第七章 光子的干涉测量 --- 第二部分

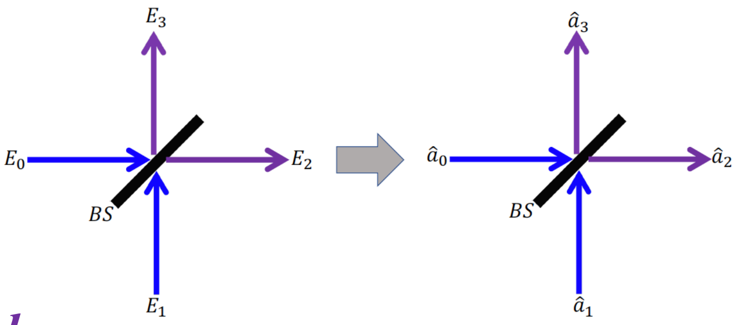
六、 量子分束

6.1 经典分束原理

6.2 量子分束原理

6.3 基于量子分束的量子态输出

6.4 其它相关内容



20241112 ygu@pku.edu.cn

6.1 经典分束器原理

☞一般分束器输入和输出间的矩阵联系

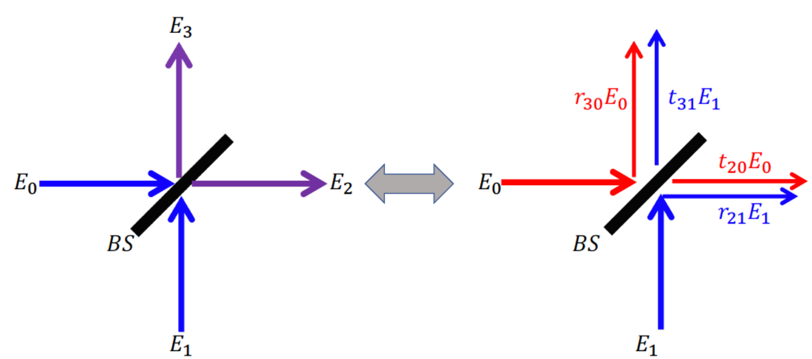
考虑两束光经过分束器，随后输出两束光

设输入光场为E_0, E_1，输出光场为E_2, E_3

其中，$r_{30}(r_{21})$代表$E_0(E_1)$场经过分束器时的反射系数，$t_{20}(t_{31})$代表$E_0(E_1)$场经过分束器时的透射系数

于是，输入场和输出场之间有如下关系

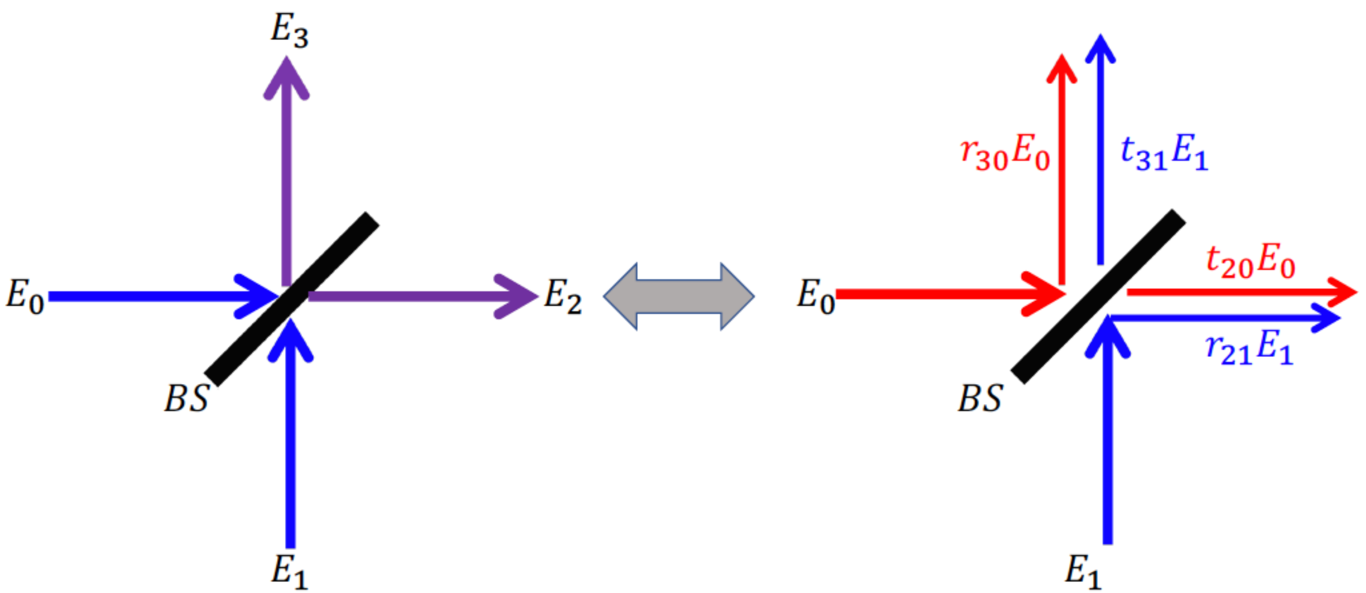
$$\begin{cases} E_2 = t_{20}E_0 + r_{21}E_1 \\ E_3 = r_{30}E_0 + t_{31}E_1 \end{cases}$$



