
鹰形陶鼎（复制品）
新石器时代晚期
原件于1958年陕西省华县太平庄M701出土
陶鼎为北京大学历史系考古专业1958年于陕西华县考古实习时所获。出土于一座成年女性墓葬，与其共出的随葬品还有十多件骨匕、数件石圭，可能与当时的祭祀活动有关。原件现藏于中国国家博物馆。

第五章 相干态和压缩态

一、相干态

二、压缩态

三、相干态和压缩态的电磁场表示

四、薛定谔猫态

五、思考题（作业）

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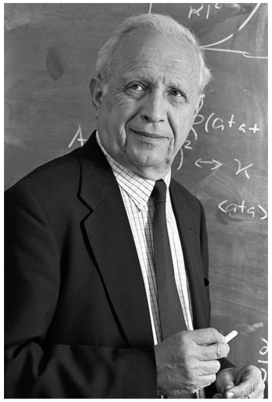
The Nobel Prize in Physics 2005

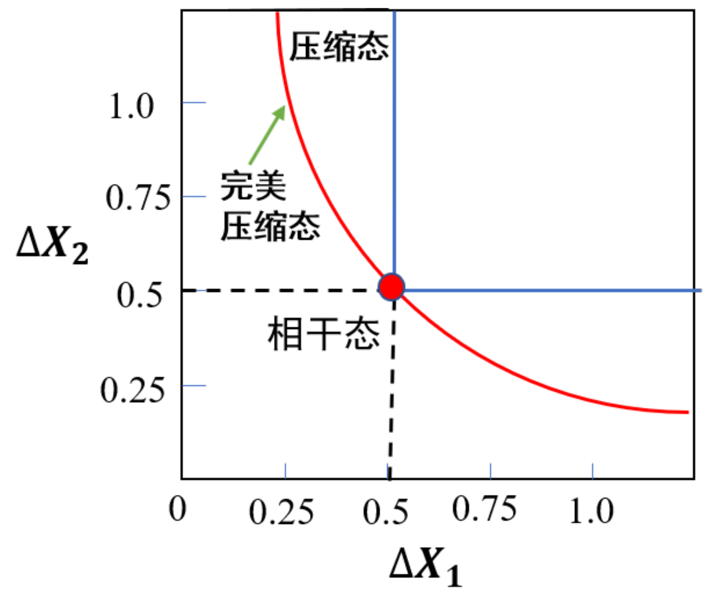


Roy J. Glauber
Born: 1925, New York, NY, USA
Died: 2018, Newton, MA, USA
Affiliation: Harvard University
Prize motivation: *"for his contribution to the quantum theory of optical coherence."*

"According to quantum physics, which was developed at the beginning of the 20th century, *light and other electromagnetic radiation appear in the form of quanta, packets with fixed energies, which can be described as both waves and as particles, photons*. However, no real in-depth theory of light based on quantum theory existed before Roy Glauber **established the foundation for quantum optics** in 1963. *This required the development of the laser. Its concentrated and coherent light gave rise to more quantum physical phenomena than regular light.*"

https://www.nobelprize.org/prizes/physics/2005/glauber/facts/

定义:



- **相干态与理想压缩态都是符合最小测不准关系的量子态**
- **令**$X_1 = (a + a^\dagger)/2$，$X_2 = (a - a^\dagger)/(2i)$，$[X_1, X_2] = i/2$。

  测不准关系允许的范围为：

$$\Delta X_1 \Delta X_1 \geq \frac{1}{4}$$

相干态是$\Delta X_1 = \Delta X_2 = 1/2$*的量子态*

而压缩态则是ΔX_1*或*$\Delta X_2 > 1/2$*的量子态*

一、相干态

1. 相干态的*几个侧面*

a) 经典电流辐射

b) 平移的真空态——$|\alpha\rangle = \mathcal{D}(\alpha)|0\rangle$

c) a的本征态——$a|\alpha\rangle = \alpha|\alpha\rangle$

其中b)和c) 的等价性可以推导

a) 经典电流辐射（相干态的一个例子）

- **如果存在随时间变化的电流$\vec{J}(\vec{r},t)$，那么它辐射出的电磁场就是相干态**
- **在Maxwell方程组中，电流出现在$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} + \vec{J}$，以及$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$中**

- 下面考虑电流与磁矢势的相互作用。在辐射规范下

$$\nabla\cdot\vec{A}=0,\varphi=0$$

于是磁矢势与电场、磁场的关系为

$$\vec{B}=\nabla\times\vec{A},\qquad \vec{E}=-\frac{\partial\vec{A}}{\partial t}$$

从量子化电场$\hat{\vec{E}}=\sum_k \overrightarrow{e_k}\epsilon_k a_k e^{-i\nu_k t+i\vec{k}\cdot\vec{r}}+H.c.$可以得到量子化的磁矢势

$$\vec{A}(\vec{r},t)=-i\sum_k\frac{1}{\nu_k}\overrightarrow{e_k}\epsilon_k a_k e^{-i\nu_k t+i\vec{k}\cdot\vec{r}}+H.c.$$

电流与磁矢势的相互作用(由于经典电流$\vec{J}(\vec{r},t)$的存在而附加的能量)可由如下哈密顿量给出：

$$\mathcal{V}(t)=\int\vec{J}(\vec{r},t)\cdot\vec{A}(\vec{r},t)d^3r$$

- 于是，量子光场态$|\psi(t)\rangle$就可以用相互作用表象下的**薛定谔方程**来描述

$$\frac{d}{dt}|\psi(t)\rangle = -\frac{i}{\hbar}\mathcal{V}(t)|\psi(t)\rangle \qquad \mathcal{V}(t) = \int \vec{J}(\vec{r},t)\cdot\vec{A}(\vec{r},t)d^3r$$

形式上有

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\int_0^t dt'\,\mathcal{V}(t')\right)|\psi(0)\rangle$$

将$\mathcal{V}(t)$具体形式代入可得 $\left(\vec{A}(\vec{r},t) = -i\sum_k \frac{1}{\nu_k}\overrightarrow{e_k}\epsilon_k a_k e^{-i\nu_k t+i\vec{k}\cdot\vec{r}} + H.c.\right)$

$$\exp\left(-\frac{i}{\hbar}\int_0^t dt'\,\mathcal{V}(t')\right) = \prod_k \exp(\alpha_k a_k^\dagger - \alpha_k^* a_k)$$

其中α_k是c数

$$\alpha_k^* = \frac{1}{\hbar\nu_k}\epsilon_k\int_0^t dt'\int d\vec{r}\,\overrightarrow{e_k}\cdot\vec{J}(\vec{r},t)e^{-i\nu_k t+i\vec{k}\cdot\vec{r}}$$

- 由上，$|\psi(t)\rangle$可以写为

$$|\psi(t)\rangle = \prod_k \exp(\alpha_k a_k^\dagger - \alpha_k^* a_k)\,|\psi(0)\rangle$$

这里取$|\psi(0)\rangle$为光场真空态$|0\rangle$，在单模光场中，就可以得到相干态：

$$|\alpha\rangle = e^{\alpha a^\dagger - \alpha^* a}|0\rangle$$

其中$e^{\alpha a^\dagger - \alpha^* a}$即为平移算符$\mathcal{D}(\alpha)$

相干态就是平移的真空态

平移算符 $\mathcal{D}(\alpha)=e^{\alpha a^{\dagger}-\alpha^{*}a}$

- **平移算符有如下性质（作业中推一下）**

$$\mathcal{D}^{-1}(\alpha)a\mathcal{D}(\alpha)=a+\alpha$$

$$\mathcal{D}^{-1}(\alpha)a^{\dagger}\mathcal{D}(\alpha)=a^{\dagger}+\alpha^{*}$$

- **下面证明**$|\alpha\rangle=D(\alpha)|0\rangle$**与**$a|\alpha\rangle=\alpha|\alpha\rangle$**是等价的:**

左乘$\mathcal{D}^{-1}(\alpha)a$

$$\mathcal{D}^{-1}(\alpha)a|\alpha\rangle=\mathcal{D}^{-1}(\alpha)a\mathcal{D}(\alpha)|0\rangle=(a+\alpha)|0\rangle=\alpha|0\rangle$$

再左乘$\mathcal{D}(\alpha)$

$$a|\alpha\rangle=\alpha\mathcal{D}(\alpha)|0\rangle=\alpha|\alpha\rangle$$

所以，$|\alpha\rangle$***是***a***的本征态，本征值为***α

2. 相干态的表示 （作业中推一下）

- 相干态用Fock态表象进行展开 $|\alpha\rangle = \sum_n c_n |n\rangle$

  c_n可以由如下得到：

$$|\alpha\rangle = \sum_n |n\rangle\langle n|\alpha\rangle = \sum_n c_n|n\rangle$$

  即$c_n = \langle n|\alpha\rangle$

- 要得到c_n具体的表达式，从$a|\alpha\rangle = \alpha|\alpha\rangle$开始。先左乘$\langle n|$，有

$$\langle n|a|\alpha\rangle = \alpha\langle n|\alpha\rangle$$

$$\sqrt{n+1}\langle n+1|\alpha\rangle = \alpha\langle n|\alpha\rangle$$

  则 $$c_{n+1} = \frac{\alpha}{\sqrt{n+1}} c_n$$

  相干态的展开系数间有递推关系： $c_n = \frac{\alpha^n}{\sqrt{n!}} c_0$

- 将 c_0 用如下方法给出：从 $|\alpha\rangle = \mathcal{D}(\alpha)|0\rangle$ 开始

平移算符$\mathcal{D}(\alpha) = e^{\alpha a^{\dagger} - \alpha^* a}$， $e^{\hat{A}+\hat{B}} = e^{\hat{A}} e^{\hat{B}} e^{-[\hat{A},\hat{B}]/2}$

$$c_0 = \langle 0|\alpha\rangle = \langle 0|\mathcal{D}(\alpha)|0\rangle = \left\langle 0\left| e^{-\frac{|\alpha|^2}{2}} e^{\alpha a^{\dagger}} e^{-\alpha^* a} \right| 0\right\rangle = e^{-\frac{|\alpha|^2}{2}}$$

其中对平移算符$\mathcal{D}(\alpha)$的展开用到了*B-H定律*

所以

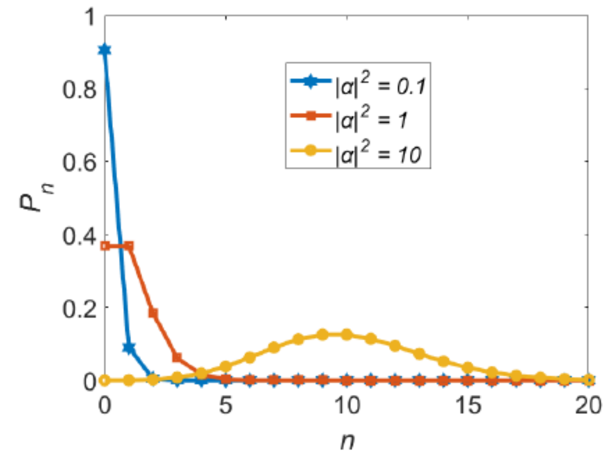
$$c_n = e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}}$$

作为相干态在Fock态上的展开系数， $|c_n|^2$ *也是相干态中有n个光子存在的几率幅*

最后，相干态可表示成

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

3. 相干态的性质



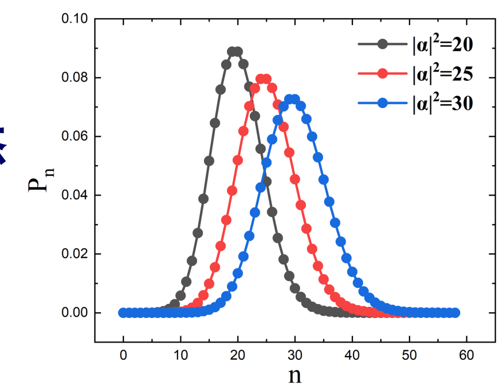
i. 相干态$|$下$\alpha\rangle$的*平均光子数*以及*光子数分布*

