

JET/PARTON MATCHING



Qiang Li

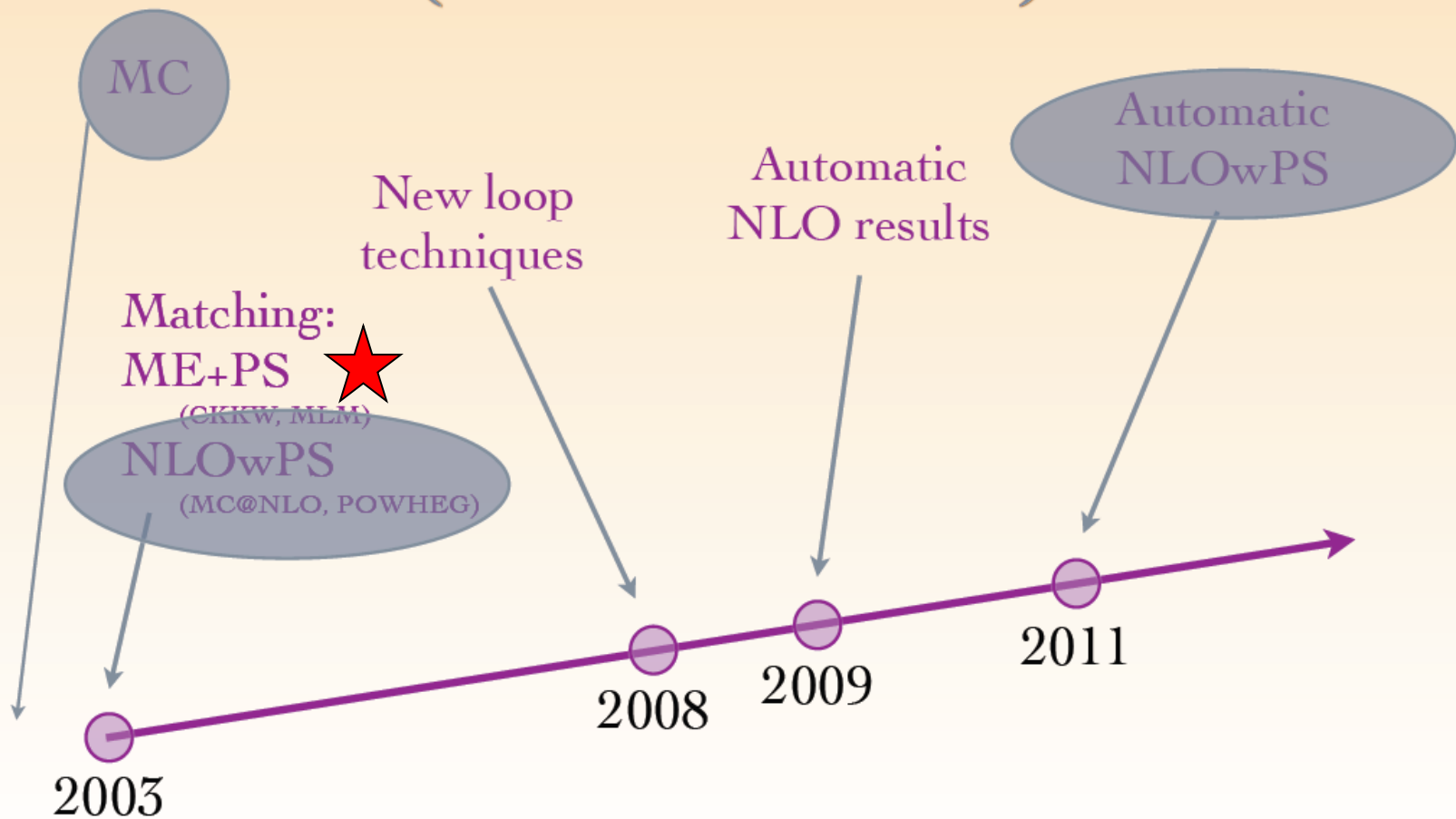
Many slides from Paolo Torrielli

1. What and why matching
2. A bit about Parton Shower
3. NLO Parton Shower Matching
4. $b\bar{b} \rightarrow h$ in POWHEG as an Example
5. Going Beyond and Summary

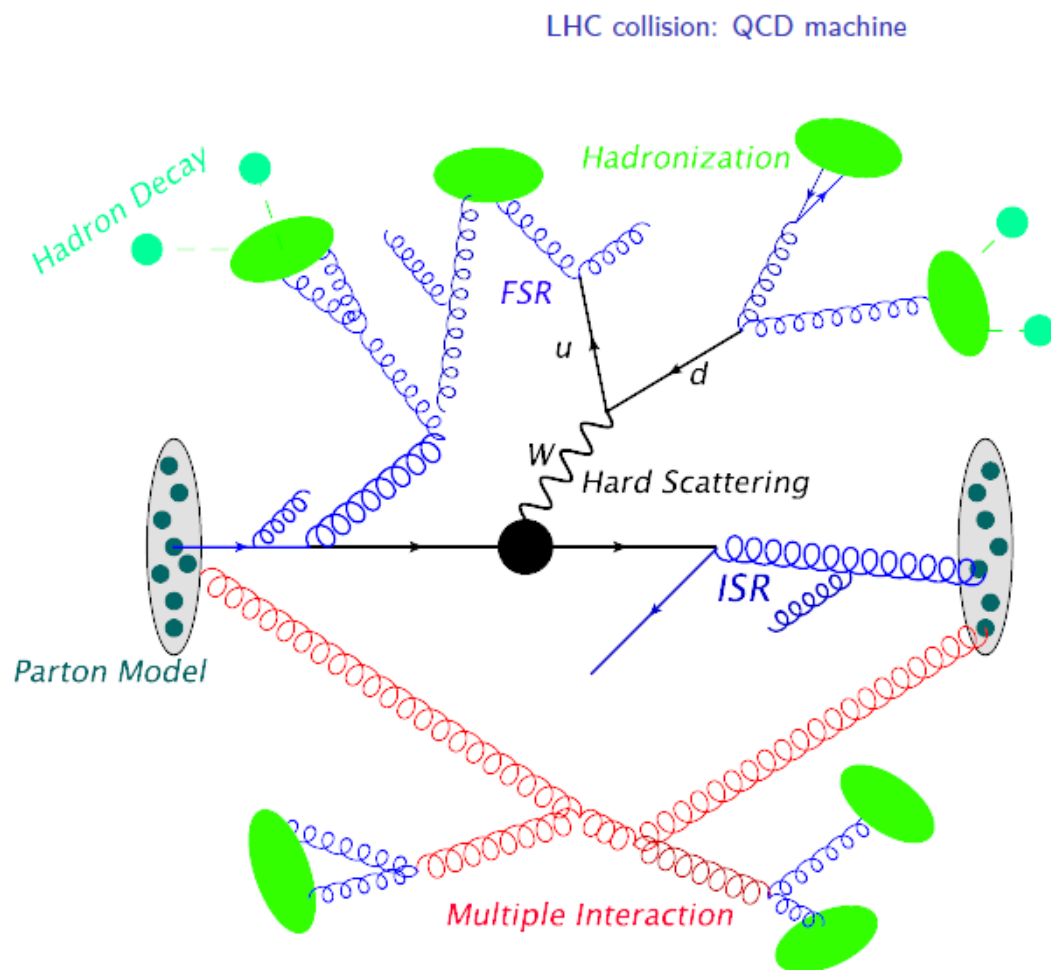
1. What and why matching



QCD AND MC PROGRESS (SIMPLIFIED)



(N+1)-th Jet can come from parton shower, can also from ME parton



Factorization Theorem:
Separate Short Distance
Physics from Soft one

Hard Scattering:
LO, NLO, NNLO QCD,
QED..

ISR, FSR Parton
Shower:
Pythia6/8, Herwig(++),
Sherpa Dipole Shower

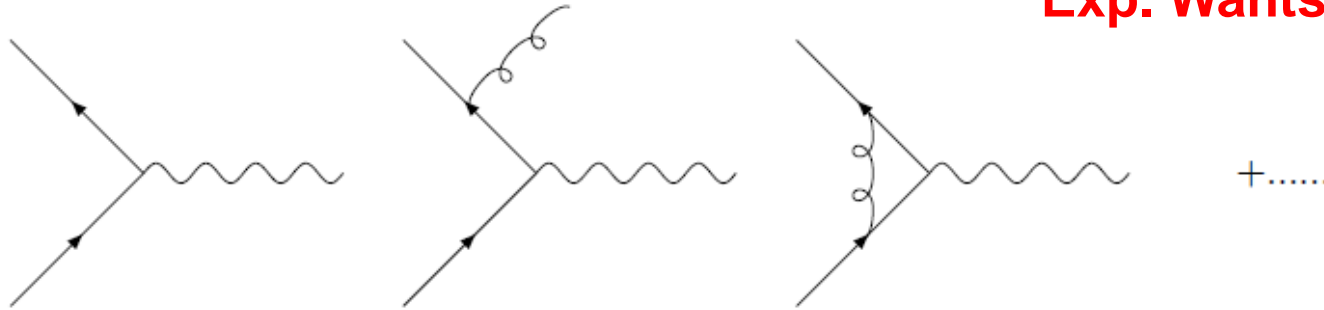
Hadronization:
String Model;
Color Cluster

**Parton Shower: capture the leading singularities, have a good description
of an arbitrary numbers of emission**

What and why matching



Exp. Wants Events!



Exclusive observables not described well in PQCD

e.g.