写成矩阵形式就是

$$\begin{pmatrix} E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} t_{20} & r_{21} \\ r_{30} & t_{31} \end{pmatrix} \begin{pmatrix} E_0 \\ E_1 \end{pmatrix} = U \begin{pmatrix} E_0 \\ E_1 \end{pmatrix}$$

U就是输入与输出之间的矩阵关系，我们称它为变换矩阵



☞变换矩阵U中各元素间的关系

$$\begin{cases} E_2 = t_{20}E_0 + r_{21}E_1 \\ E_3 = r_{30}E_0 + t_{31}E_1 \end{cases}$$

假设分束器没有能量损失，则从输入到输出的过程能量守恒，

或者等价地写为

$$|E_2|^2 + |E_3|^2 = |E_0|^2 + |E_1|^2$$

根据输入与输出之间的关系，不难写出

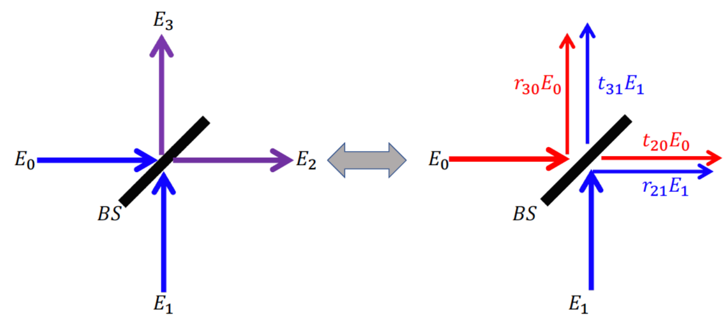
$$|E_2|^2 = E_2 \cdot E_2^* = |t_{20}|^2|E_0|^2 + |r_{21}|^2|E_1|^2 + t_{20}r_{21}^*E_0E_1^* + t_{20}^*r_{21}E_0^*E_1$$

$$|E_3|^2 = E_3 \cdot E_3^* = |r_{30}|^2|E_0|^2 + |t_{31}|^2|E_1|^2 + r_{30}t_{31}^*E_0E_1^* + r_{30}^*t_{31}E_0^*E_1$$

由于上式对任意输入输出场大小都成立

则能量守恒条件要求（*无损*）

$$
\begin{cases}
|t_{20}|^2 + |r_{30}|^2 = 1 \\
|r_{21}|^2 + |t_{31}|^2 = 1 \\
t_{20}r_{21}^* + r_{30}t_{31}^* = 0
\end{cases}
$$



其中前两个式子给出了变换矩阵U中元素之间的振幅关系

而第三式给出了各矩阵元素之间的相位关系

设矩阵U各元素的复数形式为$t_{20}=|t_{20}|e^{i\phi_{20}}$， $r_{21}=|r_{21}|e^{i\phi_{21}}$，

$r_{30}=|r_{30}|e^{i\phi_{30}}$，$t_{31}=|t_{31}|e^{i\phi_{31}}$，则相位关系式$t_{20}r_{21}^{*}+r_{30}t_{31}^{*}=0$

