粒子物理

19. 宇称和弱相互作用

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参考书：《Introduction to Elementary Particle Physics 》
by Alessandro Bettini (Chapter 03)

The Birth: Beta Decay

$$A \to B + e^-$$

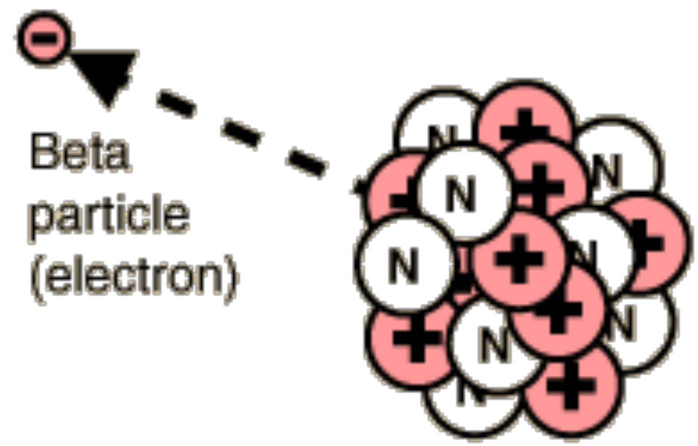
$$(Z, N) \to (Z+1, N-1) + e^-$$

$m_n = 939.5656\ MeV$

$m_p = 938.2723\ MeV$

$m_e = 0.510999\ MeV$

$0.7823\ MeV = Q\ \ for\ \ n \to p + e^-$



The conservation of Energy and momentum
requires the electron have a single value of energy.

Parity Violation

Parity conservation had been assumed, almost without question

$\theta - \tau$ puzzle (1950's)

$$\boldsymbol{\theta} \longrightarrow \underbrace{\boldsymbol{\pi}^{+}\boldsymbol{\pi}^{0}}_{\color{red}{P=+1}}$$

$$\boldsymbol{\tau} \longrightarrow \underbrace{\boldsymbol{\pi}^{+}\boldsymbol{\pi}^{+}\boldsymbol{\pi}^{0}}_{\color{red}{P=-1}}$$

Two particles with same mass, charge, spin, lifetime, but different decay modes and parity

Lee, Yang (1956)

历史

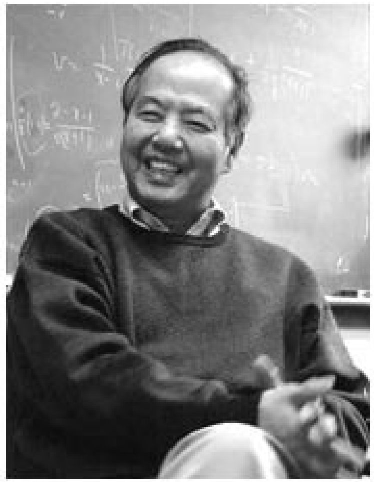
1956年4月，李政道和杨振宁在 Rochester 会议提出或许有具有两种宇称的粒子

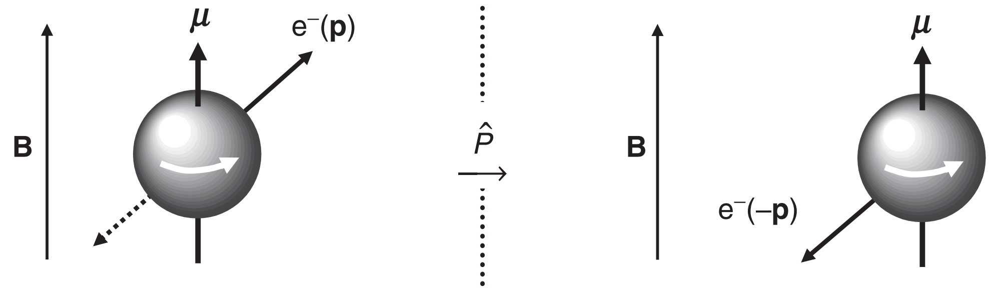
在会议上，Feynman 提出宇称不守恒的想法，但他本人并不相信。在 Lee 和 Yang 宇称破坏文章出现后，Feynman 和打赌 50 美元，押在宇称守恒上。

同次会议上，将宇称引入 QM 的 Wigner 也提出有可能的。Yang 报告后，有人提问 P 的不守性，Yang 回答说"我们仔细考虑过了，但并没有适宜表明的"

1956年10月，Lee 和 Yang 写文章提出"弱作用中的"。

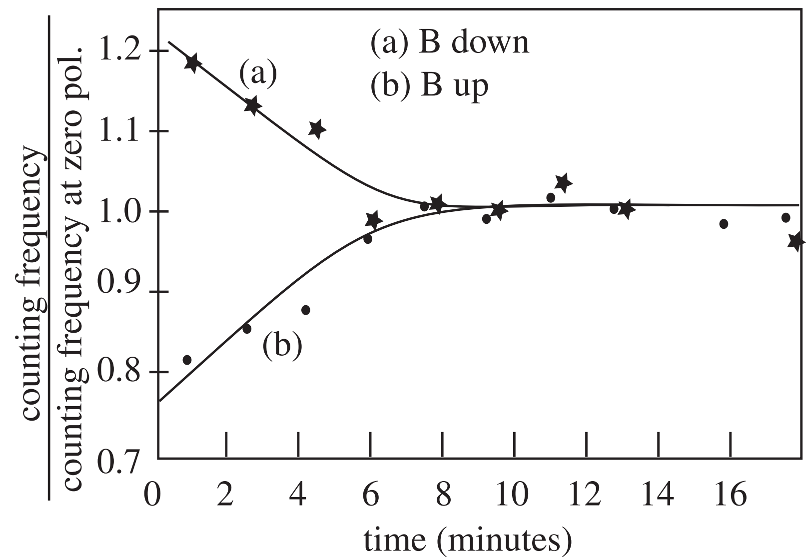
Parity Violation





$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni}^{*} + e^{-} + \bar{\nu}_{e}$$

Beta asymmetry



两组独立实验（1957）

Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson,
National Bureau of Standards, Washington, D. C.

(Received January 15, 1957)

In a recent paper[1] on the question of parity in weak interactions, Lee and Yang critically surveyed the experimental information concerning this question and reached the conclusion that there is no existing evidence either to support or to refute parity conservation in weak interactions. They proposed a number of experiments on beta decays and hyperon and meson decays which would provide the necessary evidence for parity conservation or nonconservation. In beta decay, one could measure the angular distribution of the electrons coming from beta decays of polarized nuclei. If an asymmetry in the distribution between θ and $180°-\theta$ (where θ is the angle between the orientation of the parent nuclei and the momentum of the electrons) is observed, it provides unequivocal proof that parity is not conserved in beta decay. This asymmetry effect has been observed in the case of oriented Co^{60}.

Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

Richard L. Garwin,† Leon M. Lederman, and Marcel Weinrich

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York

(Received January 15, 1957)

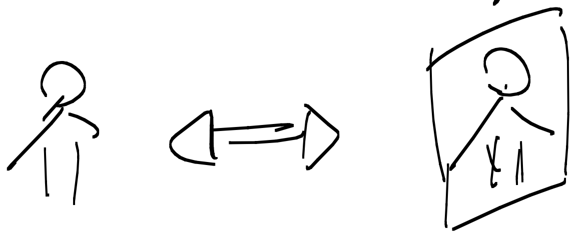
Lee and Yang[1-3] have proposed that the long held space-time principles of invariance under charge conjugation, time reversal, and space reflection (parity) are violated by the "weak" interactions responsible for decay of nuclei, mesons, and strange particles. Their hypothesis, born out of the $\tau-\theta$ puzzle,[4] was accompanied by the suggestion that confirmation should be sought (among other places) in the study of the successive reactions

$$\pi^+ \rightarrow \mu^+ + \nu, \tag{1}$$

$$\mu^+ \rightarrow e^+ + 2\nu. \tag{2}$$

They have pointed out that parity nonconservation implies a polarization of the spin of the muon emitted from stopped pions in (1) along the direction of motion and that furthermore, the angular distribution of electrons in (2) should serve as an analyzer for the muon polarization. They also point out that the longitudinal polarization of the muons offers a natural way of determining the magnetic moment.[5] Confirmation of this proposal in the form of preliminary results on β decay of oriented nuclei by Wu *et al.* reached us before this experiment was begun.[6]

Parity: inversion of the three spatial coordinate axes



mirror

The $\mathbb{P}$ operation

$$\vec{r} \longrightarrow -\vec{r} \qquad\qquad \vec{p} \longrightarrow -\vec{p}$$

$$\Longrightarrow \qquad \vec{r}\times\vec{p} \longrightarrow \vec{r}\times\vec{p}$$

$$t \longrightarrow t \qquad\qquad \vec{S} \longrightarrow \vec{S}$$

$$\begin{cases} \text{Scalar} \longrightarrow \text{scalar} \\ \text{Pseudo-scalar} \longrightarrow -\text{pseudo-scalar} \\ \text{Vector} \longrightarrow -\text{vector} \\ \text{Axial vector} \longrightarrow \text{Axial vector} \end{cases}$$

