relative shifts of levels, for example, due to a difference in chemical composition of the environment or due to weak fields that are magnetic or even gravitational.

5.10
Virtual Processes and Relativistic Effects

Because of the energy-time uncertainty relation, the precise definition of energy is possible only in the process of infinite duration. Frequently, it is convenient to subdivide the process into intermediate stages of finite duration. In our discussion of resonance fluorescence, Section 5.9, we spoke of the photon emitted by the source and absorbed by the absorber as of a real particle with energy $E = \hbar\omega$ and momentum $p = E/c$. However, if the time interval Δt between the emission and absorption acts is getting very small, the notion of the photon with certain energy becomes meaningless by virtue of the uncertainty relation. In such an approximate description, a borderline between the particle with certain energy and momentum, and a transient object that has no time to acquire the features of a particle becomes blurred, and the distinction only has quantitative rather than qualitative character.

For such situations, we speak of *virtual* processes or states of the system. If, as in the example above, a new "particle" appears for a short time, the particle can be also called virtual. This approach facilitates the quantum description of complicated states, as in the fluorescence example with two atoms (or nuclei) and an electromagnetic field. The energy conservation law is fulfilled for this system as a whole. Though during a virtual process, energy of each subsystem does not have a certain value. This is a source of the whole idea of unstable states, level width and so on. Such an approximate description is convenient due to its straightforward connection to experimental observables. Other conservation laws – momentum, angular momentum, charge and so on – are still respected at each step. Another type of description is used frequently in relativistic quantum theory with energy conservation fulfilled at each stage, but in the absence of normal relation (1.6) between energy and momentum of a virtual particle. In other words, the mass of a virtual particle can take arbitrary values.

Problem 5.15

It is known that the range of the strong forces between two nucleons is of the order $R \sim (1 \div 2) \cdot 10^{-13}$ cm. Assuming that the nuclear forces are mediated, Figure 5.12, by the exchange of *mesons*, estimate the mass of the lightest meson.

Solution A real production of a meson by a single nucleon is forbidden by the energy-momentum conservation. The interaction proceeds as a *virtual* process of some duration τ. To connect the nucleons, the virtual meson should have enough time to pass the internucleon distance R, $R < c\tau$. In the presence of the virtual meson of mass m, the uncertainty of energy of the system is $\Delta E \sim mc^2 \sim \hbar/\tau$. From here, the maximum distance the meson can travel determines the range of nuclear forces,

$$R \sim c\tau \sim \frac{\hbar}{mc} . \tag{5.85}$$

In other words, the exchange of quanta of mass m couples the particles over the distance of the Compton wave length of the mediator, recall (1.39). For the virtual photon, $m = 0$, the forces are long-range (Coulomb). In the nucleon–nucleon interaction with $R \sim 2\,\text{fm}$, (5.85) predicts $m \sim 200 m_e$. In this way, the very existence of π-mesons (*pions*) was predicted (*H. Yukawa*, 1935). The pion exchange leads to the Yukawa potential (1.50) that is not sufficiently strong for binding two nucleons, Problem 1.8. The existence of the bound proton-neutron state, the *deuteron*, is ensured by the additional exchange of other, heavier mesons.

In the more rigorous consideration, we would have, instead of separate nucleons and virtual mesons, a complicated system of interacting nucleons and the meson field; this system as a whole would have certain energy in its ground state. In fact, a similar idea can be used in the consideration of tunneling, Section 2.7. Virtual states are possible when a particle gets into the forbidden region under the barrier for a short time Δt. This leads to the energy fluctuation $\Delta E \sim \hbar/\Delta t$. For small Δt, the "instantaneous" energy of the particle may exceed the height of the barrier and the particle can overcome ("jump over") the forbidden region. This also precludes the possibility to "catch" the particle under the barrier: this would require extra energy greater than the barrier height. We need to stress again that, with the exact solution of quantum problems, one would not need the notion of a virtual state. The full wave function of the system as a whole provides complete information, allowing, in particular, one to find the probabilities of the processes as tunneling.

In Problem 5.15, the existence of the limiting velocity c of signal propagation was crucial. In general, in the relativistic domain, new restrictions emerge on top of the usual quantum uncertainty relations [7, §1]. The group velocity of the relativistic wave packet with centroid energy E and momentum p is $v = pc^2/E < c$. Since the energy and momentum spreads in the packet are related as $\Delta E \sim v\Delta p$, and the passage time satisfies $\Delta t \sim \hbar/\Delta E$, we come to the new restriction (that disappears in the limit $c \to \infty$),

$$\Delta p \cdot \Delta t \geq \frac{\hbar}{c} . \tag{5.86}$$

This determines the optimal accuracy of the momentum measurement at a given duration of the measurement process.

Another relation puts limits on the very concept of a *single-particle* state. For the notion of a given particle of mass m to be meaningful, the energy uncertainty of the state has to at least be smaller than mc^2 so that the duration of measurement $\Delta t > \hbar/mc^2$. Equation (5.86) then gives the boundary for the momentum uncertainty, $\Delta p \sim mc$, and therefore the particle cannot be localized in the rest frame better than within its Compton wave length,

$$\Delta x \geq \frac{\hbar}{\Delta p} \geq \frac{\hbar}{mc} = \lambda_C \,. \tag{5.87}$$

Attempting to achieve better localization, the uncertainties of energy and momentum grow so much that the creation of new particles becomes energetically allowed and the problem loses its single-particle character. Then, one needs to use the full relativistic quantum field theory instead of quantum mechanics.