平均光子数$\langle n\rangle$由下式给出：

$$\langle n\rangle = \langle\alpha|a^\dagger a|\alpha\rangle = |\alpha|^2$$

那么，在$|\alpha\rangle$态上发现$|n\rangle$态的概率为$P_n = |c_n|^2$

$$P_n = |c_n|^2 = \frac{|\alpha|^{2n}e^{-\langle n\rangle}}{n!} = \frac{\langle n\rangle^n e^{-\langle n\rangle}}{n!}$$



这里α可连续取值，相干态是**连续变量**的量子态

当$|\alpha|^2 = 0.1$时，P_n最大处是$n = 0$

当$|\alpha|^2 = 1$时，P_n最大处是$n = 0$和1

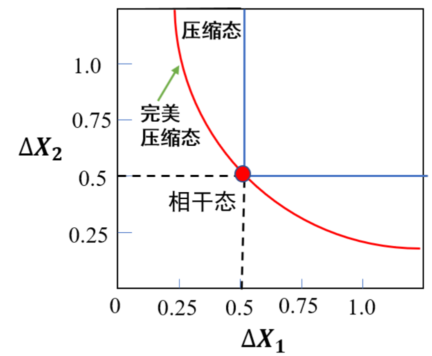
当$|\alpha|^2 = 10$时，P_n最大在$n = 10$处

激光也是相干态光场。当$|\alpha|^2$即$\langle n\rangle$很大时，就是激光的光场形式。

ii. 验证相干态$|\alpha\rangle$下的测不准关系（作业中推一下）

- **定义广义量**$X_1=\frac{a+a^\dagger}{2}, X_2=\frac{a-a^\dagger}{2i}$。**它们的涨落由**$\Delta X_i^2=\langle X_i^2\rangle-\langle X_i\rangle^2$**得到。**

  下面就要验证在相干态中$\Delta X_1=\Delta X_2=1/2$



- **在相干态$|\alpha\rangle$中**

$$\Delta X_1^2=\langle X_1^2\rangle-\langle X_1\rangle^2$$

$$=\langle\alpha|X_1^2|\alpha\rangle-\langle\alpha|X_1|\alpha\rangle^2$$

$$=\frac{1}{4}\left\langle\alpha\middle|a^2+aa^\dagger+a^\dagger a+{a^\dagger}^2\middle|\alpha\right\rangle-\frac{1}{4}\langle\alpha|a+a^\dagger|\alpha\rangle^2$$

$$=\frac{1}{4}\left(\alpha^2+{\alpha^*}^2+2|\alpha|^2+1\right)-\frac{1}{4}(\alpha+\alpha^*)^2=\frac{1}{4}$$

其中用到$a|\alpha\rangle=\alpha|\alpha\rangle$，$\langle\alpha|a^\dagger=\langle\alpha|\alpha^*$**以及**$[a,a^\dagger]=1$

同理有$\Delta X_2^2=\frac{1}{4}$**(亲自算)**

- **在Fock态$|n\rangle$中，两个广义量的涨落为$\Delta X_1 = \Delta X_2 = \frac{1}{2}\sqrt{2n+1}$**

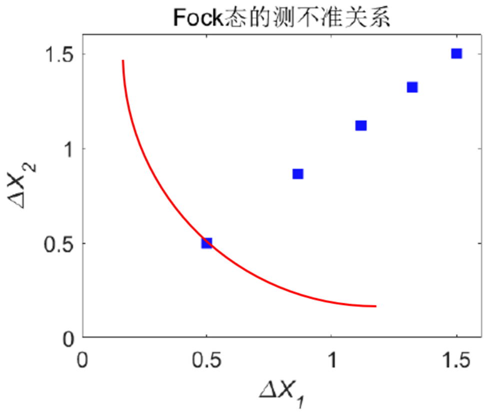
$$\Delta X_1^2 = \langle X_1^2\rangle - \langle X_1\rangle^2$$

$$= \langle n|X_1^2|n\rangle - \langle n|X_1|n\rangle^2$$

$$= \frac{1}{4}\left\langle n\middle|a^2 + aa^\dagger + a^\dagger a + a^{\dagger 2}\middle|n\right\rangle - \frac{1}{4}\langle n|a + a^\dagger|n\rangle^2$$

$$= \frac{1}{4}(2n+1)$$

同理，$\Delta X_2^2 = \frac{1}{4}(2n+1)$



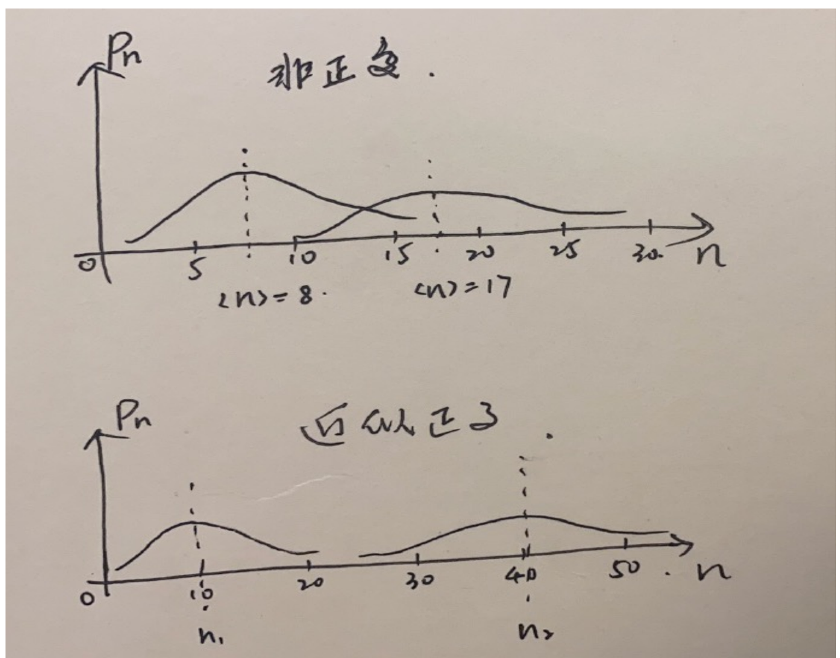
- $n = 0$**时，**$|0\rangle$**也是相干态，符合最小测不准原理**
- ***在相干态中，所有<u>quadrature量</u>都符合最小测不准关系***

iii. 相干态的正交完备性

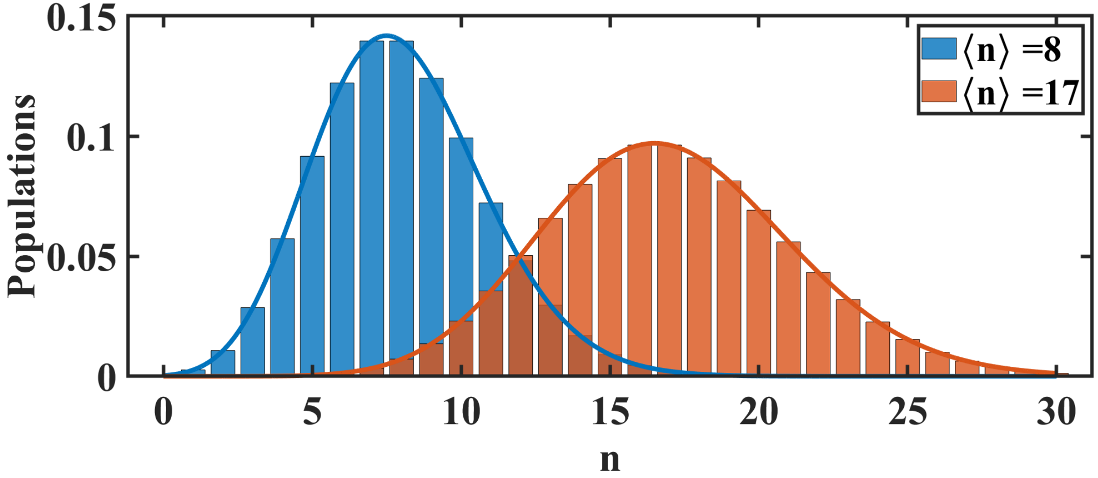
- 相干态互相之间是非正交的： $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

$$\langle\alpha|\alpha'\rangle = \exp\left(-\frac{1}{2}|\alpha|^2 + \alpha^*\alpha' - \frac{1}{2}|\alpha'|^2\right) \neq 0$$

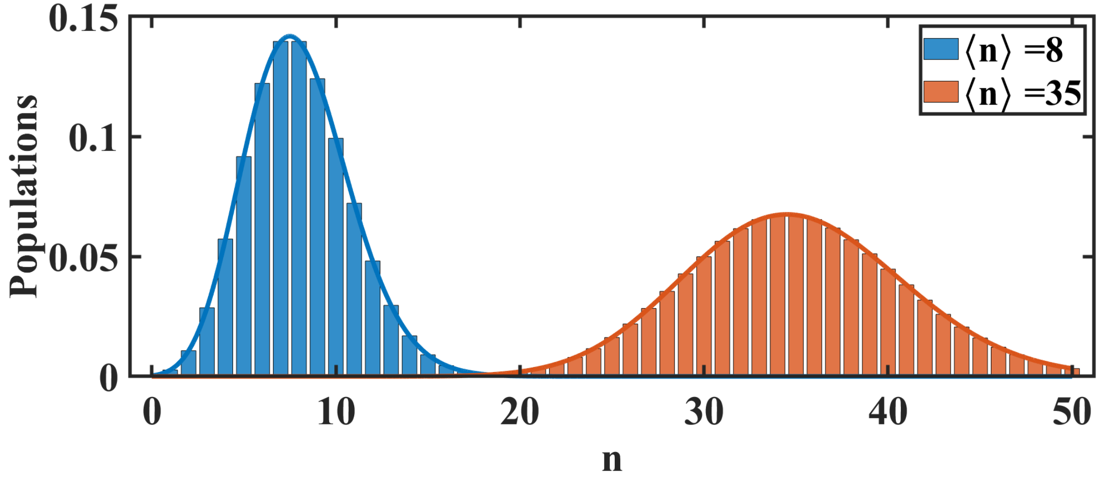
可知，当$|\alpha - \alpha'| \gg 1$时，$|\alpha\rangle$和$|\alpha'\rangle$近似正交。



两个相干态非正交



两个相干态近似正交



- **相干态是超完备的**

一个相干态$|\alpha\rangle$可以用其它相干态$|\alpha'\rangle$展开

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

利用$\int |\alpha\rangle\langle\alpha|d^2\alpha = \pi$（证明见后）有

$$|\alpha\rangle = \frac{1}{\pi}\int |\alpha'\rangle\langle\alpha'|\alpha\rangle d^2\alpha'$$

$$= \int |\alpha'\rangle \underbrace{\frac{1}{\pi}\exp\left(-\frac{1}{2}|\alpha|^2 + \alpha^*\alpha' - \frac{1}{2}|\alpha'|^2\right)}_{\text{展开系数}} d^2\alpha'$$

- **下面证明$\int |\alpha\rangle\langle\alpha| d^2\alpha = \pi$（思路）**

$$\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = \int e^{-\frac{|\alpha|^2}{2}} \cdot \sum_{n'} \frac{\alpha^{n'}}{\sqrt{n'!}} |n'\rangle\langle n| e^{-\frac{|\alpha|^2}{2}} \cdot \sum_{n} \frac{(\alpha^*)^n}{\sqrt{n!}} \mathrm{d}^2\alpha$$

$$= \int e^{-|\alpha|^2} \cdot \sum_{n'}\sum_{n} \frac{(\alpha^*)^n \cdot \alpha^{n'}}{\sqrt{n'! \cdot n!}} \mathrm{d}^2\alpha\, |n'\rangle\langle n|$$

$$\int (\alpha^*)^n \cdot \alpha^{n'} e^{-|\alpha|^2} \mathrm{d}^2\alpha \xlongequal[\mathrm{d}^2\alpha = |\alpha|\cdot \mathrm{d}|\alpha| \cdot \mathrm{d}\theta]{\alpha = |\alpha| e^{i\theta}} \int_0^\infty |\alpha|^{n+n'+1} \cdot e^{-|\alpha|^2} \mathrm{d}|\alpha| \int_0^{2\pi} e^{i(n'-n)\theta} \mathrm{d}\theta = \pi n!\, \delta_{n,n'}$$

$$\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = \pi \sum_n |n\rangle\langle n| = \pi$$