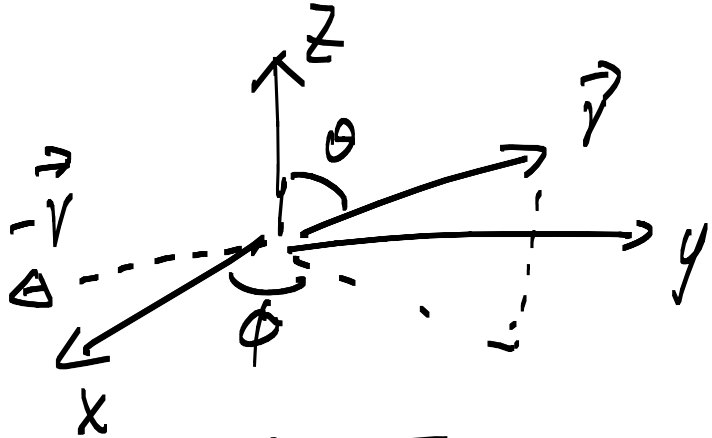
Intrinsic Parity (simple parity)

The eigenvalue p of $\mathbb{P}$ in the rest frame of a particle

either Positive (+) or Negative (−)

Parity application on a wave function

$$\hat{\mathbb{P}}\,\psi(\vec{r}) = \psi(-\vec{r})$$



$\theta \rightarrow \pi - \theta$

$\phi \rightarrow \pi + \phi$

For a system $\hat{H}$

$$[\hat{H}, \hat{P}] = 0 \Rightarrow \text{Parity conserved}$$

Eg: $H(-\vec{r}) = H(\vec{r})$

Atomic bound state (neglecting the spin)

$$\psi(r, \theta, \phi) = \chi(r)\, \underbrace{Y_l^m(\theta, \phi)}_{\Rightarrow \text{ spherical harmonics}}$$

$$Y_l^m(\theta, \phi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}\; \underbrace{P_l^m(\cos\theta)}_{\text{Legendre polynomials}}\, e^{im\phi}$$

$$\hat{P}\, e^{im\phi} = e^{im(\phi+\pi)} = e^{im\pi} e^{im\phi} = (-1)^m e^{im\phi}$$

$$\hat{P}\, P_l^m(\cos\theta) = (-1)^{l+m} P_l^m(\cos\theta)$$

$$\Rightarrow \hat{P}\, Y_l^m(\theta, \phi) = (-1)^l\, Y_l^m(\theta, \phi)$$

* $l = 0, 2, 4, \cdots$ $P = +1$
* $l = 1, 3, 5, \cdots$ $P = -1$

* Parity of boson can be determined without ambiguity
* Fermions (Spin-$\frac{1}{2}$) — produced in pair
  $\Rightarrow$ relative parities can be defined

惯例(约定) $P(\text{proton}) = P(\text{neutron}) = +1$

QFT: $P(\text{fermion}) = -P(\text{antifermion})$

$P(\text{boson}) = +P(\text{antiboson})$

Strange 超子 (Hyperon) : produced in Pair

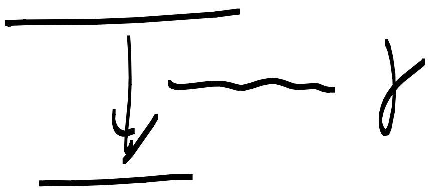
$\Lambda^0 \to p\pi^-$ (?) <u>**No!**</u> Weak Decay $\cancel{P}$

约定 $P(\Lambda^0) = +1$

At quark level, $P(\text{quarks}) = +1$, $P(\text{antiquark}) = -1$

* Parity of Photon $J^P = 1^-$

Atomic radiation



$\Delta l = \pm 1$ Electric dipole

$\Downarrow$

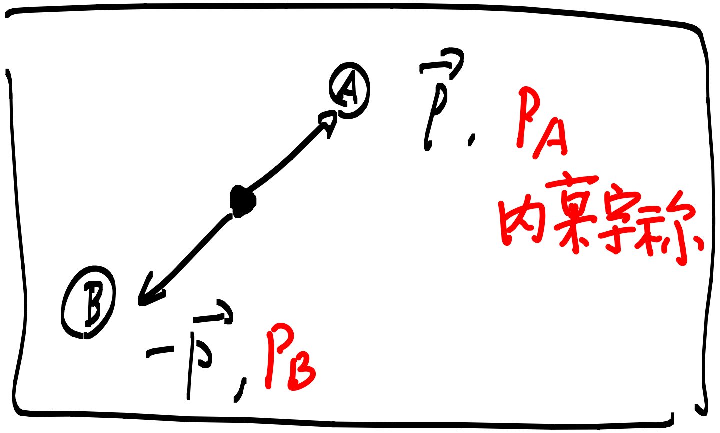
Two levels have opposite parities

$\Rightarrow$ QED (EM) conserved parity

$\Rightarrow$ $P(\text{initial}) = P(\text{final}) \times P(\gamma)$

$\Rightarrow$ $P(\gamma) = -1$

※ Parity of a two-particle system



$$\psi(\vec{p},-\vec{p}) = \psi(p,\theta,\phi) = |p,\theta,\phi\rangle$$

$$|p\,l\,m\rangle = \sum_{\theta,\phi} |p,\theta,\phi\rangle\langle p,\theta,\phi|p\,l\,m\rangle = \sum_{\theta\phi} Y_l^{*m}(\theta,\phi)\,|\vec{p},-\vec{p}\rangle$$

$$Y_l^{*m}(\theta,\phi) \xrightarrow{\mathbb{P}} Y_l^{*m}(\pi-\theta,\pi+\phi) = (-1)^l\, Y_l^{*m}(\theta,\phi)$$

$$\Rightarrow \quad \mathbb{P}\,|p,l\,m\rangle = P_A P_B \sum_{\theta,\phi} Y_l^{*m}(\pi-\theta,\pi+\phi)\,|-\vec{p},\vec{p}\rangle$$

$$= P_A P_B (-1)^l \sum_{\theta,\phi} Y_l^{*m}(\theta,\phi)\,|\vec{p},-\vec{p}\rangle$$

$$= \underline{\underline{P_A P_B (-1)^l}}\,|p,l,m\rangle$$

<u>Example.</u>

(1) Parity of two mesons with the same intrinsic parity

($\underline{\underline{m_1}}$, $\underline{\underline{m_2}}$) meson-1

$$P(m_1, m_2) = (-1)^l$$

$$\text{Spin}(\pi) = 0 \Rightarrow J = l$$

$$\Rightarrow \begin{cases} J^P(m_1, m_2) = 0^+, 1^-, 2^+, \cdots \\ \quad \text{two mesons are different} \\ J^P(m_1, m_2) = 0^+, 2^+, \cdots \end{cases}$$

two same meson

(Boson-Einstein statistics)

2) Fermion - Antifermion pair

The two intrinsic parities are opposite

$\Rightarrow P(f) \times P(\bar{f}) = -1$

If ℓ is the orbit momentum,

then $\quad P(f\bar{f}) = (-1)^{\ell+1}$

Ex 18 : Find J^P for a spin-½ particle and its antiparticle if they are in an S-wave or P-wave

$^{2S+1}L_J$

Total spin = 0 (singlet) or 1 (triplet)

S-wave ($\ell=0$) $\Rightarrow$ $\begin{cases} J=0 \ ({}^1S_0) \\ J=1 \ ({}^3S_1) \end{cases}$

$P=-1$

P-wave $\ell=1$ $\quad J = 0, 1, 2$

$P=+1$

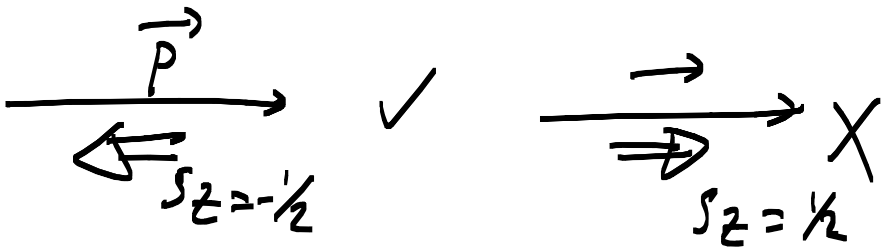
${}^1P_1$, $J^P = 1^+$ $\quad {}^3P_1$ ($J^P = 1^+$)

${}^3P_0$, $J^P = 0^+$ $\quad {}^3P_2$ ($J^P = 2^+$)

* Parity violation in weak interaction

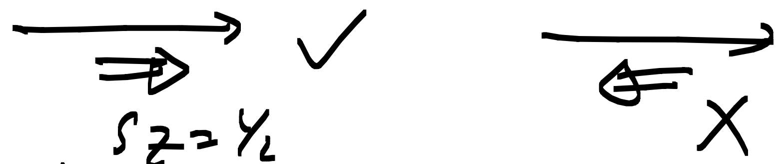
ν_e : electron neutrino $\quad S = \frac{1}{2}, \quad S_z = \pm \frac{1}{2}$

Exp. Data : ν_e



neutrino is left-hand

$\bar{\nu}_e$



right-handed

$\mathbb{P}(\nu_e) : \vec{p} \to -\vec{p}, \; S_z \to S_z \;\Rightarrow\; P(\overset{\longrightarrow}{\Leftarrow} \nu_e) = (\overset{\longleftarrow}{\Leftarrow} \nu_e)$

Not observed!