NLO V: divergence at low PT_V

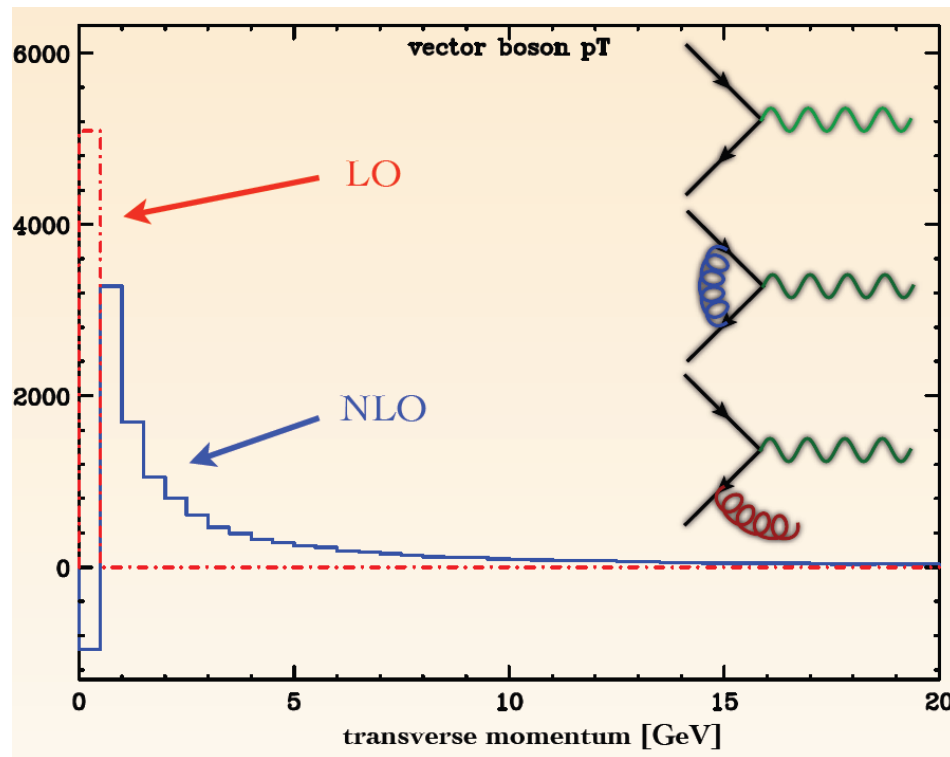
NLO V+J: 2nd Jet? Unwgt Eff?

Parton Shower

although approximation (\sim LL),
work well in collinear and soft
region; LHE events

Let's Combine them!

**Be careful of: Double counting,
matching scale dependence**



1. ME-PS Matching (Shape)

e.g. Eur.Phys.J.C53:473,2008

CKKW
MLM ...

2. NLO PS Matching (Shape & Normalization)

MC@NLO e.g. hep-ph/0204244
POWHEG e.g. arXiv:0709.2092
...

3. Multi-leg Multi-Loop Matching: (Shape & Normalization)

Merging e.g. arXiv:1311.3634

Not meant to be exhaustive, see more
In arXiv:1203.6803

QCD Matrix Elements + Parton Showers

S. Catani^{a,*}, F. Krauss^b, R. Kuhn^{c,d} and B.R. Webber^b

[arXiv:hep-ph/0109231](https://arxiv.org/abs/hep-ph/0109231)

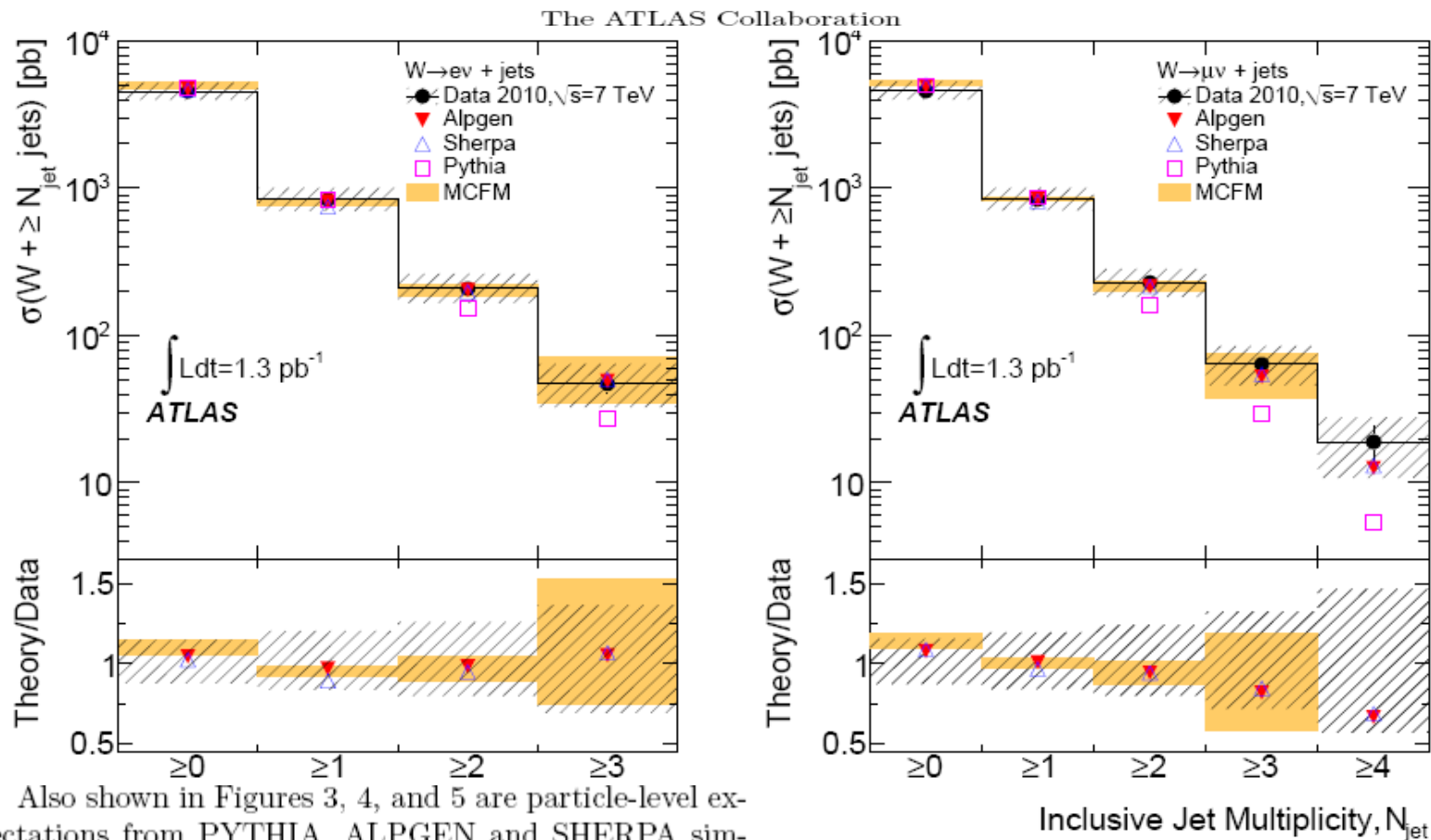
ABSTRACT: We propose a method for combining QCD matrix elements and parton showers in Monte Carlo simulations of hadronic final states in e^+e^- annihilation. The matrix element and parton shower domains are separated at some value y_{ini} of the jet resolution, defined according to the k_T -clustering algorithm. The matrix elements are modified by Sudakov form factors and the parton showers are subjected to a veto procedure to cancel dependence on y_{ini} to next-to-leading logarithmic accuracy. The method provides a leading-order description of hard multi-jet configurations together with jet fragmentation, while avoiding the most serious problems of double counting. We present first results of an approximate implementation using the event generator APACIC++.

**Sudakov
Veto
Double counting**

ME-PS Matching@LHC



Measurement of the production cross section for W-bosons in association with jets in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector



Also shown in Figures 3, 4, and 5 are particle-level expectations from PYTHIA, ALPGEN and SHERPA simulations as well as a calculation using MCFM v5.8 [35]. PYTHIA is LO, while ALPGEN and SHERPA match higher multiplicity matrix elements to a leading-logarithmic parton shower; these predictions have been normalised to the NNLO inclusive W production cross section. The version

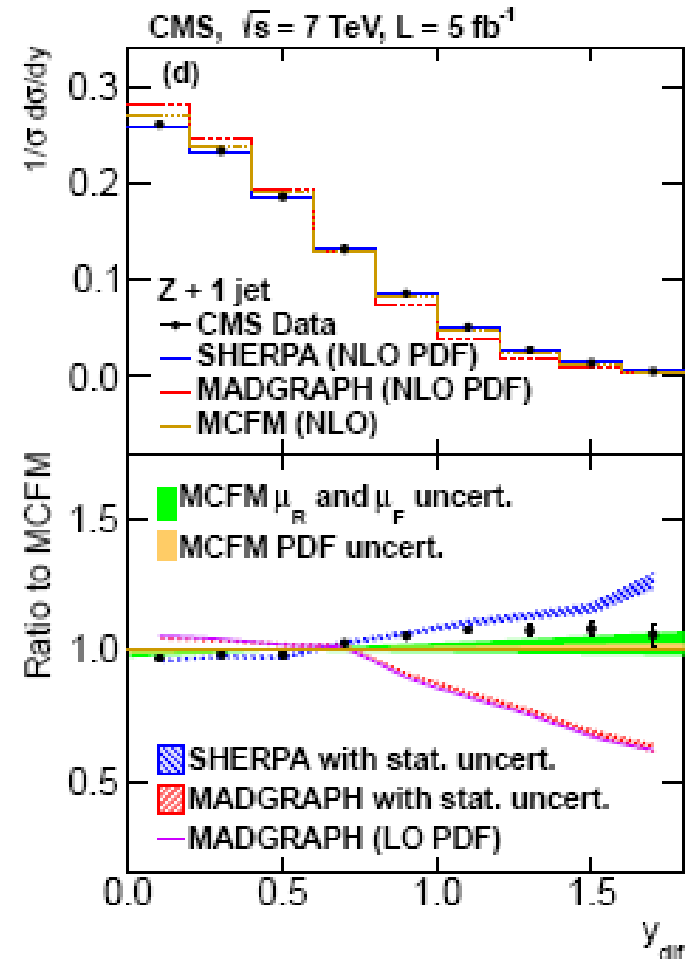
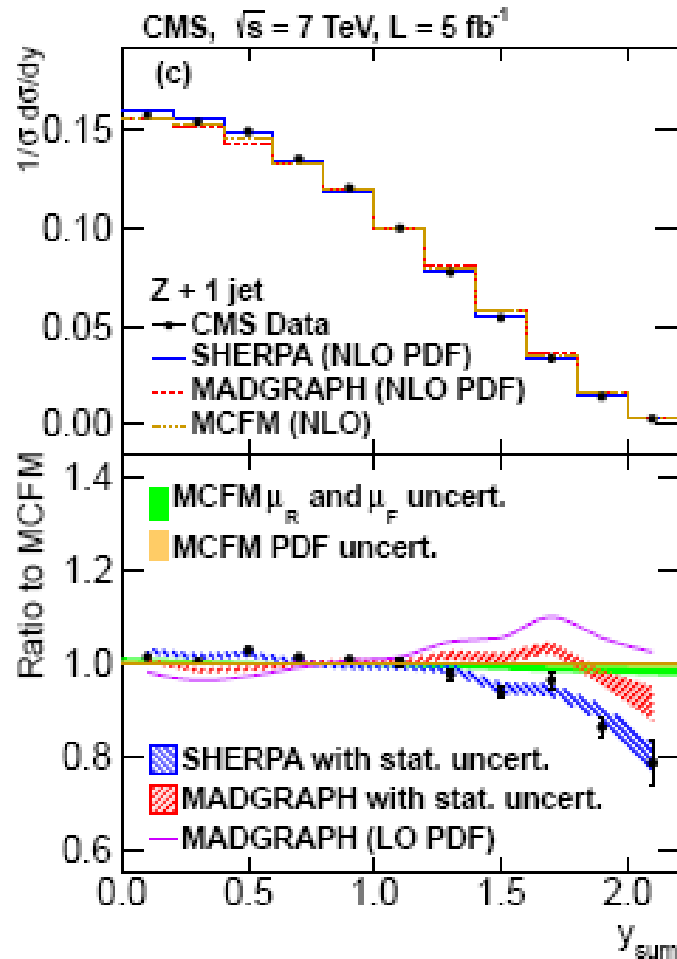
arXiv:1012.5382

