$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$S = \int \mathcal{L}\, d^n x$$

$$\frac{\delta S}{\delta \varphi} = 0$$

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial \varphi} = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}\right)$$

$$\mathcal{L} = T - V$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad D_\mu = \partial_\mu - ieA_\mu$$

标变换
$$\begin{cases} \phi(x) \to \phi'(x) = U(x)\phi(x) = e^{i\alpha(x)}\phi(x) \\ \phi^+(x) \to \phi'^+(x) = \phi^+(x)U^+(x) = \phi^+(x)e^{-i\alpha(x)} \end{cases}$$

$$\partial_\mu \phi(x) \to \partial_\mu \phi'(x) = e^{i\alpha(x)}\phi(x) + i(\partial_\mu \alpha)e^{i\alpha(x)}\phi(x)$$

$$\underbrace{\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = \frac{\partial \mathcal{L}}{\partial x} \Rightarrow m\ddot{x} = -kx$$

$\phi^+(x)\, D_\mu \phi(x)$

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{e}[D_\mu, \alpha(x)]$$

$$e^-: \psi_e(x) = \int \frac{d^3k}{(2\pi)^3 2E} \Sigma \left\{ b_{k\sigma}\, u(k,\sigma)\, e^{-ik\cdot x} + d^+_{k,\sigma}\, v(k,\sigma)\, e^{ikx} \right\}$$

$$e^+: \psi^c_e(x) = C\bar{\psi}^T = \int \frac{d^3k}{(2\pi)^3 2E} \sum_k \left\{ b^+_{k,\sigma}\, v(k,\sigma)\, e^{ikx} + d_{k\sigma}\, u(k,\sigma)\, e^{-ikx} \right\}$$

$$u = Cv^T, \quad v = Cu^T$$

$$\begin{cases} \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{cases}$$

$$\boxed{A^+ = A^{T*} \qquad \bar{A} = A^+\gamma^0}$$

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma_+^\mu \\ \sigma_-^\mu & 0 \end{pmatrix} \qquad \sigma_\pm^\mu = (1, \pm\vec{\sigma})$$

$= i\gamma^0\gamma^1\gamma^2\gamma^3$

$$d\Phi_2 = (2\pi)^4 \delta^4(Q - P_1 - P_2)\frac{d^3P_1}{(2\pi)^3 2E_1}\frac{d^4P_2}{(2\pi)^3}\delta(P_2^2 - m_2^2) = \frac{\lambda(Q^2, m_1^2, m_2^2)}{32\pi^2}\, d\cos\theta\, d\varphi$$

$$d\Phi_2 = (2\pi)^4 \delta^4(Q-P_1-P_2) \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^4P_2}{(2\pi)^3} \delta(P_2^2-m_2^2)$$

$$= \frac{1}{(2\pi)^2} \delta\left((Q-P_1)^2 - m_2^2\right) \frac{d^3p_1}{2E_1} \quad \frac{|P_1|^2 d|P_1|\, d\cos\theta\, d\varphi}{2E_1}$$

Q 质心系下

$$= \frac{1}{(2\pi)^2} \delta\left[S - 2\sqrt{S}E_1 + m_1^2 - m_2^2\right] \frac{|\vec{P}_1|^2 d|\vec{P}_1|\, d\cos\theta\, d\varphi}{2E_1}$$

$$= \frac{1}{8\pi^2} \delta(\cdots) \frac{|P_1|}{2} \frac{d|P_1|^2\, d\cos\theta\, d\varphi}{E_1} \quad (E_1^2 - m_1^2) \rightarrow 2dE_1$$

$$= \frac{1}{8\pi^2} \frac{\sqrt{E_1^2 - m_1^2}}{2 \times 2\sqrt{S}} 2 \quad d\cos\theta\, d\varphi$$

