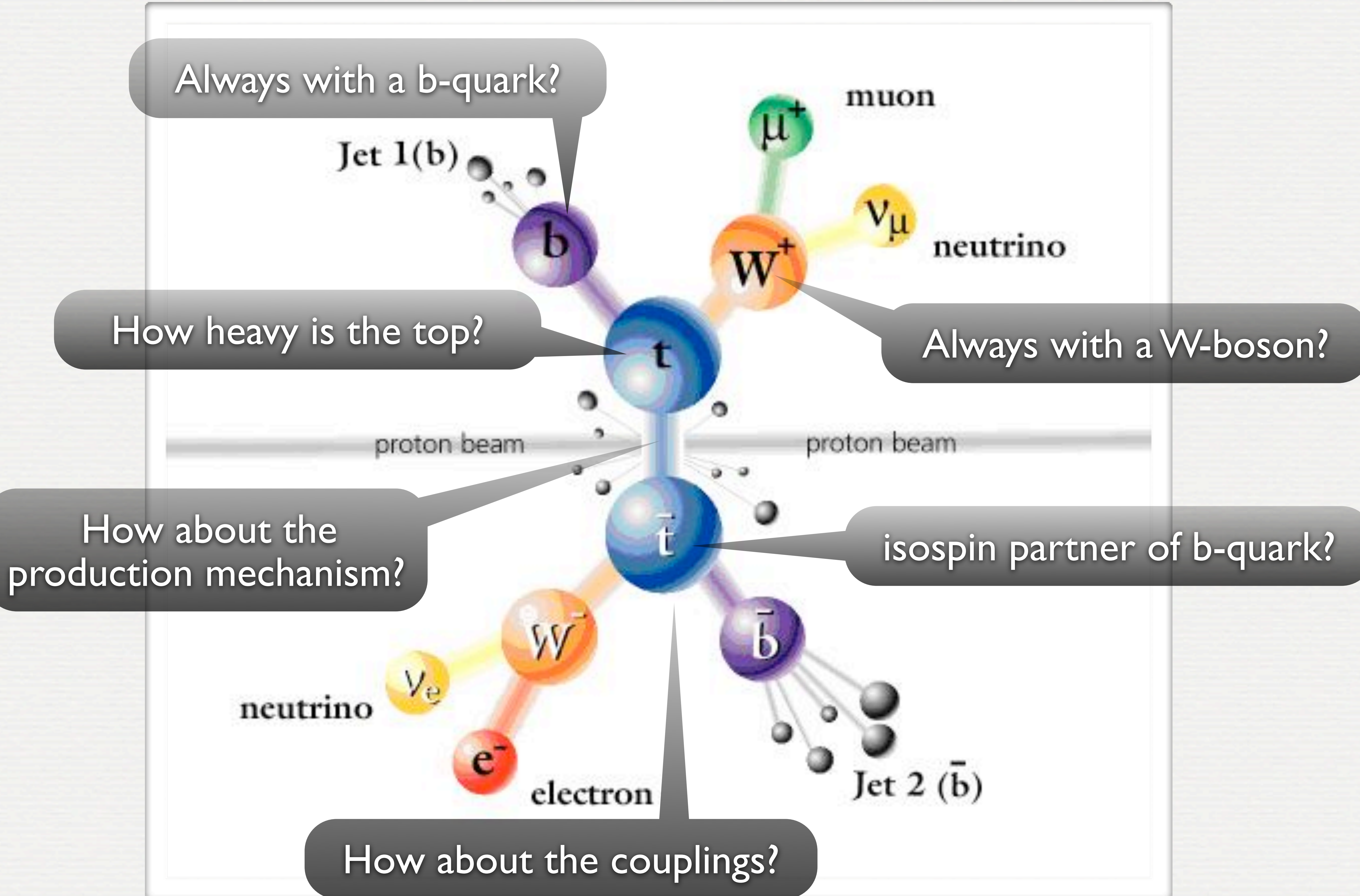
$\lambda_t = +$ right-handed

$\lambda_t = -$ left-handed



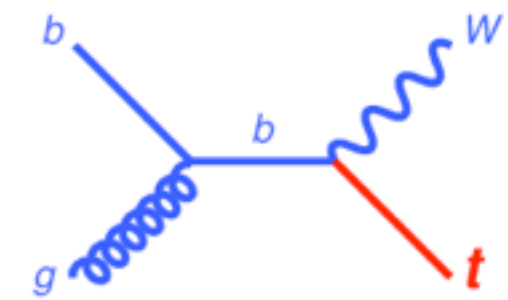


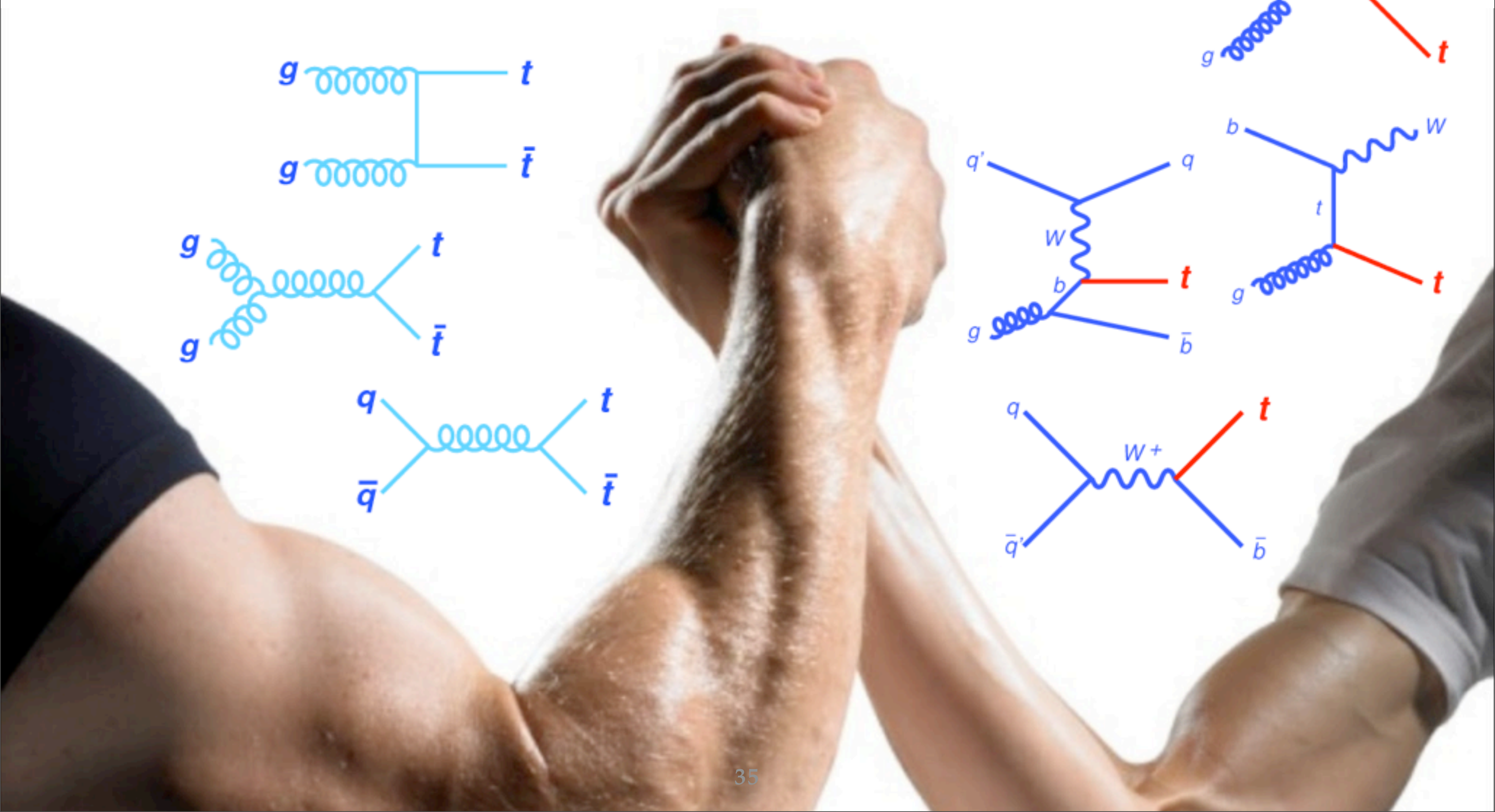
TOP PHYSICS AT LHC



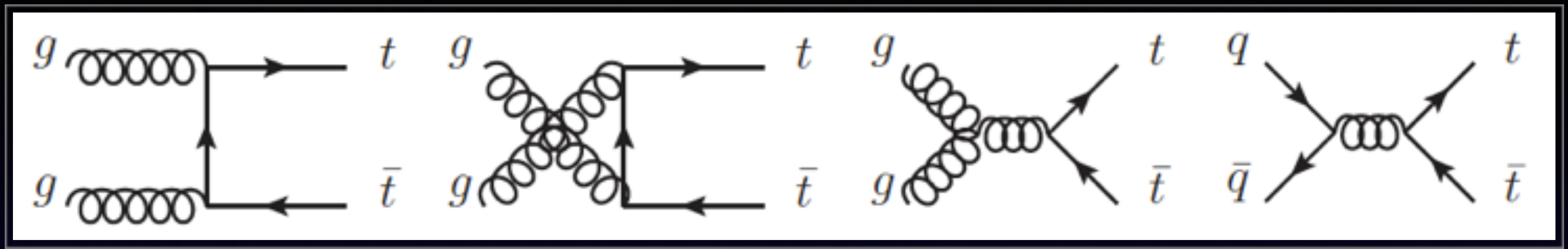
TOP PRODUCTIONS

Either **STRONG** or **ELECTROWEAK**



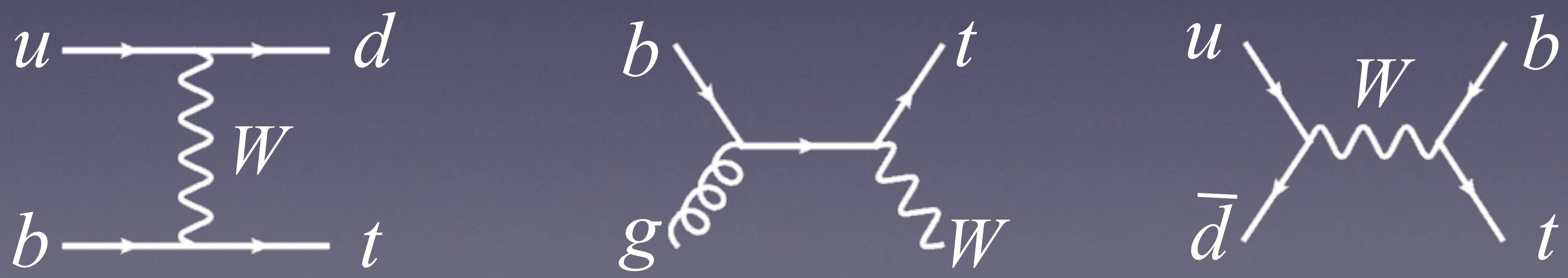


Top pair production in the SM



Tevatron:	10%	**90%**
LHC (7TeV):	80%	20%
LHC (14TeV):	**90%**	10%

Single top production in the SM

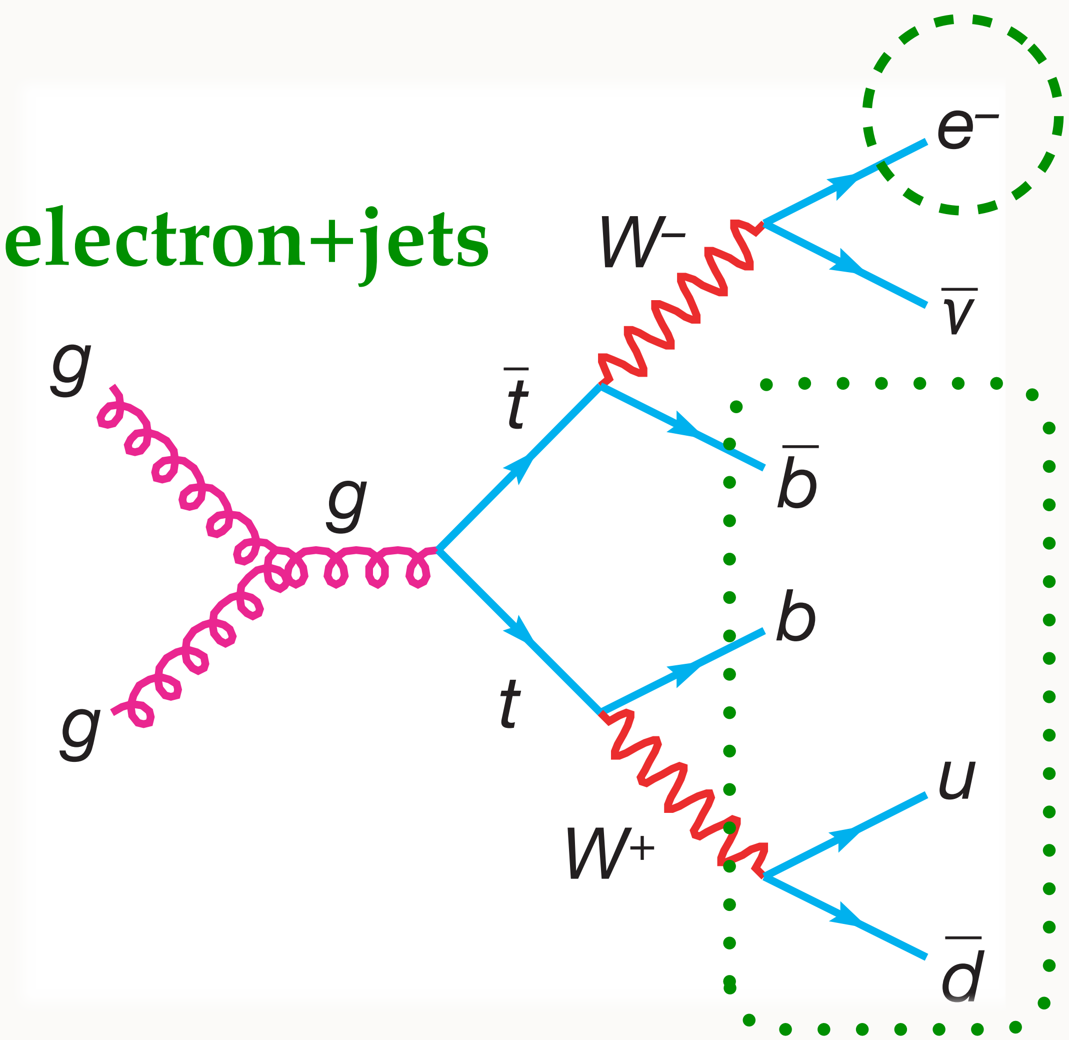


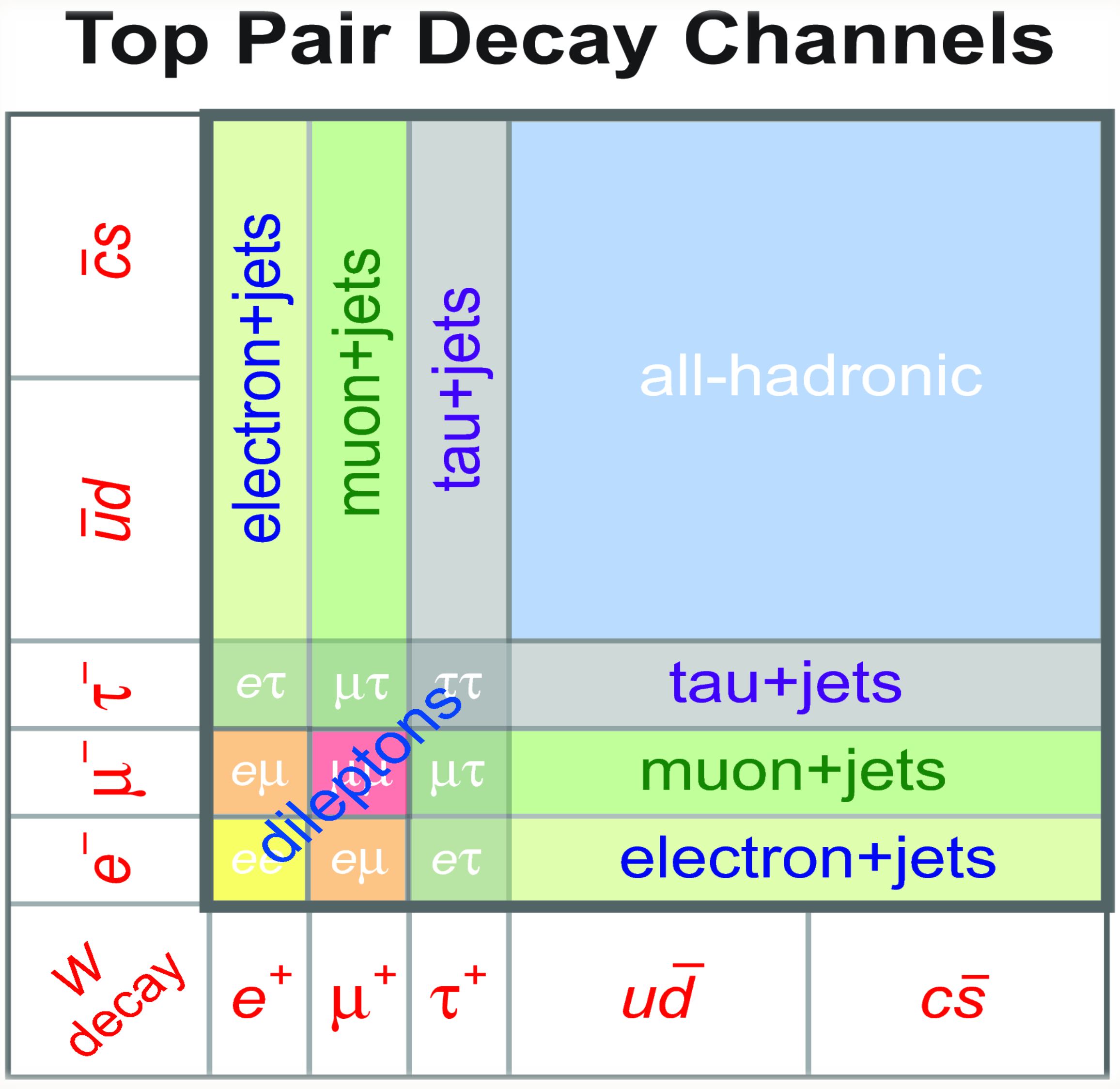
t-channel   *tW*   *s*-channel

TOP PAIR PRODUCTIONS

- **Top quark pairs are produced *strongly* with quark-antiquark annihilation or gluon-gluon fusion.**
  - Final states are categorized by W decay products: **dilepton / lepton+jets / all-hadronic jets**