- 下面证明$\int |\alpha\rangle\langle\alpha| d^2\alpha = \pi$（细节）

$$\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = \int e^{-\frac{|\alpha|^2}{2}} \cdot \sum_{n'} \frac{\alpha^{n'}}{\sqrt{n'!}} |n'\rangle\langle n| e^{-\frac{|\alpha|^2}{2}} \cdot \sum_{n} \frac{(\alpha^*)^n}{\sqrt{n!}} \mathrm{d}^2\alpha$$

$$= \int e^{-|\alpha|^2} \cdot \sum_{n'} \sum_{n} \frac{(\alpha^*)^n \cdot \alpha^{n'}}{\sqrt{n'! \cdot n!}} \mathrm{d}^2\alpha \, |n'\rangle\langle n|$$

$$= \sum_{n'} \sum_{n} \frac{|n'\rangle\langle n|}{\sqrt{n'! \cdot n!}} \int (\alpha^*)^n \cdot \alpha^{n'} e^{-|\alpha|^2} \mathrm{d}^2\alpha$$

$$\int (\alpha^*)^n \cdot \alpha^{n'} e^{-|\alpha|^2} \mathrm{d}^2\alpha \xrightarrow[\mathrm{d}^2\alpha = |\alpha| \cdot \mathrm{d}|\alpha| \cdot \mathrm{d}\theta]{\alpha = |\alpha| e^{i\theta}} \int_0^\infty |\alpha|^{n+n'+1} \cdot e^{-|\alpha|^2} \mathrm{d}|\alpha| \int_0^{2\pi} e^{i(n'-n)\theta} \mathrm{d}\theta$$

$$= 2\pi \delta_{n,n'} \int_0^\infty |\alpha|^{n+n'+1} \cdot e^{-|\alpha|^2} \mathrm{d}|\alpha|$$

$$= 2\pi \delta_{n,n'} \int_0^\infty |\alpha|^{2n+1} \cdot e^{-|\alpha|^2} \mathrm{d}|\alpha|$$

← 伽马函数的积分定义

$$= \pi n! \, \delta_{n.n'}$$

$$\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = \pi \sum_{n} |n\rangle\langle n| = \pi$$

量子态或算符的相干表象展开

基于完备性，任意量子态$|f\rangle$可以在相干表象下展开

$$|f\rangle = \frac{1}{\pi}\int d^2\alpha\ |\alpha\rangle\langle\alpha|f\rangle$$

注意：$\langle\alpha|f\rangle$涉及到复数的运算，由于超完备性，其形式可能不唯一

基于完备性，任意算符 $\hat{T}$ 可以在相干表象下展开

$$\hat{T} = I\cdot\hat{T}\cdot I = \frac{1}{\pi^2}\int d^2\alpha\, d^2\beta\ |\alpha\rangle\langle\alpha|\hat{T}|\beta\rangle\langle\beta|$$

$$= \frac{1}{\pi^2}\int d^2\alpha\, d^2\beta\ \langle\alpha|\hat{T}|\beta\rangle|\alpha\rangle\,\langle\beta|$$

同样的：$<\alpha|\hat{T}|\beta>$ 涉及到复数的运算，由于超完备性，其形式可能不唯一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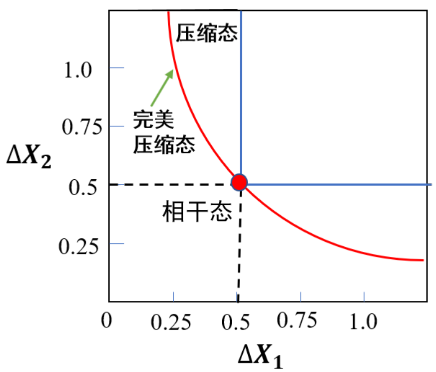
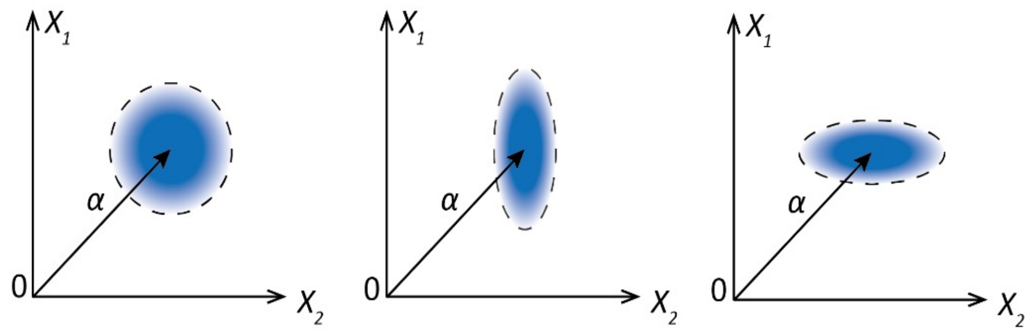
二、压缩态

1. 压缩态的定义：符合最小测不准关系的量子态

- 压缩态中涨落满足ΔX_1或$\Delta X_2 > 1/2$，但是$\Delta X_1 \Delta X_2$仍等于$1/4$。





压缩态也可以用$\Delta X_1 - \Delta X_2$的图表来表示：

最左边是相干态的图示，X_1与X_2的涨落是相同的

右边两张图则是压缩态的图示，X_1或X_2的涨落被从一个方向“压缩”

在$\Delta X_1 \Delta X_2 = 1/4$的情况下，三个图示中的面积是相等的

2. 相干态与压缩态的比较

	相干态	压缩态
哈密顿量	$\mathcal{H}_I = \int \vec{J}\cdot\vec{A}d^3r \propto (c_k^* a_k + c_k a_k^\dagger)$ 单光子过程	$\mathcal{H}_I = i\hbar(g a_k^{\dagger 2} - g^* a_k^2)$ 双光子过程
代入薛定谔方程$\frac{d}{dt}\lvert\psi(t)\rangle = -\frac{i}{\hbar}\mathcal{H}_I\lvert\psi(t)\rangle$		
算符	平移算符 $\mathcal{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$	压缩算符 $\mathcal{S}(\xi) = e^{(-\xi a^{\dagger 2} + \xi^* a^2)/2}$
算符性质	$\mathcal{D}^\dagger(\alpha) = \mathcal{D}(-\alpha) = \mathcal{D}^{-1}(\alpha)$ $\mathcal{D}^\dagger(\alpha)a\mathcal{D}(\alpha) = a + \alpha$ $\mathcal{D}^\dagger(\alpha)a^\dagger\mathcal{D}(\alpha) = a^\dagger + \alpha^*$	$\mathcal{S}^\dagger(\xi) = \mathcal{S}(-\xi) = \mathcal{S}^{-1}(\xi)$ $(\xi = re^{i\theta})$ $\mathcal{S}^\dagger(\xi)a\mathcal{S}(\xi) = a\cosh r - a^\dagger e^{i\theta}\sinh r$ $\mathcal{S}^\dagger(\xi)a^\dagger\mathcal{S}(\xi) = a^\dagger\cosh r - ae^{-i\theta}\sinh r$
算符作用规则	$\mathcal{D}^\dagger(\alpha)(X_1 + iX_2)\mathcal{D}(\alpha) = X_1 + \alpha + i(X_2 + \alpha)$ 共轭量都平移，噪声不变	$\mathcal{S}^\dagger(\xi)(Y_1 + iY_2)\mathcal{S}(\xi) = Y_1e^{-r} + iY_2e^{r}$ 共轭量一个压缩，一个拉伸。噪声随之变化
量子态	$\lvert\alpha\rangle = \mathcal{D}(\alpha)\lvert 0\rangle$	$\lvert\alpha,\xi\rangle = \mathcal{D}(\alpha)\mathcal{S}(\xi)\lvert 0\rangle = \mathcal{S}(\xi)\mathcal{D}(\alpha)\lvert 0\rangle$

证明：$S^\dagger(\xi)aS(\xi) = a\cosh r - a^\dagger e^{i\theta}\sinh r$ （作业中算一下）

$$S(\xi) = e^{(-\xi a^{\dagger 2} + \xi^* a^2)/2} \qquad \xi = re^{i\theta}$$

主要利用公式$e^{x\hat{A}}\hat{B}e^{-x\hat{A}} = \hat{B} + x[\hat{A},\hat{B}] + \frac{x^2}{2!}\left[\hat{A},[\hat{A},\hat{B}]\right] + \frac{x^3}{3!}\left[\hat{A},\left[\hat{A},[\hat{A},\hat{B}]\right]\right] + \cdots$

在这里，$x = 1$， $\hat{A} = \frac{\xi a^{\dagger 2} - \xi^* a^2}{2}$， $\hat{B} = a$

$$[\hat{A},\hat{B}] = \left[\frac{\xi a^{\dagger 2} - \xi^* a^2}{2}, a\right] = \frac{\xi}{2}\left[a^{\dagger 2}, a\right] = \frac{\xi}{2}\left(a^{\dagger 2}a - aa^{\dagger 2}\right)$$

$$= \frac{\xi}{2}\left[a^\dagger(aa^\dagger - 1) - (a^\dagger a + 1)a^\dagger\right] = -\xi a^\dagger$$

$$\left[\hat{A},[\hat{A},\hat{B}]\right] = \left[\frac{\xi a^{\dagger 2} - \xi^* a^2}{2}, -\xi a^\dagger\right] = \frac{|\xi|^2}{2}\left[a^2, a^\dagger\right] = |\xi|^2 a$$

$$\left[\hat{A},\left[\hat{A},[\hat{A},\hat{B}]\right]\right] = \left[\frac{\xi a^{\dagger 2} - \xi^* a^2}{2}, |\xi|^2 a\right] = \frac{\xi}{2}|\xi|^2\left[a^{\dagger 2}, a\right] = -\xi|\xi|^2 a^\dagger$$

遵循这一递推公式

$$S^\dagger(\xi)aS(\xi) = a - \xi a^\dagger + \frac{1}{2!}|\xi|^2 a - \frac{1}{3!}\xi|\xi|^2 a^\dagger + \cdots$$

$$= a\left(1 + \frac{1}{2!}|\xi|^2 + \frac{1}{4!}|\xi|^4 + \cdots\right) - a^\dagger\left(\xi + \frac{1}{3!}\xi|\xi|^2 + \frac{1}{5!}\xi|\xi|^4 + \cdots\right)$$

$$= a\left(1 + \frac{1}{2!}r^2 + \frac{1}{4!}r^4 + \cdots\right) - a^\dagger e^{i\theta}\left(r + \frac{1}{3!}r^3 + \frac{1}{5!}r^4 + \cdots\right)$$

$$= a\cosh r - a^\dagger e^{i\theta}\sinh r$$

变换关系得证

$$\cosh r = \frac{e^r + e^{-r}}{2} = 1 + \frac{1}{2!}r^2 + \frac{1}{4!}r^4 + \cdots$$

$$\sinh r = \frac{e^r - e^{-r}}{2} = r + \frac{1}{3!}r^3 + \frac{1}{5!}r^4 + \cdots$$

3. 压缩态对应的算符本征态 [Ref: H. P. Yuen Phys. Rev. A 13, 2226(1976)]

（作业中推一下）

- 定义$b = \mu a + \nu a^\dagger$，且$|\mu|^2 - |\nu|^2 = 1$。压缩态就是$b$的本征态。并且可以得到压缩态$|\alpha, \xi\rangle = \mathcal{S}(\xi)\mathcal{D}(\alpha)|0\rangle$。

- 由$[a, a^\dagger] = 1$，可得

$$[b, b^\dagger] = [\mu a + \nu a^\dagger, \mu^* a^\dagger + \nu^* a] = |\mu|^2 - |\nu|^2 = 1$$

- 由$b = U^\dagger a U$出发，其中幺正变换$U = \mathcal{S}(\xi) = e^{(-\xi a^{\dagger 2} + \xi^* a^2)/2}$，这里面$\xi = re^{i\phi}$，有

$$b = U^\dagger a U = a \cosh r - a^\dagger e^{i\phi} \sinh r \equiv \mu a + \nu a^\dagger$$