← δ函数中系数

$$E_1 = \frac{S + m_1^2 - m_2^2}{2\sqrt{S}}$$

$$\lambda = \sqrt{\frac{(S-m_1^2-m_2^2) - 4m_1^2 m_2^2}{S}}$$

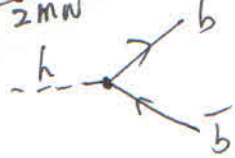
$$\Rightarrow \frac{d\cos\theta\, d\varphi}{32\pi^2} \lambda(S, m_1^2, m_2^2)$$

$$d^4p\, \delta(p^2 - m^2) = d^4p\, \delta(E^2 - p^2 - m^2)$$

$$= d^3p \quad dE\, \delta(E^2 - p^2 - m^2)$$

$$= \frac{d^3p}{2E_p}$$

$\frac{-ig m_b}{2m_W}$



$$M = \frac{g^2 m_b^2}{4m_W^2} \bar{u}_{p_1} v_{p_2}$$

$$|M|^2 = \frac{g^2 m_b^2}{4m_W^2} \mathrm{Tr}[\not{p}_1 \not{p}_2]$$

$$= \cancel{m_b^2 4 p_1 p_2}$$

$$= \cancel{2 m_b^2} m_h^2$$

$$= \frac{g^2 m_b^2 m_h^2}{2 m_W^2}$$

$$\cancel{\frac{1}{2m_H} \frac{1}{8\pi} m_b^2 \, 2 m_b^2 m_h^2}$$

$$\cancel{= \frac{2m_b^2}{16\pi}}$$

$$\frac{1}{2m_H} \frac{1}{8\pi} \times \frac{g^2 m_b^2 m_h^{\cancel{2}}}{2m_W^2}$$

$$= \boxed{\frac{\frac{e^2}{s_W^2} m_b^2 m_h}{32\pi m_W^2}}$$

$$= \frac{\alpha}{8 s_W^2} \frac{m_b^2}{m_W^2} m_h$$

Glor

$$3 \times \frac{1}{137} \times \frac{1}{8} \times \frac{1}{0.2223} \frac{4.7^2}{80^2} \times 125$$

$$\approx \boxed{5\text{–}6 \text{ MeV}}$$

n-body phase space

$$d\Phi_n(P_{CM}, P_1, P_2 \cdots P_n) = (2\pi)^4 \delta^4(P_{CM} - \sum_{i=1}^{n} P_i) \prod_{i=1}^{n} \frac{d^4 P_i}{(2\pi)^3} \delta(P_i^2 - m_i^2)$$

$$= \int d^4Q \; (2\pi)^4 \delta^4(P_{CM} - \sum_{i=1}^{n-2} P_i - Q) \prod_{i=1}^{n-2} \frac{d^4 P_i}{(2\pi)^3} \delta(P_i^2 - m_i^2)$$

$$\times \delta^4(P_{n-1} + P_n - Q) \frac{d^4 P_n}{(2\pi)^3} \frac{d^4 P_{n-1}}{(2\pi)^3} \delta(P_{n-1}^2 - m_{n-1}^2) \delta(P_n^2 - m_n^2)$$

$(d^4P_n \to d^4Q)$

$= \cancel{d\Phi_{n-1}(P_{CM}, P_1, P_2 \cdots P_{n-2}, Q)}$ [illegible] $\delta(Q^2 - t_Q)$

$$\cancel{d^4Q}\, \delta^4(P_{n-1} + P_n - Q) \; \frac{d^4 P_n}{(2\pi)^3} \delta(P_n^2 - m_n^2) \; \frac{d^4 P_{n-1}}{(2\pi)^3} \delta(P_{n-1}^2 - m_{n-1}^2)$$

$$= \int dt_Q \frac{d^4Q}{(2\pi)^3} \cancel{\delta(Q^2 - t_Q)} \; \frac{d^3 P_{n-1}}{(2\pi)^3 2E_{n-1}} \; \delta((Q - P_{n-1})^2 - m_n^2)$$

$$= \int dt_Q \frac{d^4Q}{(2\pi)^3} \cancel{\delta(Q^2 - t_Q)} \; \delta((Q - P_{n-1})^2 - m_n^2) \; \frac{|P_{n-1}|^2 d|P_{n-1}| \, d\cos\theta \, d\varphi}{(2\pi)^3 2E_{n-1}}$$