ME-PS Matching@LHC



Rapidity distributions in exclusive $Z + \text{jet}$ and $\gamma + \text{jet}$ events
in pp collisions at $\sqrt{s} = 7 \text{ TeV}$

arXiv:1310.3082



NLO PS Matching@LHC



1. Matching NLO QCD computations with Parton Shower simulations: the POWHEG method

Stefano Frixione (INFN, Genoa), Paolo Nason (INFN, Milan Bicocca), Carlo Oleari (INFN, Milan Bicocca & Milan Bicocca U.). Sep 2007. 91 pp.

Published in JHEP 0711 (2007) 070

BICOCCA-FT-07-9, GEF-TH-21-2007

DOI: [10.1088/1126-6708/2007/11/070](https://doi.org/10.1088/1126-6708/2007/11/070)

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2. Matching NLO QCD and parton showers in heavy flavor production

Stefano Frixione (INFN, Genoa), Paolo Nason (INFN, Milan), Bryan R. Webber (CERN & Cambridge U.). May 2003. 70 pp.

Published in JHEP 0308 (2003) 007

BICOCCA-FT-03-11, CAVENDISH-HEP-03-03, CERN-TH-2003-102, GEF-TH-5-2003

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3. Matching NLO QCD computations and parton shower simulations

Stefano Frixione (Annecy, LAPP), Bryan R. Webber (Cambridge U.). Apr 2002. 69 pp.

Published in JHEP 0206 (2002) 029

CAVENDISH-HEP-02-01, LAPTH-905-02, GEF-TH-2-2002

DOI: [10.1088/1126-6708/2002/06/029](https://doi.org/10.1088/1126-6708/2002/06/029)

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Until 2013/10

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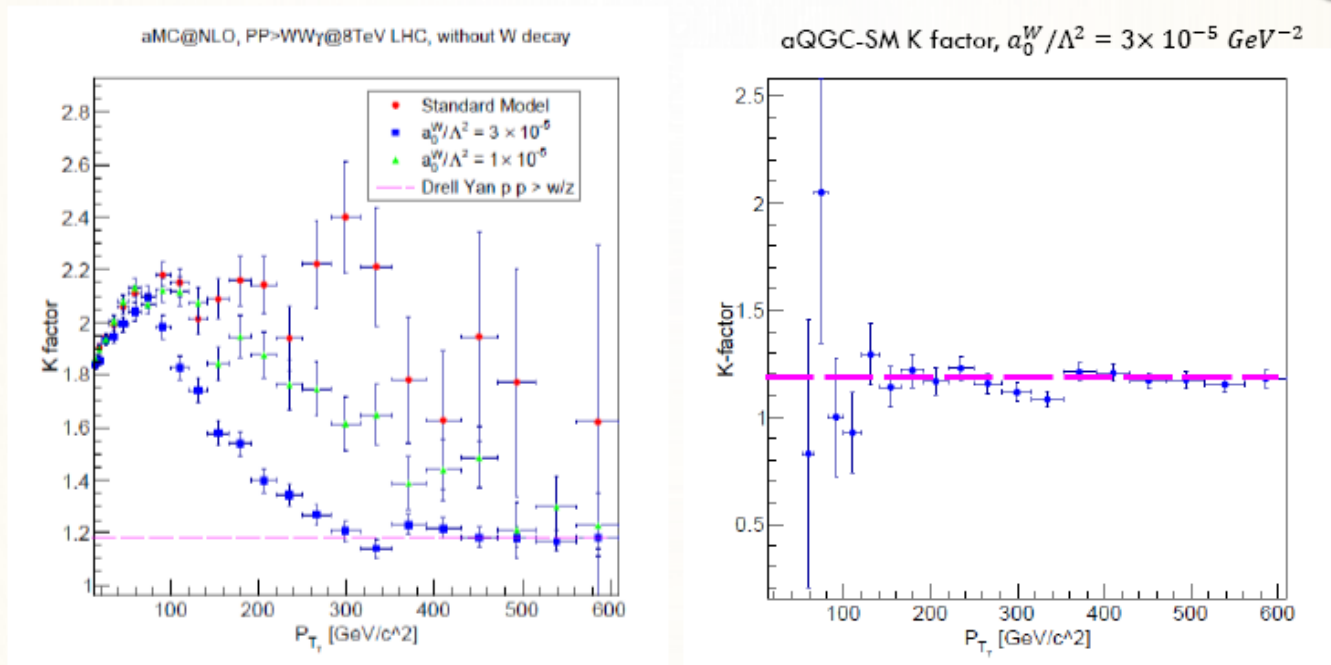
Until
2014/03/12

NLO PS Matching@LHC



aMC@NLO Used in our recent analysis **CMS-PAS-SMP-12-006**
 $WW\gamma+WZ\gamma \rightarrow l\nu jj\gamma$ for full Sim and K factors