$\Rightarrow$ weak int is not invariant under spatial inversion (parity violation)

Spin-Parity of π meson

(1) Spin of π meson

$$\pi^+ d \to pp \quad / \quad pp \to \pi^+ d$$

d: deuteron 氘核
(pn)
ground state $^3S_1$
$S_d = 1$

detailed balance principle

$$|M_{if}|^2 = |M_{fi}|^2$$

$$\Rightarrow \quad \frac{\sigma(pp \to \pi^+ d)}{\sigma(\pi^+ d \to pp)} = 2\,\frac{(2S_\pi+1)(2S_d+1)}{(2S_p+1)^2}\,\frac{P_\pi^2}{P_p^2} = \frac{3}{2}(S_\pi+1)\frac{P_\pi^2}{P_p^2}$$

$$\Rightarrow \quad \underline{S_{\pi^+} = 0}$$

In e^+e^- / $p\bar{p}$ / pp collider at high energy ($E_{cm} > 10\,\text{GeV}$)

π^+, π^-, π^0 are abundantly produced in equal number

$$\Rightarrow \quad S_{\pi^+} = S_{\pi^0} = S_{\pi^-} = 0$$

2) Parity of π meson

low-energy π^- absorption in deuterium

$$\pi^- d \to n n$$

Initial $(\pi^- d)$ $\quad S_d = 1$, $S_{\pi^-} = 0$, $l = 0$ (s-wave)

low-En capture

$$|\vec{J}_i| = |\vec{S}_\pi + \vec{S}_d + \vec{L}_{\pi d}| = 1$$

Final (nn): $\quad |\vec{J}_f| = |\vec{L}_{nn} + \vec{S}_{nn}|$

Total angular momentum conservation
$\Rightarrow |\vec{J}_i| = |\vec{J}_f|$

$$\psi_{tot} = \alpha(\text{space}) \cdot \beta(\text{spin})$$

nn identical fermion
$\Rightarrow \psi_{tot}$ is antisymmetric

$$\Downarrow$$

$$Y_l^m(\theta, \phi) \xleftrightarrow{1 \leftrightarrow 2} (-1)^l Y_l^m(\theta, \phi)$$

* β (spin)

$$\left.\begin{array}{l}\beta_1,\ S=\tfrac{1}{2},\ S_z=\pm\tfrac{1}{2} \\ \beta_2,\ S=\tfrac{1}{2},\ S_z=\pm\tfrac{1}{2}\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\beta(1,1)=\uparrow\uparrow \\ \beta(1,0)=\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow) \\ \beta(0,1)=\downarrow\downarrow\end{array}\right. \quad \text{or} \quad \begin{array}{l}\beta(0,0) \\ =\frac{1}{\sqrt{2}}(\downarrow\uparrow-\uparrow\downarrow)\end{array}$$

$S=1$ $S_z=\pm 1, 0$. $\quad S=0, S_z=0$

$$\beta(1,2) \xrightarrow{1\leftrightarrow 2} (-1)^{S+1}\beta(1,2)$$

$$\left.\begin{array}{l}\Rightarrow \psi_{tot} \xrightarrow{1\leftrightarrow 2} (-1)^{l+S+1}\psi_{tot} \\ \psi_{tot}(n,n) \xrightarrow{1\leftrightarrow 2} (-1)\psi_{tot}(n,n)\end{array}\right\} \Rightarrow (-1)^{l+S+1}=-1$$

$$\Rightarrow \boxed{l+S=\text{even}}$$

Summary: (1) $l+S=\text{even}$ (final)

(2) $|\vec{J}|=1$ (initial)

$|J| = 1$, $S = 1$ or 0, $l = 0, 1, 2$

$l = 0$	$l = 1$	$l = 1$	$l = 2$
$S = 1$	$S = 0$	$S = 1$	$S = 1$
$l + S =$ odd	$l + S =$ odd	$l + S =$ even	$l + S =$ odd
×	×	✓	×

Parity of the final state

$$P(2n) = P(n) \cdot P(n) (-1)^{l} = -1$$

$$P(p) = P(n) = +1$$

重子数守恒 ⇒ 每个核子内禀宇称值并不重要
因为重子-反重子必须成对产生
⇒ 相互抵消，仅余相对

Assume Parity is conserved in strong int

$$P(\pi^- d) = P(nn) = -1$$

$$P(\pi^- d) = P(\pi^-)\,P(d)\,(-1)^{l=0} \quad (\text{s-wave})$$

$$P(d) = P(p)\,P(n)\,(-1)^0 = +1$$

$$\Rightarrow P(\pi^- d) = P(\pi^-) = P(nn) = -1$$

the π^- has negative parity

$$P(\pi^+) = P(\pi^0) = P(\pi^-) = -1 \qquad \boxed{J^P = 0^-}$$

pseudo-scalar

System with n-pions 的内禀宇称之积为

$$P(n\pi) = (-1)^n$$

Negative pion capture Experiments

Chinowsky, Seinberger, Physical Review 95, 1561 (1954)

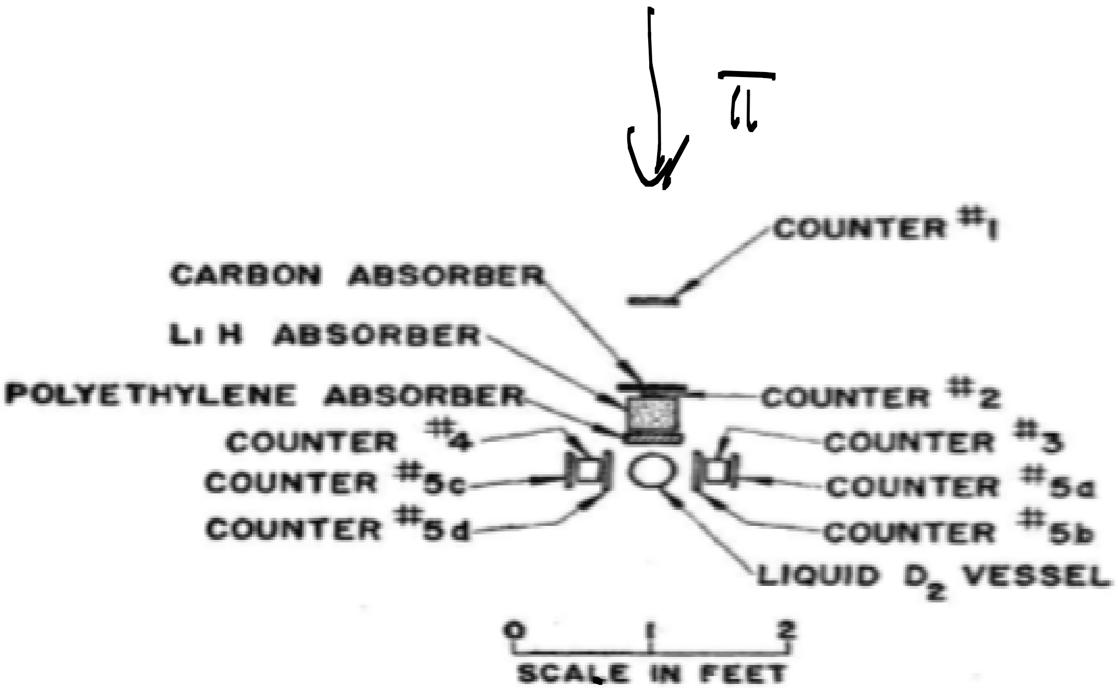
Deuterium 俘获 pion 要求

(1) Negative pion is slow

(2) Deuterium 有足够大的压强 $\Rightarrow$ 俘获反应的时间比 τ_π 短

为此，在 pion 的运动方向上摆放多个吸收能量材料将 pion 变慢

(3种不同材料)