e.g. **electron+jets**





1977年：顶夸克是存在的！

（从底夸克的实验数据推断出）

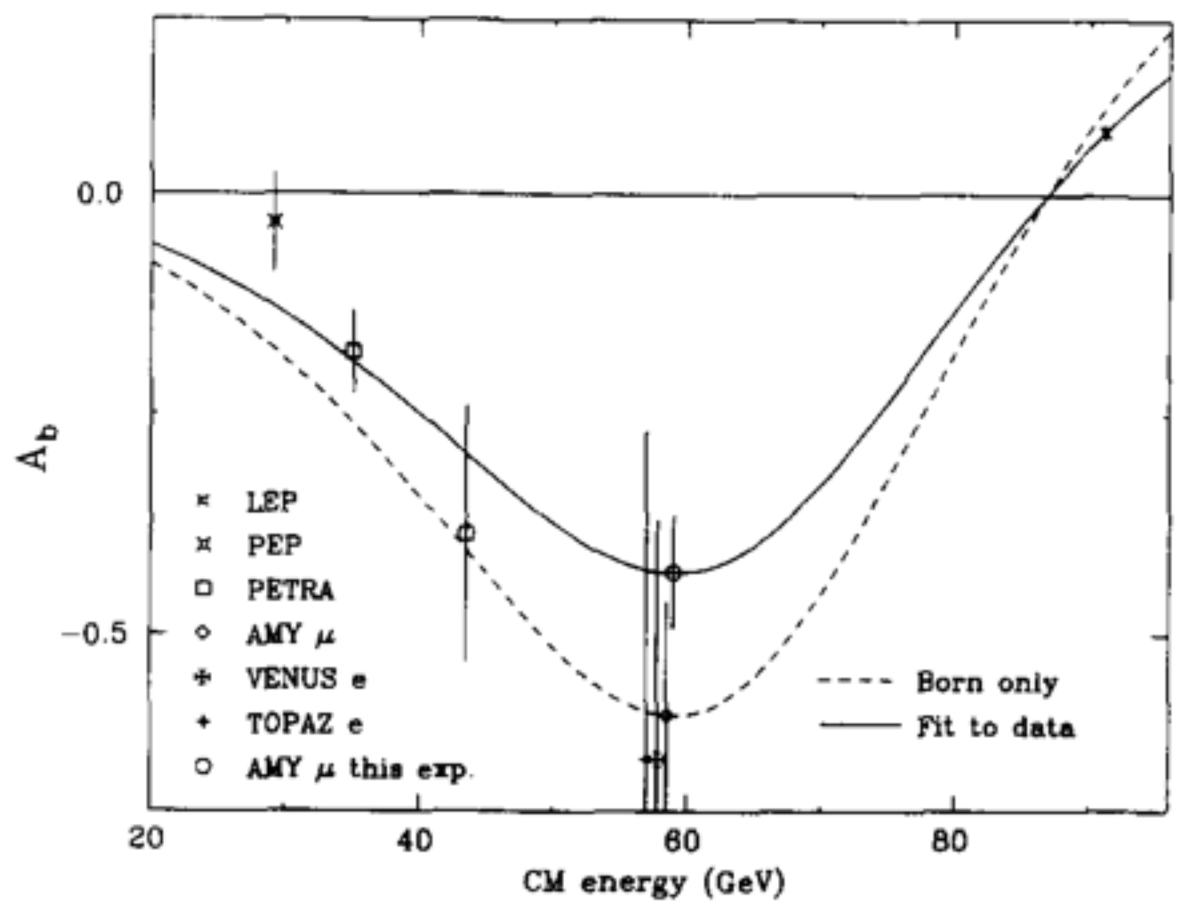


Fig. 5. The present measurement of the asymmetry A_b together with other experiments. The statistical and systematic errors are added in quadrature. The two curves are the Born term prediction without mixing (broken line) and the fit to the data (solid line) with mixing parameter χ. See the text.

Forward-Backward Asymmetry of bottom quark (A_b) in

$$e^+e^- \rightarrow b\bar{b}$$

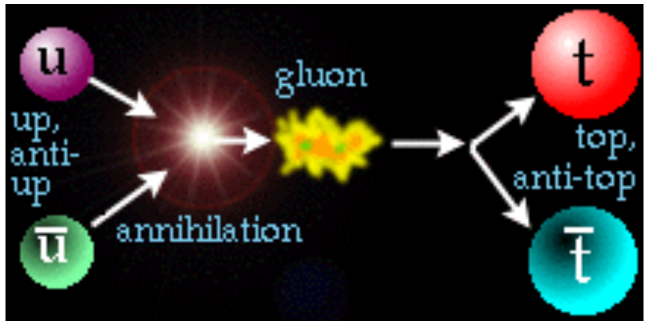
confirmed weak isospin of b

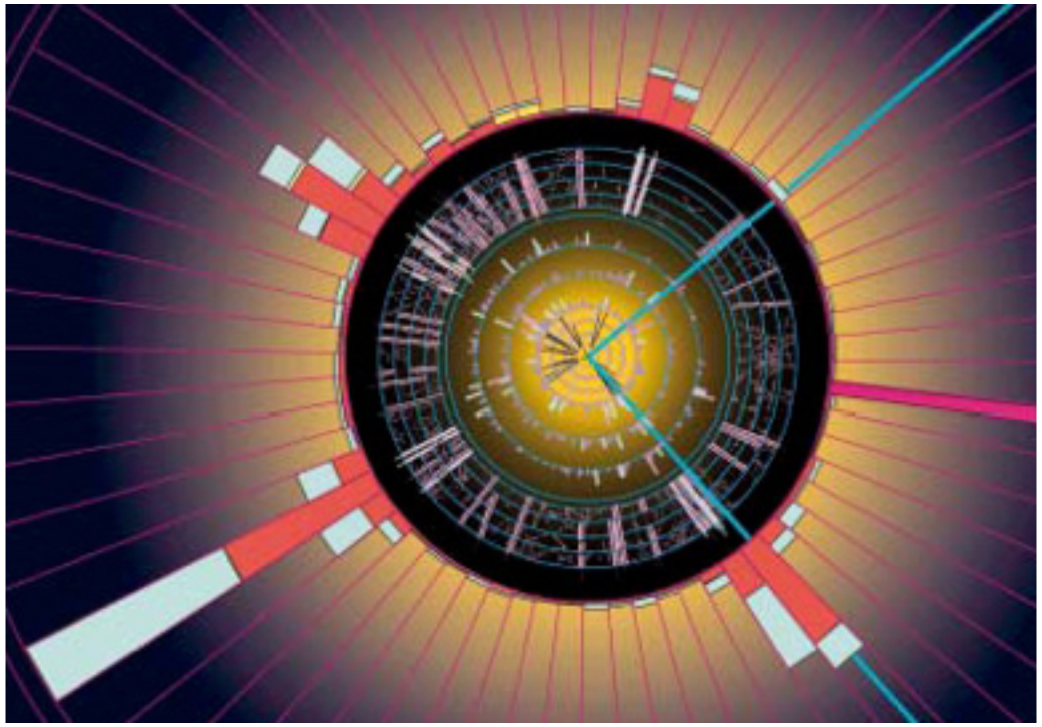
$$T_3 = -\frac{1}{2}$$

⇒ $T_3 = \frac{1}{2}$ state must exist, which is called TOP.

然而，顶夸克的发现之路却是如此漫长！

1995年3月2日





高能物理学家高举

香槟

欢庆发现顶夸克

（美国费曼国家实验室的D0和CDF实验组）

最近实验结果，

$$m_t = (173.1 \pm 1.0)\ \text{GeV}$$

顶夸克年表

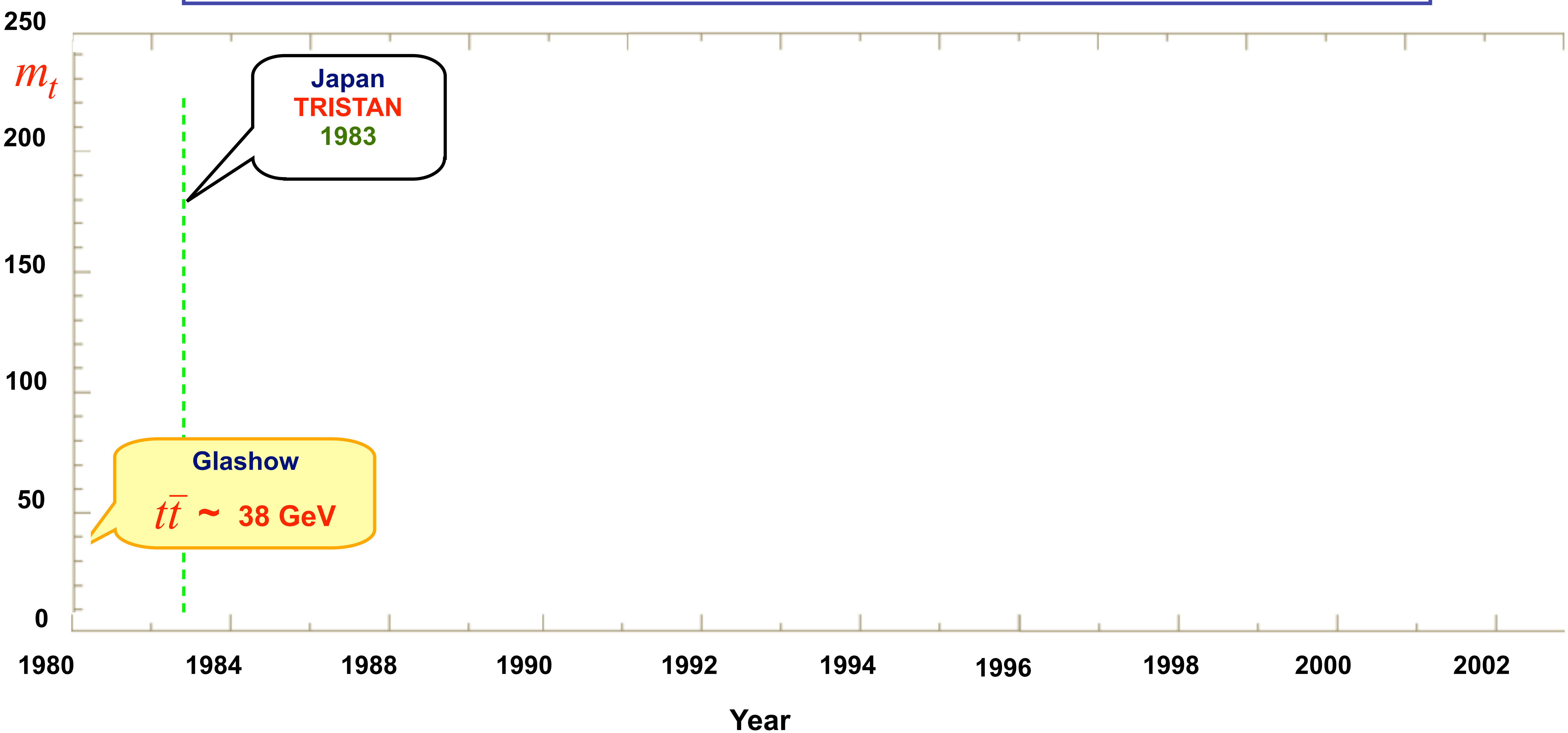
Where is the Top Quark? 1980

Sheldon L. Glashow[a]

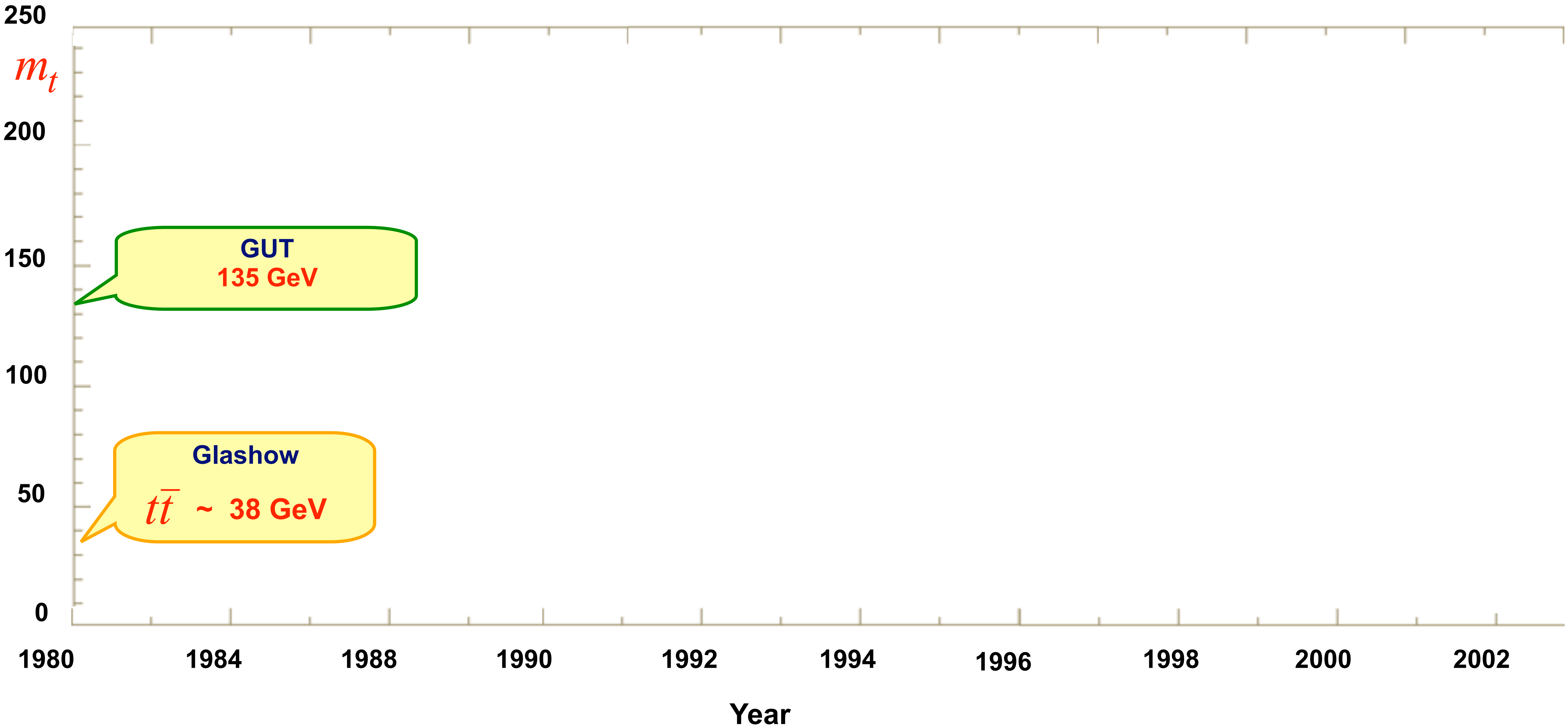
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 17 October 1980)

Arguments are presented suggesting that the top-quark analog of the J/ψ should lie at 38± 2 GeV. Should there exist a fourth $Q=\frac{2}{3}$ quark h, the first $\bar{h}h$ state must be heavier than 300 GeV.