Theoretical Uncertainties

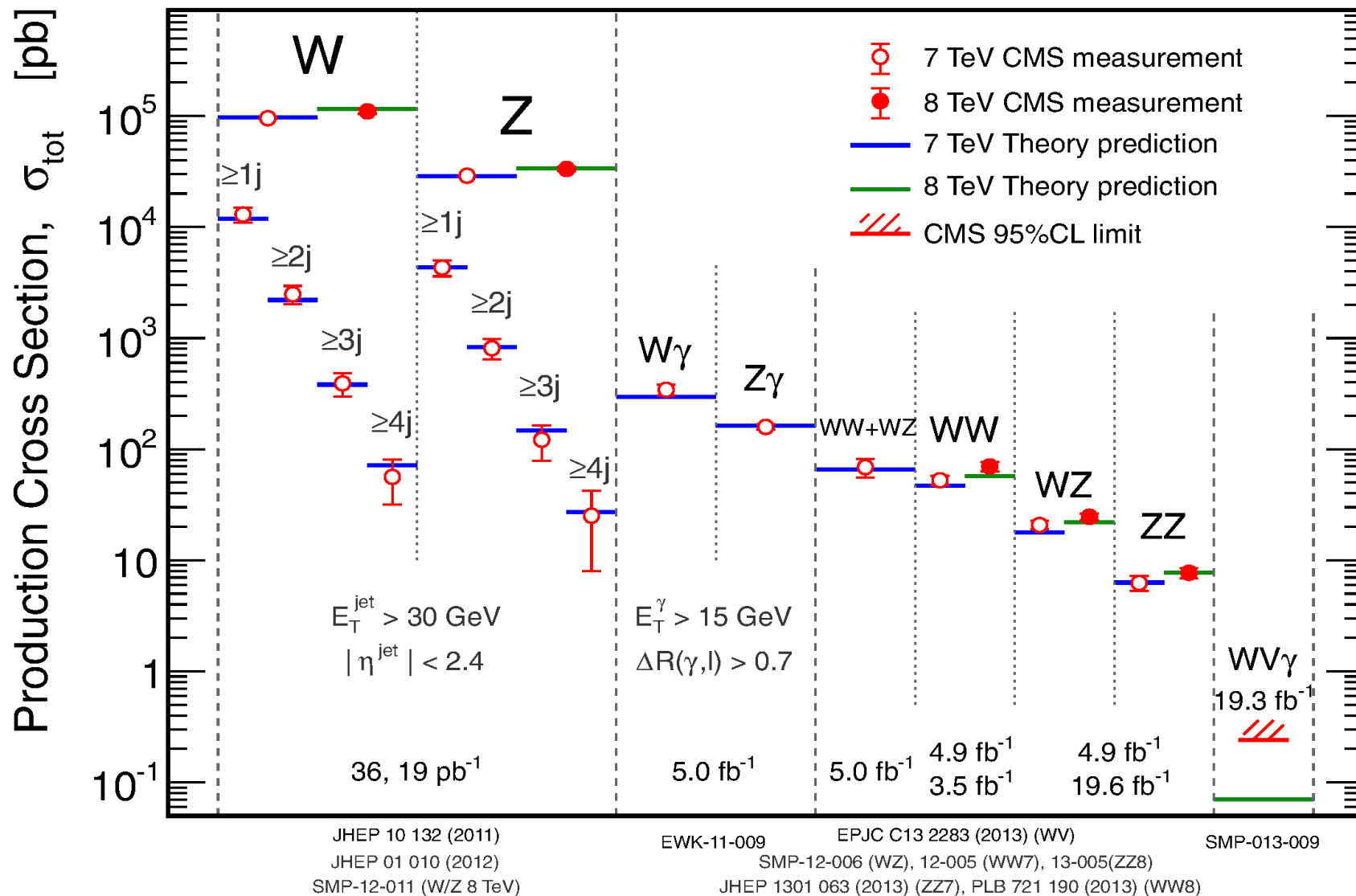


Approved by
Daneng Yang

First Triple-V
Measurement
at the LHC

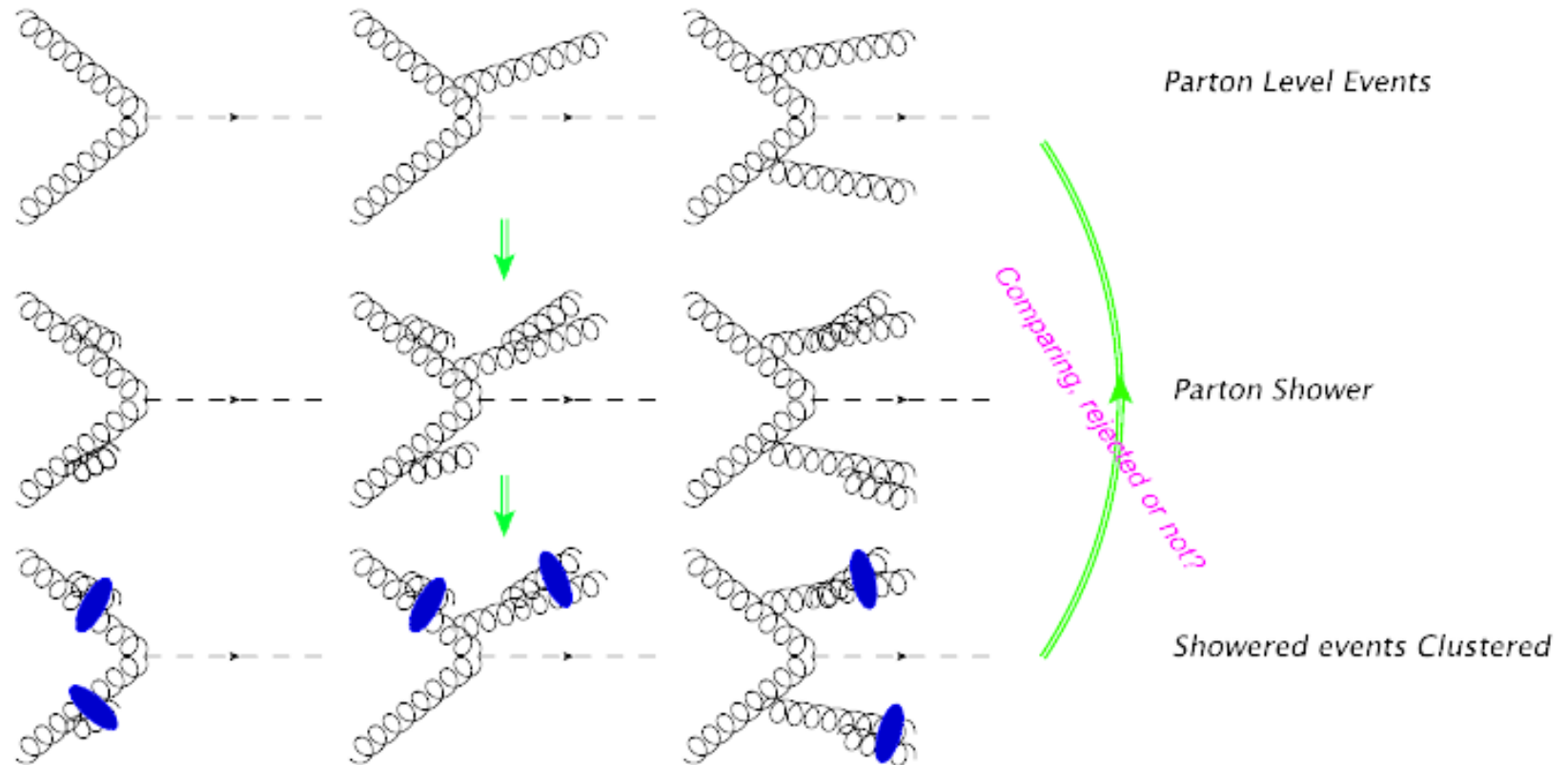
- Non-zero value of aQGCs break the flat shape
- NLO correction for aQGC excess ~ 1.2 (Drell Yan like), used for limit setter ²⁴

July 2013



PKU CMS group is now involved and leading many Triple Gauge Boson measurement: WWW , WZA , and $EWK WA+2J$, $ZA+2J$

Multi-leg Matrix Element Matching in 1 Page

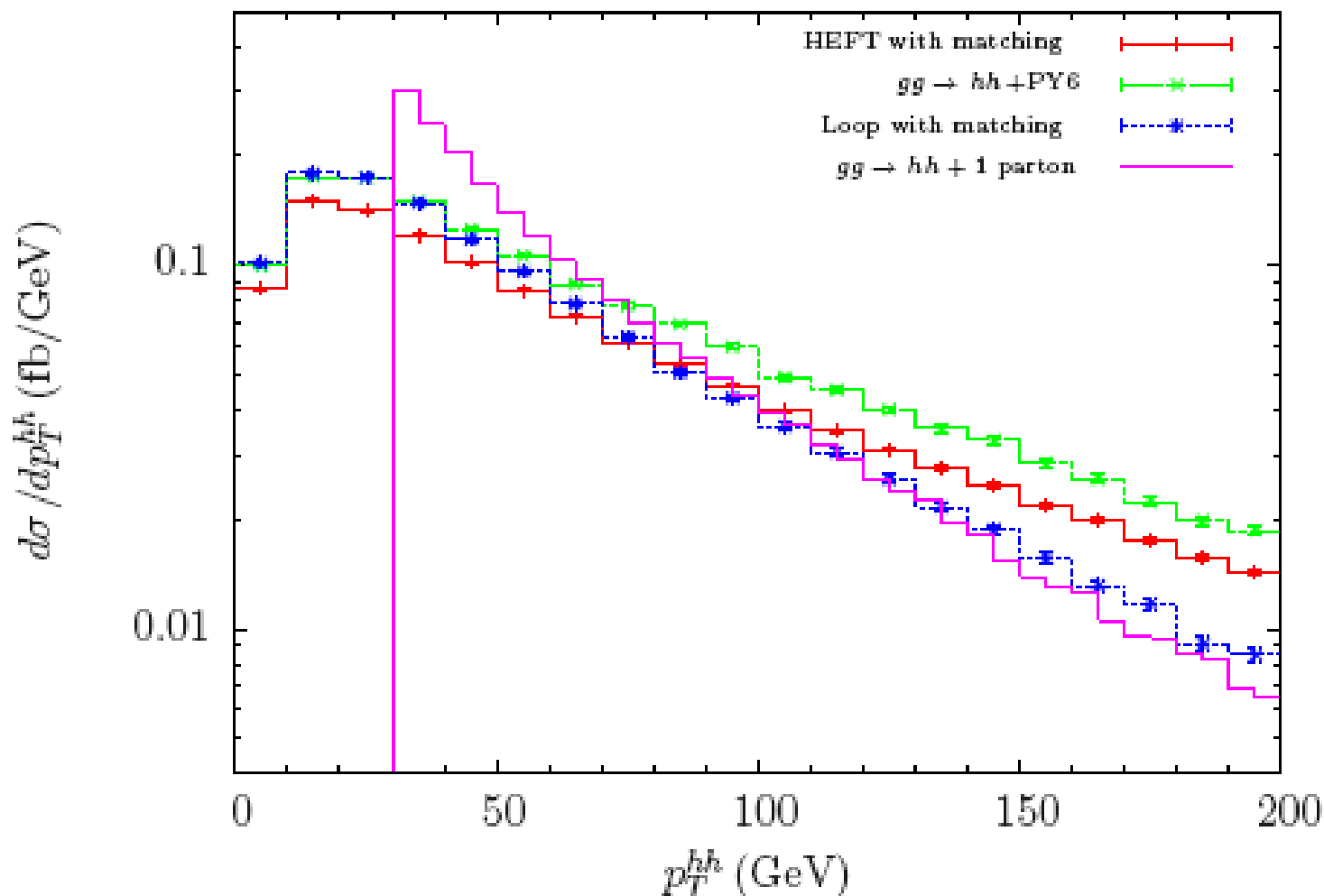


Higgs Pair Production: Improved Description by Matrix Element Matching

Qiang Li,¹ Qi-Shu Yan,² and Xiaoran Zhao³

Phys. Rev. D 89, 033015 (2014)

$gg \rightarrow HH+0,1\text{Jet}$ matching with Pythia6
Accurate Exclusive Simulation for $gg \rightarrow HH$

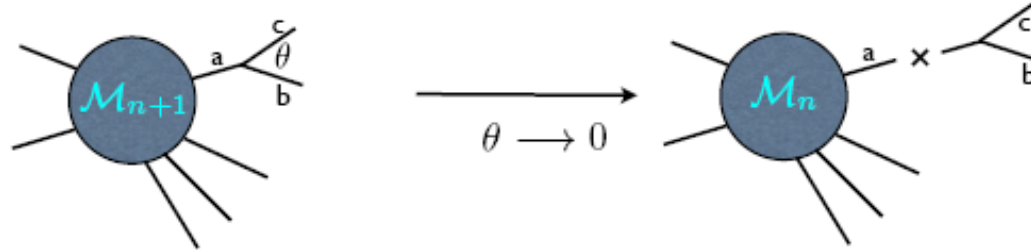


2. A bit about Parton Shower

A bit about Parton Shower



Collinear factorization



Cross section factorization in the collinear limit (universal !!):

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z).$$

$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z) \\ \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_S}{2\pi} P_{b \rightarrow de}(z').$$

- t can be called the "evolution variable" (will become clearer later): it can be the virtuality m^2 of particle a (i.e. p_a^2), its p_\perp or $\tilde{t} = E_a^2 \theta^2 \dots$
in the collinear limit one has $m^2 = z(1-z)\theta^2 E_a^2$, $p_T^2 = zm^2$.