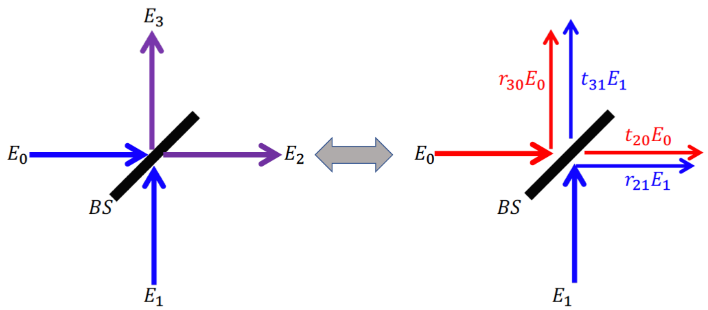
写为 $|t_{20}||r_{21}|e^{i(\phi_{20}-\phi_{21})}+|r_{30}||t_{31}|e^{i(\phi_{30}-\phi_{31})}=0$

$$\Downarrow$$

$$\frac{|t_{20}|}{|r_{30}|}=-\frac{|t_{31}|}{|r_{21}|}\cdot e^{i(\phi_{30}+\phi_{21}-\phi_{20}-\phi_{31})}$$

$$\Downarrow$$

$$\frac{|t_{20}|^2}{|r_{30}|^2}=\frac{|t_{31}|^2}{|r_{21}|^2}$$

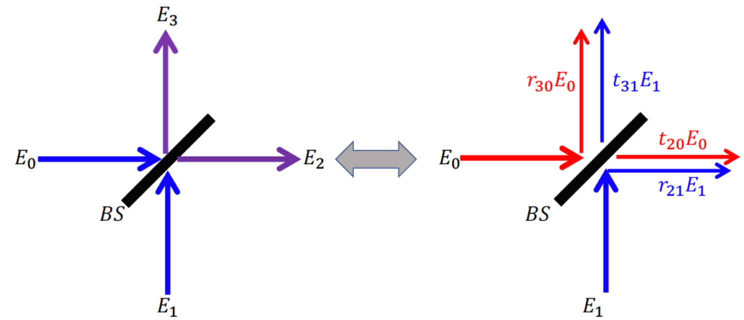


应用振幅关系$|t_{20}|^2+|r_{30}|^2=1$， $|r_{21}|^2+|t_{31}|^2=1$

$$\frac{|t_{20}|^2}{1-|t_{20}|^2}=\frac{|t_{31}|^2}{1-|t_{31}|^2}$$

$$\frac{|t_{20}|^2}{1-|t_{20}|^2}=\frac{|t_{31}|^2}{1-|t_{31}|^2}$$

注意到函数$f(x)=\frac{x}{1-x}$在$0<x<1$为单调函数，上式成立则有$|t_{20}|^2=|t_{31}|^2$，即分束器对称。所以对于无损的经典分束器，要求分束器两边的透射率（$|t_{20}|^2$，$|t_{31}|^2$）和反射率（$|r_{30}|^2$，$|r_{21}|^2$）都相等，



或等价地 $\begin{cases}|t_{20}|=|t_{31}|\\ |r_{30}|=|r_{21}|\end{cases}$

根据相位关系有$t_{20}r_{21}^*=-r_{30}t_{31}^*$，那么$t_{20}=t_{31}^*$，$r_{21}^*=-r_{30}$是一种满足相位关系的选择，记$t_{20}=t, r_{21}=r$，

那么$t_{31}=t^*, r_{30}=-r^*$

于是变换矩阵为

$$U=\begin{pmatrix} t & r \\ -r^* & t^* \end{pmatrix}$$

容易验证，$UU^\dagger=I$，即U为幺正矩阵，*并且* $|t|^2+|r|^2=1$.