顶夸克年表



250
m_t
200
150
GUT
135 GeV
100
Glashow
$t\bar{t}$ ~ 38 GeV
50
0
1980
1984
1988
1990
1992
1994
1996
1998
2000
2002
Year

顶夸克年表

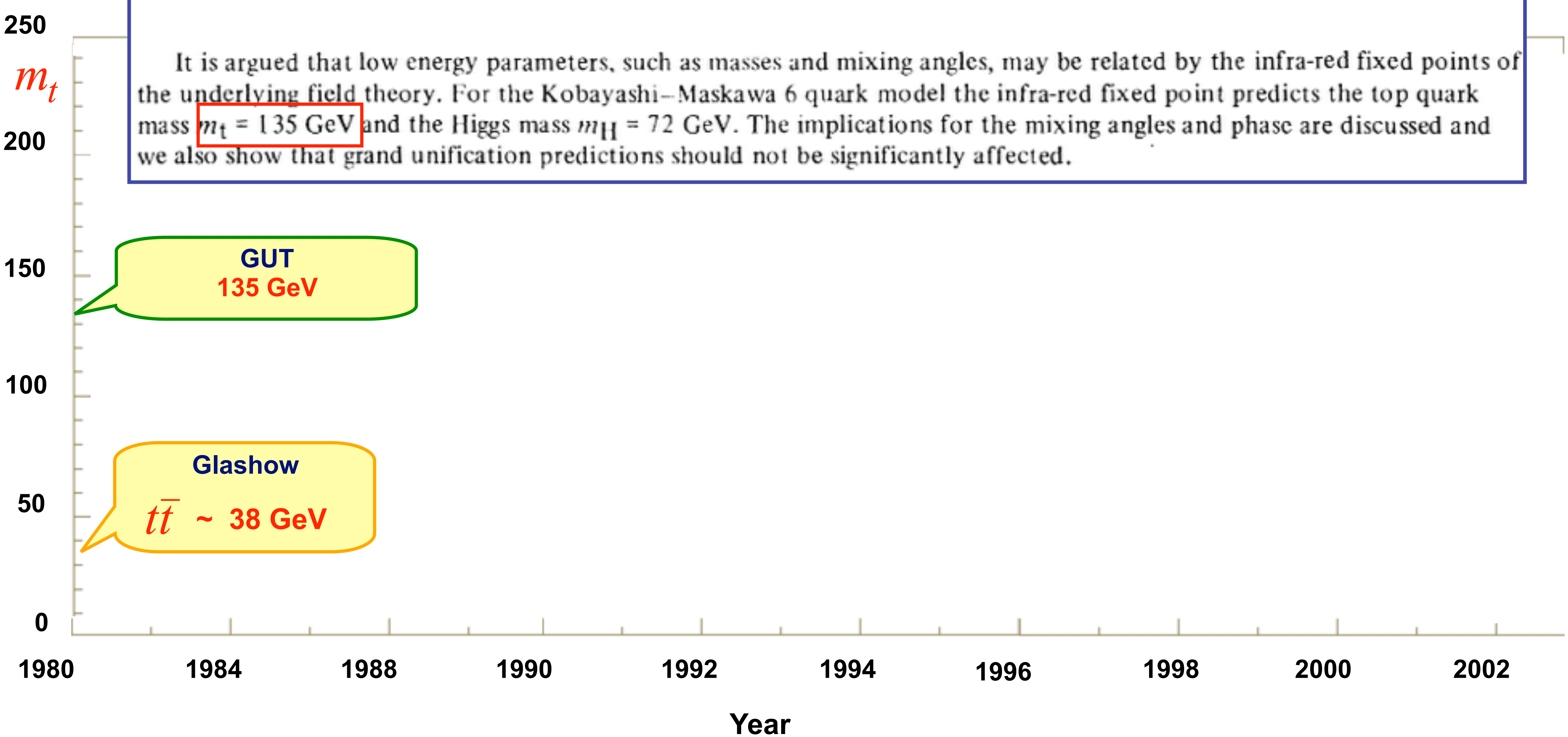
MASS AND MIXING ANGLE PREDICTIONS FROM INFRA-RED FIXED POINTS

1980

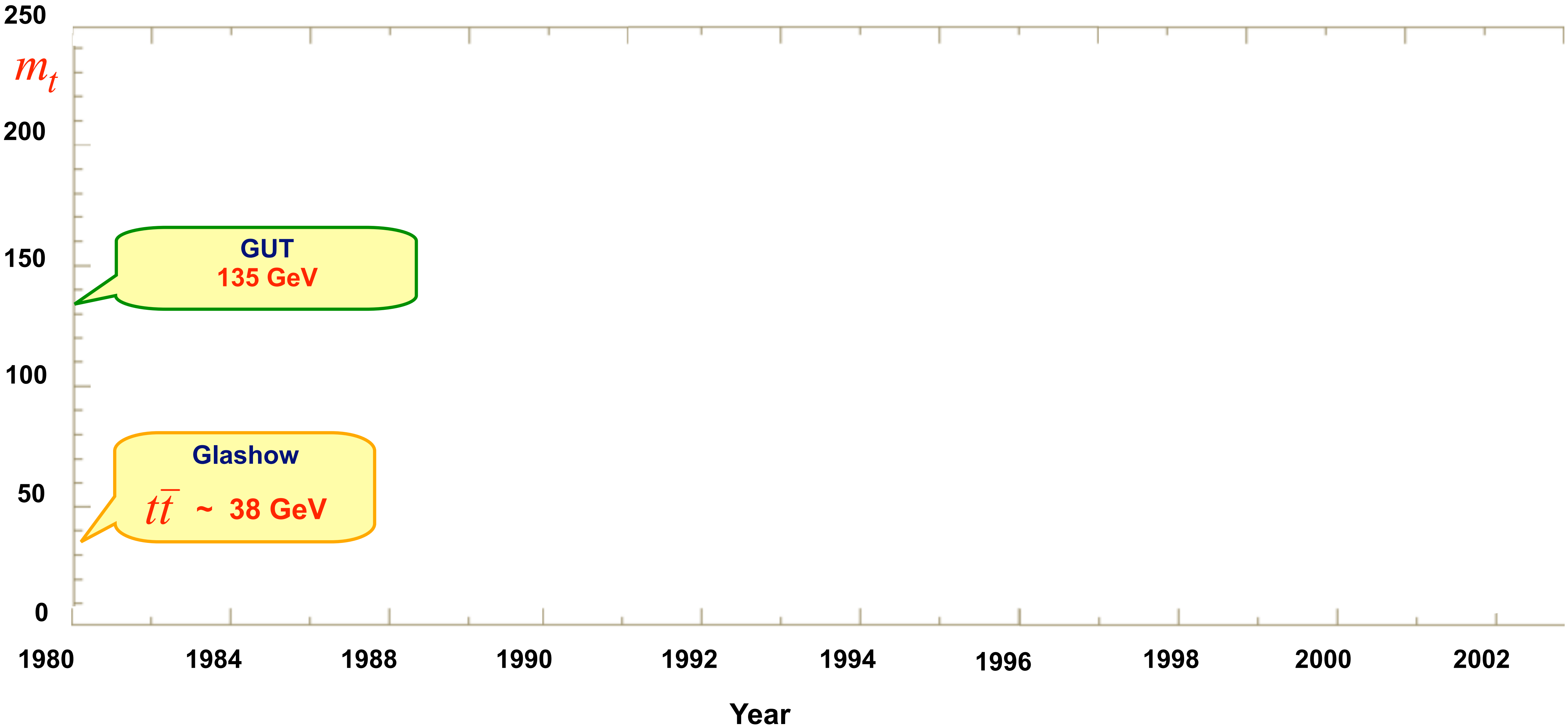
B. PENDLETON and G.G. ROSS
Theoretical Physics Department, Oxford OX1 3NP, UK

Received 15 July 1980
Revised manuscript received 3 November 1980

It is argued that low energy parameters, such as masses and mixing angles, may be related by the infra-red fixed points of the underlying field theory. For the Kobayashi–Maskawa 6 quark model the infra-red fixed point predicts the top quark mass m_t = 135 GeV and the Higgs mass m_H = 72 GeV. The implications for the mixing angles and phase are discussed and we also show that grand unification predictions should not be significantly affected.



顶夸克年表



250
m_t
200
150
100
50
0
GUT
135 GeV
Glashow
$t\bar{t}$ ~ 38 GeV
1980
1984
1988
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1992
1994
1996
1998
2000
2002
Year

顶夸克年表

TOWARDS A REALISTIC SUGRA-GUT

1983

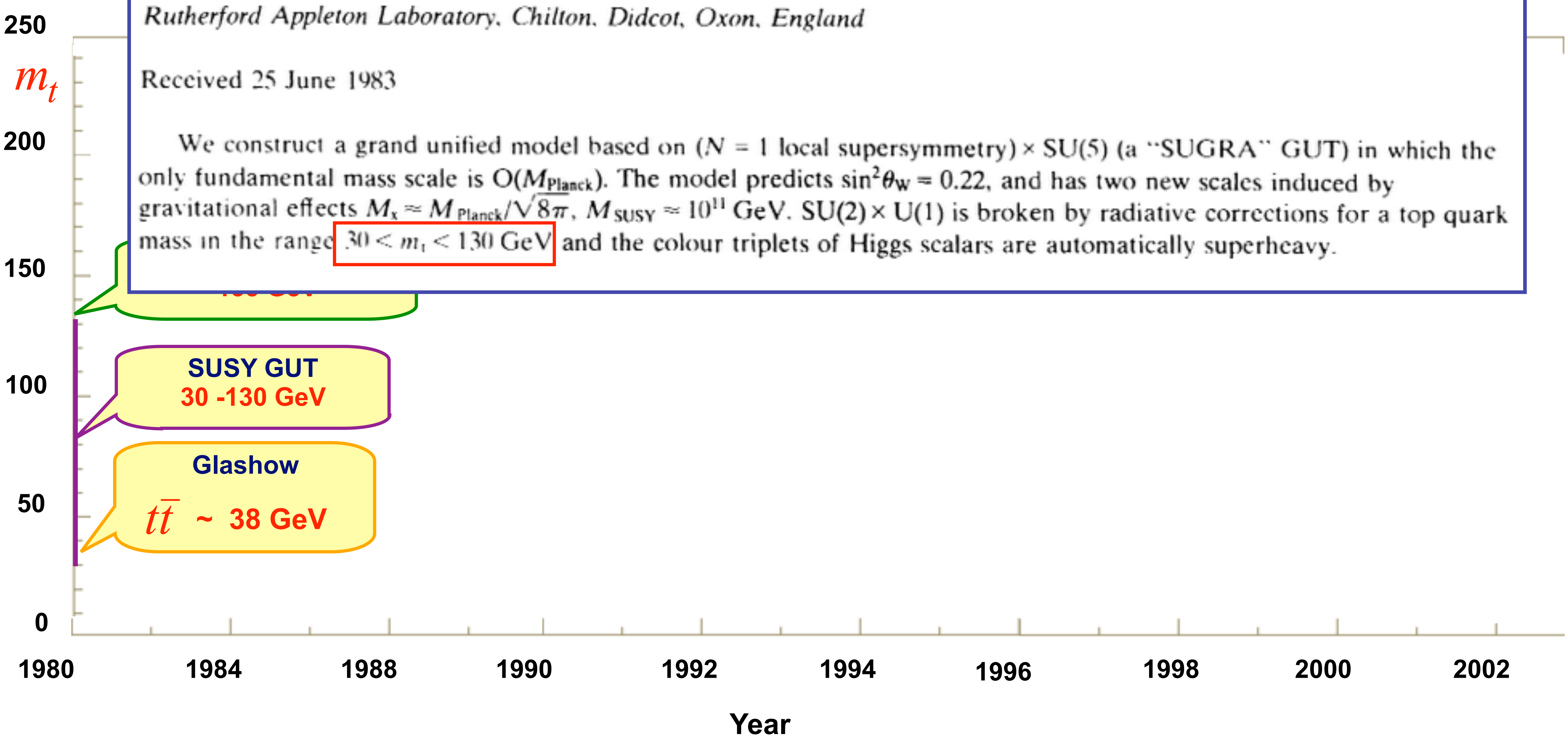
L.E. IBÁÑEZ
Departamento de Fisica Téorica C-XI, Universidad Autonoma de Madrid, Cantablanco, Madrid-34, Spain

and

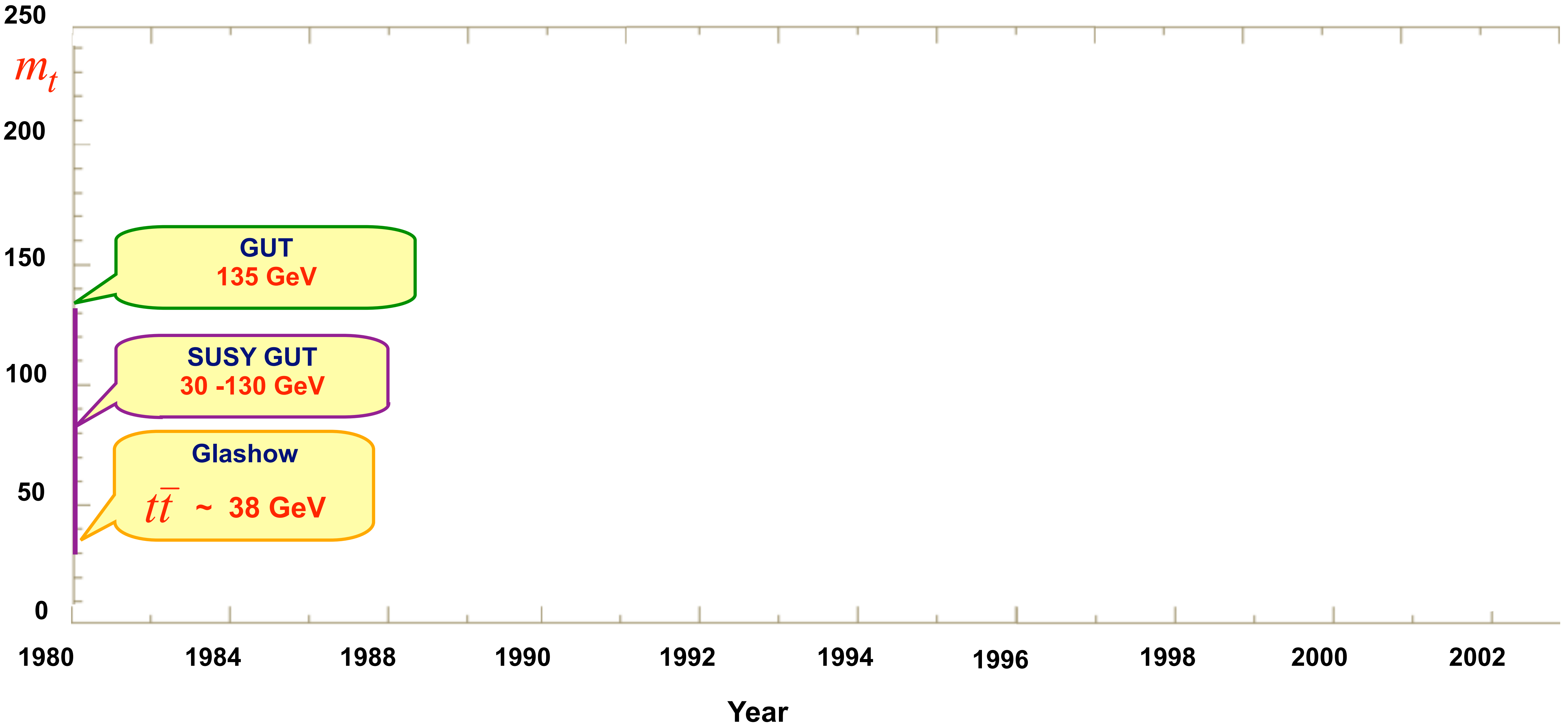
G.G. ROSS[1]
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, England

Received 25 June 1983

We construct a grand unified model based on ($N = 1$ local supersymmetry) × SU(5) (a "SUGRA" GUT) in which the only fundamental mass scale is $O(M_{\text{Planck}})$. The model predicts $\sin^2\theta_W = 0.22$, and has two new scales induced by gravitational effects $M_x \simeq M_{\text{Planck}}/\sqrt{8\pi}$, $M_{\text{SUSY}} \simeq 10^{11}$ GeV. SU(2) × U(1) is broken by radiative corrections for a top quark mass in the range $30 < m_t < 130$ GeV and the colour triplets of Higgs scalars are automatically superheavy.