Leading Log Approximation

$$\sigma_{n+k} \propto \alpha_s^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^t \frac{dt'}{t'} \dots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_s}{2\pi} \right)^k \log^k(Q^2/Q_0^2),$$

$$\begin{aligned} \frac{d\sigma}{dq_T^2 dy} \propto \frac{1}{q_T^2} [& \alpha_s(L+1) \\ & + \alpha_s^2(L^3 + L^2 + L + 1) \\ & + \alpha_s^3(L^5 + L^4 + L^3 + L^2 + L + 1) + \dots], \end{aligned}$$

where $L \equiv \ln(Q^2/q_T^2)$

► Resummation:

$$\begin{aligned} \frac{d\sigma}{dq_T^2 dy} &\propto \frac{1}{q_T^2} [\alpha_s Z_1 + \alpha_s^2 Z_2 + \alpha_s^3 Z_3 + \dots] \\ \alpha_s Z_1 &= \alpha_s(L+1) + \alpha_s^2(L^3 + L^2) + \alpha_s^3(L^5 + L^4) + \dots, \\ \alpha_s^2 Z_2 &= \alpha_s^2(L+1) + \alpha_s^3(L^3 + L^2) + \dots, \\ \alpha_s^3 Z_3 &= \alpha_s^3(L+1) \dots \end{aligned}$$

$$Z_{n+1}/Z_n = \alpha_s$$

QCD: $0.12 \cdot \ln((10/100)^2) \sim 50\%$, QED: $1/137 \cdot \ln((1/100)^2) \sim 10\%$

A bit about Parton Shower



Differential probability for the branching $a \rightarrow bc$ between scales t and $t + dt$ knowing that no emission occurred before:

$$dp(t) = \sum_{bc} \frac{dt}{t} \int dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

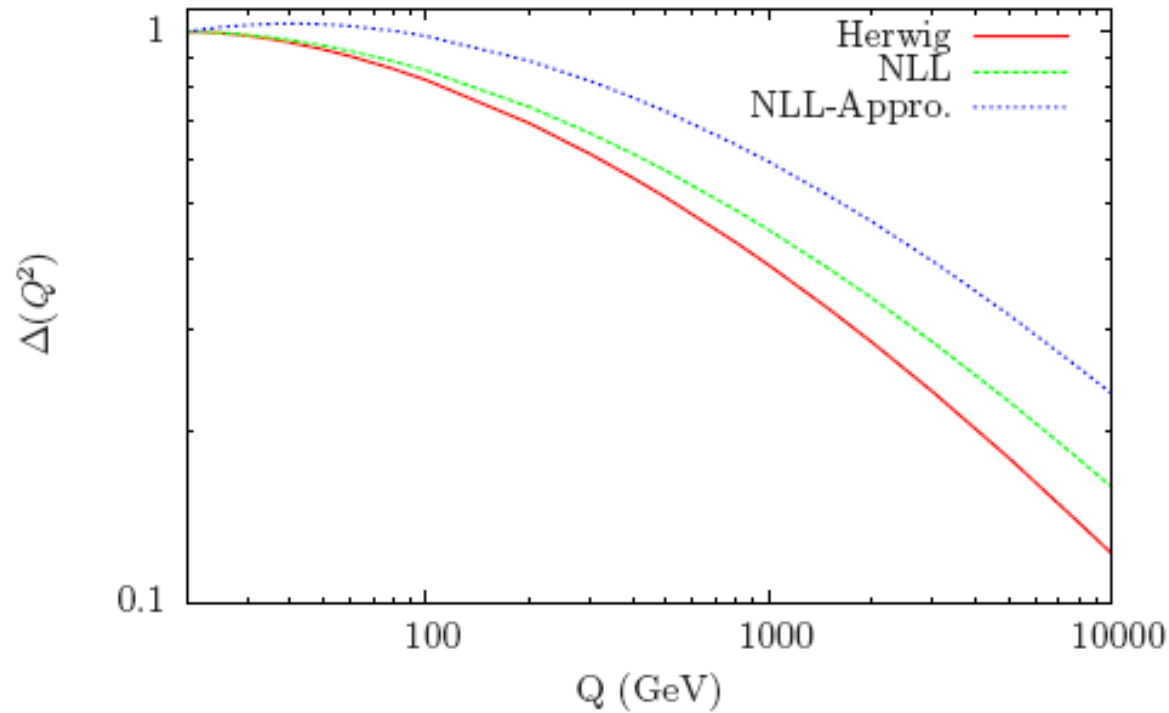
Probability that particle a does not emit between scales Q^2 and t :

$$\Delta(Q^2, t) = \prod_k \left[1 - \sum_{bc} \frac{dt_k}{t_k} \int dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) \right] = \\ \exp \left[- \sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} \int dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) \right] = \exp \left[- \int_t^{Q^2} dp(t') \right].$$

- ▶ $\Delta(Q^2, t)$ is the Sudakov form factor.
- ▶ Property: $\Delta(A, B) = \Delta(A, C)\Delta(C, B)$.

$$\Delta_{a \rightarrow bc}^{\text{HW}}(\tilde{t}) = \exp \left\{ - \int_{4t_0}^{\tilde{t}} \frac{dt'}{t'} \int_{\sqrt{\frac{t_0}{t'}}}^{1 - \sqrt{\frac{t_0}{t'}}} \frac{dz}{2\pi} \alpha_S(z^2(1-z)^2 t') \hat{P}_{ba}(z) \right\},$$

Quark Sudakov, $Q_0 = \sqrt{t_0} = 10 \text{ GeV}$



$$\Delta_{a \rightarrow bc}^{\text{NLL}}(t) = \exp \left\{ - \int_{4t_0}^t \frac{dt'}{t'} \int_{\sqrt{\frac{t'}{4t}}}^{1 - \sqrt{\frac{t'}{4t}}} \frac{dz}{2\pi} \alpha_S(t') \hat{P}_{ba}(z) \right\}$$

$$\Delta_{a \rightarrow bc}(Q) = \exp \left\{ - \int_{Q_1=2\sqrt{t_0}}^Q dq \Gamma_{a \rightarrow bc}(q, Q) \right\} \quad \Gamma_{q \rightarrow qg} = \frac{2C_F}{\pi} \frac{\alpha_S(q)}{q} \left(\ln \frac{Q}{q} - \frac{3}{4} \right)$$

A bit about Parton Shower



Cross section for 0 or 1 emissions from leg a in the parton shower:

$$\frac{d\sigma}{\sigma_n} = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

Expand at first order in α_s :

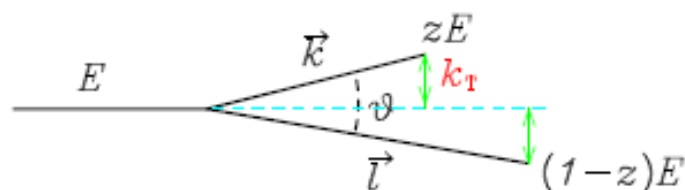
$$\frac{d\sigma}{\sigma_n} \simeq 1 - \sum_{bc} \int_{Q_0^2}^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) + \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z).$$

- ▶ Same structure of the two latter terms, with opposite signs: cancellation of divergences between the approximate virtual and approximate real emission cross sections.

$$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z} \text{ and } P_{g,gg}(z) = C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right)$$

are **singular** when $z \rightarrow 1$ ($z \rightarrow 0$).

Different choices of the ordering variables give sizeable difference in double log structure.



$$\begin{aligned} \text{virtuality} &: t \approx E^2 z(1-z)\theta^2 \\ p_T &: t \approx E^2 z^2(1-z)^2\theta^2 \\ \text{angle} &: t = E^2\theta^2 \end{aligned}$$

Mueller(1981) shows that angular ordering is the correct choice to take account for color coherence

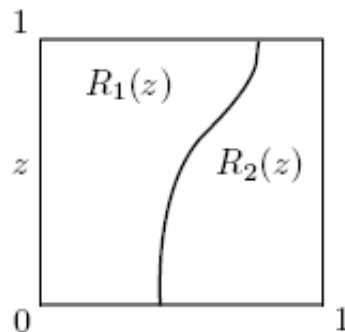
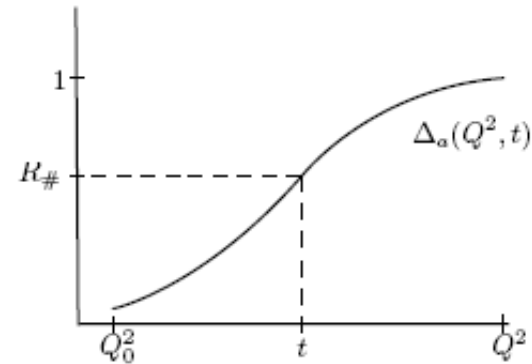
Furthermore, $\alpha_S = \alpha_S(p_T^2)$ for correctly take into account renormalization in soft and collinear regions.

A bit about Parton Shower



Implementation

- ▶ Extract the evolution variable t of the branching by solving the equation $\Delta(Q^2, t) = R_{\#}$, with $R_{\#}$ a flat random number between 0 and 1. This correctly reproduces the probability distribution since the probability of extracting a splitting scale t between t_1 and t_2 is $\Delta(Q^2, t_2) - \Delta(Q^2, t_1)$.



- ▶ Extract the energy sharing z and the daughter identities b and c according to $P_{a \rightarrow bc}(z)$. For two possible branchings $P_1(z)$ and $P_2(z)$ one can call $R_i(z) = P_i(z)/(P_1(z) + P_2(z))$, and choose z and parton identities by extracting a random point in the plane.

- ▶ Extract ϕ (flat).
- ▶ Reiterate (updating the maximum scale for the Sudakov) until all the 'external' partons are characterized by a scale smaller than a threshold $Q_0^2 \sim 1 \text{ GeV}$.
- ▶ Put partons on shell and hadronize.

A bit about Parton Shower



Main Monte Carlos available on the market: HERWIG

All HERWIG versions (Fortran and C++) implement the angular-ordering: subsequent emissions are characterized by smaller and smaller angles.

► HERWIG 6: $t = \frac{p_b \cdot p_c}{E_b E_c} \simeq 1 - \cos \theta.$

► Herwig++: $t = \frac{(p_{b\perp})^2}{z^2(1-z)^2} = t(\theta).$

Implementing angular ordering, the parton shower (without matrix element corrections) cannot populate the full phase space (without matrix element corrections): empty regions of the phase space, called "dead zones", will arise.

Note. It may seem that the presence of dead zones is a weakness, but it is not so: they implement correctly the collinear approximation, in the sense that they constrain the shower to live uniquely in the region where it is reliable.

► Hadronization: cluster model.

A bit about Parton Shower



Main Monte Carlos available on the market: PYTHIA

Choice of evolution variables for Fortran and C++ versions:

- ▶ PYTHIA 6: $t = (p_b + p_c)^2 \sim z(1-z)\theta^2 E_a^2$.
- ▶ Pythia 8: $t = (p_b)_\perp^2$.

Simpler variables, but decreasing angles not guaranteed: PYTHIA has to reject the events that don't respect the angular ordering (though this is not completely equivalent to ordering in angle).

Not implementing directly angular ordering, the phase space can be filled entirely, even without matrix element corrections, so one can have the so called "power shower" (use with a certain care).

- ▶ Hadronization: string model.

Note. Usually PYTHIA is faster than HERWIG.