$\boxed{\text{CM of } Q}$

add $\int dt_Q \, \delta(Q^2 - t_Q)$

$$= \frac{d^4Q}{(2\pi)^3} \cancel{\delta(Q^2 - t_Q)} \, dt_Q$$

$$\delta(t_Q - 2\sqrt{t_Q} E_{n-1} + m_{n-1}^2 - m_n^2) \; \frac{\frac{1}{2}|P_{n-1}| \, d(E_{n-1}^2 - m_{n-1}^2) \, d\cos\theta \, d\varphi}{(2\pi)^3 \, 2E_{n-1}}$$

$$= \frac{d^4Q}{(2\pi)^3} \cancel{\delta(Q^2 - t_Q)} \, dt_Q \; \frac{\sqrt{\left[\frac{(t_Q + m_{n-1}^2 - m_n^2)}{2\sqrt{t_Q}}\right]^2 - m_{n-1}^2}}{2 \times 2\sqrt{t_Q}} \; d\cos\theta \, d\varphi \,/ (2\pi)^3$$

$$= \frac{d^4Q}{(2\pi)^3} \cancel{\delta(Q^2 - t_Q)} \; \frac{dt_Q \, d\cos\theta \, d\varphi}{64\pi^3} \lambda(t_Q, m_{n-1}^2, m_n^2)$$

$$\lambda(t_Q, m_{n-1}^2, m_n^2) = \frac{\sqrt{(t_Q - m_n^2 - m_{n-1}^2)^2 - 4m_n^2 m_{n-1}^2}}{t_Q}$$

$$= d\Phi_{n-1}(P_{CM}, P_1, P_2 \cdots Q)\big|_{Q^2 = t_Q} \; \frac{\lambda(t_Q, m_{n-1}^2, m_n^2)}{64\pi^3} \, dt_Q \, d\cos\theta \, d\varphi$$

PS-4

$dx_a dx_b \, d\Phi_n (ab \to 12 \cdots n)$

$= dx_a dx_b \, (2\pi)^4 \delta^4 (p_a + p_b - \sum_{i=1}^n p_i) \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 2E_i}$

$= \frac{2}{s} (2\pi)^{4-3n} \delta^2 (\sum_{i=1}^n p_i) \prod_{i=1}^n \frac{1}{2} dy_i \, d^2 p_i$

$= \frac{2(2\pi)^{4-3n}}{s} \delta^2 (\sum_{i=1}^n p_{T_i}) \prod_{i=1}^n dy_i \frac{1}{2} p_{T_i} dp_{T_i} d\phi_i$

$= \frac{2(2\pi)^{4-3n}}{s} \prod_{i=2}^n dy_i \frac{1}{4} p_{T_i}^2 \, d\ln p_{T_i}^2 \, d\phi_i$

$\times \delta^2 [(p_{T_1} \cos\phi \cdots)(p_{T_1} \sin\phi \cdots)] \, dy_1 \frac{1}{2} p_{T_1} dp_{T_1} d\phi_1$

$\frac{dy_1}{2}$

$\frac{2}{4^n}$

$du_i \delta(p_i^2 - m_i^2)$

$y_i = \frac{1}{2} \ln \frac{E + p_{/\!/}}{E - p_{/\!/}}$

$u_i = (E + p_{/\!/})(E - p_{/\!/})$

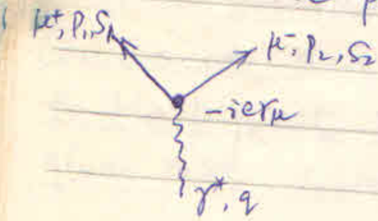
$dy_i du_i = 2 dE dp_{/\!/}$

$\frac{\partial (p_{T_1} \cos\phi, p_{T_1} \sin\phi)}{\partial p_{T_1} \, \partial \phi_1}$

