# Some Old Thoughts of Higgs Physics 

Qing-Hong Cao

Q.-H. Cao, S. Kanemura, C.-P. Yuan, Phys. Rev. D69 (2004) 115003
A. Belyaev, Q.-H. Cao, D. Nomura, K. Tobe, C.-P. Yuan, Phys. Rev. Lett. 100 (2008) 061801

## Outline

- $\mathrm{AH}^{+}$associate production
- A long ignored channel in "Higgs Hunter's Guide"

- A demo for collider simulations


## Collider Phenomenology



You MUST fully understand your results. You cannot simply say "I got them from Mad-..."

## Motivation

- Physics predictions depend strongly on the details of SUSY parameters.

A typical SUSY phenomenology study depends on at least two SUSY parameters, e.g. $\tan \beta$ and $m_{A}$, and various physics reach depends on other SUSY parameters as well.

Very often, the physics reach of a process is expressed in terms of bounds on

$$
\sigma(\text { production }) \times \operatorname{Br}(\text { decay branching ratio })
$$ where both $\sigma$ and Br depend on SUSY parameters

In general detection efficiency also depends on SUSY parameters.

## Our task is to find a SUSY process

- whose tree level $\sigma_{\text {prod }}$ depends on only ONE SUSY parameter that can be determined by kinematic variable (e.g. invariant mass).
- that is not sensitive to the detailed SUSY parameters via radiative corrections.
- that can bound the SUSY models by (product of)


## Br (decay branching ratio)

without convoluting with $\sigma_{\text {prod }}$.

- that can be used to distinguish MSSM from its alternatives, e.g. 2HDM.
- whose final state particle kinematics can be properly modeled without specifying any SUSY parameters.
$\longrightarrow$ The detection efficiency can be accurately determined.


## The promising process $p p \rightarrow W^{ \pm} \rightarrow A H^{ \pm}$



The production cross section in general depends on two masses: $m_{A}$ and $m_{H^{+}}$, e.g. in 2HDM.

But in MSSM,

$$
m_{H+}^{2}=m_{A}^{2}+m_{W}^{2}
$$

 $\binom{m_{A}$ can be determined from its decay kinematics, }{ e.g. the invariant mass $m_{b \bar{b}}$ in $A \rightarrow b \bar{b}$. }

## Production rates

- NLO QCD correction is about 20\%
- Uncertainty due to PDF is about 5\% at LHC for mA $=120 \mathrm{GeV}$.
- The one-loop electroweak correction to the production rate is smaller than the PDF uncertainty.

- The MSSM mass relation between A and $\mathrm{H}^{+}$holds well beyond tree level.


## Signal and BKGD

## Signal



Backgrounds
Veto additional lepton and jet from the parton level background events that satisfy

$$
\begin{aligned}
& \left.p_{T}(\text { lepton })>10 \mathrm{GeV}, \text { and } \mid \eta \text { (lepton }\right) \mid<3 \\
& p_{T}(\text { jet })>10 \mathrm{GeV}, \text { and } \mid \eta(\text { jet }) \mid<3.5
\end{aligned}
$$




## Model parameters and basic cuts

- The model parameters, production rates and decay BRs

| Sets | A | B | C |
| :---: | :---: | :---: | :---: |
| $m_{A} / \Gamma_{A}$ | $101 / 3.7$ | $165.7 / 5.6$ | $250 / 7.9$ |
| $m_{h} / \Gamma_{h}$ | $96.8 / 3.3$ | $112 / 0.04$ | $112 / 0.01$ |
| $m_{H} / \Gamma_{H}$ | $113 / 0.38$ | $163 / 5.5$ | $247.8 / 7.8$ |
| $m_{H} / \Gamma_{H}+$ | $126 / 0.43$ | $182 / 0.68$ | $261.4 / 4.2$ |
| $\sigma\left(A H^{+}\right)[f b]$ | 164 | 36 | 5.4 |
| $\sigma\left(H H^{+}\right)[f b]$ | 137.4 | 37.4 | 5.4 |
| $B r(A \rightarrow b \bar{b})$ | 0.91 | 0.90 | 0.89 |
| $B r(H \rightarrow b \bar{b})$ | 0.90 | 0.90 | 0.89 |
| $B r\left(H^{+} \rightarrow \tau^{+}{ }^{+}\right)$ | 0.98 | 0.90 | 0.00 |
| $B r\left(H^{+} \rightarrow t \bar{b}\right)$ | 0.00 | 0.09 | 0.79 |
| $B r\left(\tau^{+} \rightarrow \pi^{+} \nu\right)$ | 0.11 | 0.11 | 0.11 |