其中$\mu \equiv \cosh r$，$\nu \equiv -e^{i\phi} \sinh r$，满足$|\mu|^2 - |\nu|^2 = 1$

- 下面从相干态$|\beta\rangle$出发，推导出压缩算符b的本征态$|\beta\rangle_g$

相干态满足

$$a|\beta\rangle=\beta|\beta\rangle$$

左乘$U^\dagger$，并且插入单位算符$UU^\dagger$，得

$$U^\dagger aUU^\dagger|\beta\rangle=bU^\dagger|\beta\rangle=\beta U^\dagger|\beta\rangle$$

所以，$U^\dagger|\beta\rangle$是算符b的本征态

即$|\beta\rangle_g=U^\dagger|\beta\rangle=U(-\xi)\mathcal{D}(\beta)|0\rangle=|\beta,-\xi\rangle$

压缩态可以理解为先将真空态平移，再压缩得到的

也可以先压缩真空态，再平移

两种不同定义压缩态之间的变换关系

$$\mathcal{S}(\xi)=e^{(-\xi a^{\dagger 2}+\xi^* a^2)/2}$$

$$\mathcal{S}^\dagger(\xi)=\mathcal{S}(-\xi)=\mathcal{S}^{-1}(\xi)$$
$$(\xi=re^{i\theta})$$
$$\mathcal{S}^\dagger(\xi)a\mathcal{S}(\xi)=a\cosh r-a^\dagger e^{i\theta}\sinh r$$
$$\mathcal{S}^\dagger(\xi)a^\dagger\mathcal{S}(\xi)=a^\dagger\cosh r-ae^{-i\theta}\sinh r$$

定义1: $|\alpha,\xi\rangle=\mathcal{S}(\xi)\mathcal{D}(\alpha)|0\rangle$　　定义2: $|\beta,\xi\rangle=\mathcal{D}(\beta)\mathcal{S}(\xi)|0\rangle$

当二者对应于同一压缩态时： $\mathcal{S}(\xi)\mathcal{D}(\alpha)=\mathcal{D}(\beta)\mathcal{S}(\xi)$

$$\mathcal{D}(\beta)=\mathcal{S}(\xi)\mathcal{D}(\alpha)\mathcal{S}^\dagger(\xi)$$

$$\mathcal{S}(\xi)a^\dagger\mathcal{S}^\dagger(\xi)$$
$$=\mathcal{S}^\dagger(-\xi)a^\dagger\mathcal{S}(-\xi)$$
$$=a^\dagger\cosh r+ae^{-i\theta}\sinh r$$
$$\xi=re^{i\theta}\,,\qquad -\xi=-re^{i\theta}$$

$$\mathcal{S}(\xi)\mathcal{D}(\alpha)\mathcal{S}^\dagger(\xi)=\mathcal{S}(\xi)\Big[\sum\frac{1}{n!}(\alpha a^\dagger-\alpha^* a)^n\Big]\mathcal{S}^\dagger(\xi)$$

$$=\sum\frac{1}{n!}\left[\mathcal{S}(\xi)(\alpha a^\dagger-\alpha^* a)\mathcal{S}^\dagger(\xi)\right]^n$$

$$=\sum\frac{1}{n!}\left[\alpha\, a^\dagger\cosh r+\alpha\, ae^{-i\theta}\sinh r-\alpha^* a\cosh r-\alpha^* a^\dagger e^{i\theta}\sinh r\right]^n$$

$$=\sum\frac{1}{n!}\left[a^\dagger(\alpha\cosh r-\alpha^* e^{i\theta}\sinh r)-a(\alpha^*\cosh r-\alpha e^{-i\theta}\sinh r)\right]^n$$

$$=exp(\beta a^\dagger-\beta^* a)=\mathcal{D}(\beta)$$

$$\mathcal{D}(\alpha)=e^{\alpha a^\dagger-\alpha^* a}$$

$$\therefore\ \mathcal{S}(\xi)\mathcal{D}(\alpha)|0\rangle=\mathcal{D}(\alpha\cosh r-\alpha^* e^{i\theta}\sinh r)\mathcal{S}(\xi)|0\rangle$$

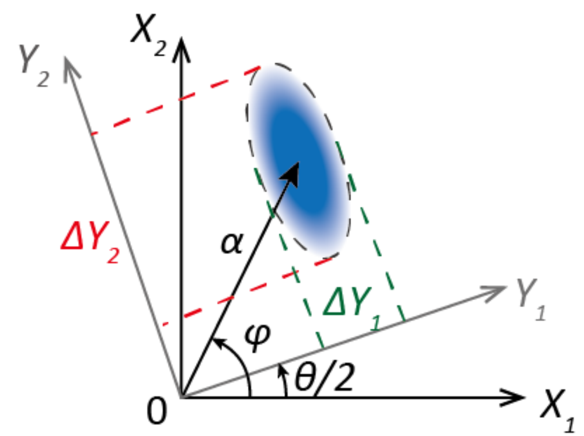
4. 压缩态的性质

- 压缩态是非正交超完备的
- 压缩态测不准关系在一个方向上被压缩

  而另一个方向则被拉伸
- 在相干态中，定义正交广义量为$X_1 = (a + a^\dagger)/2$，$X_2 = (a - a^\dagger)/(2i)$，而在压缩态中，正交广义量被定义为

  $Y_1 = (ae^{-i\theta/2} + a^\dagger e^{i\theta/2})/2$，$Y_2 = (ae^{-i\theta/2} - a^\dagger e^{i\theta/2})/(2i)$

  $[Y_1, Y_2] = i/2$



- 广义量的涨落为
- $\Delta Y_i^2 = \langle Y_i^2 \rangle - \langle Y_i \rangle^2$

- **计算涨落时需要如下的结果（定义1）：** $|\alpha,\xi\rangle = \mathcal{S}(\xi)\mathcal{D}(\alpha)|0\rangle$

$$
\begin{aligned}
\langle a\rangle &= \langle\alpha,\xi|a|\alpha,\xi\rangle \\
&= \langle 0|\mathcal{D}^\dagger(\alpha)\mathcal{S}^\dagger(\xi)a\mathcal{S}(\xi)\mathcal{D}(\alpha)|0\rangle \\
&= \langle\alpha|a\cosh r - a^\dagger e^{i\theta}\sinh r\,|\alpha\rangle \\
&= \alpha\cosh r - \alpha^* e^{i\theta}\sinh r
\end{aligned}
$$

$$\langle a^\dagger\rangle = \alpha^*\cosh r - \alpha e^{-i\theta}\sinh r$$

$$\mathcal{S}^\dagger(\xi) = \mathcal{S}(-\xi) = \mathcal{S}^{-1}(\xi)$$
$$(\xi = re^{i\theta})$$
$$\mathcal{S}^\dagger(\xi)a\mathcal{S}(\xi) = a\cosh r - a^\dagger e^{i\theta}\sinh r$$
$$\mathcal{S}^\dagger(\xi)a^\dagger\mathcal{S}(\xi) = a^\dagger\cosh r - ae^{-i\theta}\sinh r$$

以及：

$$
\begin{aligned}
\langle a^2\rangle &= \langle 0|\mathcal{D}^\dagger(\alpha)\mathcal{S}^\dagger(\xi)a^2\mathcal{S}(\xi)\mathcal{D}(\alpha)|0\rangle \\
&= \langle 0|\mathcal{D}^\dagger(\alpha)\,\mathcal{S}^\dagger(\xi)a\mathcal{S}(\xi)\,\mathcal{S}^\dagger(\xi)a\mathcal{S}(\xi)\,\mathcal{D}(\alpha)|0\rangle \\
&= \langle\alpha|\mathcal{S}^\dagger(\xi)a\mathcal{S}(\xi)\mathcal{S}^\dagger(\xi)a\mathcal{S}(\xi)|\alpha\rangle \\
&= \left\langle\alpha\middle|\left(a\cosh r - a^\dagger e^{i\theta}\sinh r\right)^2\middle|\alpha\right\rangle \\
&= \alpha^2\cosh^2 r + \alpha^{*2}e^{2i\theta}\sinh^2 r - (2|\alpha|^2+1)e^{i\theta}\cosh r\sinh r
\end{aligned}
$$

- **计算涨落时需要如下的结果** $|\alpha,\xi\rangle = \mathcal{S}(\xi)\mathcal{D}(\alpha)|0\rangle$

$$\langle a^{\dagger 2}\rangle = \left\langle\alpha\middle|\left(a^\dagger\cosh r - ae^{-i\theta}\sinh r\right)^2\middle|\alpha\right\rangle$$

$$= \alpha^{*2}\cosh^2 r + \alpha^2 e^{-2i\theta}\sinh^2 r -$$

$$(2|\alpha|^2+1)e^{-i\theta}\cosh r\sinh r$$

$$\mathcal{S}^\dagger(\xi) = \mathcal{S}(-\xi) = \mathcal{S}^{-1}(\xi)$$

$$(\xi = re^{i\theta})$$

$$\mathcal{S}^\dagger(\xi)a\mathcal{S}(\xi) = a\cosh r - a^\dagger e^{i\theta}\sinh r$$

$$\mathcal{S}^\dagger(\xi)a^\dagger\mathcal{S}(\xi) = a^\dagger\cosh r - ae^{-i\theta}\sinh r$$

还有（自己推一下）

$$\langle a^\dagger a\rangle = \langle n\rangle$$

$$= |\alpha|^2(\cosh^2 r + \sinh^2 r) + \sinh^2 r$$

$$-\left(\alpha^{*2}e^{i\theta} + \alpha^2 e^{-i\theta}\right)\sinh r\cosh r$$

压缩度$r = 0$时，结果回归到相干态

- $Y_1 = (ae^{-i\theta/2} + a^\dagger e^{i\theta/2})/2$，$Y_2 = (ae^{-i\theta/2} - a^\dagger e^{i\theta/2})/(2i)$
- **代入涨落表达式可以得到（自己推一下）**

$$\Delta Y_1^2 = \langle Y_1^2 \rangle - \langle Y_1 \rangle^2$$
$$= \frac{1}{4}\langle e^{-i\theta}a^2 + e^{i\theta}a^{\dagger 2} + 2a^\dagger a + 1\rangle - \frac{1}{4}\langle ae^{-i\theta/2} + a^\dagger e^{i\theta/2}\rangle^2$$
$$= \frac{1}{4}e^{-2r}$$

$$|\alpha,\xi\rangle = \mathcal{S}(\xi)\mathcal{D}(\alpha)|0\rangle$$

和

$$\Delta Y_2^2 = \langle Y_2^2 \rangle - \langle Y_2 \rangle^2$$
$$= -\frac{1}{4}\langle e^{-i\theta}a^2 + e^{i\theta}a^{\dagger 2} - 2a^\dagger a - 1\rangle + \frac{1}{4}\langle ae^{-i\theta/2} - a^\dagger e^{i\theta/2}\rangle^2$$
$$= \frac{1}{4}e^{2r}$$

于是压缩态quadrature广义量的测不准关系为

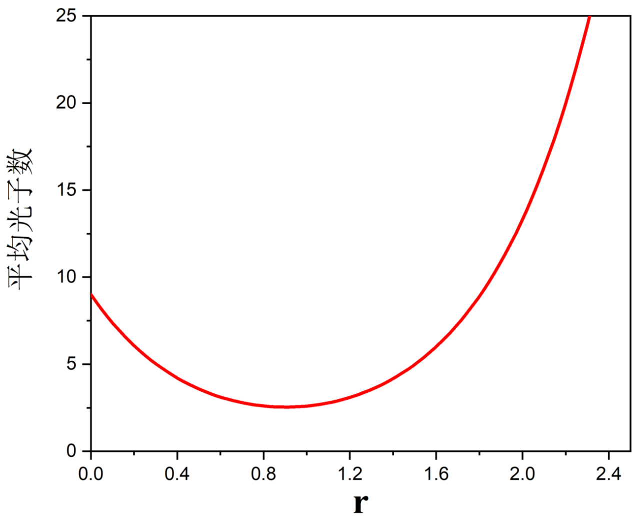
$$\Delta Y_1 \Delta Y_2 = 1/4$$

定义1: $|\alpha,\xi\rangle = \mathcal{S}(\xi)\mathcal{D}(\alpha)|0\rangle$，$\xi = re^{i\theta}$