$= \begin{vmatrix} \cos\phi, & -p_{T_1} \sin\phi \\ \sin\phi, & p_{T_1} \cos\phi \end{vmatrix}$

$= p_{T_1}$

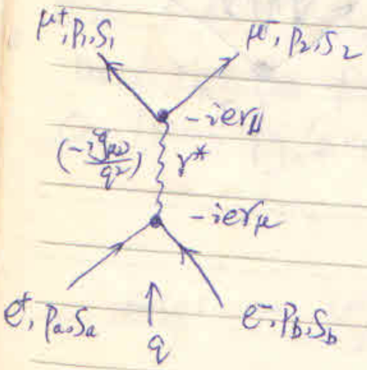
2.1. The Naive Parton Model.



A. ①. $e^+e^- \to \mu^+\mu^-$

$$A_\mu(P_2 S_2; P_1, S_1) = -ie\, \bar{u}(P_2, S_2)\, \gamma_\mu\, v(P_1, S_1)$$

$A_\mu(P_2 S_2; P_1, S_1)$



$$M = A_\mu(P_a, S_a; P_b, S_b) \cdot \frac{-ig_{\mu\nu}}{q^2} \cdot A_\nu(P_2 S_2; P_1, S_1)$$

$$= \frac{ie^2}{q^2}\, \bar{u}(P_2, S_2)\, \gamma^\mu\, v(P_1 S_1)\; \bar{u}(P_b, S_b)\, \gamma_\mu\, v(P_a, S_a)$$

对末态求和，初态平均 极化求和

$$\overline{|M|}^2 = \sum_{S_1 S_2 S_a S_b} \frac{1}{4} |M|^2$$

$$= \frac{1}{4} \frac{e^4}{q^4} \sum_{\substack{S_1 S_2 \\ S_a S_b}} M_0 M_0^+$$

$$M_0 = \bar{u}(P_2, S_2)\, \gamma^\mu\, v(P_1, S_1)\; \bar{u}(P_b, S_b)\, \gamma_\mu\, v(P_a, S_a)$$

$$\begin{cases} \sum_{S_2} u(P_2 S_2)\, \bar{u}(P_2, S_2) = \not{P}_2 + m = \not{P}_2 \\ \sum_{S_1} v(P_1 S_1)\, \bar{v}(P_1, S_1) = \not{P}_1 - m = \not{P}_1 \end{cases} \qquad m = 0$$

$$\begin{cases} (\bar{u}\gamma^\mu v)^+ = (\bar{u}\gamma^\mu v)^* = \bar{v}\gamma^\mu u \\ (\bar{v}\gamma^\mu u)^+ = (\bar{v}\gamma^\mu u)^* = \bar{u}\gamma^\mu v \end{cases}$$

$$\sum_{\substack{S_1 S_2 \\ S_a S_b}} M_0 M_0^+ = tr(\not{P}_2 \gamma^\mu \not{P}_1 \gamma^\nu)\; tr(\not{P}_b \gamma_\mu \not{P}_a \gamma_\nu)$$

$g^{\mu\nu} g_{\mu\nu} = 4$

$$= 16[P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - g^{\mu\nu} P_1 \cdot P_2]\,[P_{a\mu} P_{b\nu} + P_{a\nu} P_{b\mu} - g_{\mu\nu} P_a \cdot P_b]$$

$$= 32[(P_1 \cdot P_a)(P_2 \cdot P_b) + (P_1 \cdot P_b)(P_2 \cdot P_a)]$$

$$|\overline{M}|^2 = \frac{8e^4}{q^4}[(p_a\cdot p_1)(p_b\cdot p_2)+(p_a\cdot p_2)(p_b\cdot p_1)]$$