where $\tan \beta=40, \mu=M=500 \mathrm{GeV}$.

- Imposing basic cuts

$$
p_{T}\left(b, \bar{b}, \pi^{+}\right)>15 \mathrm{GeV},\left|\eta\left(b, \bar{b}, \pi^{+}\right)\right|<3.5, \Delta R\left(b, \bar{b}, \pi^{+}\right)>0.4
$$

# Set $A\left(m_{A}=101 \mathrm{GeV}\right)$ Kinematics Distributions 



## Significance

- Numbers of signal and background events at LHC with $100 \mathrm{fb}^{-1}$. The b-tagging efficiency ( $50 \%$, for tagging both $b$ and $\bar{b}$ jets) is included, and the kinematic cuts listed in each column are applied sequentially.
- Signal: $A H^{+}$

|  | Basic Cuts | $E_{T}>50$ | $P_{T}^{\pi}>40$ | $90<M_{b \bar{b}}<110[\mathrm{GeV}]$ |
| :---: | :---: | :---: | :---: | :---: |
| $A H^{+}$ | 507 | 391 | 241 | 216 |
| $H H^{+}$ | 48 | 38 | 24 | 0 |
| $W b \bar{b}$ | 11555 | 3111 | 864 | 67 |
| $t \bar{b}$ | 1228 | 614 | 163 | 12 |
| $W g$ | 567 | 236 | 68 | 11 |
| $t \bar{t}$ | 110 | 80 | 17 | 2 |
| Signal $(S)$ | 507 | 391 | 241 | 216 |
| Bckg $(B)$ | 13507 | 4078 | 1135 | 92 |
| $S / B$ | 0.038 | 0.095 | 0.212 | 2.35 |
| $S / \sqrt{B}$ | 4.36 | 6.12 | 7.14 | 22.5 |
| $\sqrt{S+B / S}$ | 0.23 | 0.17 | 0.15 | 0.08 |

## Constraint on MSSM



Constraints on the product of branching ratios

$$
B(A \rightarrow b \vec{b}) \times B\left(H^{+} \rightarrow \tau^{+} \nu_{\tau}\right)
$$

as a function of $\mathrm{M}_{\mathrm{A}}$ for Case A and Case B, and

$$
B(A \rightarrow b \bar{b}) \times B\left(H^{+} \rightarrow t \bar{b}\right)
$$

for Case C, at the LHC, where $\tau^{+}$decays into $\pi^{+} \bar{\nu}_{\tau}$ channel.

## So far so good, BUT

- Why are all the $P_{T}$ distributions of the signal events much harder than those of the SM backgrounds?


Answer: Matrix element
(spin correlations)

## A quick question

- What does the matrix element square look like in the c.m. frame?

(a) $(1+\cos \theta)^{2}$
(b) $(1-\cos \theta)^{2}$
(c) $\sin ^{2} \theta$
(d) $\sin ^{2} \frac{\theta}{2}$
(e) $\cos ^{2} \frac{\theta}{2}$
(f) $\sin ^{2} \theta \cos ^{2} \theta$