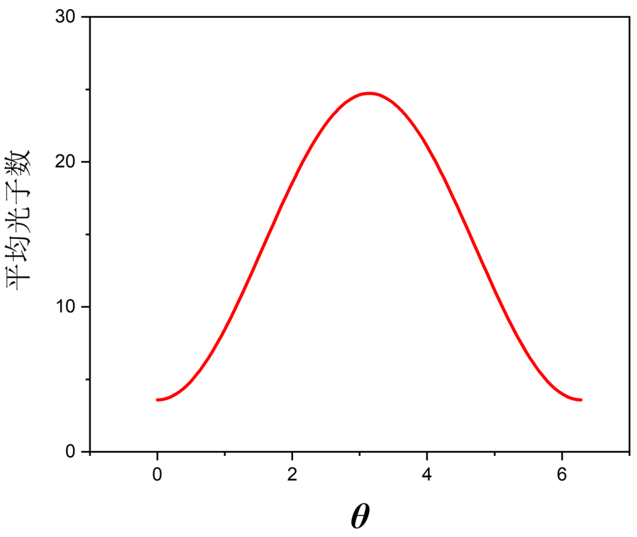
压缩态$|\alpha,\xi\rangle$的平均粒子数

$$\langle a^\dagger a\rangle = \langle n\rangle$$

$$= |\alpha|^2(\cosh^2 r + \sinh^2 r) + \sinh^2 r - \left(\alpha^{*2}e^{i\theta} + \alpha^2 e^{-i\theta}\right)\sinh r \cosh r$$



$\alpha = 3, \theta = 0$



$\alpha = 3, r = 0.5$

可以看出：*平均光子数随压缩度r 和 θ（0～2π） 变化*

定义2: $|\beta,\xi\rangle = \mathcal{D}(\beta)\mathcal{S}(\xi)|0\rangle$，$\xi = re^{i\theta}$

$$\mathcal{S}^{\dagger}(\xi) = \mathcal{S}(-\xi) = \mathcal{S}^{-1}(\xi)$$
$$(\xi = re^{i\theta})$$
$$\mathcal{S}^{\dagger}(\xi)a\mathcal{S}(\xi) = a\cosh r - a^{\dagger}e^{i\theta}\sinh r$$
$$\mathcal{S}^{\dagger}(\xi)a^{\dagger}\mathcal{S}(\xi) = a^{\dagger}\cosh r - ae^{-i\theta}\sinh r$$

压缩态$|\beta,\xi\rangle$的平均粒子数

$$\langle a^{\dagger}a\rangle = \langle n\rangle = \langle 0|\mathcal{S}^{\dagger}(\xi)\mathcal{D}^{\dagger}(\beta)a^{\dagger}a\mathcal{D}(\beta)\mathcal{S}(\xi)|0\rangle$$

$$= \langle 0|\mathcal{S}^{\dagger}(\xi)\mathcal{D}^{\dagger}(\beta)a^{\dagger}\mathcal{D}(\beta)\mathcal{D}^{\dagger}(\beta)\, a\mathcal{D}(\beta)\mathcal{S}(\xi)|0\rangle$$
$$= \langle 0|\mathcal{S}^{\dagger}(\xi)(a^{\dagger}+\beta^{*})(a+\beta)\mathcal{S}(\xi)|0\rangle$$
$$= \langle 0|(a^{\dagger}coshr - ae^{-i\theta}sinhr)(a\,coshr - a^{\dagger}\,e^{i\theta}sinhr)|0\rangle$$
$$+\beta\langle 0|a^{\dagger}coshr - ae^{-i\theta}sinhr|0\rangle + \beta^{*}\langle 0|a\,coshr - a^{\dagger}\,e^{i\theta}sinhr|0\rangle + \beta^{*}\beta$$
$$= |\beta|^{2} + sinh^{2}r = \langle n\rangle$$

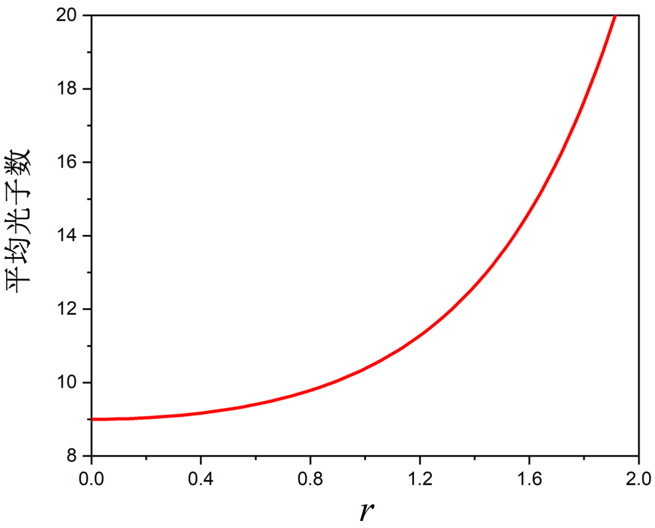
$$\langle n \rangle = |\boldsymbol{\beta}|^2 + \boldsymbol{sinh^2 r}$$

定义2: $|\boldsymbol{\beta}, \boldsymbol{\xi}\rangle = \mathcal{D}(\boldsymbol{\beta})\mathcal{S}(\boldsymbol{\xi})|\mathbf{0}\rangle$

$$\boldsymbol{\xi} = \boldsymbol{re^{i\theta}}$$

$\boldsymbol{\beta}, r$增大，平均光子数增大

且ξ相位不会影响平均光子数



$$\beta = 3, \theta = 0$$









5. 压缩态的粒子数分布

压缩态在Fock态表象展开：$|\beta,\xi\rangle = \sum_n |n\rangle\langle n|\beta,\xi\rangle = \sum_n c_n|n\rangle$

压缩态的粒子数分布$P_n = |c_n|^2$，核心在于求解展开系数c_n

这里用定义2：$|\beta,\xi\rangle = \mathcal{D}(\beta)\mathcal{S}(\xi)|0\rangle$

求解过程如下：

- 整体思路：李代数做准备，以相干态$|\alpha\rangle$为中间状态，

首先（1）得到$\langle\alpha|0,\xi\rangle = \langle\alpha|\xi\rangle$

然后（2）得到 $\langle\alpha|\beta,\xi\rangle$这一复数的值

之后（3）利用相干态的粒子数展开$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$

可以得到$\langle\alpha|\beta,\xi\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_n \frac{\alpha^{*n}}{\sqrt{n!}}\langle n|\beta,\xi\rangle$，最后得到$\langle n|\beta,\xi\rangle$的值

即可以获得压缩态的粒子数展开

- 李代数的BCH-like relation：对于满足 $\left[\hat{T}_-,\hat{T}_+\right]=2\hat{T}_c,\left[\hat{T}_c,\hat{T}_\pm\right]=\pm\hat{T}_\pm$ 的算符，

$$e^{\lambda_+\hat{T}_++\lambda_c\hat{T}_c+\lambda_-\hat{T}_-}=e^{\sigma_+\hat{T}_+}\,e^{\ln(\sigma_c)\hat{T}_c}\,e^{\sigma_-\hat{T}_-}$$

其中$\sigma_c=[\cosh(v)-\frac{\lambda_c}{2v}\sinh(v)]^{-2},\sigma_\pm=\frac{2\lambda_\pm\sinh(v)}{2v\cosh(v)-\lambda_c\sinh(v)},\ v^2=(\frac{\lambda_c}{2})^2-\lambda_+\lambda_-$

- 压缩算符：$S(\xi)=e^{(-\xi a^{\dagger 2}+\xi^* a^2)/2}$，令$\hat{T}_+=\frac{a^{\dagger 2}}{2},\hat{T}_-=\frac{a^2}{2},\ \hat{T}_c=\frac{aa^\dagger+a^\dagger a}{4}$

则$\hat{T}_+,\hat{T}_-,\hat{T}_c$满足上述对易关系

- 根据BCH-like relation，$\lambda_+=-\xi,\ \ \lambda_-=\xi^*,\lambda_c=0$

得到$\sigma_+=-\exp(i\theta)\tanh(r),\sigma_-=\exp(-i\theta)\tanh(r),\ \sigma_c=[\cosh(r)]^{-2}$

得到压缩算符的展开形式（自己推一下）

$$S(\xi)=\exp\left[-\frac{1}{2}a^{\dagger 2}e^{i\theta}\tanh(r)\right]\exp\left\{-\frac{1}{2}\ln[\cosh(r)]\left(aa^\dagger+a^\dagger a\right)\right\}\exp[\frac{1}{2}a^2e^{-i\theta}\tanh(r)]$$

$$\mathcal{S}(\xi) = \exp\left[-\frac{1}{2}a^{\dagger 2}e^{i\theta}\tanh(r)\right]\exp\left\{-\frac{1}{2}\ln[\cosh(r)]\left(aa^{\dagger} + a^{\dagger}a\right)\right\}\exp[\frac{1}{2}a^{2}e^{-i\theta}\tanh(r)]$$

（1）压缩真空态和相干态内积，求 $\langle\alpha|\xi\rangle$

$$\langle\alpha|\xi\rangle = \langle\alpha|\mathcal{S}(\xi)|0\rangle$$

$$= \left\langle\alpha\middle|\exp\left[-\frac{1}{2}a^{\dagger 2}e^{i\theta}\tanh(r)\right]\exp\left\{-\frac{1}{2}\ln[\cosh(r)]\left(2a^{\dagger}a + 1\right)\right\}\exp[\frac{1}{2}a^{2}e^{-i\theta}\tanh(r)]\middle|0\right\rangle$$

$$= \left\langle\alpha\middle|\exp\left[-\frac{1}{2}a^{\dagger 2}e^{i\theta}\tanh(r)\right]\exp\left\{-\frac{1}{2}\ln[\cosh(r)]\left(2a^{\dagger}a + 1\right)\right\}\middle|0\right\rangle$$

$$= \left\langle\alpha\middle|\exp\left[-\frac{1}{2}a^{\dagger 2}e^{i\theta}\tanh(r)\right]\exp\{-\ln[\cosh(r)a^{\dagger}a]\}\,exp\{-\frac{1}{2}\ln[\cosh(r)]\}\middle|0\right\rangle$$

$$= \frac{1}{\sqrt{\cosh(r)}}\left\langle\alpha\middle|\exp\left[-\frac{1}{2}a^{\dagger 2}e^{i\theta}\tanh(r)\right]\middle|0\right\rangle$$

$$= \frac{1}{\sqrt{\cosh(r)}}\exp\left[-\frac{1}{2}\alpha^{*2}e^{i\theta}\tanh(r)\right]\langle\alpha|0\rangle$$

$$= \frac{1}{\sqrt{\cosh(r)}}\exp\left[-\frac{1}{2}\alpha^{*2}e^{i\theta}\tanh(r) - \frac{1}{2}|\alpha|^{2}\right]$$

（2）$\langle\alpha|\beta,\xi\rangle$这一复数的值

$$\langle\alpha|\xi\rangle = \langle\alpha|\mathcal{S}(\xi)|0\rangle = \frac{1}{\sqrt{\cosh(r)}}\exp\left[-\frac{1}{2}\alpha^{*2}e^{i\theta}\tanh(r) - \frac{1}{2}|\alpha|^2\right]$$

对一般的压缩态，可以得到$\langle\alpha|\beta,\xi\rangle$

$$\langle\alpha|\beta,\xi\rangle = \langle\alpha|D(\beta)\mathcal{S}(\xi)|0\rangle = \langle 0|D(-\alpha)D(\beta)\mathcal{S}(\xi)|0\rangle$$

$$= \left\langle 0\middle|D(\beta-\alpha)\exp\left[-\frac{1}{2}(\alpha^*\beta-\alpha\beta^*)\right]\mathcal{S}(\xi)\middle|0\right\rangle$$

$$= \left\langle \alpha-\beta\middle|\exp\left[\frac{1}{2}(\alpha^*\beta-\alpha\beta^*)\right]\mathcal{S}(\xi)\middle|0\right\rangle$$

$$= \frac{1}{\sqrt{\cosh(r)}}\exp\left\{-\frac{1}{2}\left[(\alpha^*-\beta^*)^2 e^{i\theta}\tanh(r) + |\alpha-\beta|^2 + \alpha\beta^* - \alpha^*\beta\right]\right\}$$

$$D(\alpha)D(\beta) = D(\alpha+\beta)exp[\frac{1}{2}(-\alpha^*\beta+\alpha\beta^*)]$$

$$\langle\alpha|\beta,\xi\rangle=\frac{1}{\sqrt{\cosh(r)}}\exp\left\{-\frac{1}{2}\left[(\alpha^*-\beta^*)^2e^{i\theta}\tanh(r)+|\alpha-\beta|^2+\alpha\beta^*-\alpha^*\beta\right]\right\}$$

$$\langle\alpha|\beta,\xi\rangle=e^{-\frac{|\alpha|^2}{2}}\sum\nolimits_n\frac{\alpha^{*n}}{\sqrt{n!}}\langle n|\beta,\xi\rangle \qquad |\alpha\rangle=e^{-\frac{|\alpha|^2}{2}}\sum\nolimits_n\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