3. NLO Parton Shower Matching

Naive matching at the NLO

Naive matching definition

$$\frac{d\sigma^{\text{"MC@NLO"}}}{dO} = [d\Phi_B(B + V)] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} R] I_{\text{MC}}^{(n+1)}(O).$$

This simple approach does NOT work:

- ▶ **Instability:** weights associated to $I_{\text{MC}}^{(n)}(O)$ and $I_{\text{MC}}^{(n+1)}(O)$ are separately divergent. Remember from the KLN theorem that only the sum of V and $\int d\Phi_{(+1)} R$ is finite, so it is hopeless to treat n - body and $n + 1$ -body configurations separately. One could regulate the divergence by means of some cut-off but then
 - ▶ one should prove the independence upon this cutoff,
 - ▶ the unweighting of real-like configurations would be highly inefficient (hard to extract events from a singular function).
- ▶ **Double counting:** this $d\sigma^{\text{"MC@NLO"}}$, expanded at the NLO does not coincide with the NLO differential rate. One must indeed avoid overcounting the exact virtual with the approximated contribution from the Sudakov, otherwise some Born-like configurations, to be passed to the shower are spurious (not there in the exact NLO computation). Some configurations are thus accounted for by both the parton shower and the NLO.

Modified subtraction

Modify the naive formula:

$$\frac{d\sigma_{\text{MC@NLO}}}{dO} = \left[d\Phi_B (B + V + \int d\Phi_{(+1)} \text{MC}) \right] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] I_{\text{MC}}^{(n+1)}(O),$$

as opposed to the naive

$$\frac{d\sigma_{\text{"MC@NLO"}}}{dO} = [d\Phi_B (B + V)] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} R] I_{\text{MC}}^{(n+1)}(O).$$

The term MC is called the Monte Carlo counterterm, and its rough structure is

$$\text{MC} = \left| \frac{\partial(t^{\text{MC}}, z^{\text{MC}}, \phi)}{\partial\Phi_{(+1)}} \right| \frac{1}{t^{\text{MC}}} \frac{\alpha_s}{2\pi} \frac{1}{2\pi} P(z^{\text{MC}}) B.$$

- ▶ It is the cross section for the first emission in the parton shower: it is the Born matrix-element squared times the differential emission probability in a given point of the extra-parton phase space.
- ▶ It has the same collinear and soft singularities as the real and virtual emission amplitudes squared, so it acts as a local counterterm for them (subtlety on soft poles, see below).
- ▶ It is basically process independent.
- ▶ It essentially depends on the Monte Carlo one is interfacing to.

Properties

Avoiding of double counting:

$$\frac{d\sigma_{\text{MC@NLO}}}{dO} = \left[d\Phi_B (B + V + \int d\Phi_{(+1)} \text{MC}) \right] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] I_{\text{MC}}^{(n+1)}(O)$$

where, recalling slides 48 and 50, and using a simple notation for the Sudakov form factor Δ , we have

$$I_{\text{MC}}^{(k)}(O) dO = \Delta + \Delta d\Phi_{(+1)} \frac{MC}{B} + \dots,$$

$$\Delta = \exp \left(- \int d\Phi_{(+1)} \frac{MC}{B} \right).$$

Expand at the NLO (recall that MC , R and V have one α_s more than B):

$$I_{\text{MC}}^{(k)}(O) dO = 1 - \int d\Phi_{(+1)} \frac{MC}{B} + d\Phi_{(+1)} \frac{MC}{B} + \dots$$

$$\begin{aligned} d\sigma_{\text{MC@NLO}} &= \left[d\Phi_B (B + V + \int d\Phi_{(+1)} \text{MC}) \right] \left[1 - \int d\Phi_{(+1)} \frac{MC}{B} + d\Phi_{(+1)} \frac{MC}{B} + \dots \right] \\ &+ [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] [1 + \dots] \\ &= d\Phi_B (B + V + d\Phi_{(+1)} R) = d\sigma_{\text{NLO}} \end{aligned}$$

Properties

$$\frac{d\sigma_{\text{MC@NLO}}}{dO} = \left[d\Phi_B (B + V + \int d\Phi_{(+1)} \text{MC}) \right] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] I_{\text{MC}}^{(n+1)}(O)$$

Smooth matching:

- ▶ In the soft / collinear region $R - \text{MC} \sim 0$ and one gets

$$d\sigma_{\text{MC@NLO}} \propto I_{\text{MC}}^{(n)}(O) dO.$$

The shape of the spectrum is identical to the underlying Monte Carlo, while the normalization in that region takes into account real and virtual emission, at variance with the pure parton shower. This is a way to include all the aspects of an NLO computation consistently in a parton shower.

- ▶ In the hard region, pure shower effects are suppressed (recall that matrix element corrections are excluded, so the maximum scale imposed on the shower implies MC to be zero far from the singular boundaries): $\text{MC} \sim 0$, $I_{\text{MC}}^{(n)}(O) dO \sim 1$, $B = V = 0$, and then

$$d\sigma_{\text{MC@NLO}} \sim d\Phi_B d\Phi_{(+1)} R.$$

Normalization:

- ▶ Based on the unitarity property of the parton shower

$$\int I_{\text{MC}}^{(k)}(O) dO = 1 \implies \int d\sigma_{\text{MC@NLO}} = \int d\sigma_{\text{NLO}}.$$

Implementation

$$\frac{d\sigma_{\text{MC@NLO}}}{dO} = \left[d\Phi_B (B + V + \int d\Phi_{(+1)} \text{MC}) \right] I_{\text{MC}}^{(n)}(O) + [d\Phi_B d\Phi_{(+1)} (R - \text{MC})] I_{\text{MC}}^{(n+1)}(O)$$

Integrands associated with n - and $n + 1$ - parton multiplicities are called \mathbb{S} (for "standard Monte Carlo") and \mathbb{H} (for "hard").

They can be negative somewhere: MC@NLO is not positive definite (more on this later).

- ▶ Compute the integrals of \mathbb{S} - and \mathbb{H} - integrands ($I_{\mathbb{S}}$, and $I_{\mathbb{H}}$) and the integrals of the absolute values of \mathbb{S} - and \mathbb{H} -integrands ($J_{\mathbb{S}}$ and $J_{\mathbb{H}}$).
- ▶ Generate \mathbb{S} - or \mathbb{H} - kinematical configurations (events) distributed according to $J_{\mathbb{S}}$ and $J_{\mathbb{H}}$ (probability distributions are positive definite), but assign them a weight with sign \pm depending on the sign of the \mathbb{S} - and \mathbb{H} - integrand in that particular configuration (unweighting up to a sign).

$$d\sigma^{\text{"POWHEG"}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[\Delta + \Delta \int d\Phi_{(+1)} \frac{MC}{B} \right].$$

This naive definition does not work, since if one expands it at the NLO, it does not coincide with the differential NLO cross section (**double counting**). The integral implicit in the Sudakov form factor definition does not contain any R term to cancel the one in the first parenthesis, and the piece in $d\Phi_{(+1)}$ is not R/B .

$$\tilde{\Delta}(Q^2, Q_0^2) = \exp \left[- \int_{Q_0^2}^{Q^2} d\Phi_{(+1)} \frac{R}{B} \right],$$

1. Generating hardest emission with full tree level matrix element and virtual corrections.
2. The shower generates subsequent emissions, performing (N)LL resummation of collinear/soft logs.
3. Vetoring emissions harder than the first.

POWHEG

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[\tilde{\Delta}(Q^2, Q_0^2) + \tilde{\Delta}(Q^2, t) d\Phi_{(+1)} \frac{R}{B} \right],$$

► Expand at the first-emission level:

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)} R \right] \left[1 - \int d\Phi_{(+1)} \frac{R}{B} + d\Phi_{(+1)} \frac{R}{B} \right] = d\sigma_{\text{NLO}},$$

so **double counting is avoided**.

$$\begin{aligned} R &= R \times F + R \times (1 - F) \\ &= R_{\bar{B}} + R_{\text{reg}} \end{aligned}$$

$$F < 1,$$

$$F \rightarrow 1 \text{ when } p_T \rightarrow 0,$$

$$F \rightarrow 0 \text{ when } p_T \rightarrow \infty$$

$$F = \frac{h^2}{p_T^2 + h^2}$$

Freedom to define the R part:

Dumping factor

- ▶ \uparrow MC@NLO, \downarrow POWHEG: MC@NLO does not exponentiate of the non-singular part of the real emission amplitude.
- ▶ \uparrow MC@NLO, \downarrow POWHEG: MC@NLO does not require tricks for treating Born zeros ($MC \propto B$).
- ▶ \uparrow POWHEG, \downarrow MC@NLO: POWHEG is independent from the parton shower one is interfacing the computation to.
- ▶ \uparrow POWHEG, \downarrow MC@NLO: POWHEG has not negative weights. A slightly smaller statistics is required to POWHEG than to MC@NLO in order to get equally smooth plots.