顶夸克年表



250
m_t
200
150
100
50
0
GUT
135 GeV
SUSY GUT
30 -130 GeV
Glashow
$t\bar{t}$ ~ 38 GeV
1980
1984
1988
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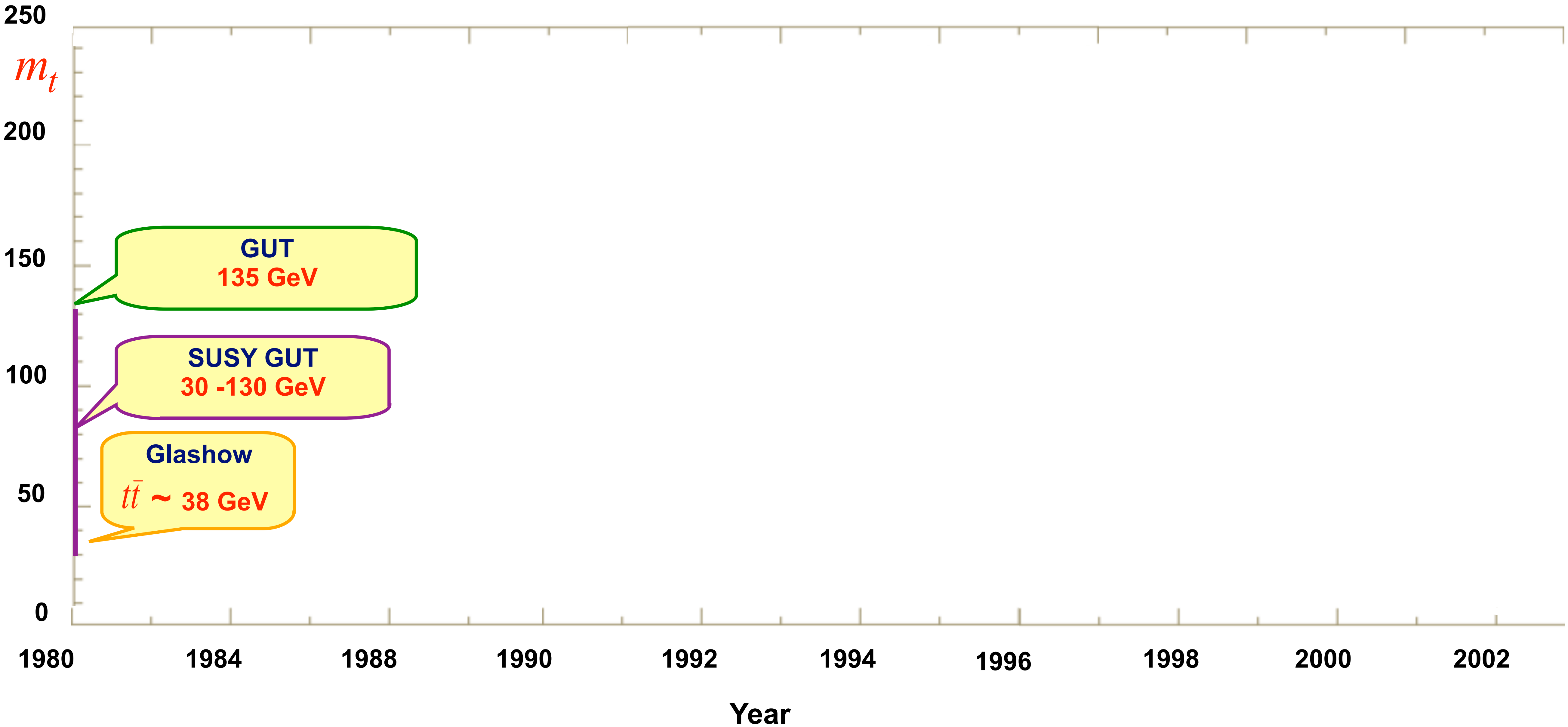
1998
2000
2002
Year

顶夸克年表



250
m_t
200
150
100
50
0
GUT
135 GeV
SUSY GUT
30 -130 GeV
Glashow
$t\bar{t}$ ~ 38 GeV
1980
1984
1988
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1992
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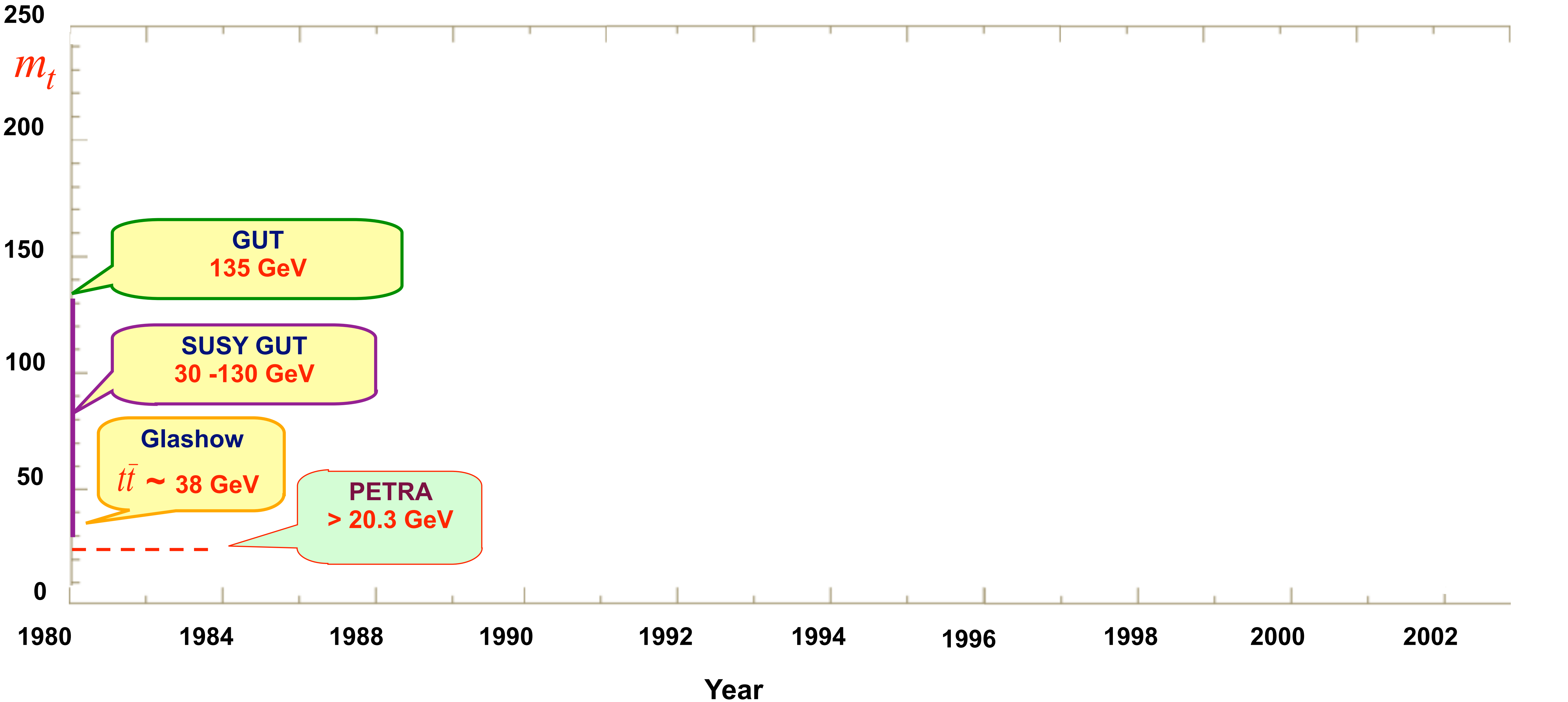
1996
1998
2000
2002
Year

顶夸克年表


250
m_t
200
150
100
50
0
GUT
135 GeV
SUSY GUT
30 -130 GeV
Glashow
$t\bar{t}$ ~ 38 GeV
PETRA
> 20.3 GeV
1980
1984
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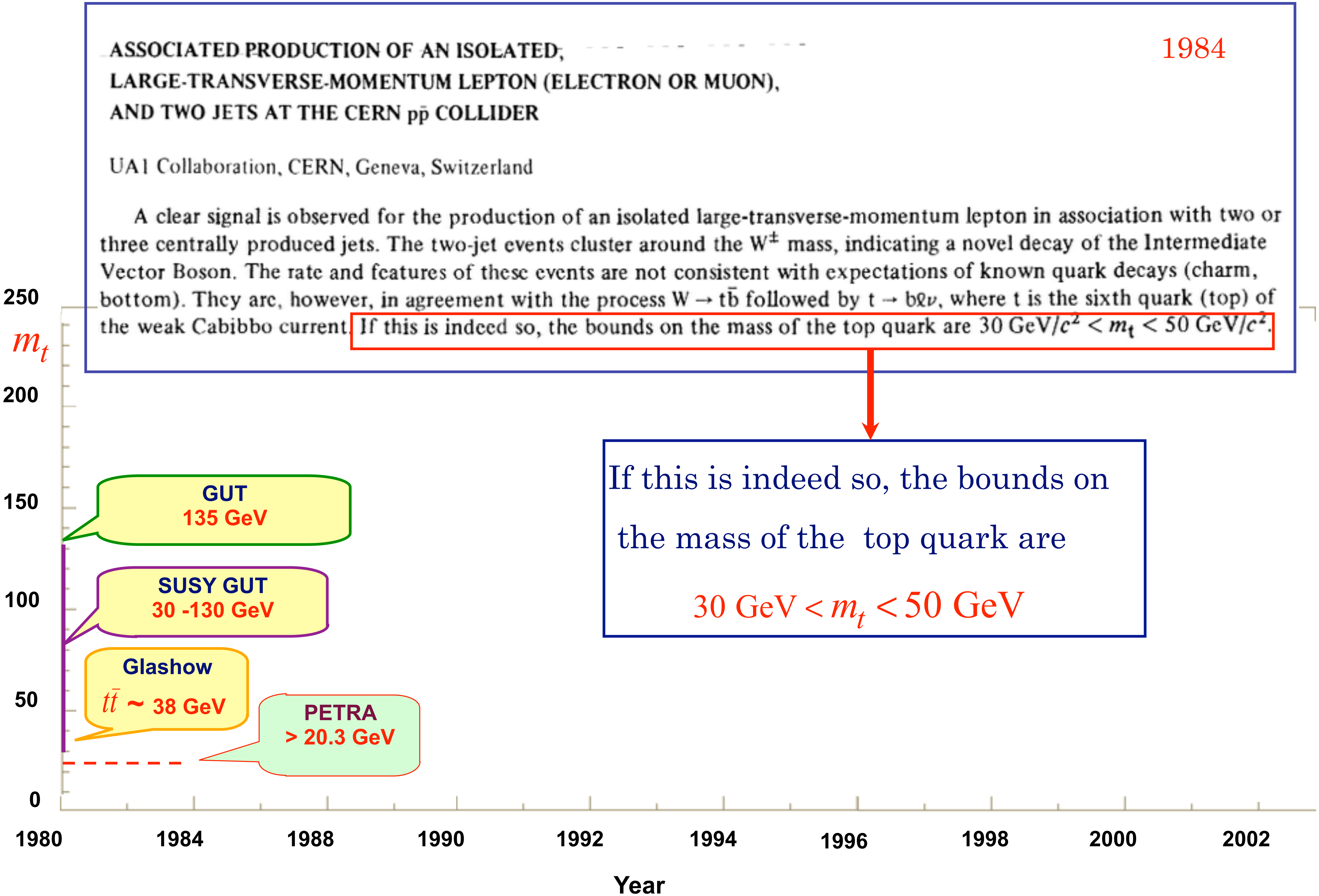
1996
1998
2000
2002
Year

顶夸克年表



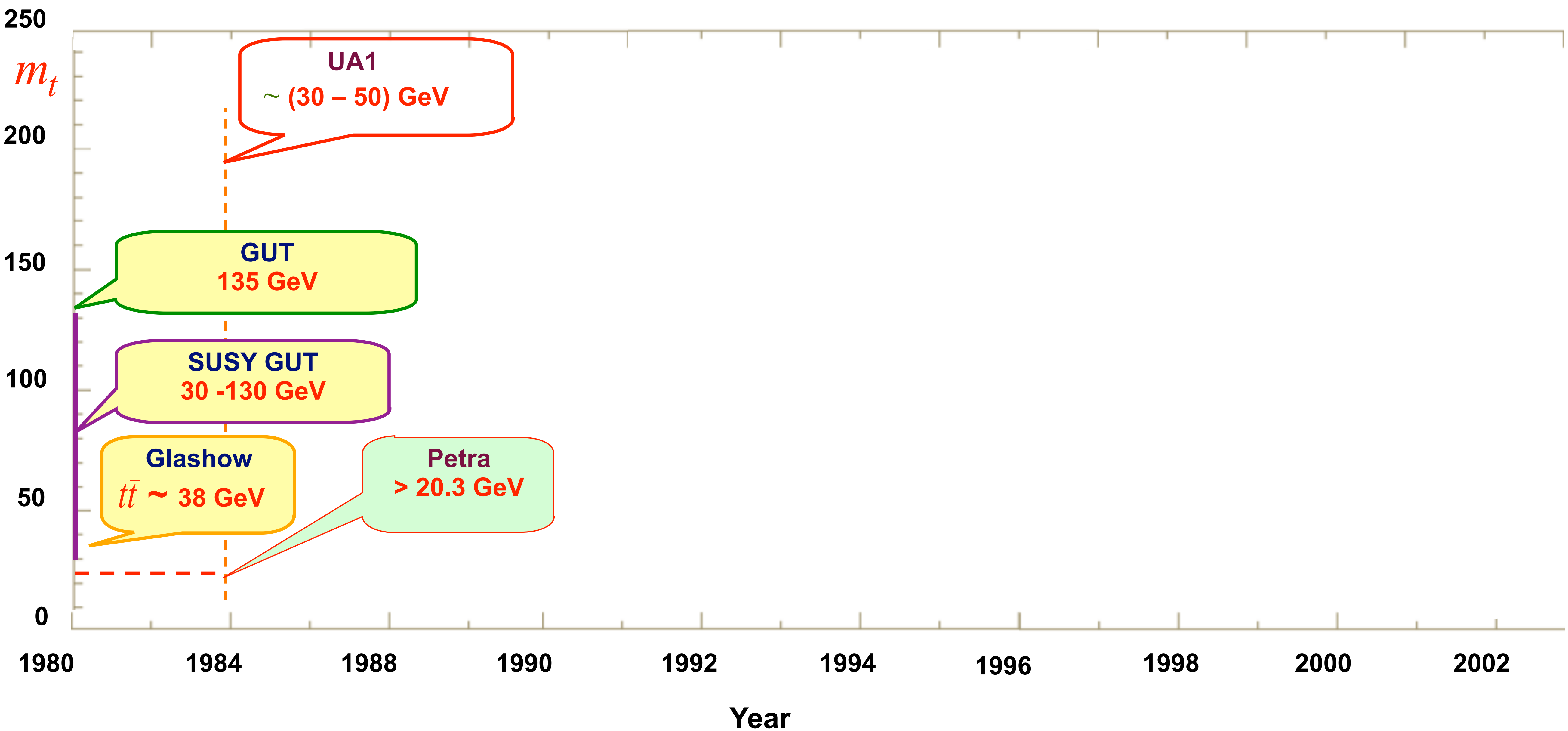
1984

ASSOCIATED PRODUCTION OF AN ISOLATED, LARGE-TRANSVERSE-MOMENTUM LEPTON (ELECTRON OR MUON), AND TWO JETS AT THE CERN $p\bar{p}$ COLLIDER

UA1 Collaboration, CERN, Geneva, Switzerland

A clear signal is observed for the production of an isolated large-transverse-momentum lepton in association with two or three centrally produced jets. The two-jet events cluster around the $W^{\pm}$ mass, indicating a novel decay of the Intermediate Vector Boson. The rate and features of these events are not consistent with expectations of known quark decays (charm, bottom). They are, however, in agreement with the process $W \to t\bar{b}$ followed by $t \to b\ell\nu$, where t is the sixth quark (top) of the weak Cabibbo current. If this is indeed so, the bounds on the mass of the top quark are $30\ \mathrm{GeV}/c^2 < m_t < 50\ \mathrm{GeV}/c^2$.