## Matrix element square

 In the c.m. frame

$$
\begin{aligned}
& p_{1}=(E, 0,0, E) \\
& p_{2}=(E, 0,0,-E) \\
& p_{3}=\left(E_{3}, P s_{\theta}, 0, P c_{\theta}\right) \\
& p_{4}=\left(E_{4},-P s_{\theta}, 0,-P c_{\theta}\right)
\end{aligned}
$$

$$
\begin{aligned}
|\mathcal{M}|^{2}= & \left(\frac{g}{\sqrt{2}}\right)^{2}\left(\frac{g}{2}\right)^{2} \frac{1}{\left(\hat{s}-m_{W}^{2}\right)^{2}+M_{W}^{2} \Gamma_{W}^{2}}=\left(\frac{g}{\sqrt{2}}\right)^{2}\left(\frac{g}{2}\right)^{2} \frac{1}{\left(\hat{s}-m_{W}^{2}\right)^{2}+M_{W}^{2} \Gamma_{W}^{2}} \\
& \times \operatorname{Tr}\left[\left(\not p_{3}-\not p_{4}\right) \not p_{1}\left(\not p_{3}-\not p_{4}\right) \not p_{2} P_{R}\right] \quad
\end{aligned}
$$

$$
=\left(\frac{g}{\sqrt{2}}\right)^{2}\left(\frac{g}{2}\right)^{2} \frac{1}{\left(\hat{s}-m_{W}^{2}\right)^{2}+M_{W}^{2} \Gamma_{W}^{2}}\left(4 E^{2}\right) \times\left\{4 P^{2}\right\}
$$

## Matrix element square

 In the c.m. frame

$$
\begin{aligned}
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& p_{2}=(E, 0,0,-E) \\
& p_{3}=\left(E_{3}, P s_{\theta}, 0, P c_{\theta}\right) \\
& p_{4}=\left(E_{4},-P s_{\theta}, 0,-P c_{\theta}\right)
\end{aligned}
$$

$$
=\left(\frac{g}{\sqrt{2}}\right)^{2}\left(\frac{g}{2}\right)^{2} \frac{1}{\left(\hat{s}-m_{W}^{2}\right)^{2}+M_{W}^{2} \Gamma_{W}^{2}}\left(4 E^{2}\right) \times\left\{4 P^{2}\right\}
$$

Can we get the angular dependence directly without any lengthy calculation?

$$
\begin{aligned}
& |\mathcal{M}|^{2}=\left(\frac{g}{\sqrt{2}}\right)^{2}\left(\frac{g}{2}\right)^{2} \frac{1}{\left(\hat{s}-m_{W}^{2}\right)^{2}+M_{W}^{2} \Gamma_{W}^{2}}=\left(\frac{g}{\sqrt{2}}\right)^{2}\left(\frac{g}{2}\right)^{2} \frac{1}{\left(\hat{s}-m_{W}^{2}\right)^{2}+M_{W}^{2} \Gamma_{W}^{2}} \\
& \times \operatorname{Tr}\left[\left(\not p_{3}-\not p_{4}\right) \not p_{1}\left(\not p_{3}-\not{ }_{4}\right) \not p_{2} P_{R}\right] \quad \times\left[4 \hat{t} \hat{u}-4 m_{A}^{2} m_{H^{+}}^{2}\right]
\end{aligned}
$$

## Helicity amplitude

- 2 to 2 scattering

$$
, c(\theta, \phi)
$$

$$
\begin{aligned}
& a+b \rightarrow \underset{\lambda_{a}}{a+b} \underset{\lambda_{b}}{c}+\underset{\lambda_{d}}{d}
\end{aligned}
$$

- Jacob-Wick formalism (partial wave decomposition)

$$
\begin{aligned}
& S_{f i}=\delta_{f i}+i(2 \pi)^{4} \delta^{4}\left(p_{f}-p_{i}\right) \mathcal{M}_{f i} \\
& \mathcal{M}_{f i}=\frac{8 \pi}{\sqrt{\beta_{i} \beta_{f}}} \sum_{J=0}^{\infty}(2 J+1) T_{\lambda_{a} \lambda_{b} ; \lambda_{c} \lambda_{d}}^{J}\left(E_{c m}\right) d_{\lambda_{i} \lambda_{f}}^{J}(\theta) e^{i\left(\lambda_{i}-\lambda_{f}\right) \phi}
\end{aligned}
$$

$$
\begin{array}{ll}
\lambda_{i}=\lambda_{a}-\lambda_{b} & d_{\lambda_{i} \lambda_{f}}^{J}(\theta) \text { Wigner d-function } \\
\lambda_{f}=\lambda_{c}-\lambda_{d} & \phi \text { angle is trivial in general. }
\end{array}
$$