不变量:

$$\begin{cases} s=(p_a+p_b)^2=(p_1+p_2)^2=E_{cm}^2=q^2=Q^2 \\ t=(p_1-p_a)^2=-2p_1\cdot p_a=-\frac{s}{2}(1-\cos\theta_{cm}) \\ t=(p_b-p_2)^2=-2p_2\cdot p_b=-\frac{s}{2}(1-\cos\theta_{cm}) \\ u=(p_1-p_b)^2=-2p_1\cdot p_b=-\frac{s}{2}(1+\cos\theta_{cm}) \\ u=(p_a-p_2)^2=-2p_2\cdot p_a=-\frac{s}{2}(1+\cos\theta_{cm}) \end{cases}$$

$\left(\frac{\sqrt{s}}{2}, 0, \frac{\sqrt{s}}{2}\sin\theta, \frac{\sqrt{s}}{2}\cos\theta\right)$

$\left(\frac{\sqrt{s}}{2},0,0,\frac{\sqrt{s}}{2}\right)$ $\vec{p}_a$, $\vec{p}_1$, θ_{cm}, $\vec{p}_b$ $\left(\frac{\sqrt{s}}{2},0,0,-\frac{\sqrt{s}}{2}\right)$

$\vec{p}_2$ cm系 $|\vec{p}_a|=|\vec{p}_b|=|\vec{p}_1|=|\vec{p}_2|$

$\left(\frac{\sqrt{s}}{2}, 0, -\frac{\sqrt{s}}{2}\sin\theta, -\frac{\sqrt{s}}{2}\cos\theta\right)$

$$p_a+p_b=p_1+p_2$$

$$(p_a\cdot p_1)(p_b\cdot p_2)+(p_a\cdot p_2)(p_b\cdot p_1)=\frac{1}{4}(t^2+u^2)$$
$$=\frac{1}{8}s^2(1+\cos^2\theta_{cm})$$

$$\frac{8e^4}{s^2}\quad \frac{1}{25}\ \frac{1}{8}s^2(1+\cos\theta)\ \frac{1}{32\pi^2}\ d\cos\theta\, d\phi$$

$$=\frac{e^4}{64\pi^2 s}(1+\cos\theta)$$

$\vec{p}_a$ $\vec{p}_b$ $v_a > v_b$

$$\frac{d\sigma}{d\Omega_{cm}}=\frac{1}{64\pi^2E_{cm}^2}|\overline{M}(e^+e^-\to\mu^+\mu^-)|^2$$

$$=\frac{1}{4}\frac{\alpha^2}{Q^2}(1+\cos^2\theta_{cm})$$

$$\alpha^2=e^2/4\pi$$

$$d\Omega_{cm}=d\cos\theta_{cm}\,d\phi_{cm}$$ $\sin\theta_{cm}\,d\theta_{cm}\,d\phi_{cm}$

$$\sigma(e^+e^-\to\mu^+\mu^-)=\int\frac{d\sigma}{d\Omega_{cm}}d\Omega_{cm}$$

$$=\int_{-1}^{1}\int_0^{2\pi}\frac{1}{4}\frac{\alpha}{Q^2}(1+\cos\theta_{cm}^2)\,d\cos\theta_{cm}\,d\phi_{cm}$$ $\int_{\pi}^{0}$ -1 1

$$=\frac{4\pi}{3}\frac{\alpha^2}{Q^2}\sim 2.2\times10^{-4}/Q^2$$

$1\,\mathrm{GeV}^{-2}=0.389379\times10^{9}\,\mathrm{pb}$

$Q=100\,\mathrm{GeV}$

② $\sigma(e^+e^-\to q\bar{q})$ 可由 $\sigma(e^+e^-\to\mu^+\mu^-)$ 得到

$$\sigma(e^+e^-\to q\bar{q})=(3)\frac{4\pi}{3}\frac{\alpha^2}{Q^2}e_q^2$$

$\downarrow$ color number

$$\sigma\simeq\frac{2.2\times10^{-4}}{10^4}\,0.4\times10^9\,\mathrm{pb}\approx 9\,\mathrm{pb}$$