顶夸克年表



250
200
150
100
50
0
m_t
UA1
~ (30 – 50) GeV
GUT
135 GeV
SUSY GUT
30 -130 GeV
Glashow
$t\bar{t}$ ~ 38 GeV
Petra
> 20.3 GeV
1980
1984
1988
1990
1992
1994
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2000
2002
Year

顶夸克年表

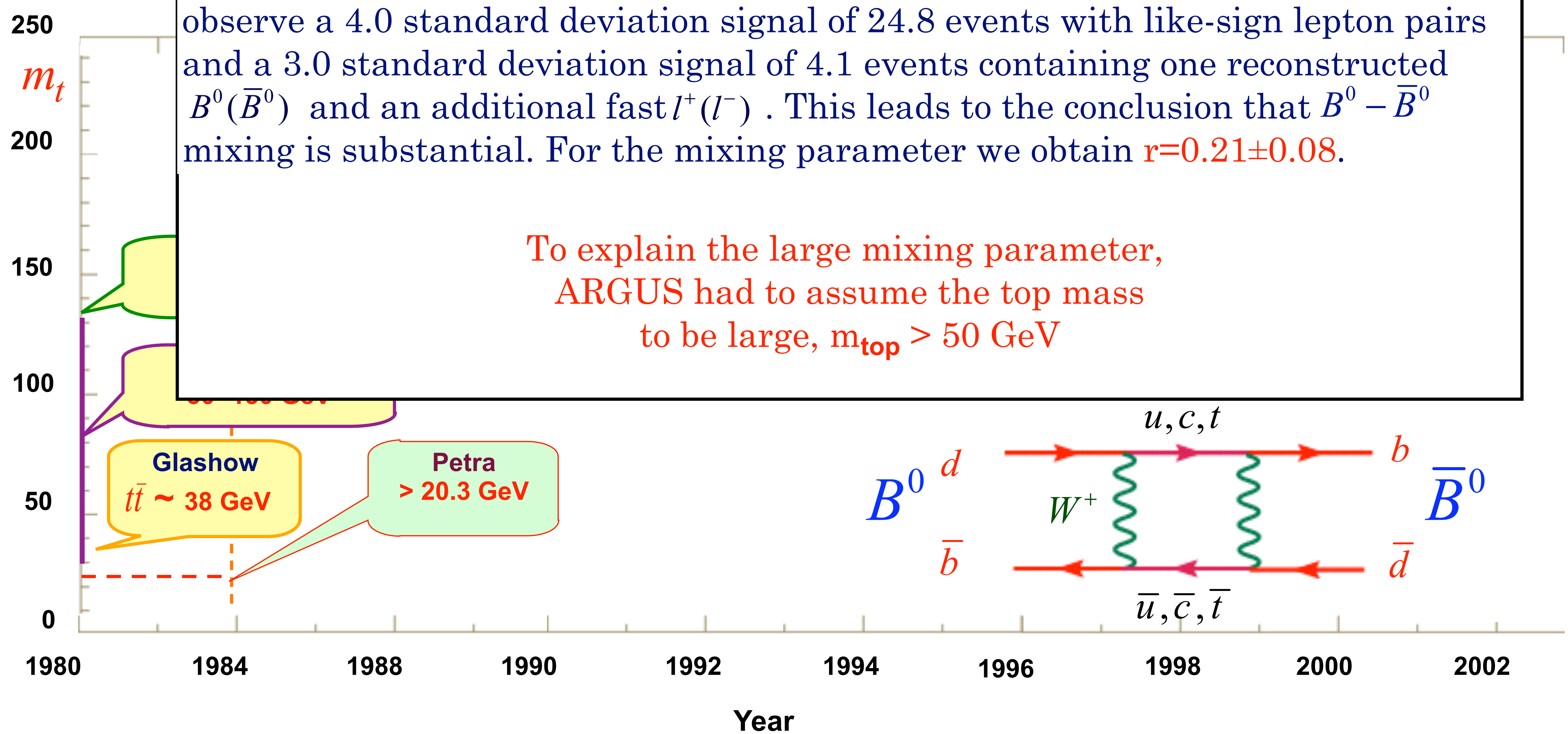
Observation of $B^0 - \bar{B}^0$ mixing

ARGUS Collaboration

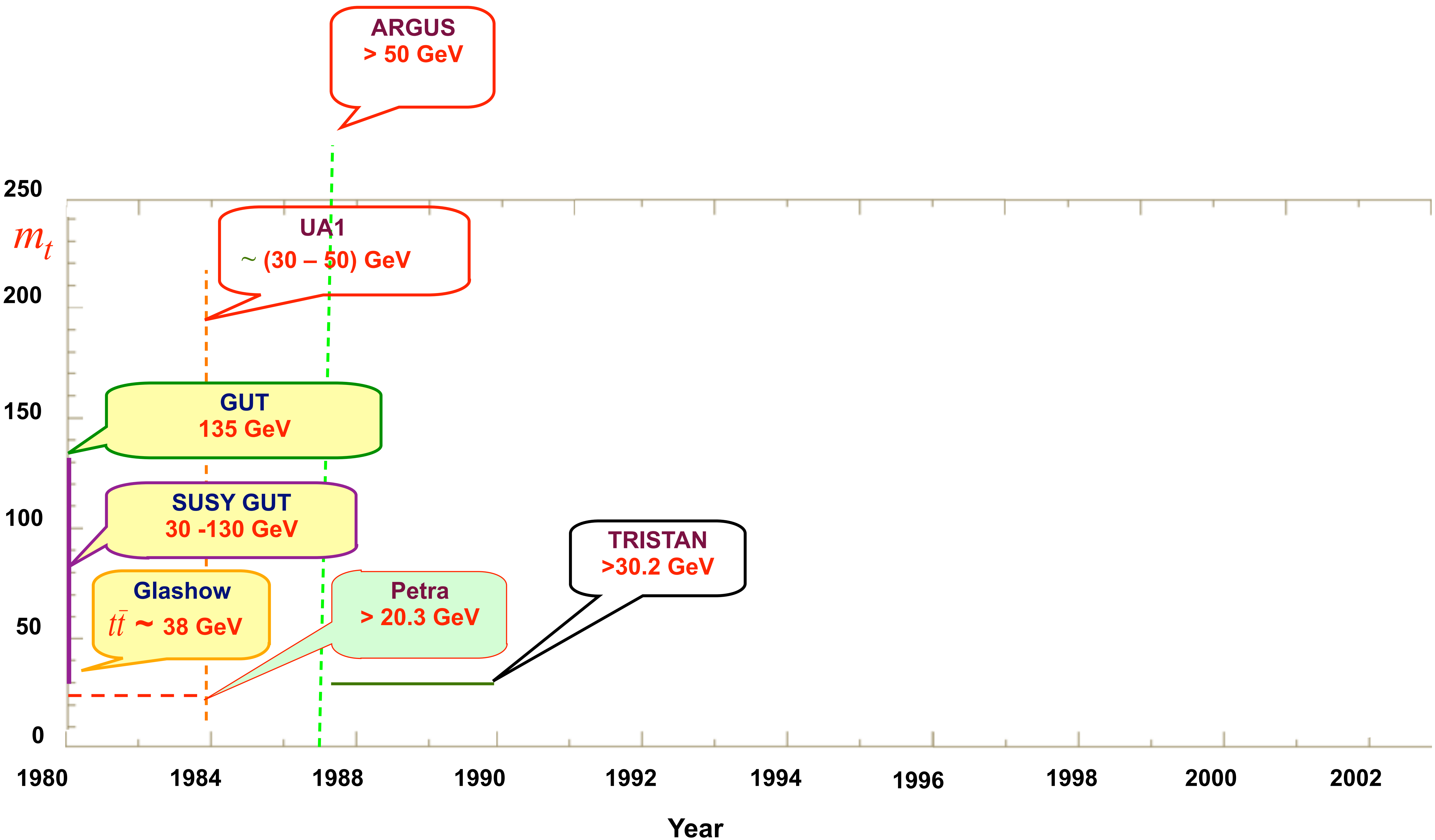
Received 9 April 1987

Using the ARGUS detector at the DORIS II storage ring we have searched in three different ways for $B^0 - \bar{B}^0$ mixing in $\Upsilon(4S)$ decays. One explicitly mixed event, a decay $\Upsilon \to B^0 \bar{B}^0$, has been completely reconstructed. Furthermore, we observe a 4.0 standard deviation signal of 24.8 events with like-sign lepton pairs and a 3.0 standard deviation signal of 4.1 events containing one reconstructed $B^0(\bar{B}^0)$ and an additional fast $l^+(l^-)$. This leads to the conclusion that $B^0 - \bar{B}^0$ mixing is substantial. For the mixing parameter we obtain r=0.21±0.08.

To explain the large mixing parameter, ARGUS had to assume the top mass to be large, m_{top} > 50 GeV



顶夸克年表



ARGUS
> 50 GeV
250
m_t
UA1
~ (30 – 50) GeV
200
150
GUT
135 GeV
100
SUSY GUT
30 -130 GeV
TRISTAN
>30.2 GeV
Glashow
$t\bar{t}$ ~ 38 GeV
Petra
> 20.3 GeV
50
0
1980
1984
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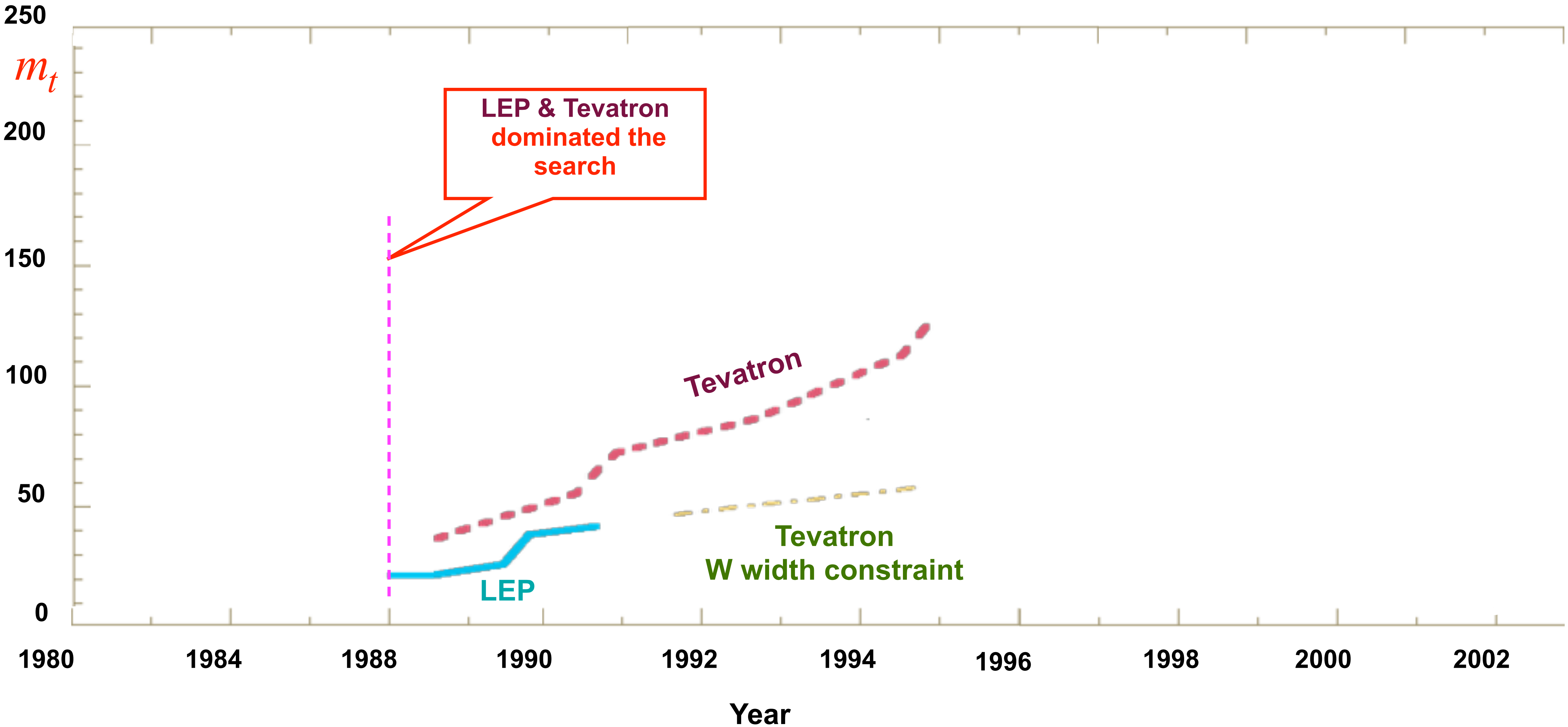
Year

顶夸克年表



250
m_t
200
150
100
50
0
LEP & Tevatron dominated the search
Tevatron
LEP
Tevatron
W width constraint
1980
1984
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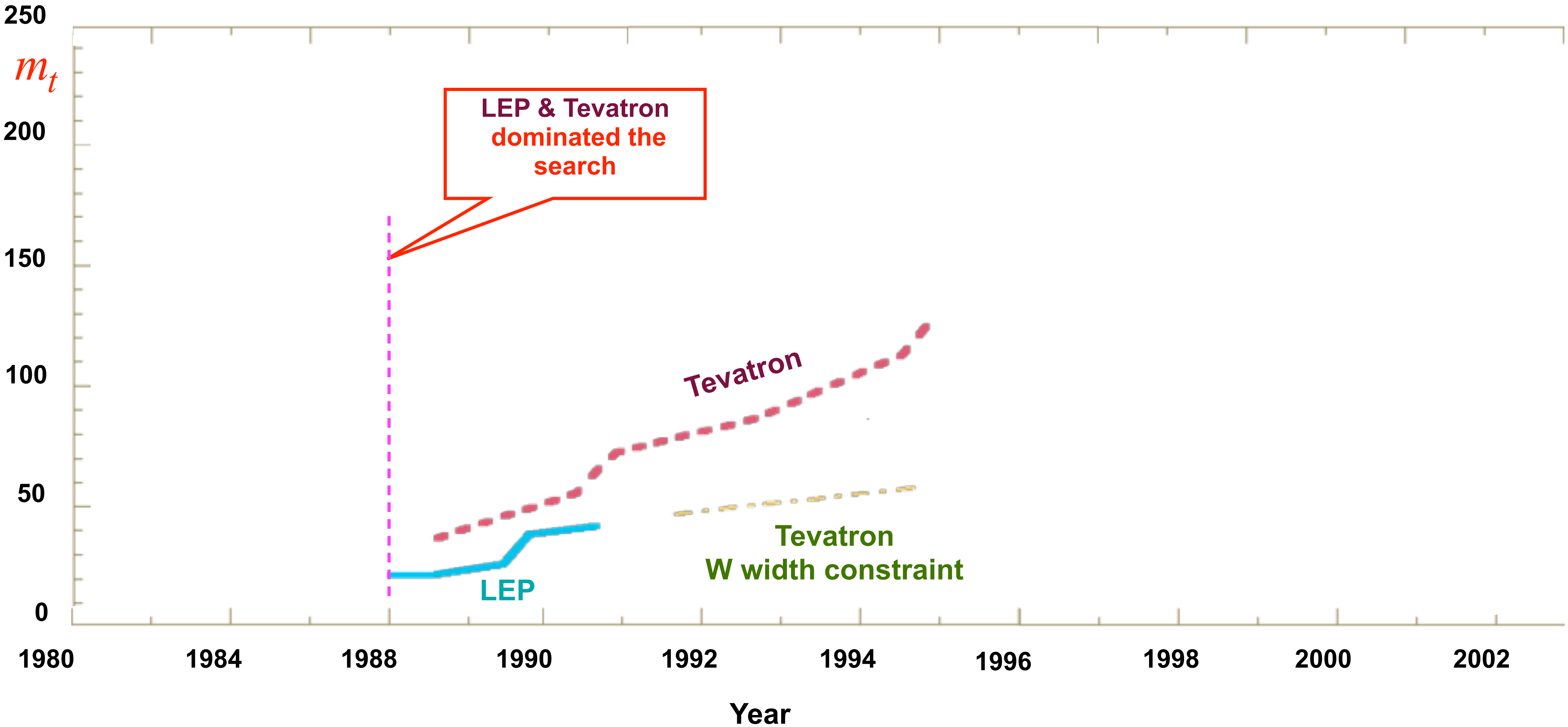
Year

顶夸克年表



250
m_t
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50
0
LEP & Tevatron dominated the search
Tevatron
Tevatron
W width constraint
LEP
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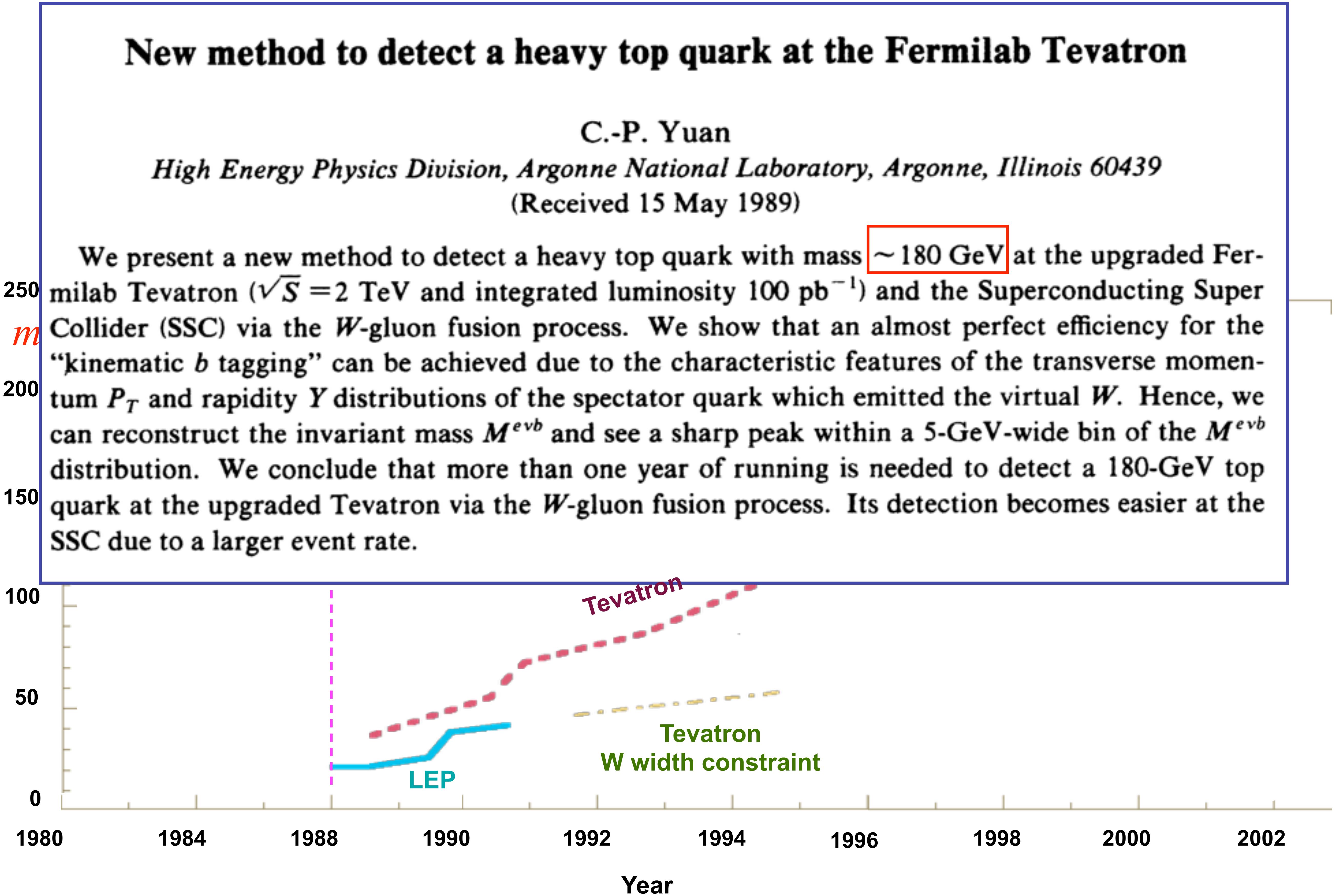
Year