## PDG Book: d-Functions



## Angular momentum in QM

- Consider a vector $|j, m\rangle \quad J^{2} J_{3}$



$$
R_{y}(\theta)=e^{i \theta J_{y}}
$$

Rotation matrices

$$
d_{m \rightarrow m^{\prime}}^{j}(\theta) \equiv d_{m, m^{\prime}}^{j}(\theta)=\left\langle j m^{\prime}\right| R_{y}(\theta)|j m\rangle
$$

The modulus squared is the probability that a particle $J_{3}=m$ will have $J_{3}=m^{\prime}$ after the rotation to the new frame.

## $\mathrm{AH}^{+}$Production

- Rotation matrices of spin-I

$$
\begin{aligned}
& J_{y}=\frac{i}{\sqrt{2}}\left(\begin{array}{ccc}
0 & -1 & 0 \\
1 & 0 & -1 \\
0 & 1 & 0
\end{array}\right) \\
& d_{-1,0}^{1}=-\frac{1}{\sqrt{2}} \sin \theta \\
& \lambda_{i}=\lambda_{q}-\lambda_{\bar{q}^{\prime}} \\
& \quad=-1 / 2-1 / 2=-1 \\
& \lambda_{f}=\lambda_{A}-\lambda_{H^{ \pm}}=0
\end{aligned}
$$

(I) Only longitudinal W-boson contributes.
(2) $A$ and $\mathrm{H}^{+}$stay in $p$-wave.

## $\mathrm{AH}^{+}$Production

- Rotation matrices of spin-I

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& \lambda_{i}=\lambda_{q}-\lambda_{\bar{q}^{\prime}} \\
&=-1 / 2-1 / 2=-1 \\
& \lambda_{f}=\lambda_{A}-\lambda_{H^{ \pm}}=0
\end{aligned}
$$

(I) Only longitudinal W-boson contributes.
(2) A and $\mathrm{H}^{+}$stay in $p$-wave. What does that mean?

## $\mathrm{AH}^{+}$Production

- Rotation matrices of spin-I

(2) A and $\mathrm{H}^{+}$stay in p-wave.


## $A H^{+}$Production

- Rotation matrices of spin-I

$$
J_{y}=\frac{i}{\sqrt{2}}\left(\begin{array}{ccc}
0 & -1 & 0 \\
1 & 0 & -1 \\
0 & 1 & 0
\end{array}\right)
$$

$d_{-1,0}^{1}=-\frac{1}{\sqrt{2}} \sin \theta$
(I) Only longitudinal

W-boson contributes.

(2) $A$ and $\mathrm{H}^{+}$stay in p-wave. Large PT's of $A$ and $\mathrm{H}^{+}$ (and their decay products)

## Tau is polarized

- Tau-lepton from $\mathrm{H}^{+}$decay is left-handedly polarized

- Tau-lepton from $\mathrm{W}^{+}$decay is right-handedly polarized



## $p_{\pi^{+}}$depends on $\tau^{+}$polarization

- A left-handed $\tau^{+}$produces a harder $\pi^{+}$


Homework:
Verify the above angular dependence with the following effective interaction

$$
\left(\partial^{\mu} \pi^{-}\right) \overline{\tau^{+}} \gamma_{\mu} P_{L} \nu
$$



## Interim summary

- Spin correlations force the scalars and its decay products in the signal events being highly boosted while those of the backgrounds are anti-boosted.



$p_{T}^{\pi^{+}}(\mathrm{GeV})$


## Mass measurement of $\mathrm{H}^{+}$

- Experimental difficulty:

Given a measured MET, can we tell it is one or two neutrinos?