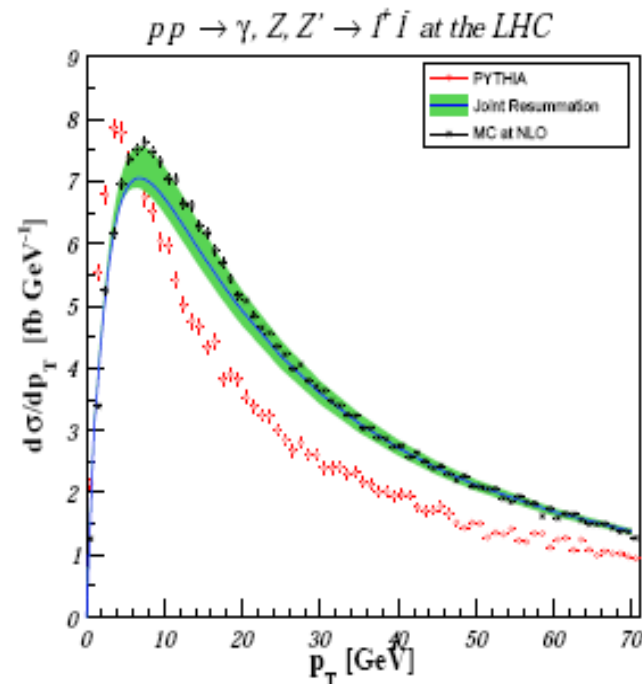
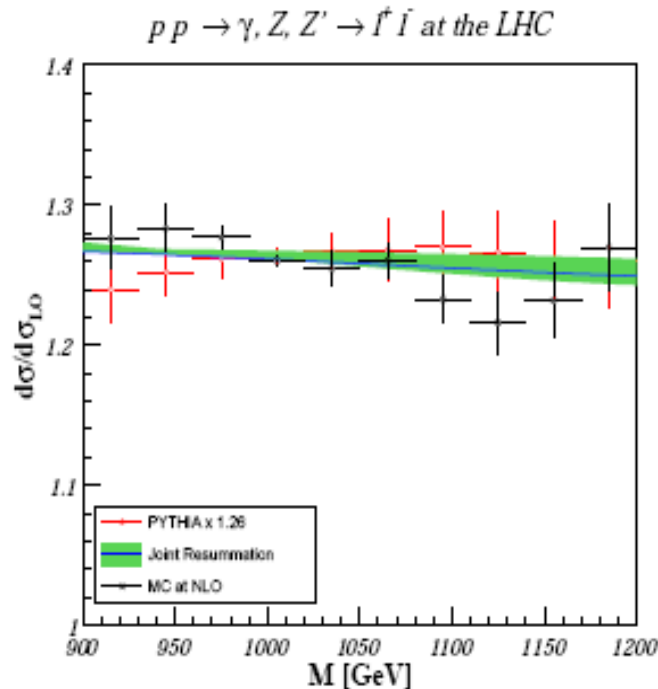
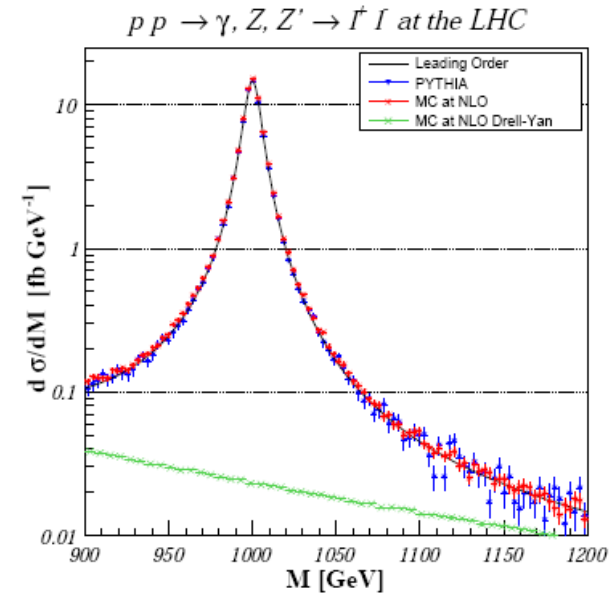
Z' in MC@NLO



Precision predictions for Z' -production at the CERN LHC: QCD matrix elements, parton showers, and joint resummation

[B. Fuks](#) (Freiburg U.), [M. Klasen](#) (LPSC Grenoble), [F. Ledroit](#) (LPSC Grenoble), [Q. Li](#) (Karlsruhe U.), [J. Morel](#) (LPSC Grenoble)

Nucl.Phys.B797:322-339,2008



4. $b\bar{b} \rightarrow h$ in POWHEG as an Example

<http://powhegbox.mib.infn.it/>

- ▶ the POWHEG BOX output are partonic events [.lhe files];
hardest emission is already included, physical results only *after* parton shower
- ▶ POWHEG BOX can be considered a large library, with many SM processes available, and some BSM ones:

`svn://powhegbox.mib.infn.it`

- ▶ often the matrix elements for complex processes have been implemented by (or with the help of) the original authors of the NLO computations.
- ▶ public interfaces to Madgraph4, GoSam and OpenLoops are available, and have been tested and used.
- ▶ main focus mostly (N)NLO QCD corrections matched to PS.
- ▶ for a bunch of processes: also NLO EW corrections [and photon radiation].

$$\begin{aligned}
d\sigma &= \bar{B}(\bar{\Phi}_1) d\bar{\Phi}_1 \left\{ \Delta(\bar{\Phi}_1, p_T^{\min}) + \Delta(\bar{\Phi}_1, p_T) \frac{R(\bar{\Phi}_1, \Phi_{\text{rad}})}{B(\bar{\Phi}_1)} d\Phi_{\text{rad}} \right\} \\
\Delta(\bar{\Phi}_1, p_T) &= \exp \left\{ - \int d\Phi_{\text{rad}} \frac{R(\bar{\Phi}_1, \Phi_{\text{rad}})}{B(\bar{\Phi}_1)} \theta(k_T - p_T) \right\}, \\
\bar{B}^{f_b}(\Phi_n) &= [B(\Phi_n) + V(\Phi_n)]_{f_b} + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int \left[d\Phi_{\text{rad}} \hat{R}(\Phi_{n+1}) \right]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n} \\
&\quad + \sum_{\alpha_{\oplus} \in \{\alpha_{\oplus} | f_b\}} \int \frac{dz}{z} G_{\oplus}^{\alpha_{\oplus}}(\Phi_{n,\oplus}) + \sum_{\alpha_{\ominus} \in \{\alpha_{\ominus} | f_b\}} \int \frac{dz}{z} G_{\ominus}^{\alpha_{\ominus}}(\Phi_{n,\ominus}).
\end{aligned}$$

POWHEG-BOX is semi-automatic for users to include new process

- a list of all flavor structures contributing to the Born process,
- the Born phase space,
- the Born amplitudes squared for all partonic subprocesses and the color correlated and the spin correlated Born amplitudes,
- the Born color structure in the limit of a large number of colors,

- the finite part of the virtual corrections,
- a list of all flavor structures contributing to the real-emission process,
- the real-emission matrix elements squared for all partonic subprocesses.

Given the ingredients listed above, the POWHEG BOX does all the rest. It automatically finds all the singular regions, builds the soft and collinear counterterms and the soft and collinear remnants, and then generates the radiation using the POWHEG Sudakov form factor.

```
find_regions(nleg,rflav,nregions,iregions)
```

Init_processes.f

```
i1=5  
i2=-5  
i3=25  
flst_nborn=1  
do k=1,3  
  flst_born(k,flst_nborn)=ii(k)  
enddo
```

```
i1=-5  
i2=5  
i3=25  
flst_nborn=2  
do k=1,3  
  flst_born(k,flst_nborn)=ii(k)  
enddo
```

nlegborn.h

parameter (nlegborn=3)

Init_processes.f

```
i1=5
i2=-5
i3=25
flst_nborn=1
do k=1,3
  flst_born(k,flst_nborn)=ii(k)
enddo

i1=-5
i2=5
i3=25
flst_nborn=2
do k=1,3
  flst_born(k,flst_nborn)=ii(k)
enddo
```

```
flst_nreal=0
condition=.false.
do i1=-5,5
  do i2=-5,5
    do i4=-5,5
c*****Lj*****
condition=((abs(i1).eq.5).and.((i1+i2).eq.0).
and.(i4.eq.0)).or.
$
((abs(i4).eq.5).and.((i1+i2).eq.i4).and.((i1*i
2).eq.0))
      if(condition) then
        flst_nreal=flst_nreal+1
        if(flst_nreal.gt.maxprocreal)
goto 998
        do k=1,4
          flst_real(k,flst_nreal)=ii(k)
        enddo
10      continue
      endif
    enddo
  enddo
enddo
enddo
```

Born.f

```
subroutine M2_bb_h(pphy,amp2)
...
real *8 pphy(0:3,nlegs)
s=(pphy(0,1)+pphy(0,2))**2-(pphy(1,1)+pphy(1,2))**2
$  -(pphy(2,1)+pphy(2,2))**2 -(pphy(3,1)+pphy(3,2))**2

xnorm=1/(2.d0*pi)*6.d0*pi/4.d0/9.d0*ph_bmass**2*ph_GF/v2
amp2=xnorm*2d0*s
end
```

```
subroutine borncolour_lh
if((idup(1).gt.0).and.(idup(2).lt.0)) then
  icolup(1,1)=501
  icolup(2,1)=0
  icolup(1,2)=0
  icolup(2,2)=501
elseif((idup(1).lt.0).and.(idup(2).gt.0)) then
  icolup(1,1)=0
  icolup(2,1)=501
  icolup(1,2)=501
  icolup(2,2)=0
```

```
subroutine M2_qaq_hg(pphy,amp2)
```

```
.....
```

```
subroutine M2_qg_hq(pphy,amp2)
```

```
.....
```

```
subroutine M2_gq_hq(pphy,amp2)
```

```
    s=(pphy(0,1)+pphy(0,2))**2-(pphy(1,1)+pphy(1,2))**2
```

```
    #  -(pphy(2,1)+pphy(2,2))**2 -(pphy(3,1)+pphy(3,2))**2
```

```
    t=(pphy(0,1)-pphy(0,3))**2-(pphy(1,1)-pphy(1,3))**2
```

```
    #  -(pphy(2,1)-pphy(2,3))**2 -(pphy(3,1)-pphy(3,3))**2
```

```
    u=(pphy(0,1)-pphy(0,4))**2-(pphy(1,1)-pphy(1,4))**2
```

```
    #  -(pphy(2,1)-pphy(2,4))**2 -(pphy(3,1)-pphy(3,4))**2
```

```
    m2=(pphy(0,3))**2-(pphy(1,3))**2-(pphy(2,3))**2 -(pphy(3,3))**2
```

```
    xnorm=4.d0/(96.d0)
```

```
    tmp=-4.d0*(m2**2+t**2)/s/u
```

```
    amp2=xnorm*tmp*(st_alpha*4.0d0*dacos(-1.d0))
```

```
    #  *ph_bmass**2*ph_GF*dsqrt(2.d0)
```

```
subroutine setvirtual(p,vflav,virtual)
```

```
.....
```

```
subroutine M2_qaq_hg(pphy,amp2)
```

```
.....
```

```
subroutine M2_qg_hq(pphy,amp2)
```

```
.....
```

```
subroutine M2_gq_hq(pphy,amp2)
```

```
    s=(pphy(0,1)+pphy(0,2))**2-(pphy(1,1)+pphy(1,2))**2
```

```
    #  -(pphy(2,1)+pphy(2,2))**2 -(pphy(3,1)+pphy(3,2))**2
```

```
    t=(pphy(0,1)-pphy(0,3))**2-(pphy(1,1)-pphy(1,3))**2
```

```
    #  -(pphy(2,1)-pphy(2,3))**2 -(pphy(3,1)-pphy(3,3))**2
```

```
    u=(pphy(0,1)-pphy(0,4))**2-(pphy(1,1)-pphy(1,4))**2
```

```
    #  -(pphy(2,1)-pphy(2,4))**2 -(pphy(3,1)-pphy(3,4))**2
```

```
    m2=(pphy(0,3))**2-(pphy(1,3))**2-(pphy(2,3))**2 -(pphy(3,3))**2
```

```
    xnorm=4.d0/(96.d0)
```

```
    tmp=-4.d0*(m2**2+t**2)/s/u
```

```
    amp2=xnorm*tmp*(st_alpha*4.0d0*dacos(-1.d0))
```

```
    #  *ph_bmass**2*ph_GF*dsqrt(2.d0)
```