Couter 1 & 2: π^- 通过

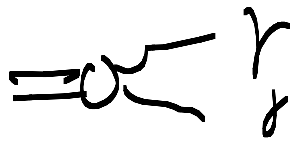
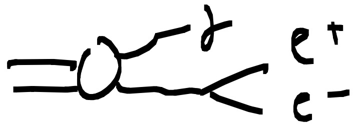
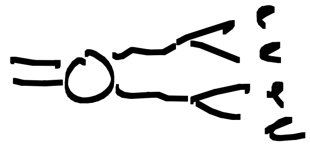
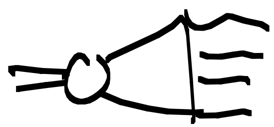
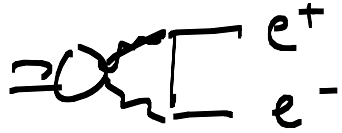
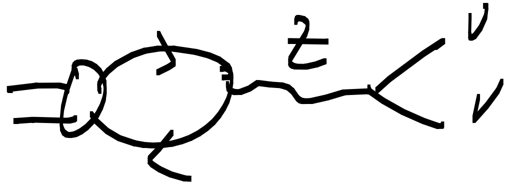
Couter 3 & 4: neutron

Couter 5a.b.c.d. photon detector

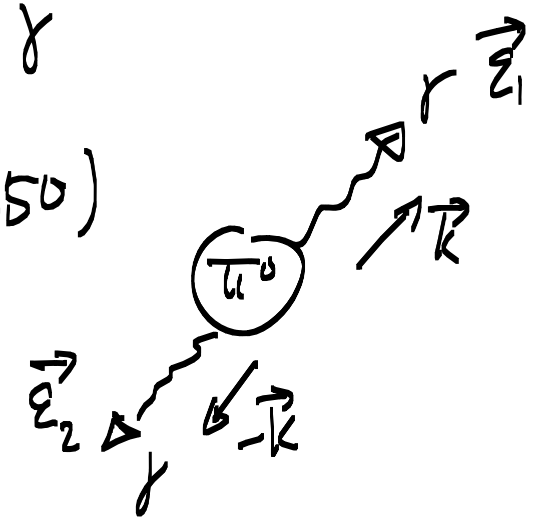
信号: 1 + 2 + 3 + 4 ✓

not 5a.b.c.d (×)

Even though one can verify the parity of π^0 indirectly from other experiments, one still needs to test it directly!

π^0 – Decay	Br	Diagram	Strength of $\lvert m \rvert^2$ normalized to $\pi \to \gamma\gamma$
$\pi^0 \to 2\gamma$	99%		1
$\pi^0 \to \gamma e^+ e^-$	1.2%		$\alpha \sim \frac{1}{137}$
$\pi^0 \to e^+ e^- e^+ e^-$	3×10^{-5}		α^2
$\pi^0 \to \gamma\gamma\gamma\gamma$	$< 1.6 \times 10^{-6}$		α^2
$\pi^0 \to e^+ e$	10^{-7}		$\alpha^2 \frac{1}{16\pi^2}$
$\pi^0 \to \nu\nu$			

中性pion (π^0) 的自旋和宇称 $\pi^0 \to \gamma\gamma$



(1) π^0 自旋不为1 (Landau-Yang theorem, 1950)

⊛ 末态波函数可由两个光子的极化矢量 $\vec{\varepsilon}_1$ 和 $\vec{\varepsilon}_2$ 来构造，另外一个矢量为光子动量 $\vec{k}$

⊛ 末态波函数是 $\vec{\varepsilon}_1$, $\vec{\varepsilon}_2$ 和 $\vec{k}$ 的线性函数
如果 $S_{\pi^0}=1$ (或pion是矢量玻色子)，那么

⊛ 双光子是全同粒子 $\Longrightarrow$ 末态波函数遵从玻色-爱因斯坦统计
有三种可能性

$1 \longleftrightarrow 2$

$\vec{k} \longleftrightarrow -\vec{k}$

(1) $\vec{\varepsilon}_1 \times \vec{\varepsilon}_2$

(2) $(\vec{\varepsilon}_1 \cdot \vec{\varepsilon}_2)\,\vec{k}$

] not symmetric

(3) $\vec{k} \times (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2) = \vec{\varepsilon}_1(\vec{k}\cdot\vec{\varepsilon}_2) - \vec{\varepsilon}_2(\vec{k}\cdot\vec{\varepsilon}_1)$

B-E. ok, but $\vec{k}\cdot\vec{\varepsilon}_1 = \vec{k}\cdot\vec{\varepsilon}_2 = 0$
光子的横波性

所以，π^0的自旋不为1

Landau-Yang theorem.　　　　$e^+e^-(^3S_1) \not\to 2\gamma$

自旋为1的矢量波色子不能衰变到两个光子，例如 $Z^0 \not\to 2\gamma$

2) π^0宇称 ($\pi^0 \to \gamma\gamma$)

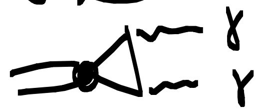
末态波函数必须是 $J=0$ 系统，并且此波函数必须是每个光子极化矢量的线性组合

⊛ $\vec{\varepsilon}_1 \cdot \vec{\varepsilon}_2$ even Parity $\Rightarrow$ π^0是scalar　　($\vec{\varepsilon}_1 \parallel \vec{\varepsilon}_2$)

⊛ $(\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)\cdot\vec{k}_1$ odd Parity $\Rightarrow$ π^0是pseudo-scalar　　($\vec{\varepsilon}_1 \perp \vec{\varepsilon}_2$)

所以，我们可以通过光子极化矢量来测量π^0的宇称，并验证π^0是赝标介子，并不是标量介子

$\Rightarrow$ π^0的内禀宇称为(-1)　　($\pi \to 2\gamma$ 是电磁相互作用

π^0 

)

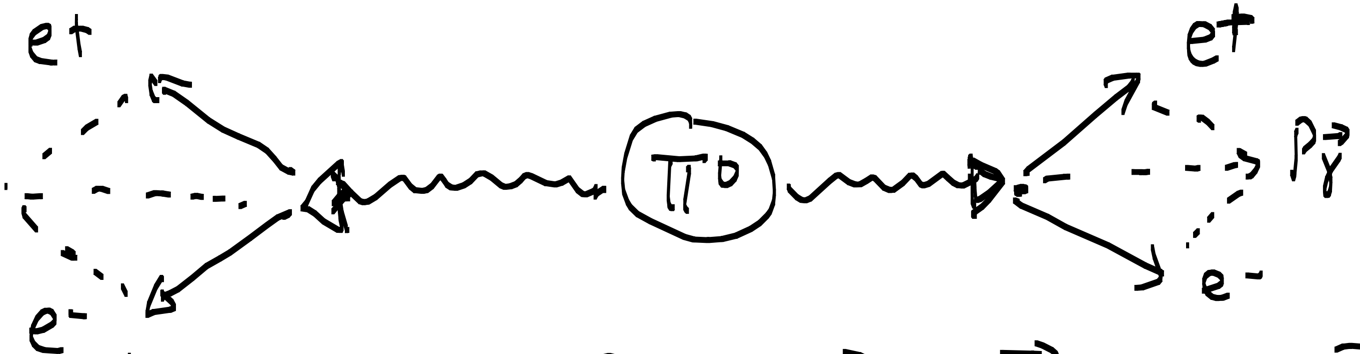
$\pi^0 \to \gamma^* \gamma^* \to e^+e^- \, e^+e^-$ (Decay plane correlation)

*) 双光子极化测量具有挑战性

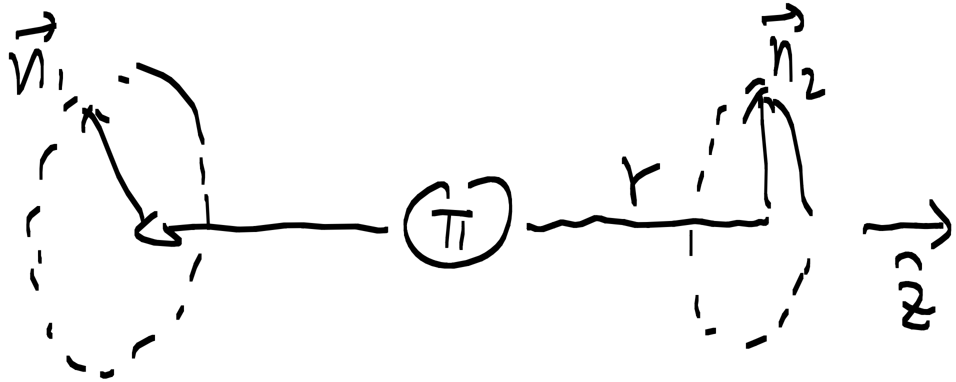
*) Steinberger (Plano et al, PRL 3, 525 (1959); Samios et al, Phys. Rev. 126, 1844 (1962))