- Four exceptions:
- $\mathrm{H}^{+}: H^{+} \rightarrow \tau^{+} \nu \rightarrow \pi^{+} \nu \bar{\nu} \rightarrow \pi^{+} Ð_{T}$
spin corr.
, $t \bar{t}: t \bar{t} \rightarrow b \bar{b} \ell^{+} \ell^{\prime-} \nu \bar{\nu} \rightarrow b \bar{b} \ell^{+} \ell^{\prime-} \bar{E}_{T}$
on mass shell conditions
, $h: h \rightarrow W^{+} W^{-} \rightarrow \ell^{+} \ell^{\prime-} \nu \bar{\nu} \rightarrow \ell^{+} \ell^{\prime-} \Phi_{T}$ spin corr.
, $h: h \rightarrow \tau^{+} \tau^{-} \rightarrow \pi^{+} \pi^{-} \nu \bar{\nu} \rightarrow \pi^{+} \pi^{-} \risingdotseq_{T}$


## Transverse Mass



* TM: measuring the mass of the W-boson in the leptonic decay channel

$$
\begin{aligned}
m_{T}^{2} & =2\left(E_{T}^{\ell} झ_{T}-p_{T}^{\ell} \not p_{T}\right) \\
& =2 p_{T}^{\ell} \not \nabla_{T}(1-\cos \phi)
\end{aligned}
$$

$\star$ The true mass of the W boson satisfies

$$
m_{T}^{2} \leq m_{W}^{2}
$$

* The end point of the transverse mass distribution is the W boson mass.


$$
\frac{\mathrm{d} \sigma}{\mathrm{~d} m_{T}^{2}} \sim \frac{1}{\sqrt{1-m_{T}^{2} / \hat{s}}}
$$

## $H^{+}$reconstruction

- Spin correlation dominates


Neutrino from tau decay

$$
m_{T}=\sqrt{2 p_{T}\left(\pi^{+}\right) E_{T}(1-\cos \Delta \phi)}
$$

is anti-boosted such that it tends to be very soft.
$\Delta \phi$ is the azimuthal angle between $\pi^{+}$and

## Transverse mass of $\pi^{+}$and $\equiv_{T}$

 - Transverse mass of $\mathrm{H}+$ after imposing the mass window cut on the two b-jets$$
|M(b \bar{b})-100|<10 \mathrm{GeV}
$$



Set-A

## Transverse mass of $\pi^{+}$and $\equiv_{T}$

 - Transverse mass of $\mathrm{H}+$ after imposing the mass window cut on the two b-jets$$
|M(b \bar{b})-165|<10 \mathrm{GeV}
$$



Set-B

## Constraint on MSSM



Constraints on the product of branching ratios

$$
B(A \rightarrow b \stackrel{\rightharpoonup}{b}) \times B\left(H^{+} \rightarrow \tau^{+} \nu_{\tau}\right)
$$

as a function of $\mathrm{M}_{\mathrm{A}}$ for Case A and Case B, and

$$
B(A \rightarrow b \bar{b}) \times B\left(H^{+} \rightarrow t \bar{b}\right)
$$

for Case C, at the LHC, where $\tau^{+}$decays into $\pi^{+} \bar{\nu}_{\tau}$ channel.

## Auxiliary material (I) Two neutrinos

- $\mathrm{H}^{+}: H^{+} \rightarrow \tau^{+} \nu \rightarrow \pi^{+} \nu \bar{\nu} \rightarrow \pi^{+} Ð_{T}$ spin corr.
$t \bar{t}: t \bar{t} \rightarrow b \bar{b} \ell^{+} \ell^{\prime-} \nu \bar{\nu} \rightarrow b \bar{b} \ell^{+} \ell^{\prime-} \bar{L}_{T}$
$h: \quad h \rightarrow W^{+} W^{-} \rightarrow \ell^{+} \ell^{\prime-} \nu \bar{\nu} \rightarrow \ell^{+} \ell^{-} \mp_{T} \quad$ spin corr.
$h: h \rightarrow \tau^{+} \tau^{-} \rightarrow \pi^{+} \pi^{-} \nu \bar{\nu} \rightarrow \pi^{+} \pi^{-} \unrhd_{T}$