顶夸克年表



New method to detect a heavy top quark at the Fermilab Tevatron

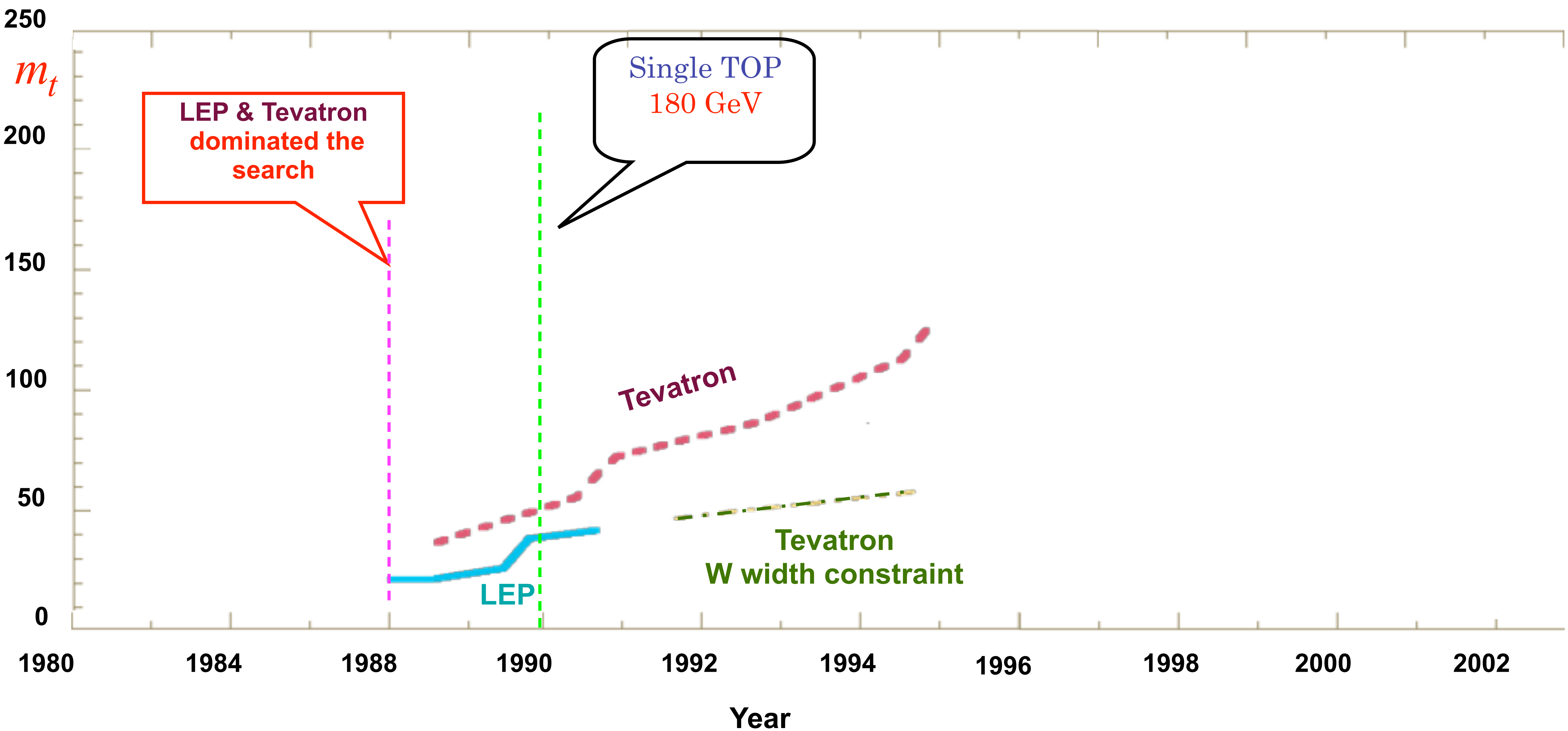
C.-P. Yuan

High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

(Received 15 May 1989)

We present a new method to detect a heavy top quark with mass ~ 180 GeV at the upgraded Fermilab Tevatron ($\sqrt{S} = 2$ TeV and integrated luminosity 100 pb^{-1}) and the Superconducting Super Collider (SSC) via the W-gluon fusion process. We show that an almost perfect efficiency for the "kinematic b tagging" can be achieved due to the characteristic features of the transverse momentum P_T and rapidity Y distributions of the spectator quark which emitted the virtual W. Hence, we can reconstruct the invariant mass $M^{e\nu b}$ and see a sharp peak within a 5-GeV-wide bin of the $M^{e\nu b}$ distribution. We conclude that more than one year of running is needed to detect a 180-GeV top quark at the upgraded Tevatron via the W-gluon fusion process. Its detection becomes easier at the SSC due to a larger event rate.

顶夸克年表



250
m_t
200
150
100
50
0
LEP & Tevatron dominated the search
Single TOP
180 GeV
Tevatron
LEP
Tevatron
W width constraint
1980
1984
1988
1990
1992
1994
1996
1998
2000
2002
Year

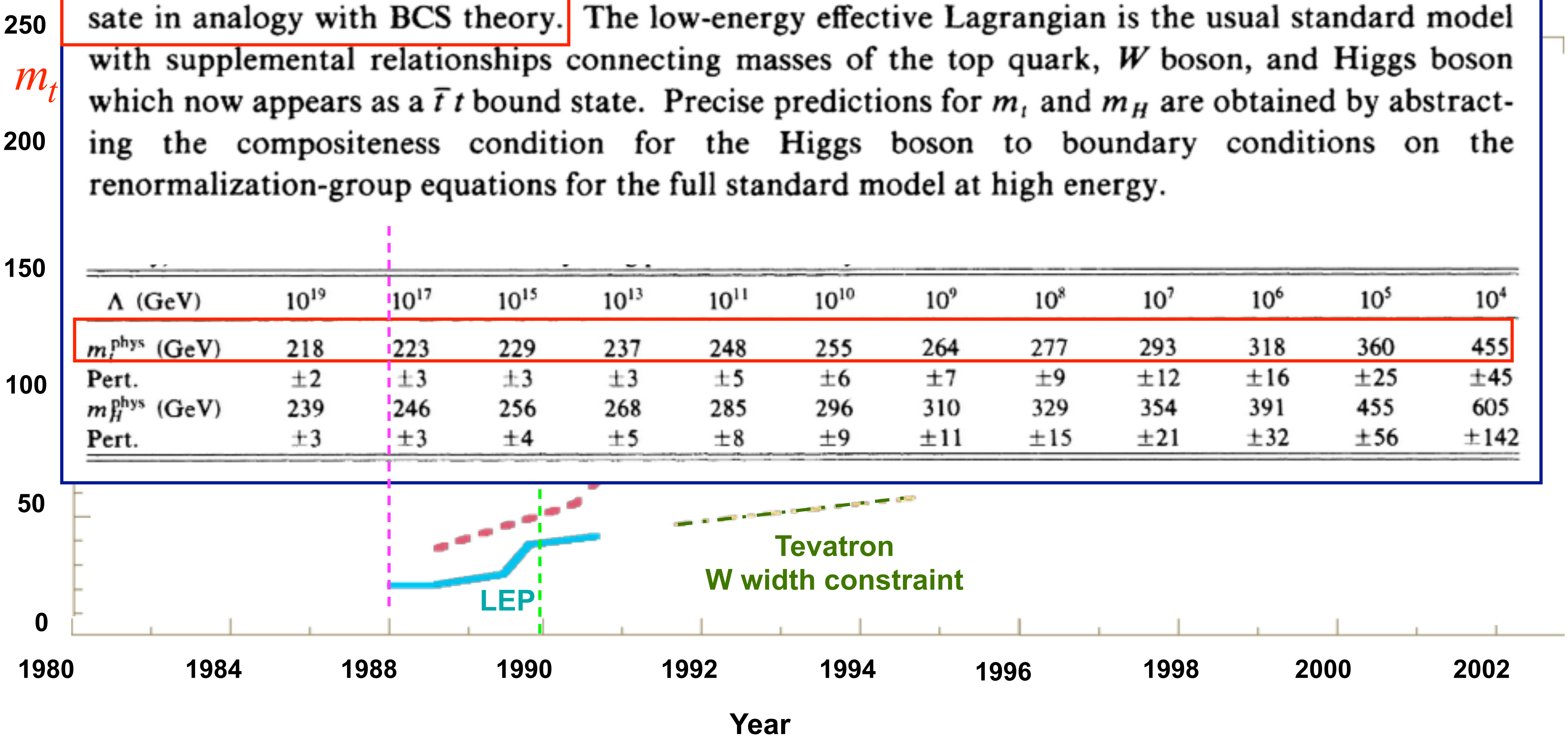
顶夸克年表

Minimal dynamical symmetry breaking of the standard model

William A. Bardeen, Christopher T. Hill, and Manfred Lindner
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510
(Received 21 July 1989; revised manuscript received 2 November 1989)

We formulate the dynamical symmetry breaking of the standard model by a top-quark condensate in analogy with BCS theory. The low-energy effective Lagrangian is the usual standard model with supplemental relationships connecting masses of the top quark, W boson, and Higgs boson which now appears as a $\bar{t}t$ bound state. Precise predictions for m_t and m_H are obtained by abstracting the compositeness condition for the Higgs boson to boundary conditions on the renormalization-group equations for the full standard model at high energy.

Λ (GeV)	10^{19}	10^{17}	10^{15}	10^{13}	10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}
m_t^{phys} (GeV)	218	223	229	237	248	255	264	277	293	318	360	455
Pert.	±2	±3	±3	±3	±5	±6	±7	±9	±12	±16	±25	±45
m_H^{phys} (GeV)	239	246	256	268	285	296	310	329	354	391	455	605
Pert.	±3	±3	±4	±5	±8	±9	±11	±15	±21	±32	±56	±142



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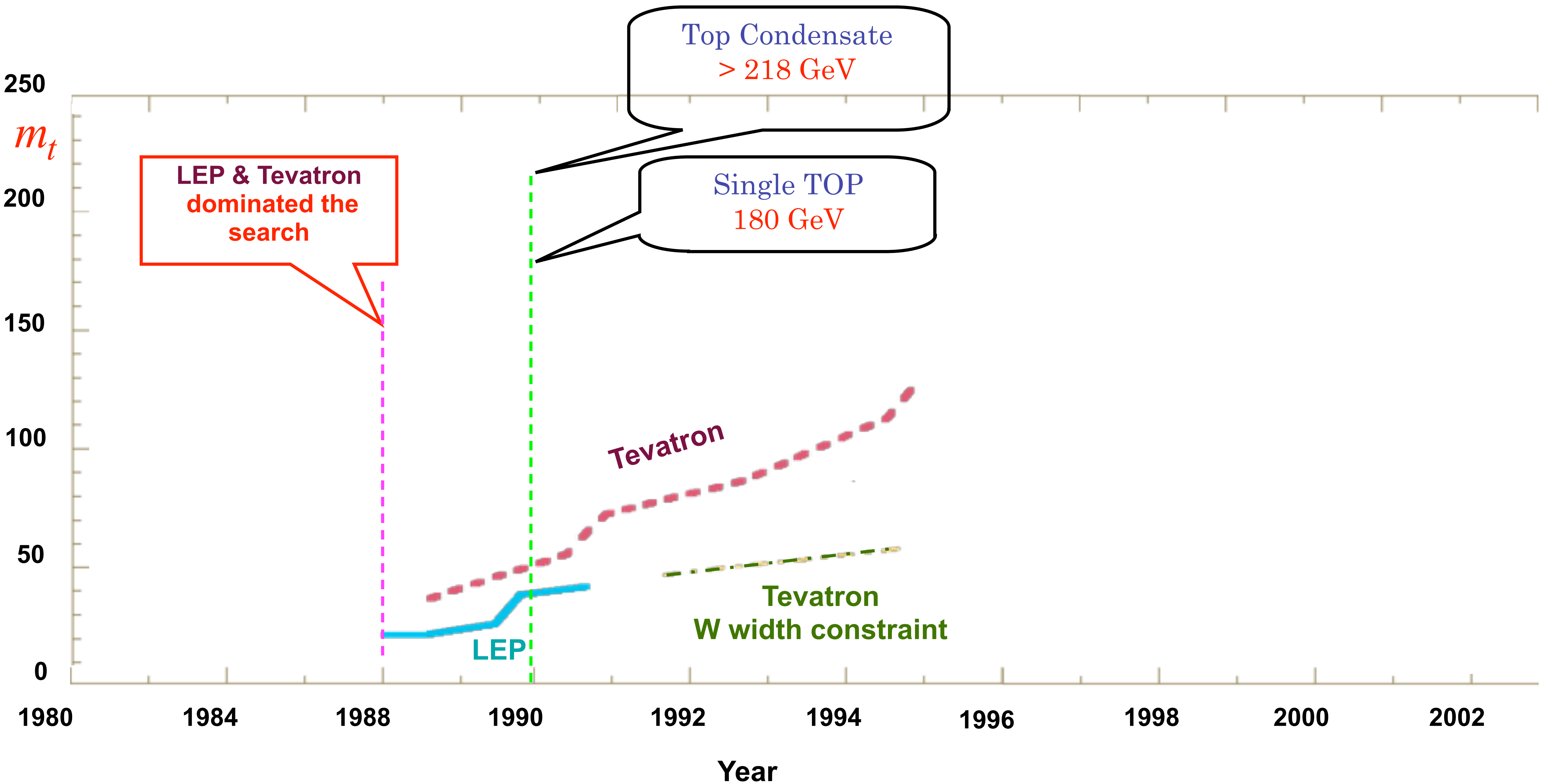
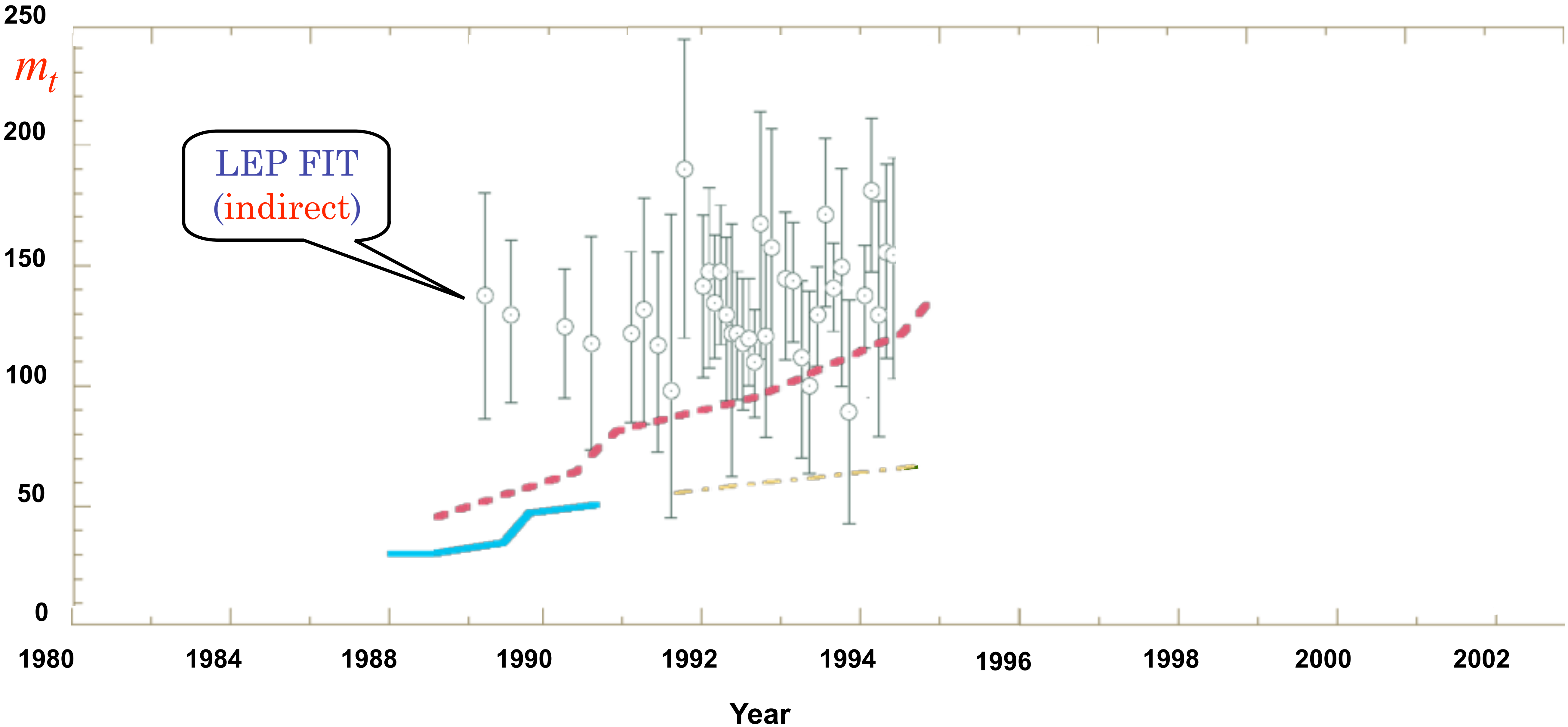