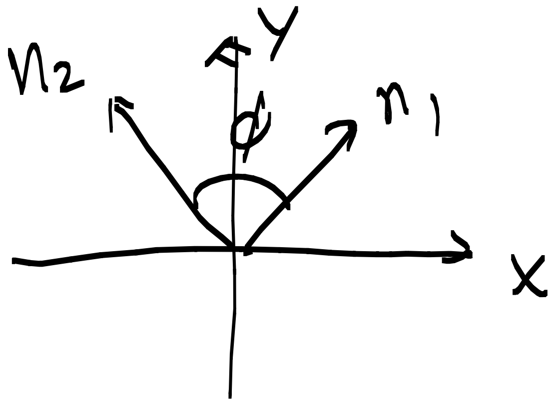
$\frac{1}{30000}$ 几率 $\pi^0 \to \gamma^* \gamma^* \to e^+e^- \, e^+e^-$ (internal conversion)

$\Longrightarrow$ 两个正负电子对的平面"记住"中间态的光子的极化信息



正负电子平面方向定义为 $(\vec{p}_{e^+} \times \vec{p}_{e^-})$ 方向. 因为 $\vec{p}_{e^+} + \vec{p}_{e^-} = \vec{p}_\gamma$,
所以 $\vec{p}_\gamma$ 位于 $(e^+ - e^-)$ 平面。这也意味着 $(e^+ - e^-)$ 平面的法向方向一定垂直于 $\vec{p}_\gamma$。另一侧 $(e^+ e^-)$ 平面法向也垂直于 $\vec{p}_\gamma$。





设两个光子运动方向为 z 轴，那么 $\vec{n}_1$ 和 $\vec{n}_2$ 位于 $(x\text{-}y)$ 平面

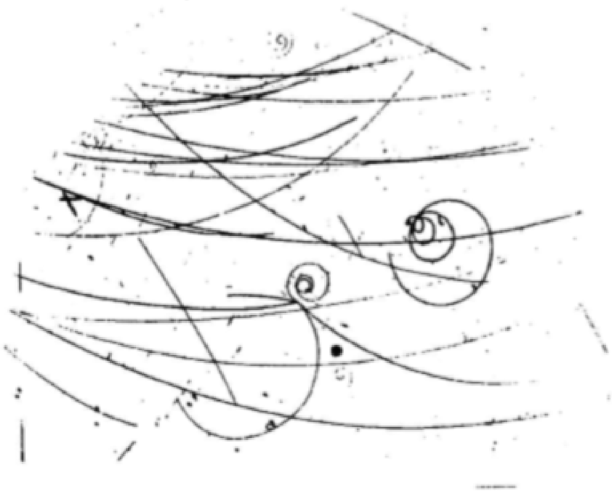
定义 $\vec{n}_1$ 和 $\vec{n}_2$ 之间夹角为 ϕ （方位角之差）

因为 scalar: $\vec{\varepsilon}_1 \cdot \vec{\varepsilon}_2$ 即 $\vec{\varepsilon}_1 \parallel \vec{\varepsilon}_2$

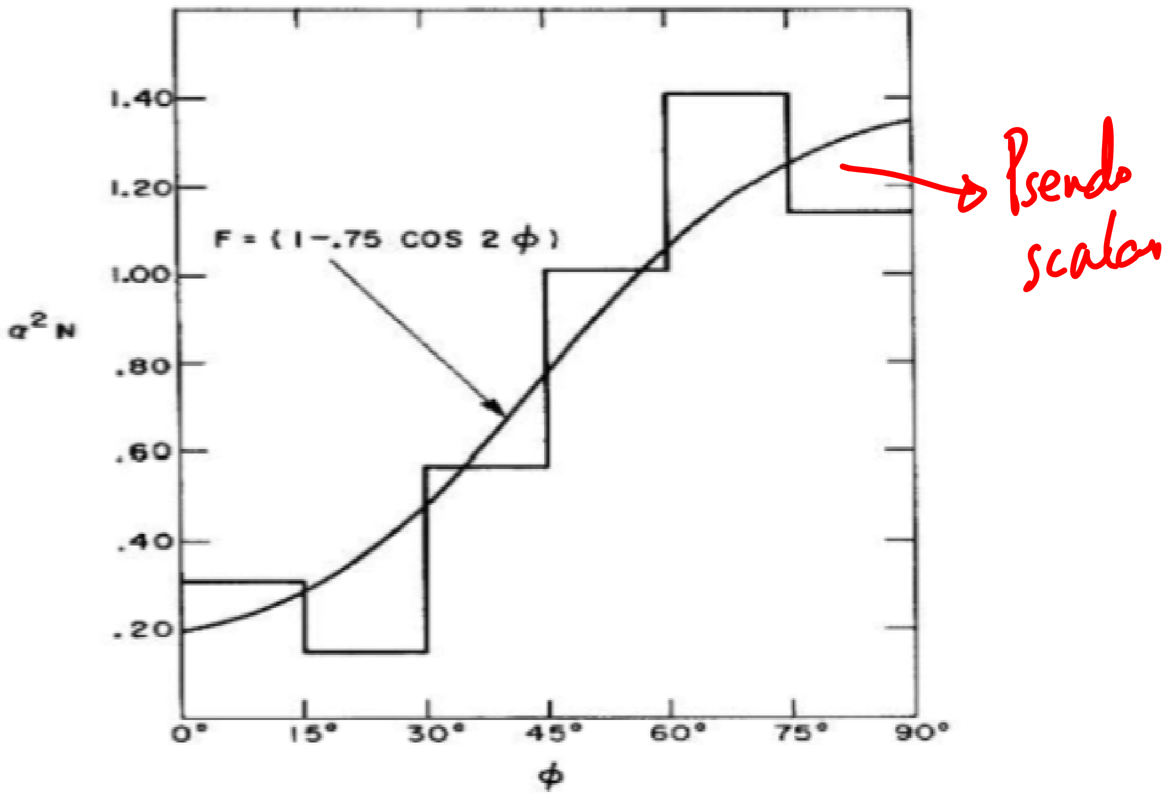
$$\Longrightarrow \frac{d\sigma}{d\phi} \propto \cos^2\phi$$

pseudo-scalar: $(\vec{\varepsilon}_1 \times \vec{\varepsilon}_2) \cdot \vec{k}_1$ 即 $\vec{\varepsilon}_1 \perp \vec{\varepsilon}_2$

$$\Longrightarrow \frac{d\sigma}{d\phi} \propto \sin^2\phi$$



$\pi^0 \to \gamma^*\gamma^* \to e^+e^-\, e^+e^-$
in a hydrogen Bubble
chamber at Nevis LAB



$(e^+e^-)_1$ 和 $(e^+e^-)_2$ 平面夹角分布

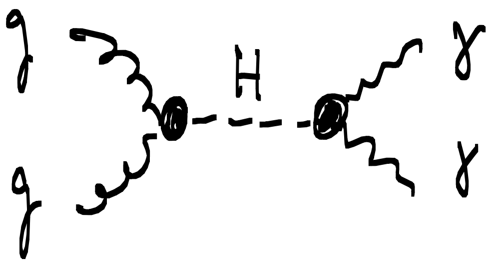
直方图为实验数据

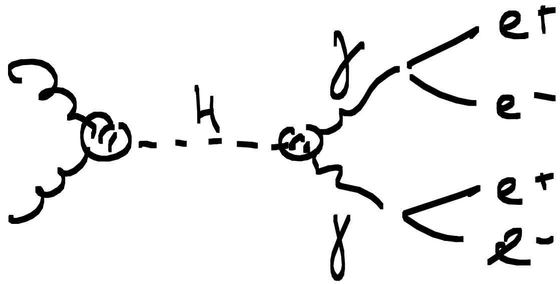
平滑曲线为理论预言

(Scalar 曲线峰值在 $\phi=0$ 处)

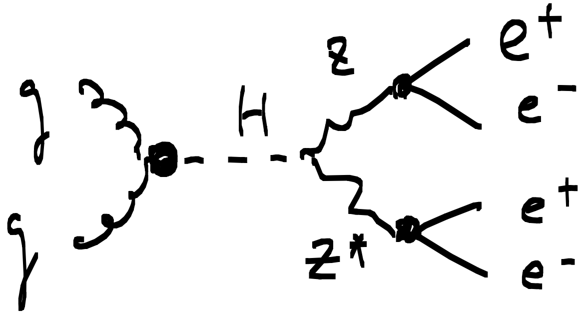
希格斯粒子的CP性质测量

(125 GeV, $H \to \gamma\gamma$)