（3）利用相干态的粒子数展开，可以得到$\langle\alpha|\beta,\xi\rangle=e^{-\frac{|\alpha|^2}{2}}\sum_n\frac{\alpha^{*n}}{\sqrt{n!}}\langle n|\beta,\xi\rangle$**，**

及其$\langle \boldsymbol{n}|\boldsymbol{\beta},\xi\rangle$**的值 （自己课下推导）**

$$e^{1/2|\alpha|^2}\langle\alpha|\beta,\xi\rangle=\sum\nolimits_n\frac{\alpha^{*n}}{\sqrt{n!}}\langle n|\beta,\xi\rangle=$$

$$\frac{1}{\sqrt{\cosh(r)}}\exp\left\{-\frac{1}{2}\alpha^{*2}e^{i\theta}\tanh(r)+\alpha^*\left[\beta+\beta^*e^{i\theta}\tanh(r)\right]\right\}\times\exp\{-\frac{1}{2}[\beta^{*2}e^{i\theta}\tanh(r)+|\beta|^2]\}$$

$$e^{1/2|\alpha|^2}\langle\alpha|\beta,\xi\rangle = \sum_n \frac{\alpha^{*n}}{\sqrt{n!}}\langle n|\beta,\xi\rangle = \sum_n \frac{\alpha^{*n}}{\sqrt{n!}} c_n =$$

$$\frac{1}{\sqrt{\cosh(r)}}\exp\left\{-\frac{1}{2}\alpha^{*2}e^{i\theta}\tanh(r) + \alpha^*\left[\beta + \beta^* e^{i\theta}\tanh(r)\right]\right\}\times\exp\{-\frac{1}{2}[\beta^{*2}e^{i\theta}\tanh(r) + |\beta|^2]\}$$

利用展开式：$\exp\left(-t^2 + 2zt\right) = \sum_{n=0}^{\infty}\frac{t^n}{n!}H_n(z)$　　$H_n(z)$为n-阶厄米多项式

令$t = \sqrt{\frac{1}{2}e^{i\theta}\tanh(r)}\,\alpha^*$, $z = \frac{\beta+\beta^* e^{i\theta}\tanh(r)}{e^{i\theta/2}\sqrt{2\sinh(r)/\cosh(r)}}$

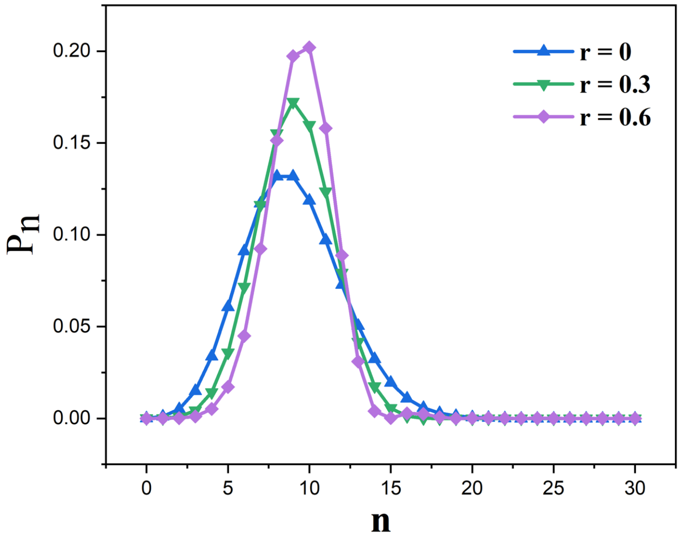
得到一般压缩态的Fock态表象展开系数c_n表达式：

$$c_n = \langle n|\beta,\xi\rangle = \frac{\left(e^{\frac{i\theta}{2}}\tanh r\right)^{\frac{n}{2}}}{2^{\frac{n}{2}}\sqrt{n!\cosh r}}\exp[-\frac{1}{2}\left(\beta^{*2}e^{i\theta}\tanh r + |\beta|^2\right)]$$

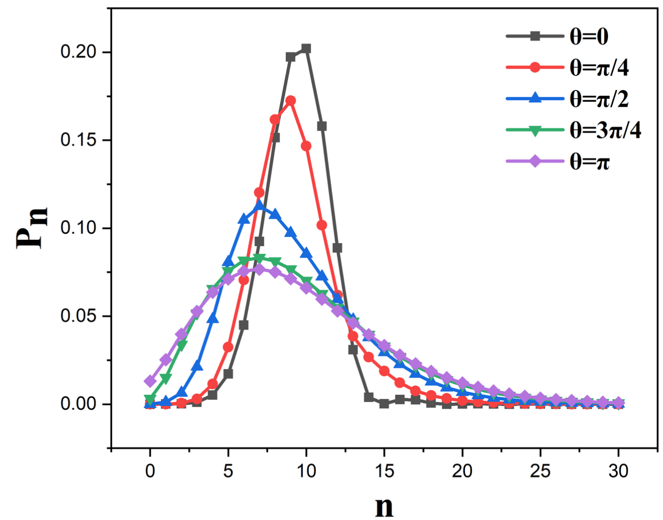
$$\times H_n\left(\frac{\beta e^{-i\theta/2}\cosh r + \beta^* e^{i\theta/2}\sinh r}{\sqrt{2\sinh r.\cosh r}}\right)$$

由此可以得到一般压缩态的粒子数分布$P_n = |c_n|^2$

1. 压缩态$|\beta,\xi>$的粒子数分布



$\boldsymbol{\beta = 3, \theta = 0}$



$\boldsymbol{\beta = 3, r = 0.6}$

压缩度r越大，P_n的峰值对应的n越大

ξ的相位会影响平光子数分布

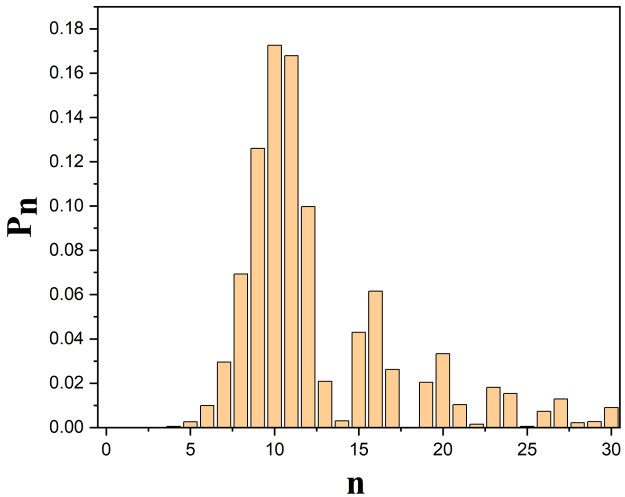
$$|\beta,\xi\rangle = \mathcal{D}(\beta)\mathcal{S}(\xi)|0\rangle$$

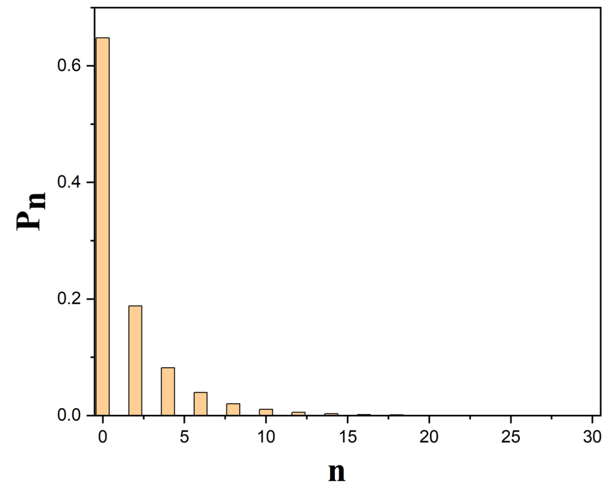
$$\xi = re^{i\theta}$$

$|\beta, \xi\rangle = \mathcal{D}(\beta)\mathcal{S}(\xi)|0\rangle$

2. 压缩度 r 很大时，压缩态$|\beta, \xi>$的粒子数分布的振荡

$\xi = re^{i\theta}$





$\boldsymbol{\beta = 3, r = 1.5, \theta = 0}$

$\boldsymbol{\beta = 0, r = 1, \theta = 0}$

3. 特别的，当$\beta = 0$时，只存在偶数粒子分布，不存在奇数粒子分布





6.双模压缩态

$$\mathcal{S}(\xi) = e^{(-\xi a^{\dagger 2} + \xi^* a^2)/2}$$

- 定义：双模压缩算符作用于双模真空态得到双模压缩态

- 形式：$|\xi\rangle = S_{ab}(\xi)|0,0\rangle$, $S_{ab}(\xi) = exp\left(-\xi a^\dagger b^\dagger + \xi^* ab\right)$, $\xi = re^{i\theta}$

$$[a, a^\dagger] = [b, b^\dagger] = 1, \qquad [a, b^\dagger] = 0$$

- 算符性质：（作业推一下）

$$S_{ab}{}^\dagger(\xi) = S_{ab}(-\xi) = S_{ab}{}^{-1}(\xi)$$

$$S_{ab}{}^\dagger(\xi) a S_{ab}(\xi) = a\cosh r - b^\dagger e^{i\theta} \sinh r$$

$$S_{ab}{}^\dagger(\xi) a^\dagger S_{ab}(\xi) = a^\dagger \cosh r - b\, e^{-i\theta} \sinh r$$

$$S_{ab}{}^\dagger(\xi) b S_{ab}(\xi) = b\cosh r - a^\dagger e^{i\theta} \sinh r$$

$$S_{ab}{}^\dagger(\xi) b^\dagger S_{ab}(\xi) = b^\dagger \cosh r - a\, e^{-i\theta} \sinh r$$

$$\mathcal{S}^\dagger(\xi) = \mathcal{S}(-\xi) = \mathcal{S}^{-1}(\xi)$$

$$(\xi = re^{i\theta})$$

$$\mathcal{S}^\dagger(\xi) a \mathcal{S}(\xi) = a\cosh r - a^\dagger e^{i\theta} \sinh r$$

$$\mathcal{S}^\dagger(\xi) a^\dagger \mathcal{S}(\xi) = a^\dagger \cosh r - a e^{-i\theta} \sinh r$$

$$S_{ab}(\xi) = exp\left(-a^\dagger b^\dagger e^{i\theta} tanh\, r\right) exp\left[-(ln\, cosh\, r)\left(a^\dagger a + b^\dagger b + 1\right)\right] exp\left(abe^{-i\theta} tanh\, r\right)$$

- **Fock表象展开：** $|\xi\rangle = S_{ab}(\xi)|0,0\rangle =$

$$exp\left(-a^\dagger b^\dagger e^{i\theta} tanh\, r\right) exp\left[-(ln\, cosh\, r)\left(a^\dagger a + b^\dagger b + 1\right)\right] exp\left(abe^{-i\theta} tanh\, r\right)|0,0\rangle$$

$$= exp\left(-a^\dagger b^\dagger e^{i\theta} tanh\, r\right) exp\left[-(ln\, cosh\, r)\left(a^\dagger a + b^\dagger b + 1\right)\right]|0,0\rangle$$

$$= exp\left(-a^\dagger b^\dagger e^{i\theta} tanh\, r\right) exp\left[-ln\, coshr\; a^\dagger a\right] exp\left[-ln\, coshr\; b^\dagger b\right] exp\left[-ln\, coshr\right]|0,0\rangle$$

$$= \frac{1}{cosh\, r} exp\left(-a^\dagger b^\dagger e^{i\theta} tanh\, r\right)|0,0\rangle$$

$$= \frac{1}{cosh\, r} \sum_0^\infty \frac{1}{n!} (-e^{i\theta} tanh\, r)^n (a^\dagger b^\dagger)^n\, |0,0\rangle$$

$$= \frac{1}{cosh\, r} \sum\nolimits_{n=0}^\infty (-e^{i\theta} tanh\, r)^n |\, n, n\rangle$$