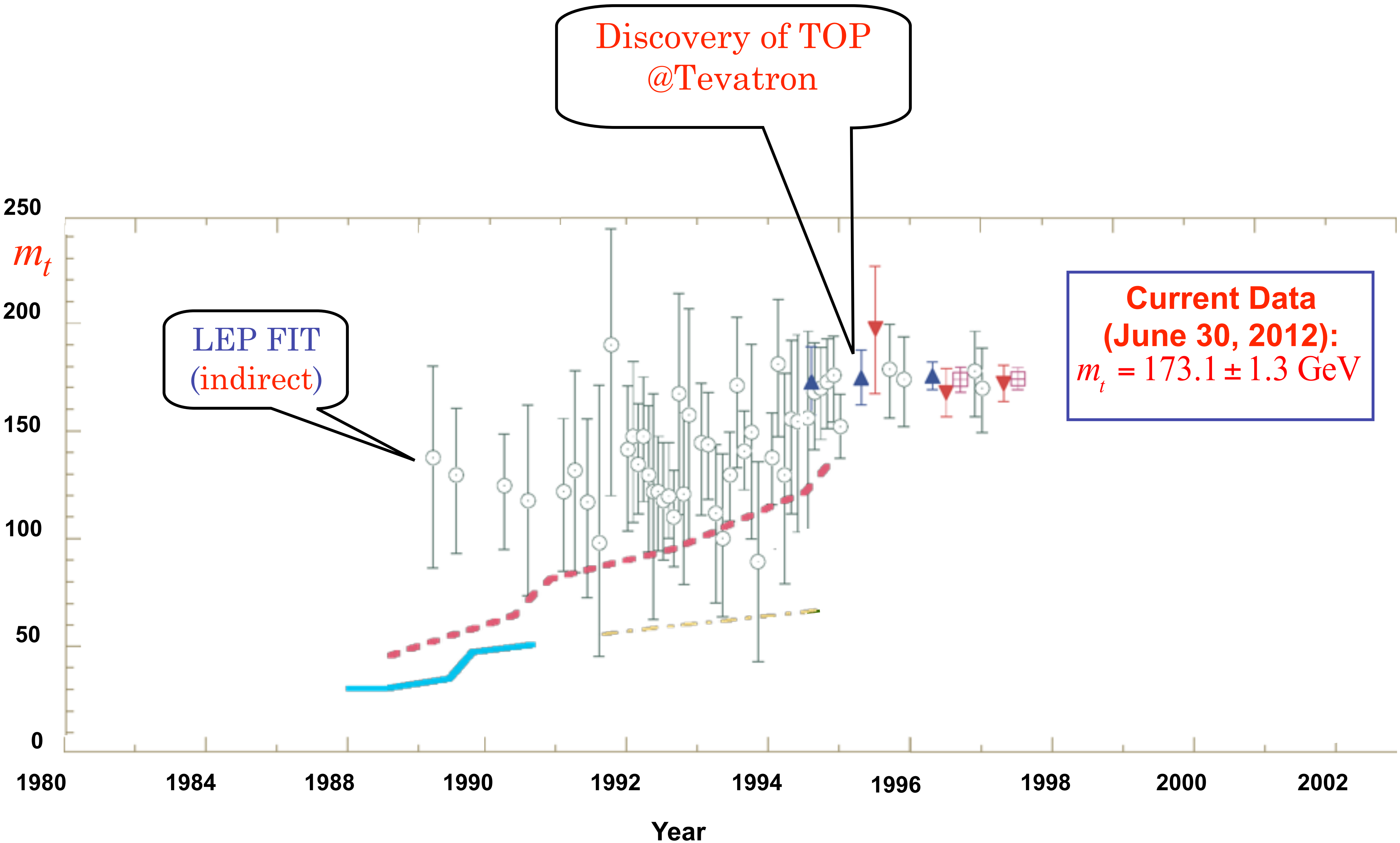
Top Condensate
> 218 GeV
Single TOP
180 GeV
LEP & Tevatron dominated the search
250
200
150
100
50
0
m_t
Tevatron
LEP
Tevatron
W width constraint
1980
1984
1988
1990
1992
1994
1996
1998
2000
2002
Year

顶夸克年表



250
m_t
200
150
100
50
0
LEP FIT
(indirect)
1980
1984
1988
1990
1992
1994
1996
1998
2000
2002
Year

顶夸克年表



Discovery of TOP
@Tevatron
LEP FIT
(indirect)
Current Data
(June 30, 2012):
$m_t = 173.1 \pm 1.3$ GeV
m_t
250
200
150
100
50
0
1980
1984
1988
1990
1992
1994
1996
1998
2000
2002
Year

Top discovery: EW theory tests at Loop level

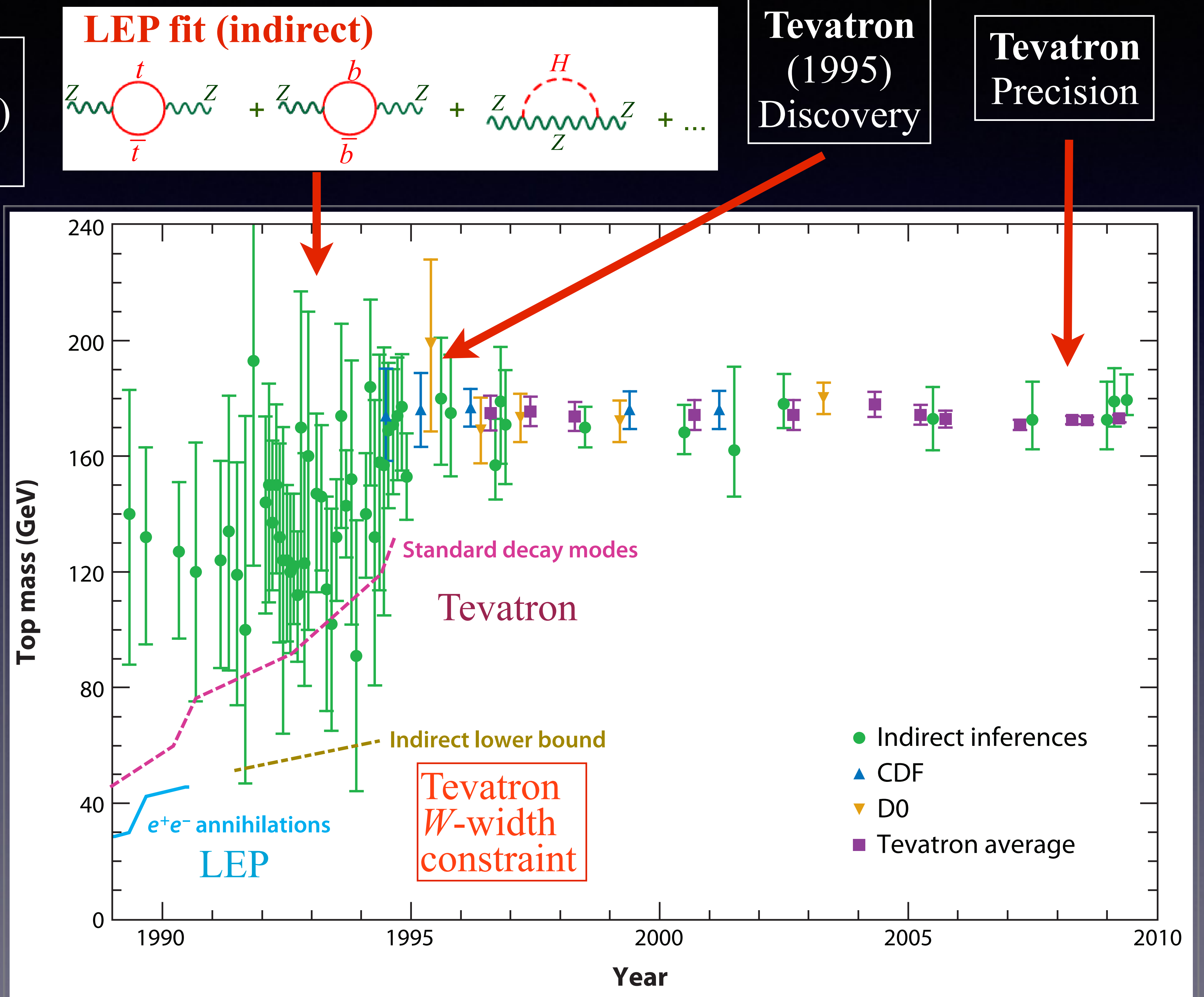
Bardeen, Hill, Lindner
Top-condensation (1989)
m_t >218GeV