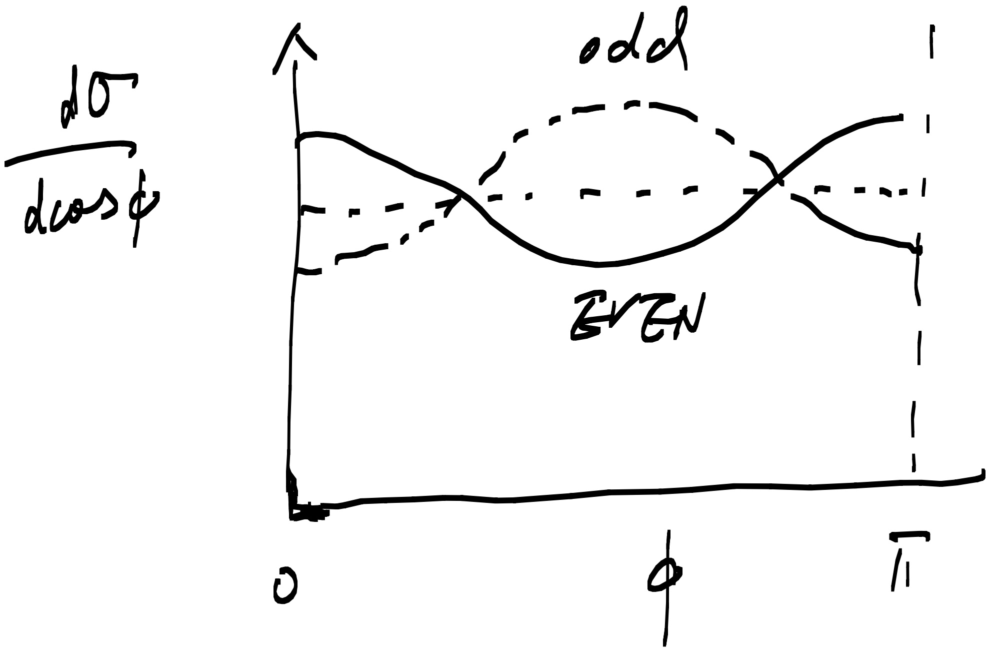




(rate too small)



(ZZ^* decay plane corelation)



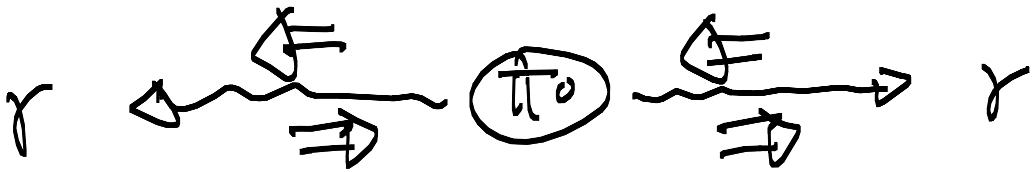
$$\frac{d\sigma}{d\cos\phi} \propto 1 + a\cos^2\phi$$

(CP-even)

$$\propto 1 + b\sin^2\phi$$

(CP-odd)

*) Puzzle: 光子的横波性 $\Rightarrow$ π^0 是偶宇称?



自旋耦合 $\vec{s}_1 \oplus \vec{s}_2$ 给出 $S_\pi = -2,\ 0,\ 2$

从 $\vec{J} = \vec{l} + \vec{s}$ $\Rightarrow$ $\vec{l}$ 必须是偶数

所以 2γ 系统的宇称是 $(-1)^l \underbrace{(-1)(-1)}_{P_\gamma \text{ intrinsic}} = (-1)^l =$ 偶数 (???)

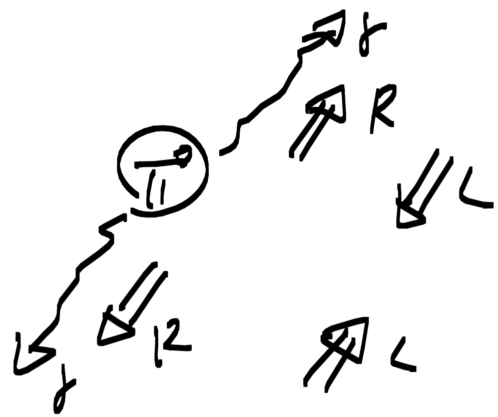
what is wrong?

原因在于 the transversality of the photon does NOT imply that in the 2γ system, the spin state has to be even under parity

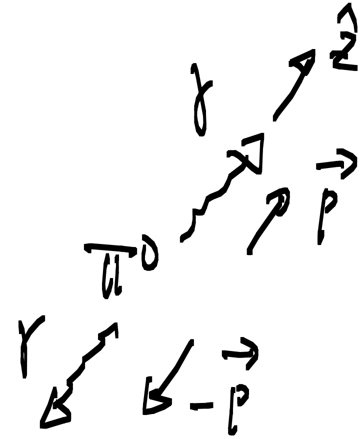
角动量守恒要求双光子的 Helicity 态为 RR 或 LL

但宇称本征态为 RR + LL (parity even)

RR − LL (parity odd)



$\pi^0 \to 2\gamma$：场论语言



2γ state can be written as

$$|2\gamma\rangle = \int d^3p\, \chi_{ij}(\vec{p})\, \underbrace{a_i^\dagger(\vec{p})}_{\gamma\text{产生标符}}\, a_j^\dagger(-\vec{p})\, \underbrace{|0\rangle}_{\text{vaccum}}$$

为保证π^0无自旋，$\chi_{ij}(\vec{p})$必须是空间旋转变换的二阶张量

又因为χ_{ij}仅依赖于$\vec{p}$，所以它的形式只能是

$$\chi_{ij}(\vec{p}) = A\,\delta_{ij} + B\,\epsilon_{ijk}\,p_k + C\,\underline{p_i p_j}$$

贡献为0，$p_i a_i^\dagger(\vec{p}) = 0$（光子横波性）

（A, B, C 都是$|\vec{p}|$的函数）

$\mathbb{P}|0\rangle = |0\rangle$

因为 $\mathbb{P}\, a_i^\dagger(\vec{p})\, \mathbb{P}^\dagger = -\,a_i^\dagger(-\vec{p})$

所以 $\mathbb{P}|2\gamma\rangle = \int d^3p\ \chi_{ij}(\vec{p})\ \mathbb{P}a_i^\dagger(\vec{p})\mathbb{P}^\dagger\mathbb{P}a_j^\dagger(-\vec{p})\mathbb{P}^\dagger\underline{\mathbb{P}|0\rangle}$

$$= \int d^3p\ \chi_{ij}(\vec{p})\ a_i^\dagger(-\vec{p})\ a_j^\dagger(\vec{p})|0\rangle$$

$$= \int d^3p\ \chi_{ij}(-\vec{p})\ a_i^\dagger(\vec{p})\ a_j^\dagger(-\vec{p})|0\rangle$$

这意味着对于宇称 $P=\pm1$ 态，$\chi_{ij}(-\vec{p}) = \pm\chi_{ij}(\vec{p})$

$$\Rightarrow \quad \chi_{ij}(\vec{p}) = \begin{cases} A\,\delta_{ij}, & P=+1 \\ B\,\epsilon_{ijk}p_k, & P=-1 \end{cases}$$

因为 $\vec{a}^\dagger$ 的矢量方向是 $\vec{E}$ 的方向，

所以上面方程表明 $\begin{cases} P=+1，\text{双光子极化矢量平行} \\ P=-1，\text{双光子极化矢量垂直} \end{cases}$

$\pi^0 \to \gamma\gamma$ in an Effective Lagrangian

Denote π^0 as ϕ

From Lorentz invariance

$$\mathcal{L}_{eff}^{(1)} = \phi F_{\mu\nu}^{(a)} F^{\mu\nu(b)} \sim \phi(\vec{E}_a\cdot\vec{E}_b - \vec{B}_a\cdot\vec{B}_b) \quad \text{scalar}$$

$$\mathcal{L}_{eff}^{(2)} = \phi\, \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu(a)} F^{\rho\sigma(b)} \sim \phi(\vec{E}_a\cdot\vec{B}_b + \vec{B}_a\cdot\vec{E}_b) \quad \text{pseudo scalar}$$

在宇称变换下 $\vec{E} \xrightarrow{P} -\vec{E}$, $\vec{B} \xrightarrow{P} +\vec{B}$

所以 $\mathcal{L}_{eff}^{(1)}$ 是偶宇称，$\mathcal{L}_{eff}^{(2)}$ 是奇宇称

如果相互作用不遵从宇称，则我们无法确定中宇称

$\mathcal{L} \ni \mathcal{L}_{eff}^{(1)} + \mathcal{L}_{eff}^{(2)} \Rightarrow$ parity 破坏