MH=126.0 7TeV LHC
yukawa Mb=4.6
mur=muf=MH
CTEQ6M

Validated with bbh@nnlo 0304035

POWHEG

LO 0.65906748055878517 pb
NLO 0.56681231885198458 +-
1.00631133811981922E-003 pb

bbh@nnlo 0304035

	0.312285431E+01	# mb(muR)
LO	0.303755716E+00	# sigma(0) [pb]
NLO	0.260989891E+00	# sigma(1) [pb]
NNLO	0.207439506E+00	# sigma(2) [pb]

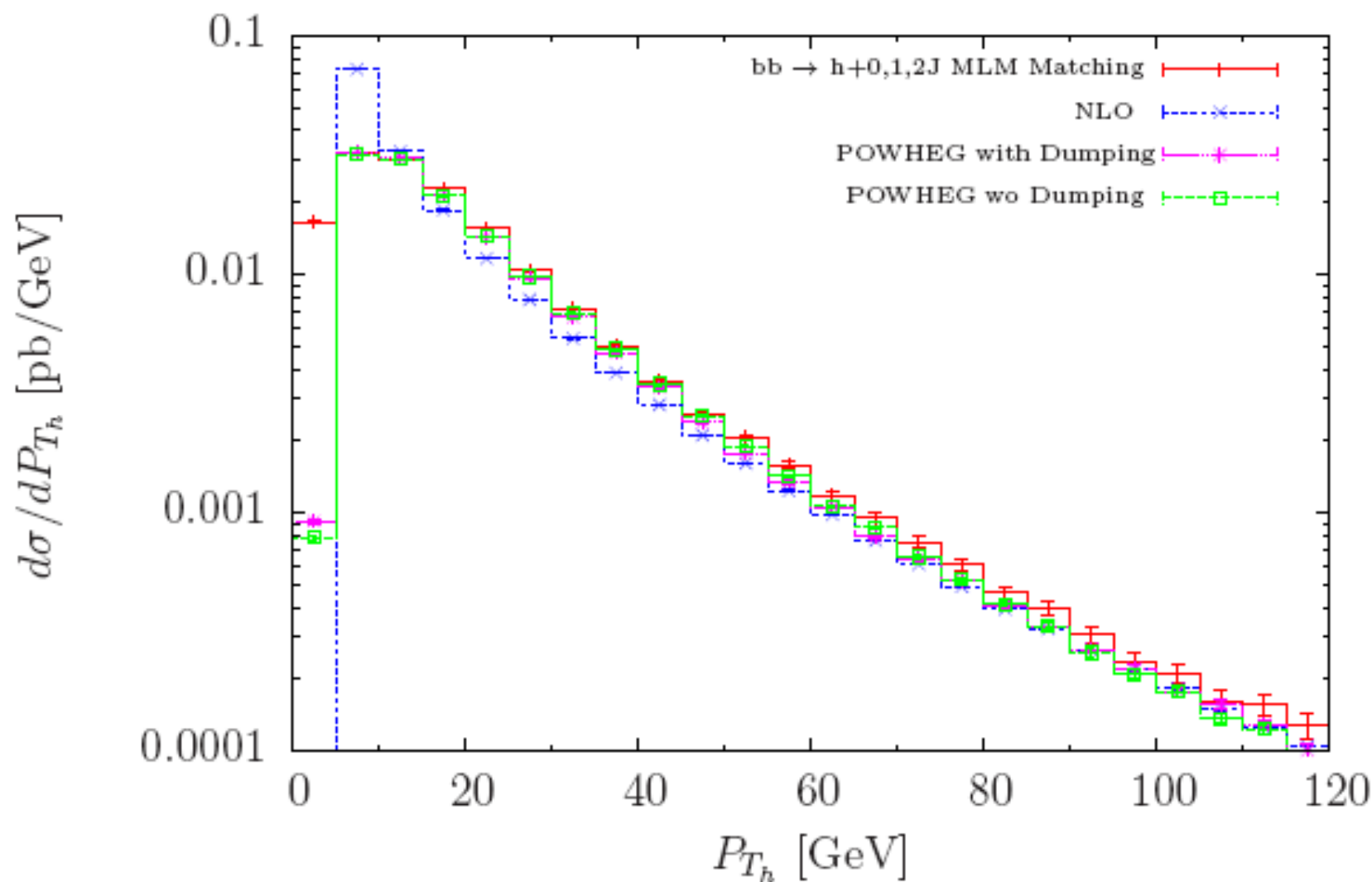
note running mb used!!

$(4.6/3.12285431)^{**2} \rightarrow$

LO 0.659077787652

NLO 0.566286100637

bbh with mh=120GeV@7TeV-LHC



1. Matrix Element Matching with Parton Shower
2. Multi-leg/loop merging with PS
3. NLO QCD matching with Parton Shower
4. Next QED matching ?

Thank you!

A bit about Parton Shower



Absence of interference

An angular ordered algorithm include a certain class of interference

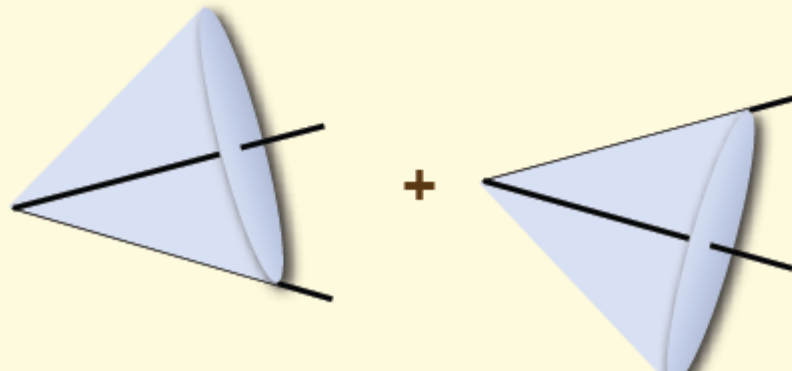
Angular ordering

(slide by M. Mangano)

Angular ordering

$$\left| \begin{array}{c} \text{wavy line} \\ \text{blob} \\ \text{two lines} \end{array} \right|^2 = \left| \begin{array}{c} \text{wavy line} \\ \text{cone } \varphi_1 \\ \text{cone } \varphi \end{array} \right|^2 \Theta(\varphi - \varphi_1) + \left| \begin{array}{c} \text{wavy line} \\ \text{cone } \varphi_2 \\ \text{cone } \varphi \end{array} \right|^2 \Theta(\varphi - \varphi_2)$$

Radiation inside the cones is allowed, and described by the eikonal probability, radiation outside the cones is suppressed and averages to 0 when integrated over the full azimuth



A bit about Parton Shower



Initial state radiation

Infinitesimal change in $f_b(z, t)$:

$$df_b(z, t) = \frac{dt}{t} \sum_{ac} \int_0^1 \frac{dw}{w} \frac{\alpha_S}{2\pi} [P_{a \rightarrow bc}(w)]_+ f_a\left(\frac{z}{w}, t\right).$$

Differential emission probability in backwards evolution = infinitesimal change in $f_b(z, t)$ normalized to $f_b(z, t)$:

$$d\hat{p}(t) = \frac{df_b(z, t)}{f_b(z, t)} = \sum_{ac} \frac{dt}{t} \int \frac{dw}{w} \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(w) \frac{f_a(z/w, t)}{f_b(z, t)},$$

as opposed to the final state radiation probability (averaged over azimuth)

$$dp(t) = \sum_{bc} \frac{dt}{t} \int dz \frac{\alpha_S}{2\pi} P_{a \rightarrow bc}(z).$$

Thus, the Sudakov form factor for initial state radiation is

$$\hat{\Delta}(Q^2, t) = \exp \left[- \int_t^{Q^2} d\hat{p}(t') \right].$$

A bit about Parton Shower



Main Monte Carlos available on the market: SHERPA

- ▶ A new and completely different kind of shower not based on the collinear $1 \rightarrow 2$ branching, but on more complex $2 \rightarrow 3$ elementary process: emission of the daughter off a color dipole.
- ▶ The real emission matrix element squared is decomposed into a sum of terms $D_{ij,k}$ (dipoles) that capture the soft and collinear singularities in the limits i collinear to j , i soft (k is the spectator), and a factorization formula is deduced in the leading color approximation:

$$D_{ij,k} \rightarrow B \frac{\alpha_s}{p_i \cdot p_j} K_{ij,k}.$$

- ▶ The shower is developed from a Sudakov form factor

$$\Delta = \exp \left(- \int \frac{dt}{t} \int dz \alpha_s K_{ij,k} \right).$$

- ▶ It treats correctly the soft gluon emission off a color dipole, so angular ordering is built in.
- ▶ Hadronization: cluster model.

aMC@NLO: detailed structure of the MC counterterm

$$MC = \sum_{pq, c, l \in c} \frac{\delta_{p \in l}}{N_p} \frac{\alpha_s}{(2\pi)^2} \left| \frac{\partial(t_p^{(l)}, z_p^{(l)}, \phi)}{\partial \Phi_{(+1)}} \right| \Theta(DZ) d\Phi_B \frac{P_{p \rightarrow qr}(z_p^{(l)}) |\overline{\mathcal{M}}_c|_B^2 + Q_{p \rightarrow qr}(z_p^{(l)}) |\overline{\mathcal{M}}_c|^2}{t_p^{(l)}}$$

- ▶ c, l = color flow / color line: shower variables and scales may depend on it.
- ▶ $|\overline{\mathcal{M}}_c|_B^2 \equiv B |\mathcal{M}_c|_B^2 / \sum_{c'} |\mathcal{M}_{c'}|_B^2$ = 'barred' Born amplitude squared, to recover the full Born summing only on leading color.
- ▶ $Q_{p \rightarrow qr}(z_p^{(l)})$ = azimuthal kernel.
- ▶ $|\overline{\mathcal{M}}_c|^2$ = 'barred' azimuthal amplitude squared.
- ▶ $\Theta(DZ)$ = dead zone (built-in for HERWIG, imposed to PYTHIA).

Steps performed in a fully automatic and process-independent way:

- ▶ Assignment of color flow and color partner.
- ▶ Assignment of the splitting type (initial state radiation from leg 1 or 2, final state radiation from massive or massless leg).
- ▶ Shower variables definitions and computation of the jacobian.
- ▶ Computation of barred amplitudes and Altarelli-Parisi kernels.