## t-tbar in dilepton mode

$\star$ Four unknowns and four on-shell conditions


$$
\begin{aligned}
m_{W_{1}}^{2} & =\left(p_{\mu_{1}}+p_{\nu_{1}}\right)^{2} \\
m_{W_{2}}^{2} & =\left(p_{\mu_{2}}+p_{\nu_{2}}\right)^{2} \\
m_{t_{1}}^{2} & =\left(p_{W_{1}}+p_{b_{1}}\right)^{2} \\
m_{t_{2}}^{2} & =\left(p_{W_{2}}+p_{b_{2}}\right)^{2}
\end{aligned}
$$

Quartic equation (correct paring is necessary)
$p_{x}^{4}\left(\nu_{1}\right)+a p_{x}^{3}\left(\nu_{1}\right)+b p_{x}^{2}\left(\nu_{1}\right)+c p_{x}\left(\nu_{1}\right)+d=0$
Two complex, two real solutions

## Higgs search in WW dilepton mode

- Spin correlation demands both leptons moving in parallel


Rainwater, Zeppenfeld,
Phys. Rev. D6I (2000) 093005
Q.-H. Cao and C.-R. Chen,

Phys. Rev. D76 (2007) 075007


- Transverse cluster mass

$$
\begin{aligned}
& M_{T}=\sqrt{2 p_{T}^{L L} E_{T}\left(1-\cos \Delta \phi\left(p_{T}^{L L}, E_{T}\right)\right)} \\
& M_{C}=\sqrt{p_{T}^{L L^{2}}+m_{L L}^{2}}+E_{T}
\end{aligned}
$$



## Higgs search in tau-tau mode

- Collinear approximation

$$
p_{\tau^{+}}=x p_{\pi^{+}}+(1-x) p_{\nu_{1}}
$$



further demands

$$
x>0, y>0
$$

## Plehn, Rainwater, Zeppenfeld, Phys. Rev. D6I (2000) 093005

## Auxiliary material (II)

W-boson Helicity as a measure of the chirality structure of the $W$ - $t-b$ coupling

## W-boson helicity

- can be measured from the charged-lepton angular distribution

$$
d_{d_{-1,1}^{1}(\theta)=\frac{1-\cos \theta}{2}}^{l^{l^{+}}}
$$



## W Helicity from Top Decay

- A good probe of the handness of W-t-b coupling

$$
\frac{1}{\Gamma_{t}} \frac{\mathrm{~d} \Gamma_{t}}{\mathrm{~d} \cos \theta}=f_{0} \frac{3}{4} \sin ^{2} \theta+f_{-} \frac{3}{8}(1-\cos \theta)^{2}+f_{+} \frac{3}{8}(1+\cos \theta)^{2}
$$

$$
\cos \theta \simeq \frac{2 m_{b e}^{2}}{m_{t}^{2}-m_{W}^{2}}-1
$$

In the SM at the tree level:


$$
\begin{aligned}
f_{0} & =0.7, f_{-}=0.3, f_{+}=0 \\
f_{0} & =\frac{\Gamma\left(t \rightarrow b W_{\text {Long }}\right)}{\Gamma\left(t \rightarrow b W_{\text {Long }}\right)+\Gamma\left(t \rightarrow b W_{T}\right)} \\
& \simeq \frac{m_{t}^{2}}{m_{t}^{2}+2 m_{W}^{2}}
\end{aligned}
$$



## Quizzes

- Angular distribution of the Drell-Yan process $u \bar{d} \rightarrow e^{+} \nu$



## Quizzes

- Angular distribution of the Drell-Yan process $u \bar{d} \rightarrow e^{+} \nu$


$$
d_{1,1}^{1}=1+\cos \theta
$$

## Quizzes

- Angular distribution of the single-top process $u \bar{d} \rightarrow t \bar{b}$



## Quizzes

- Angular distribution of the single-top process $u \bar{d} \rightarrow t \bar{b}$



## Quizzes

- Angular distribution of the single-top process $u \bar{d} \rightarrow t \bar{b}$

t
$d_{1,1}^{1}=1+\cos \theta$

$$
\begin{array}{r}
d_{1,0}^{1}=\sin \theta \\
\mathcal{M} \propto m_{t}
\end{array}
$$