Parity of Particles & antiparticles

* 费米子的内禀宇称取决于约定，但费米子和反费米子之间的相对宇称却是可以观测的物理量
* Dirac 的费米子理论指出费米子和反费米子具有相反的内禀宇称，

$\Longrightarrow$ 被吴健雄和 Shaknov 在正负电子偶素中验证 (1950)

* 正负电子偶素 (positronium)：e^+ 和 e^- 组成的类原子的束缚态

Wu & Shaknov
Phys. Rev 77, 136 (1950)

基态为 $^1S_0$ (S-wave，总自旋 $=0$，$\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)$)

$$e^+e^-(^1S_0) \longrightarrow 2\gamma \qquad \text{类似于} \quad \pi^0 \to 2\gamma$$

因为
$$P(e^+e^-) = (-1)^l \underbrace{P(e^+)P(e^-)}_{\text{intrinsic Parity}} \xlongequal{^1S_0} P(e^+)P(e^-) = P(2\gamma)$$

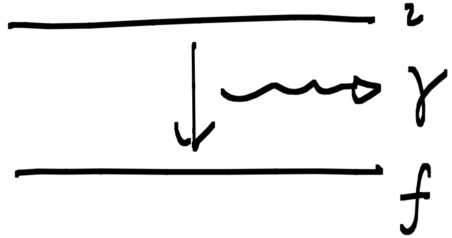
所以，实验上检验两个光子的极化矢量关联可以帮助我们测量 $P(2\gamma)$

$$\vec{\varepsilon}_1 \parallel \vec{\varepsilon}_2 \Longrightarrow P(2\gamma)=+1 \Longrightarrow P(e^+)=P(e^-) \quad \times$$

$$\vec{\varepsilon}_1 \perp \vec{\varepsilon}_2 \Longrightarrow P(2\gamma)=-1 \Longrightarrow P(e^+)=-P(e^-) \quad \checkmark$$

宇称不守恒

1) 1924年, O. Laporte 注意到



$P(i) = -P(f)$

从而提出 Laporte rule:

原子单光子辐射过程中的宇称守恒原则

2) 1927年, Eugene Wigner 证明 Laporte rule 起源于电磁相互作用遵守左右手对称性

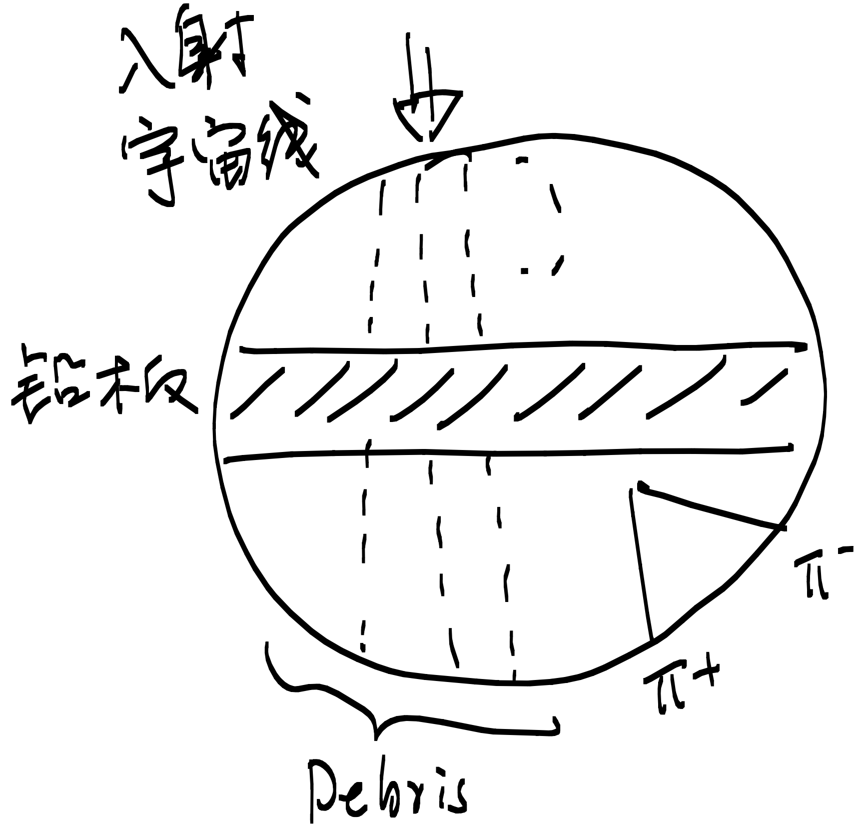
⇒ 将经典的左右对称性应用在量子过程中

3) 1949年, Weak force 被提出时, 人们自然地认为弱相互作用也遵守左右对称性

奇异粒子

*) 1947年G. Rochester和C. Butler在云室研究宇宙线时发现V型事例

(1943年, L. Leprince-Ringuet和M. l'Héritier已发现这个粒子
并测得 $m \simeq 506 \pm 61$ MeV, Comp. Rend. 219 (1944) 618
但因二战, 并未引起人们注意)



中性粒子飞行一段距离后衰变为两个带电粒子

分析显示: 两条V型径迹为 π^+ 和 π^-

$\Rightarrow$ 衰变前的中性粒子质量介于

π介子和质子之间

(重介子)

$\Rightarrow$ V^0 粒子, 后改为 θ^0 粒子

$$\theta^0 \rightarrow \pi^+\pi^-$$

更多实验表明，有些V型事例中的中间粒子是带电的，称作为 τ^+ 粒子

$$\tau^+ \to \pi^+\pi^+\pi^-$$

今天我们知道 θ^+ 和 τ^+ 是同一种粒子（kaon）

（见 Braibant 书中第3.14图形，$K^+ \to \pi^+\pi^+\pi^-$，$\pi^+ \to \mu^+\nu$

为何将之称作为奇异粒子？

它们行为很奇怪：产生很快（10^{-23} s） strange
衰变很慢（10^{-10} s）

1952年，A. Pais 提出奇异粒子通过强相互作用产生，但通过弱相互作用衰变，并预言奇异粒子应成对产生。
（不久即被加速器实验验证）

3) $\theta-\tau$ puzzle

1956年实验测量表明θ和τ介子具有的质量和寿命

Decay	mass	lifetime	Branching ratio
$\theta^+ \to \pi^+ + \pi^0$	$(966.7 \pm 2.0)\, m_e$	$(1.21 \pm 0.02) \times 10^{-8}$ sec	$\sim 20\%$
$\tau^+ \to 2\pi^+ + \pi^-$	$(966.3 \pm 2.0)\, m_e$	$(1.19 \pm 0.05) \times 10^{-8}$ sec	$\sim 5\%$

现在我们知道θ^+和τ^+是同一个粒子，命名为k^-

($k^+ \to \pi^+ \pi^0$, $B_r = 21.17 \pm 0.15\%$　　$m_{k^+} = 493.646 \pm 0.009$ MeV
$k^+ \to \pi^+ \pi^+ \pi^-$, $B_r = 5.589 \pm 0.028\%$　　$I(J^P) = \frac{1}{2}(0^-)$)

令人们困惑的是 θ 和 τ 具有不同的宇称.

1) θ^+粒子:

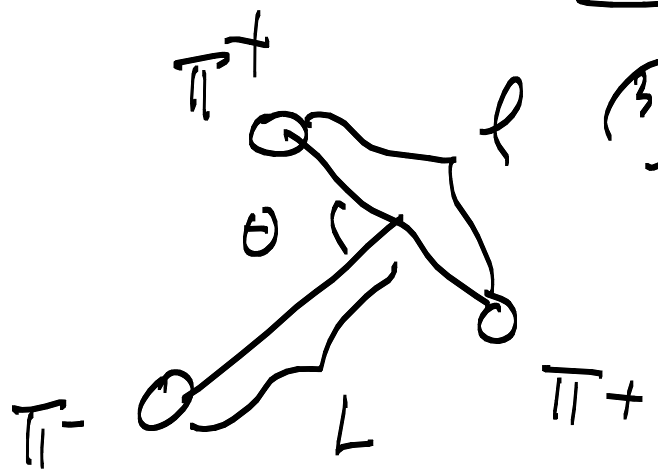
设θ^+粒子自旋为S，则π^+和π^0间的轨道角动量$L=S$，因为$S_\pi = 0$

$\theta^+ \to \pi^+ \pi^0$　　$P(\theta^+) = P(\pi^+\pi^0) = (-1)^L (-1)(-1) = (-1)^L = (-1)^S$

└→ Assume pairty is conserved

$\Longrightarrow J^P(\theta^+) = 0^+, 1^-, 2^+, 3^-, \cdots \Longrightarrow J^P(\theta^+) = 0^+$ Parity even