- **算符平均值：** $\langle a^{\dagger}a\rangle = \langle b^{\dagger}b\rangle = sinh^2 r,\ \langle ab\rangle = -sinh\, r\, cosh\, r\, e^{i\theta}$

$$\langle a^{\dagger}b\rangle = \langle a^2\rangle = \langle b^2\rangle = \langle a\rangle = \langle b\rangle = 0$$

其中，双模压缩态的平均光子数：

$$\langle a^{\dagger}a\rangle = \langle 0,0|S_{ab}^{\dagger}(\xi)a^{\dagger}a\, Sab(\xi)|0,0\rangle$$

$$= \langle 0,0|S_{ab}^{\dagger}(\xi)a^{\dagger}S_{ab}(\xi)S_{ab}^{\dagger}(\xi)aS_{ab}(\xi)|0,0\rangle$$

$$= \langle 0,0|(a^{\dagger}cosh\, r - b\, e^{-i\theta}\, sinh\, r)(a\, cosh\, r - b^{\dagger}\, e^{i\theta}\, sinh\, r)\,|0,0\rangle = sinh^2 r$$

自己算一下：

$$\langle ab\rangle = \langle 0,0|S_{ab}^{\dagger}(\xi)abS_{ab}(\xi)|0,0\rangle$$

$$= \langle 0,0|S_{ab}^{\dagger}(\xi)aS_{ab}(\xi)S_{ab}^{\dagger}(\xi)\mathrm{b}S_{ab}(\xi)|0,0\rangle$$

$$= \langle 0,0|(a\, cosh\, r - b^{\dagger}\, e^{i\theta}\, sinh\, r)\,(b\, cosh\, r - a^{\dagger}\, e^{i\theta}\, sinh\, r)|0,0\rangle$$

$$= -sinh\, r\, cosh\, r\, e^{i\theta}$$

- **直积形式：双模压缩真空态可表示为两个独立单模压缩真空态的直积形式**

定义另一组算符： $c=\frac{1}{\sqrt{2}}\left(a+e^{i\delta}b\right)\quad d=\frac{1}{\sqrt{2}}\left(a-e^{-i\delta}b\right)$ δ**为实数相位因子**

满足对易关系： $[c,c^{\dagger}]=\left[d,d^{\dagger}\right]=1,\ \left[c,d^{\dagger}\right]=[c,d]=\left[c^{\dagger},d^{\dagger}\right]=\left[c^{\dagger},d\right]=0$

逆变换为： $a=\frac{1}{\sqrt{2}}\left(c-e^{i\delta}d\right)\quad b=\frac{1}{\sqrt{2}}\left(d+e^{-i\delta}c\right)$

压缩算符重写： $S_{ab}(\xi)=exp\left(-\xi a^{\dagger}b^{\dagger}+\xi^{*}ab\right)$

$$S_{ab}(\xi)=exp\left[-\frac{1}{2}\xi\left(c^{\dagger 2}e^{i\delta}-d^{\dagger 2}e^{-i\delta}\right)+\frac{1}{2}\xi^{*}\left(c^{2}e^{-i\delta}-d^{2}e^{i\delta}\right)\right]$$

$$=S_{c}(\xi e^{i\delta})S_{d}(-\xi e^{-i\delta})$$

- **最一般的双模压缩相干态定义：**

$$|\alpha,\beta,\xi\rangle=D_{a}(\alpha)D_{b}(\beta)S_{ab}(\xi)|0,0\rangle$$

$D_{a}(\alpha),D_{b}(\beta)$**分别对应模式**$a,b$**的平移算符**

三、相干态和压缩态下的电磁场表示

- 这里*只计算相干态下的电磁场*表示，压缩态的情况只在图中给出
- 从电场$\hat{E}(t)$在量子光场中是算符形式可知：Fock态$|n\rangle$，相干态$|\alpha\rangle$，压缩态$|\alpha,\xi\rangle$都不是电场算符的本征态，所以一定存在电场涨落
- 考虑一维单模光场

（作业中算一下，大家可以算算Fock态$|n\rangle$下的情况）

从$\hat{E} = \mathcal{E} a e^{-i\nu t} \sin kz + H.c.$开始

在某个$kz = \frac{\pi}{2}$处，且假设$\mathcal{E} = \mathcal{E}^*$，有

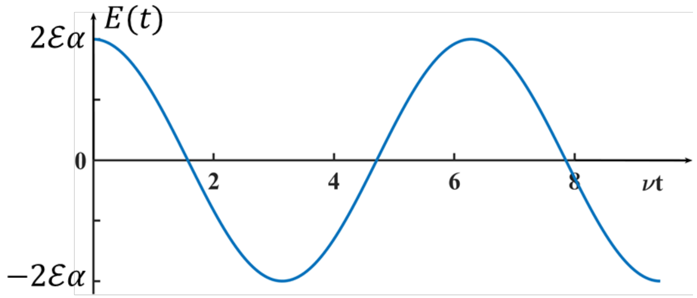
$$\hat{E}(t) = \mathcal{E}(a e^{-i\nu t} + a^{\dagger} e^{i\nu t})$$

$\hat{E}(t)=\mathcal{E}(ae^{-i\nu t}+a^{\dagger}e^{i\nu t})$

在相干态$|\alpha\rangle$下

$$\langle\hat{E}(t)\rangle=\langle\alpha|\hat{E}(t)|\alpha\rangle=\mathcal{E}(\alpha e^{-i\nu t}+\alpha^{*}e^{i\nu t})$$

若$\alpha=\alpha^{*}$，则 $\langle\hat{E}(t)\rangle=\mathcal{E}\alpha\left(e^{-i\nu t}+e^{i\nu t}\right)=2\mathcal{E}\alpha\cos\nu t$



若$X_1=(a+a^{\dagger})/2$，$X_2=(a-a^{\dagger})/(2i)$，则

$\hat{E}(t)=2\mathcal{E}(X_1\cos\nu t+X_2\sin\nu t)$

- **相干态下对应的电场涨落**$\Delta\hat{E}(t)^2 = \langle\alpha|\hat{E}^2(t)|\alpha\rangle - \langle\alpha|\hat{E}(t)|\alpha\rangle^2$

下面开始计算，利用 $\hat{E}(t) = 2\mathcal{E}(X_1\cos\nu t + X_2\sin\nu t)$

$\langle\alpha|\hat{E}^2(t)|\alpha\rangle$

$= 4\mathcal{E}^2\langle\alpha|X_1^2\cos^2\nu t + X_2^2\sin^2\nu t + (X_1X_2 + X_2X_1)\cos\nu t\sin\nu t\,|\alpha\rangle$

$= 4\mathcal{E}^2(\langle X_1^2\rangle\cos^2\nu t + \langle X_2^2\rangle\sin^2\nu t + \langle X_1X_2 + X_2X_1\rangle\cos\nu t\sin\nu t)$

$\langle\alpha|\hat{E}(t)|\alpha\rangle^2$

$= 4\mathcal{E}^2(\langle X_1\rangle^2\cos^2\nu t + \langle X_2\rangle^2\sin^2\nu t + 2\langle X_1\rangle\langle X_2\rangle\cos\nu t\sin\nu t)$

所以

$\Delta\hat{E}(t)^2 = \langle\alpha|\hat{E}^2(t)|\alpha\rangle - \langle\alpha|\hat{E}(t)|\alpha\rangle^2$

$= 4\mathcal{E}^2[(\langle X_1^2\rangle - \langle X_1\rangle^2)\cos^2\nu t + (\langle X_2^2\rangle - \langle X_2\rangle^2)\sin^2\nu t$

$+ (\langle X_1X_2 + X_2X_1\rangle - 2\langle X_1\rangle\langle X_2\rangle)\cos\nu t\sin\nu t]$

由于在相干态中 $\Delta X_1^2 = \Delta X_2^2 = 1/4$，$\langle X_1X_2 + X_2X_1\rangle - 2\langle X_1\rangle\langle X_2\rangle = 0$

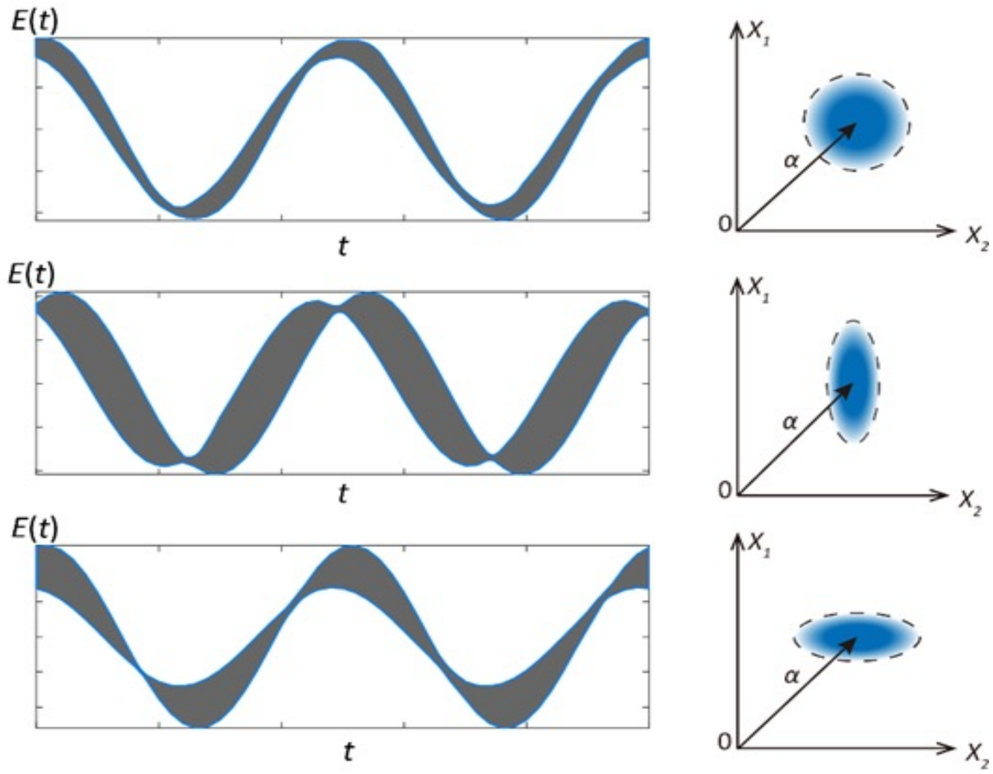
所以 $\Delta\hat{E}(t)^2 = \mathcal{E}^2$

- **下面用图示来刻画相干态与压缩态的电磁场表示**

  对于相干态，X_1和X_2的噪声相等

  对于压缩态有两种情况，$\Delta X_1 < 1/2$，$\Delta X_2 > 1/2$和$\Delta X_1 > 1/2$，$\Delta X_2 < 1/2$

  X_1可以理解为与振幅相关的量，X_2是与相位相关的量，与振幅、相位对应

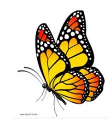


补充计算：

相干态和压缩态中 $\langle X_1X_2+X_2X_1\rangle-2\langle X_1\rangle\langle X_2\rangle=0$

$X_1=\frac{1}{2}(a+a^\dagger)$ $\quad X_2=\frac{1}{2i}(a-a^\dagger)$ $\quad$ **满足** $[X_1,X_2]=\frac{i}{2}$

$$\langle X_1X_2+X_2X_1\rangle=\left\langle 2X_1X_2-\frac{i}{2}\right\rangle$$



相干态 $|\alpha\rangle$ **下，**

$$\left\langle 2X_1X_2-\frac{i}{2}\right\rangle=\left\langle\alpha\left|\frac{1}{2i}(a+a^\dagger)(a-a^\dagger)-\frac{i}{2}\right|\alpha\right\rangle=\frac{1}{2i}(\alpha^2-\alpha^{*2})$$

$$2\langle X_1\rangle\langle X_2\rangle=\frac{1}{2i}(\alpha+\alpha^*)(\alpha-\alpha^*)=\frac{1}{2i}(\alpha^2-\alpha^{*2})$$

$$\therefore\ \langle X_1X_2+X_2X_1\rangle-2\langle X_1\rangle\langle X_2\rangle=0$$