Ibanez, Ross
SUGRA-GUT (1983)
30<m_t<150GeV

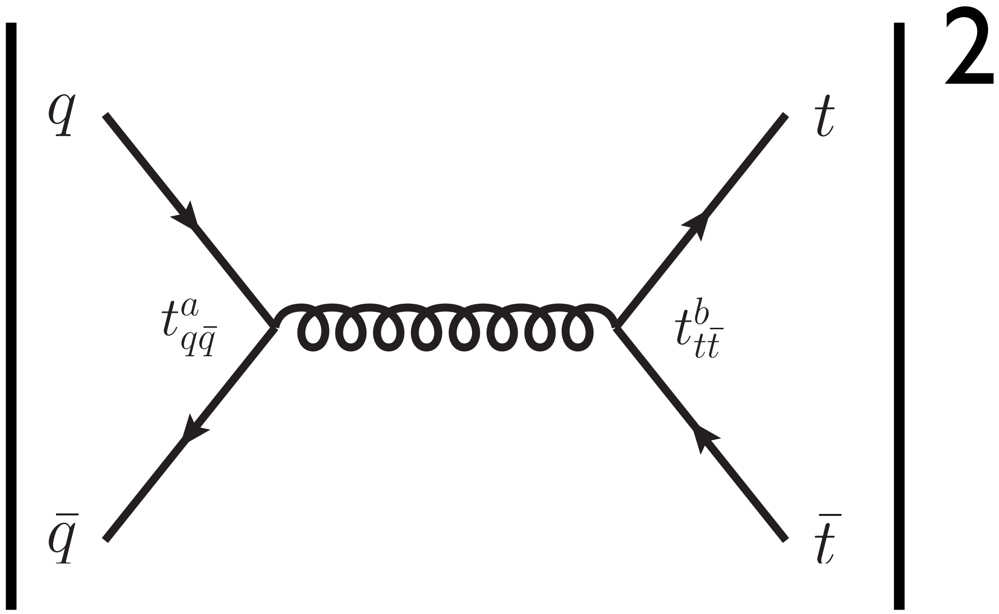
Pendleton, Ross
GUT (1980)
m_t=130GeV

Glashow (1980)
m_{tt}>38GeV

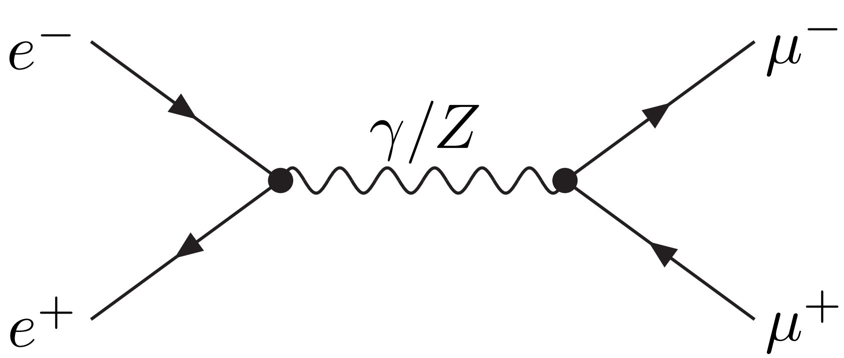
Tristan
1983



Top-quark Production

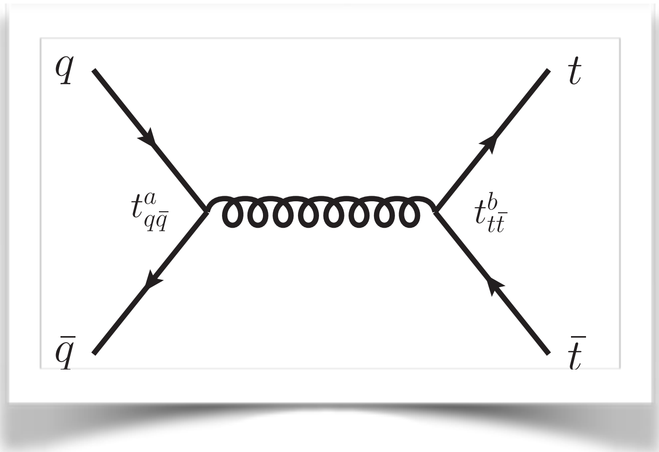


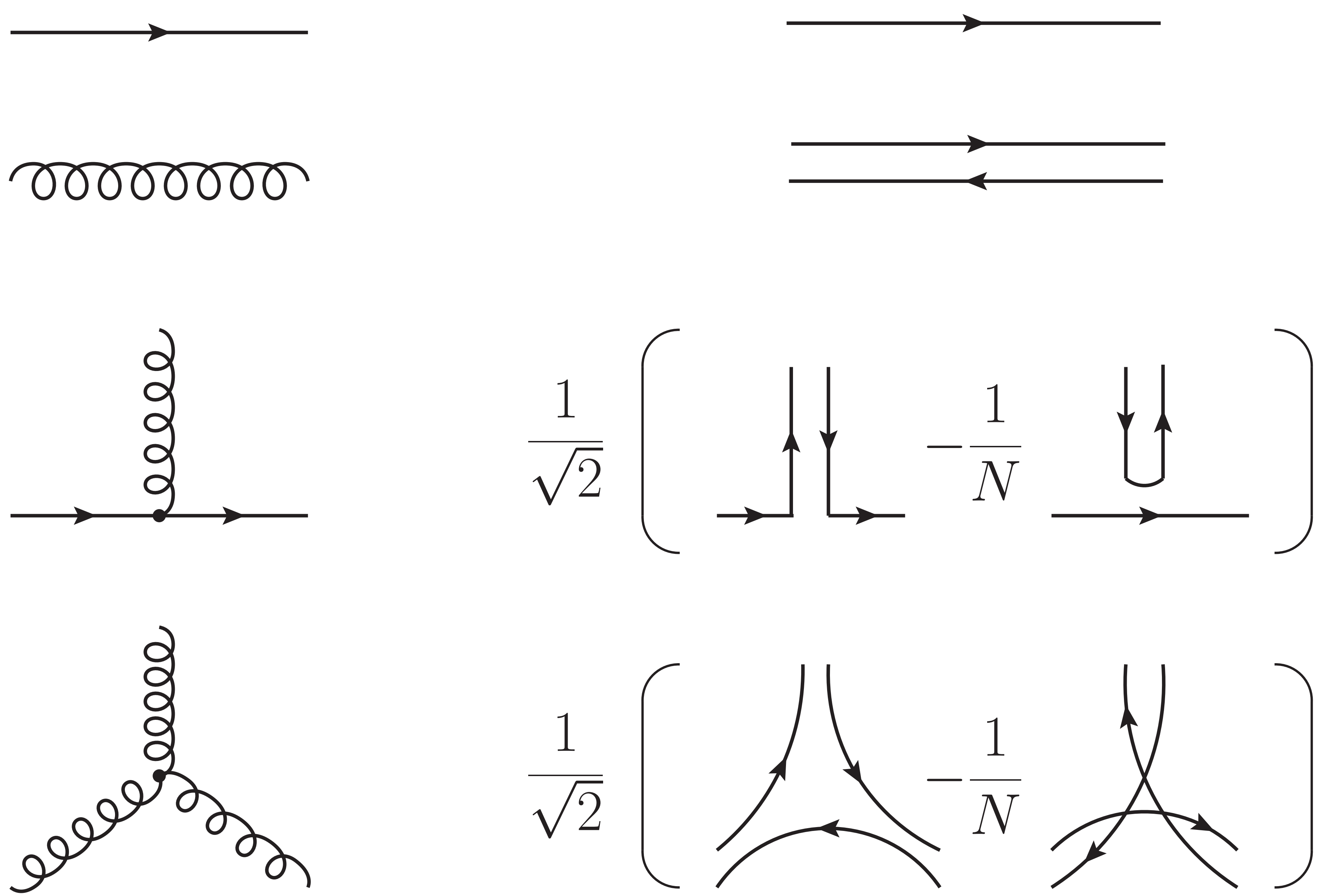
color factor



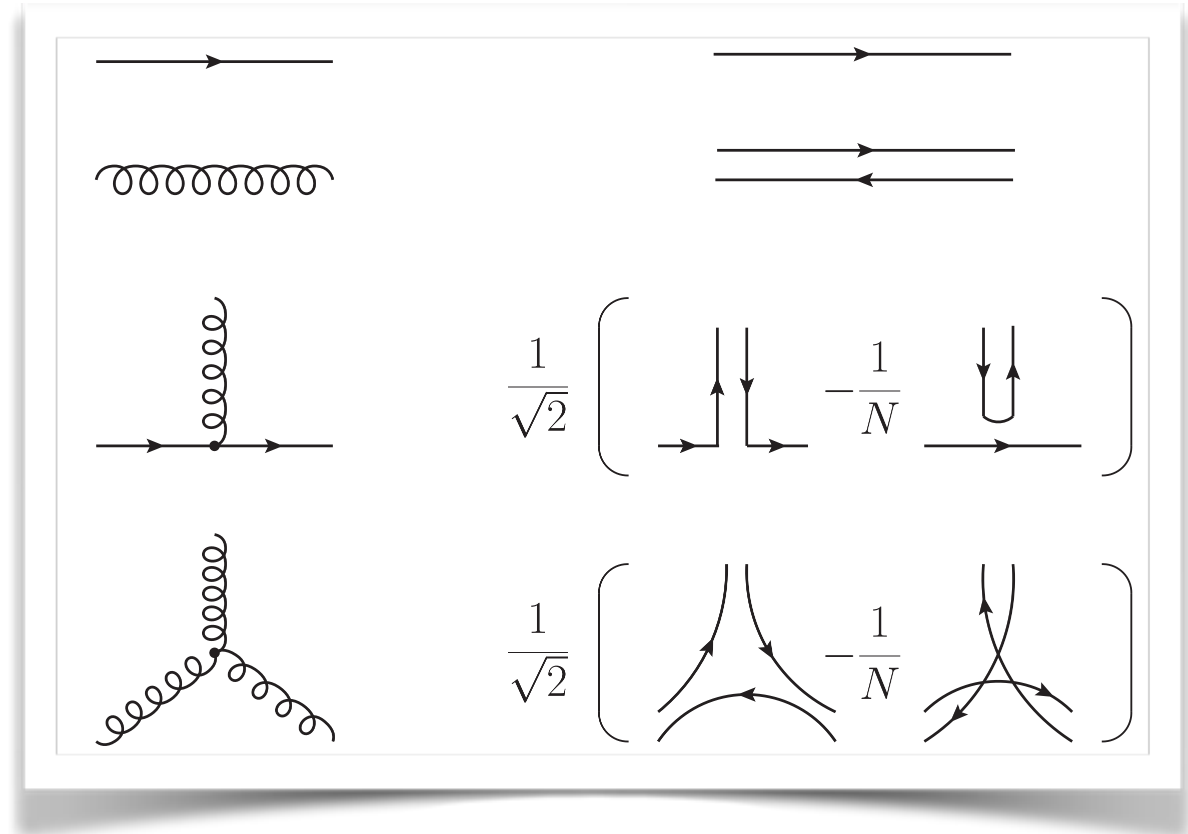
$$
\begin{aligned}
&\sum_{i,j,k,l} \left| \sum_{a,b} \delta_{ab} \left(\frac{\lambda^a}{2}\right)^i_j \left(\frac{\lambda^b}{2}\right)^l_k \right|^2 \\
&= \sum_{i,j,k,l} \left[\left(\frac{\lambda^a}{2}\right)^i_j \left(\frac{\lambda^a}{2}\right)^l_k \right]^* \left[\left(\frac{\lambda^b}{2}\right)^i_j \left(\frac{\lambda^b}{2}\right)^l_k \right] \\
&= \sum_{i,j,k,l} \left[\left(\frac{\lambda^a}{2}\right)^k_l \left(\frac{\lambda^a}{2}\right)^j_i \right] \left[\left(\frac{\lambda^b}{2}\right)^i_j \left(\frac{\lambda^b}{2}\right)^l_k \right] \\
&= \sum_{i,j,k,l} \frac{1}{4} \left(\delta^k_i \delta^l_j - \frac{1}{N} \delta^k_l \delta^j_i \right) \left(\delta^i_k \delta^j_l - \frac{1}{N} \delta^i_j \delta^k_l \right) \\
&= \frac{1}{4}\left(N^2 - \frac{N}{N} - \frac{N}{N} + \frac{1}{N^2} N^2\right) = \frac{1}{4}(N^2 - 1).
\end{aligned}
$$

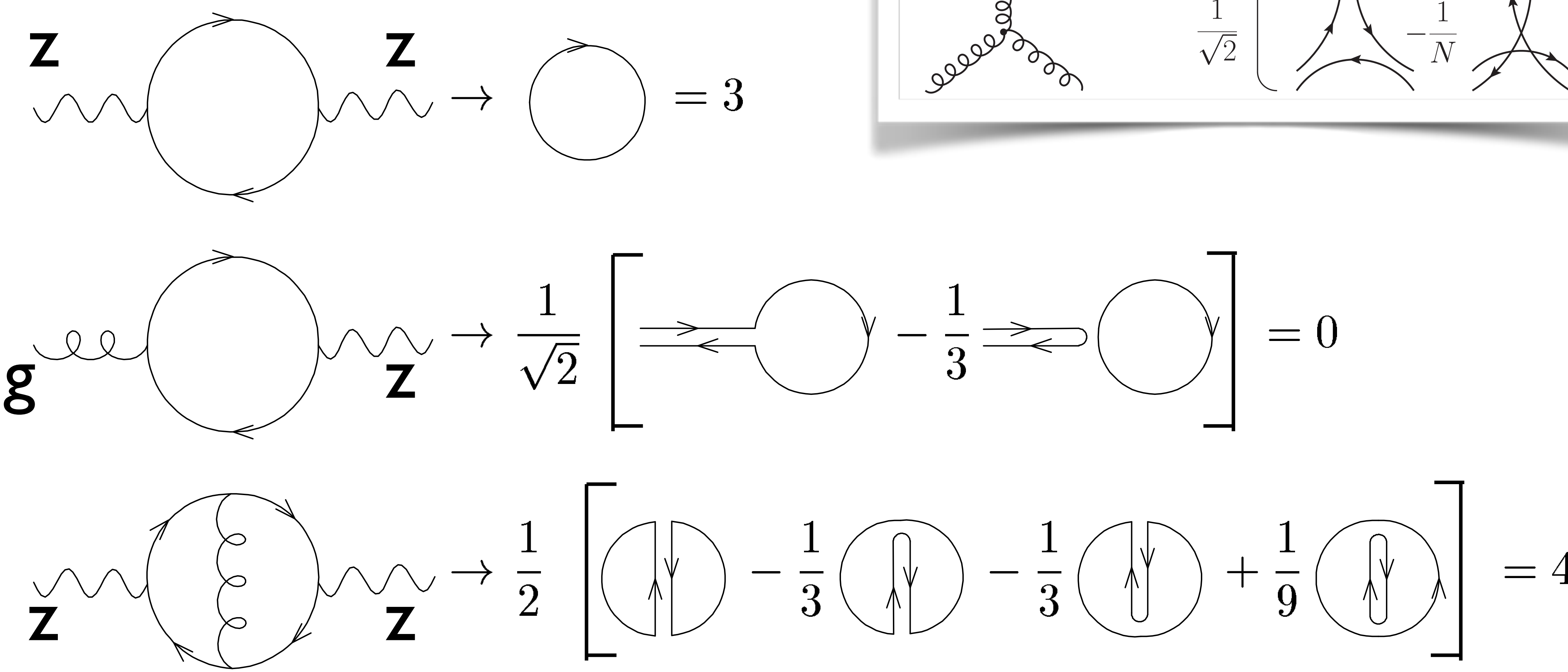
Color Feynman Rule



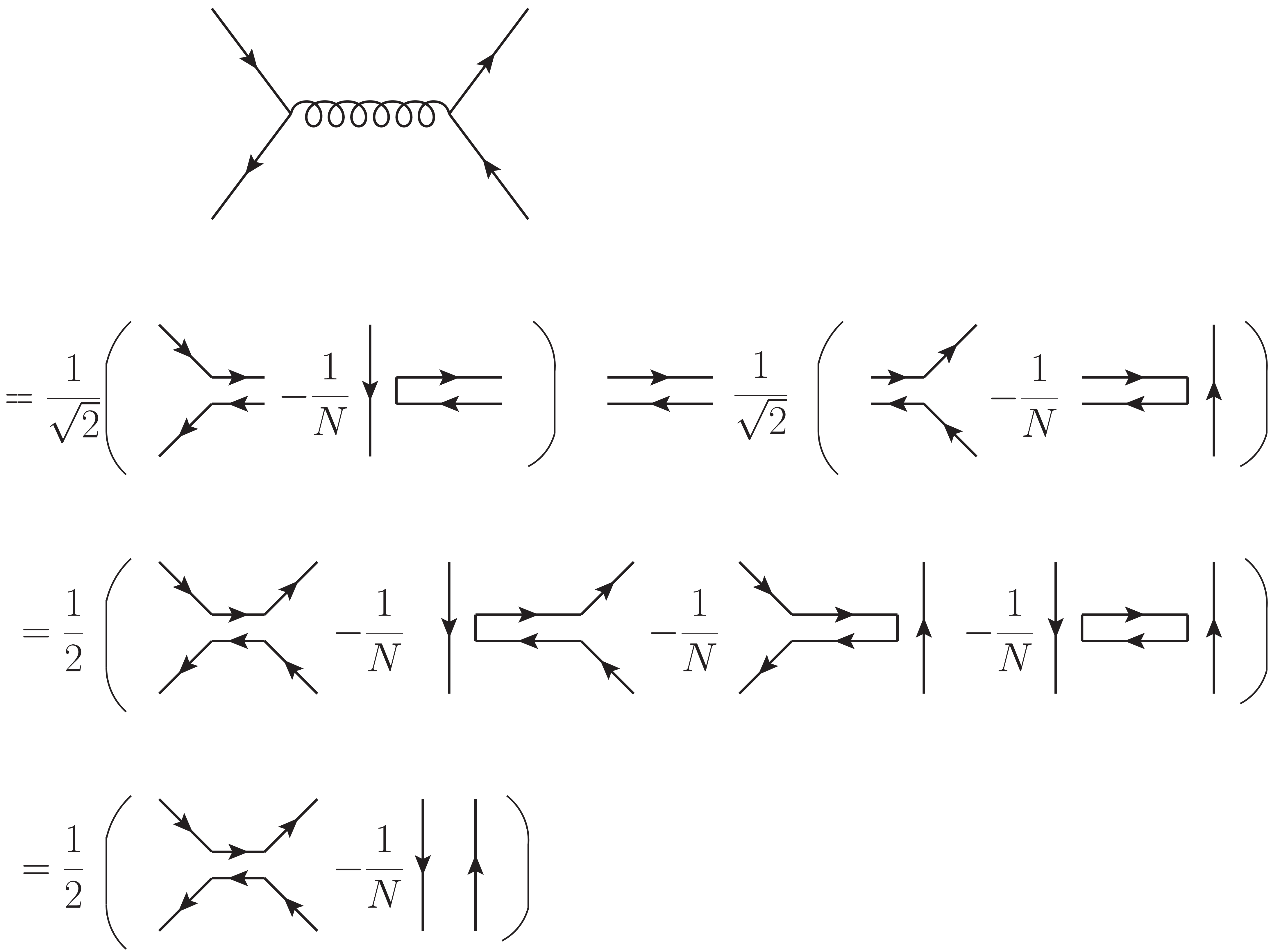


Color Factor





Color Flow (Large Nc)



Color Flow (Large Nc)

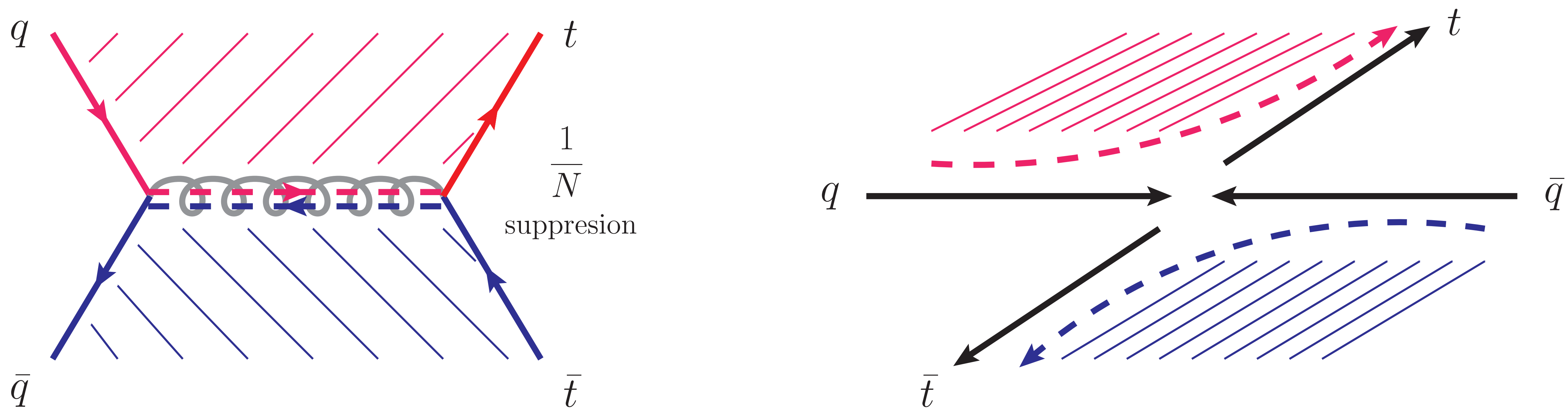
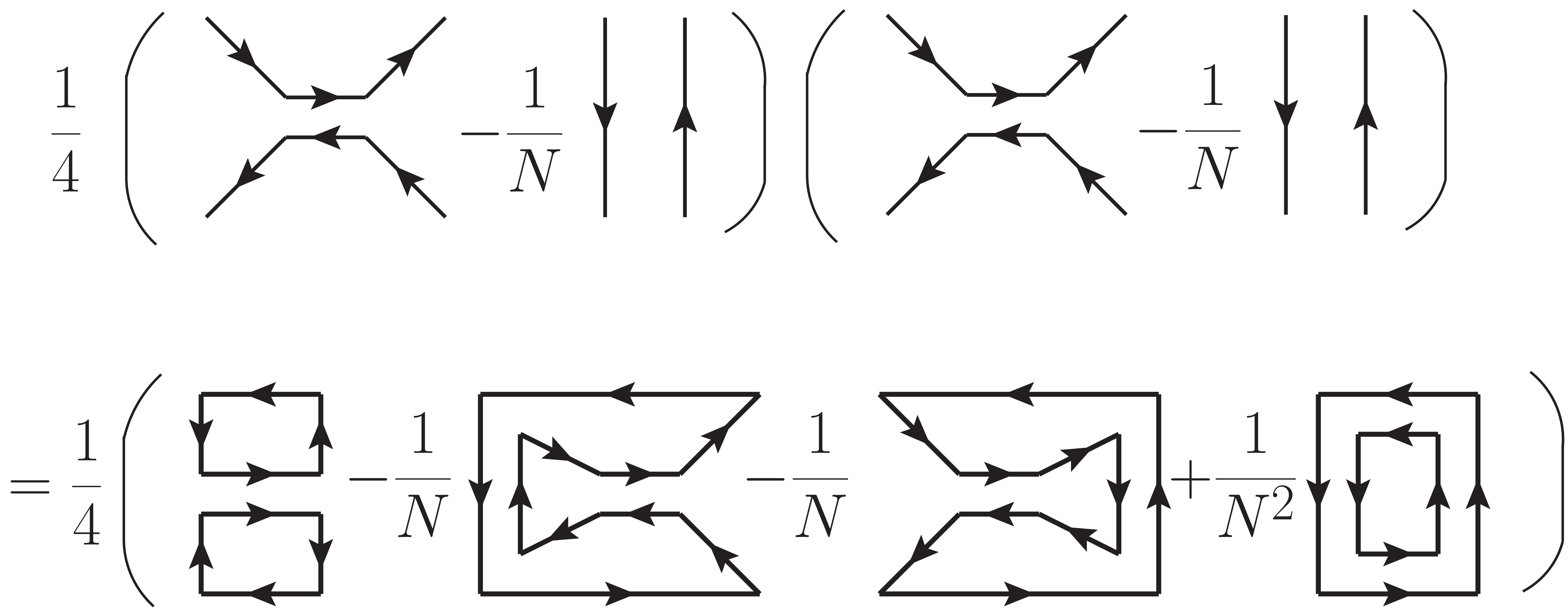


图 8.4: $q\bar{q} \to t\bar{t}$散射过程的色偶极子和额外胶子辐射区域。阴影区间是额外胶子辐射出现的主要区域，而空白区间的胶子辐射被色因子$1/N$压低。

Color Factor



$$= \frac{1}{4}\left(N^2 - \frac{N}{N} - \frac{N}{N} + \frac{N^2}{N^2}\right) = \frac{1}{4}(N^2 - 1)$$

Summary

There are so many things I cannot cover here. Sorry!

The shortcut of learning collider physics is to practice, to test, to play with it.

For theorists and phenomenologists, automation tools make your life easy but you should know what you are doing.