2) Consider $\tau^+ \to \pi^+\pi^+\pi^-$



我们将末态分作为$(\pi^+\pi^+)$子系统和π^-

令$\pi^+\pi^+$之间的轨道角动量为l

π^-和$(\pi^+\pi^+)$质心系的相对轨道角动量为L

因为τ^+自旋为0 且 $\vec{S}_{\tau^+} = \vec{l} + \vec{L}$,

$$|L-l| \leq S \leq L+l$$

所以 $l = L$，否则无法得到零自旋

因此末态宇称为

$$(-1)^{l+L}(-1)^3 = -1 \implies P(\tau^+) = -1$$

1953年 Dalitz 提出 $P(\tau^+)=-1$，$P(\Theta^+)=+1$

综上所述，如果宇称守恒，那么Θ^+和τ^+不可能是同一个粒子。

李政道和杨振宁在1956年提出“弱相互作用破坏宇称” Θ和τ是一个粒子 现称作为 Kaon

显然我们无法通过 K^+ 的衰变产物来确定其内禀宇称

$$K^+ \to \pi^+\pi^0 \ / \ \pi^+\pi^+\pi^-$$

因为强相互作用遵守宇称守恒，考虑如下强作用散射过程

$$\pi^- + p \to K^0 + \Lambda^0$$

我们可以确定 K-Λ 之间的相对宇称。

我们约定： $P(\Lambda) = P(p) = +1$

实验观测到 $\pi^- + p \to K^0 + \Lambda^0$ 在 $S_{1/2} \to S_{1/2}$ 散射中发生，所以

$$\begin{array}{ccccccc} \pi^- & + & p & \to & K^0 & + & \Lambda^0 \\ (-1) & & (+1) & & & & (+1) \to \text{约定} \\ & & & & \to (-1) & & \end{array}$$

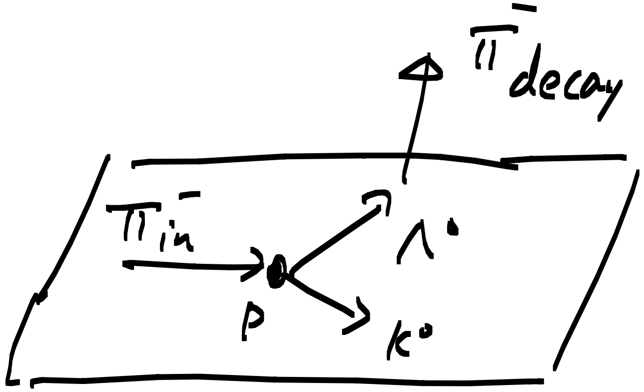
$$\Rightarrow P(K^0) = -1$$

Λ 的宇称：无法通过其衰变产物得到

$$\Lambda^0 \to p + \pi^- , \quad \Lambda^0 \to n + \pi^0 \qquad (\text{弱相互作用})$$

为测量 Λ^0 衰变过程中的宇称破坏效应，我们需要构造宇称为奇的物理观测量

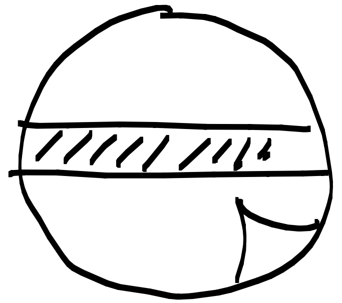
$$(\vec{p}_{\pi_{in}} \times \vec{p}_{\Lambda}) \cdot \vec{p}_{\pi_{decay}}$$



检验

$$\langle (\vec{p}_{\pi_{in}} \times \vec{p}_{\Lambda}) \cdot \vec{p}_{\pi_{decay}} \rangle \neq 0$$

$\Rightarrow$ Λ^0 衰变过程中宇称破坏！



1950年 Anderson发现另一类中性V事例

$$\Lambda \longrightarrow p + \pi^-$$

因为Λ比质子重，属于超子（Hyperon）。

其后，人们在宇宙线中观测到许多新粒子 Σ^+ Σ^-，K^-

$$\Sigma^- \longrightarrow n + \pi^-, \quad \Sigma^+ \longrightarrow n + \pi^+$$
$$\qquad\qquad\qquad\qquad\qquad \hookrightarrow p + \pi^0$$

$$K^- \longrightarrow \pi^- + \pi^0$$

但宇宙线中粒子的种类、数量和能量无法控制，人们只能被动地记录所发生的事例 $\Longrightarrow$ 实验室中人工制备产生高能量粒子束

1937，Lawrence（Cyclotron），之后 Synchrocyclotron

1953，Brookhaven，Cosmotron．1.5GeV π^- $\longrightarrow$ 质子靶

1955，Berkeley，BevaTron

Cosmotron 上人们发现 $\pi^- + p \rightarrow \Lambda + K^0$

$K^0 \rightarrow \pi^+\pi^-$

$\Lambda \rightarrow p + \pi^-$

(Λ^0 和 K^0 协同产生)

$$\pi^- + p \rightarrow \Sigma^- + K^+$$

$K^+ \rightarrow \pi^+ + \pi^0$

$\Sigma^- \rightarrow n + \pi^-$

不久人们发现双V事例

$$\left.\begin{array}{l} \pi^- + p \rightarrow \Xi^- + K^+ + K^0 \\ \Xi^- \rightarrow \Lambda + \pi^- \\ \Lambda \rightarrow p + \pi^- \end{array}\right\} \text{三个奇异粒子}$$

1952年 Gell-mann
1953年 Kazuhiro Nishijima

独立提出"奇异数"概念。

奇异数（strangeness）

*）每个粒子都具有"奇异数"量子数 $\left\}\begin{array}{ll} \text{普通粒子} & S=0 \\ \text{奇异粒子} & S=\text{整数} \end{array}\right.$

*）奇异数的规定是相对的

*）强相互作用中奇异数守恒

约定：

$$\pi^0 + p \to \underset{S=-1}{\Lambda} + \underset{S=+1}{K^0} \qquad S(K^0)=+1$$

$$\pi^+ + n \to \Lambda + K^+ \Rightarrow S(K^+)=+1,\ S(K^-)=-1$$

$$\left.\begin{array}{l} \pi^{\pm} + p \to \Sigma^{\pm} + K^+ \\ \pi^- + p \to \Sigma^0 + K^0 \end{array}\right\} \Rightarrow S(\Sigma^+)=S(\Sigma^-)=S(\Sigma^0)=-1$$

$$K^- + p \to \Xi^0 + K^0 \Rightarrow S(\Xi^0)=-2$$

*) Exp 显示: EM 中 S 守恒, 但 WEAK INT 中 $\cancel{S}$

一般的奇异粒子衰变中 $\Delta S = -1$

例 $\Xi^{-} \to \Lambda + \pi^{-}$

$\Lambda \to p + \pi$

也有 $\Delta S = -2$ 过程

$\Xi^{0} \to p + \pi^{-}$, $\Xi^{-} \to n + \pi^{-}$

但几率很小,

$P_{|\Delta S|=2} \ll P_{|\Delta S|=1}$

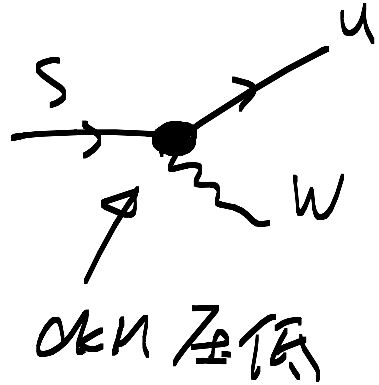


表 1.1: 奇异粒子的奇异数 S、质量 m、$c\tau$ 及其反粒子。

奇异粒子	S	m (MeV/c^2)	$c\tau$	反粒子
K^+	+1	493.677 (16)	3.712 m	K^-
K^0	+1	497.614 (24)	K^0_S: 2.6842 cm K^0_L: 15.34 cm	$\overline{K}^0$
Λ	−1	1115.683 (6)	7.89 cm	$\overline{\Lambda}$
Σ^+	−1	1189.37 (7)	2.404 cm	$\overline{\Sigma}^+$
Σ^0	−1	1192.642 (24)	2.22×10^{-11} m	$\overline{\Sigma}^0$
Σ^-	−1	1197.449 (30)	4.434 cm	$\overline{\Sigma}^-$
Ξ^-	−2	1321.71 (7)	4.91 cm	Ξ^+
Ξ^0	−2	1314.86 (20)	8.71 cm	$\overline{\Xi}^0$
Ω^-	−3	1672.45 (29)	2.461 cm	Ω^+

*) $S(\Sigma^+) = S(\Sigma^-) = -1 \Rightarrow$ Σ^+和Σ^-不是正反粒子

*) $\left.\begin{array}{l} m(K) \sim m(\Sigma) \sim m(\Xi) \\ S = +1, \quad -1, \quad -2 \end{array}\right\} \Rightarrow$ 这些粒子组成多重态

1962年 Gell-mann 预言 Ω^-，1964年被实验证实