压缩态$|\alpha,\xi\rangle$下

$$\left\langle 2X_1X_2-\frac{i}{2}\right\rangle=\left\langle\alpha\middle|\mathcal{S}^\dagger(\xi)\left[\frac{1}{2i}(a+a^\dagger)(a-a^\dagger)--\frac{i}{2}\right]\mathcal{S}(\xi)\middle|\alpha\right\rangle$$

$$=\frac{1}{2i}\{[\alpha^2\cosh^2 r+\alpha^{*2}e^{2i\theta}\sinh^2 r-\left(2|\alpha|^2+1\right)e^{i\theta}\cosh r\sinh r]$$

$$-[\alpha^{*2}\cosh^2 r+\alpha^2 e^{-2i\theta}\sinh^2 r-\left(2|\alpha|^2+1\right)e^{-i\theta}\cosh r\sinh r]\}$$

$$2\langle X_1\rangle\langle X_2\rangle=\frac{1}{2i}\langle\alpha|\mathcal{S}^\dagger(\xi)\left(a+a^\dagger\right)\mathcal{S}(\xi)|\alpha\rangle\langle\alpha|\mathcal{S}^\dagger(\xi)\left(a-a^\dagger\right)\mathcal{S}(\xi)|\alpha\rangle$$

$$=\frac{1}{2i}\left[\left(\alpha\cosh r-\alpha^* e^{i\theta}\sinh r\right)+\left(\alpha^*\cosh r-\alpha e^{-i\theta}\sinh r\right)\right]\times$$

$$\left[\left(\alpha\cosh r-\alpha^* e^{i\theta}\sinh r\right)-\left(\alpha^*\cosh r-\alpha e^{-i\theta}\sinh r\right)\right]$$

$$=\frac{1}{2i}\{[\alpha^2 cosh^2 r+\alpha^{*2}e^{2i\theta} sinh^2 r-\left(2|\alpha|^2+1\right)e^{i\theta} cosh\, r\, sinh\, r]$$

$$-[\alpha^{*2}cosh^2 r+\alpha^2 e^{-2i\theta} sinh^2 r-\left(2|\alpha|^2+1\right)e^{-i\theta} cosh\, r\, sinh\, r]\}$$

$$\therefore\langle X_1X_2+X_2X_1\rangle-2\langle X_1\rangle\langle X_2\rangle=0$$

四、薛定谔猫态

- **定义**：诸如$|\psi\rangle = N\left(|\alpha\rangle + e^{i\phi}|-\alpha\rangle\right)$ $|\psi\rangle$形式的叠加相干态即为光场的“薛定谔猫态”，其中N为归一化因子，ϕ为叠加态的相位差，当$\phi = 0$时，为**偶猫态**；当$\phi = \pi$时，为**奇猫态**。$|\alpha|$越大，猫态越大，态越宏观。
- **奇、偶猫态形式：**

$$|\alpha\rangle_o = N_o(|\alpha\rangle - |-\alpha\rangle) = (\sinh|\alpha|^2)^{-1/2}\sum_{n=0}^{\infty}\frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle$$

$$|\alpha\rangle_e = N_e(|\alpha\rangle + |-\alpha\rangle) = (\cosh|\alpha|^2)^{-1/2}\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{\sqrt{(2n)!}}|2n\rangle$$

N_o, N_e为归一化因子， $N_o = \frac{1}{2}e^{\frac{|\alpha|^2}{2}}(\sinh|\alpha|^2)^{-\frac{1}{2}}$ ， $N_e = \frac{1}{2}e^{\frac{|\alpha|^2}{2}}(\cosh|\alpha|^2)^{-\frac{1}{2}}$

奇、偶猫态相互正交。相干态可以写作奇、偶猫态的组合:（自己推一下）

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\left[\left(\sinh|\alpha|^2\right)^{\frac{1}{2}}|\alpha\rangle_o + \left(\cosh|\alpha|^2\right)^{\frac{1}{2}}|\alpha\rangle_e\right]$$

$$|\alpha\rangle_o = N_o(|\alpha\rangle - |-\alpha\rangle) = (sinh\,|\alpha|^2)^{-1/2}\sum_{n=0}^{\infty}\frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle$$

$$|\alpha\rangle_e = N_e(|\alpha\rangle + |-\alpha\rangle) = (cosh\,|\alpha|^2)^{-1/2}\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{\sqrt{(2n)!}}|2n\rangle$$

$$N_o = \frac{1}{2}e^{\frac{|\alpha|^2}{2}}(sinh|\alpha|^2)^{-\frac{1}{2}}$$

$$N_e = \frac{1}{2}e^{\frac{|\alpha|^2}{2}}(cosh|\alpha|^2)^{-\frac{1}{2}}$$

- **性质：**

奇、偶猫态都是a^2的本征态

$$a^2|\alpha\rangle_{o(e)} = a^2 N_{o(e)}(|\alpha\rangle \mp |-\alpha\rangle) = \alpha^2|\alpha\rangle_{o(e)}$$

奇、偶猫态可以相互转化

$$a|\alpha\rangle_o = \alpha\,(coth|\alpha|^2)^{1/2}|\alpha\rangle_e$$

$$a|\alpha\rangle_e = \alpha\,(tanh|\alpha|^2)^{1/2}|\alpha\rangle_e$$

$$a|\alpha\rangle_o = aN_o(|\alpha\rangle - |-\alpha\rangle) = \alpha(|\alpha\rangle + |-\alpha\rangle)N_o$$

$$= \alpha\frac{N_o}{N_e}|\alpha\rangle_e = \alpha\,(coth|\alpha|^2)^{\frac{1}{2}}|\alpha\rangle_e$$

$$|\alpha\rangle_o = N_o(|\alpha\rangle - |-\alpha\rangle)$$
$$|\alpha\rangle_e = N_e(|\alpha\rangle + |-\alpha\rangle)$$

$$N_o = \frac{1}{2} e^{\frac{|\alpha|^2}{2}} (sinh|\alpha|^2)^{-\frac{1}{2}}$$
$$N_e = \frac{1}{2} e^{\frac{|\alpha|^2}{2}} (cosh|\alpha|^2)^{-\frac{1}{2}}$$

- **正交完备性：**

$$\langle\alpha|\alpha'\rangle = \exp\left(-\frac{1}{2}|\alpha|^2 + \alpha^*\alpha' - \frac{1}{2}|\alpha'|^2\right) \neq 0$$

奇猫态之间非正交、偶猫态之间非正交

$${}_o\langle\alpha|\alpha'\rangle_o = \left(sinh|\alpha|^2\, sinh|\alpha'|^2\right)^{-\frac{1}{2}} sinh(\alpha^*\alpha')$$

$${}_e\langle\alpha|\alpha'\rangle_e = \left(cosh|\alpha|^2\, cosh|\alpha'|^2\right)^{-\frac{1}{2}} cosh(\alpha^*\alpha')$$

$${}_o\langle\alpha|\alpha'\rangle_o = N_o(\alpha)N_o(\alpha')\big(\langle\alpha| - \langle-\alpha|\big)(|\alpha'\rangle - |-\alpha'\rangle)$$

$$= N_o(\alpha)N_o(\alpha')[2\exp\left(-\frac{1}{2}|\alpha|^2 + \alpha^*\alpha' - \frac{1}{2}|\alpha'|^2\right) - 2\exp\left(-\frac{1}{2}|\alpha|^2 - \alpha^*\alpha' - \frac{1}{2}|\alpha'|^2\right)]$$

$$= \frac{1}{4} e^{\frac{|\alpha|^2}{2}+\frac{|\alpha'|^2}{2}} (sinh|\alpha|^2 sinh|\alpha'|^2)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}|\alpha|^2 - \frac{1}{2}|\alpha'|^2\right)[2\exp(\alpha^*\alpha') - 2\exp(-\alpha^*\alpha')]$$

$$= \left(sinh|\alpha|^2\, sinh|\alpha'|^2\right)^{-\frac{1}{2}} sinh(\alpha^*\alpha')$$

$$|\alpha\rangle_o = N_o(|\alpha\rangle - |-\alpha\rangle)$$
$$|\alpha\rangle_e = N_e(|\alpha\rangle + |-\alpha\rangle)$$

$$N_o = \frac{1}{2} e^{\frac{|\alpha|^2}{2}} (sinh|\alpha|^2)^{-\frac{1}{2}}$$
$$N_e = \frac{1}{2} e^{\frac{|\alpha|^2}{2}} (cosh|\alpha|^2)^{-\frac{1}{2}}$$

$$\int |\alpha\rangle\langle\alpha| d^2\alpha = \pi$$

奇猫态和偶猫态可以共同组成超完备的基

但是奇猫态或者偶猫态自身不完备

$$\boldsymbol{I = \frac{1}{\pi}\int d\alpha^2 e^{-|\alpha|^2}\left[sinh|\alpha|^2 |\alpha\rangle_{oo}\langle\alpha| + cosh|\alpha|^2|\alpha\rangle_{ee}\langle\alpha|\right]}$$

$$\boldsymbol{= \frac{1}{\pi}\int d\alpha^2\, e^{-|\alpha|^2}[N_o^2 sinh\,|\alpha|^2(|\alpha\rangle - |-\alpha\rangle)\big(\langle\alpha| - \langle-\alpha|\big) + N_e^2 cosh|\alpha|^2(|\alpha\rangle}$$

$$\boldsymbol{+ |-\alpha\rangle)\big(\langle\alpha| + \langle-\alpha|\big)] = \frac{1}{\pi}\int d\alpha^2 \frac{1}{2}\big(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|\big) = I}$$

（自己推一下）

- **压缩猫态**

定义：压缩算符作用于薛定谔猫态得到压缩猫态

具体形式：$|\psi\rangle_s = \mathcal{S}(\xi)\left(|\alpha\rangle + e^{i\phi}|-\alpha\rangle\right)$，

$$\mathcal{S}(\xi) = e^{(-\xi a^{\dagger 2} + \xi^* a^2)/2}$$ 为单模压缩算符

这里可以接入前沿文献。。。。。。

五、思考题

1. 相干态与压缩态的定义是什么？
2. 证明相干态的两个定义$|\alpha\rangle = \mathcal{D}(\alpha)|0\rangle$和$a|\alpha\rangle = \alpha|\alpha\rangle$是等价的。
3. 平移算符$\mathcal{D}(\alpha)$和压缩算符$\mathcal{S}(\xi)$的具体表达式是什么？
4. 相干态用Fock态表象展开$|\alpha\rangle = \sum_n c_n |n\rangle$，展开系数的形式是什么？如果用相干态表象展开Fock态呢？
5. 相干态$|\alpha\rangle$中的平均光子数$\langle n\rangle$和光子数分布$P_n = |\langle n|\alpha\rangle|^2$是怎样的？
6. Fock态$|n\rangle$，相干态$|\alpha\rangle$，压缩态$|\alpha, \xi\rangle$中测不准关系？噪声或涨落？
7. 相干态、压缩态的正交完备性？
8. 电场$\hat{E}(t)$在相干态和压缩态中的表示？
9. 压缩态是哪个算符的本征态？
10. 压缩态的平均光子数$\langle n\rangle$和光子数分布$P_n = |\langle n|\alpha, \xi\rangle|^2$的特点？
11. 薛定谔猫态及其特点？

作业：小报告的形式

说说对相干态、压缩态及其猫态的理解？

（ 有题目、摘要、章节、总结、文献等）

用自己的语言完整表述,

有公式地方要说清楚来历并亲自推一下

背景、定义、算符、性质。。。

态表示和特点、电磁场表示。。。

此次作业重在公式推导

或者，也可以写个前沿小报告