一般地，若令$t=\cos\theta, r=\sin\theta\, e^{-i\phi}$，则变换矩阵写成

$$\boldsymbol{U}=\begin{pmatrix} \cos\boldsymbol{\theta} & \sin\boldsymbol{\theta}\,\boldsymbol{e}^{-i\boldsymbol{\phi}} \\ -\sin\boldsymbol{\theta}\,\boldsymbol{e}^{i\boldsymbol{\phi}} & \cos\boldsymbol{\theta} \end{pmatrix}$$

当$\theta=\frac{\pi}{4},\phi=-\frac{\pi}{2}$时，对应分束器的变换矩阵 $U=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

当$\theta=\frac{\pi}{4},\phi=\pi$时，对应分束器的变换矩阵 $U=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$$\begin{pmatrix} E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} t_{20} & r_{21} \\ r_{30} & t_{31} \end{pmatrix} \begin{pmatrix} E_0 \\ E_1 \end{pmatrix} = U \begin{pmatrix} E_0 \\ E_1 \end{pmatrix}$$

思考：

① 存在增益或者损耗时，经典分束器的变换矩阵如何？

② 量子光场输入的情况下变换矩阵是怎样的？

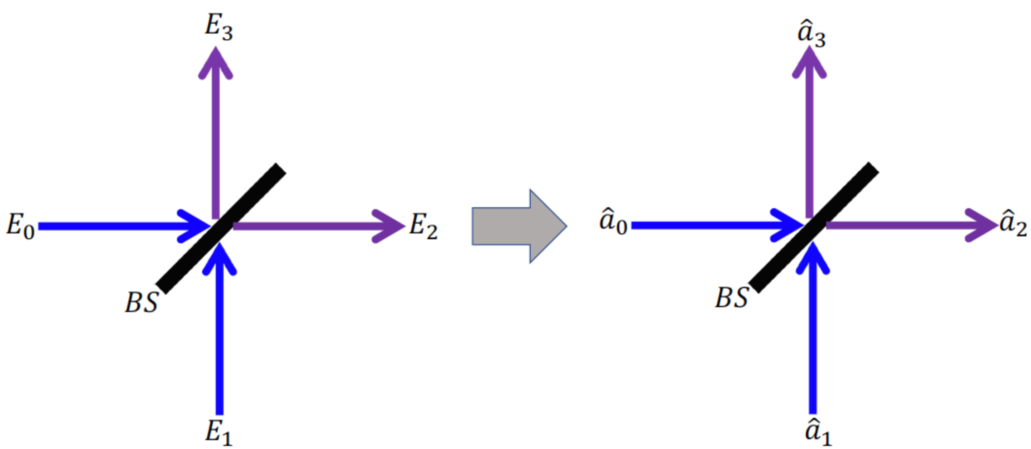
$$\boldsymbol{U} = \begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ -\sin\theta\, e^{i\phi} & \cos\theta \end{pmatrix}$$

6.2 量子分束原理

☞经典分束到量子分束的对应

在 量 子 描 述 中 ， 将 光 场 换 成 相 应 量 子 态 的 湮 灭 算 符

$\hat{a}_0, \hat{a}_1, \hat{a}_2, \hat{a}_3$，如下图



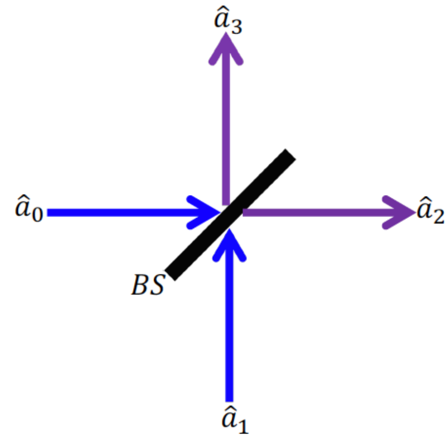
输入和输出之间的变换关系保持不变

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = U \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} = \begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ -e^{i\phi}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix}$$

与经典情况不同，量子中算符要满足对易关系，光子是玻色子，对于输入态算符$\hat{a}_0, \hat{a}_1$满足玻色对易关系：

$$[\hat{a}_0, \hat{a}_0^\dagger] = 1, [\hat{a}_1, \hat{a}_1^\dagger] = 1$$

输入态算符间的其它对易关系都为0



$$U = \begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ -\sin\theta\, e^{i\phi} & \cos\theta \end{pmatrix}$$

对于输出态对应的算符，对易关系也应成立：

$$[\hat{a}_2, \hat{a}_2^\dagger] = 1, [\hat{a}_3, \hat{a}_3^\dagger] = 1$$

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ -e^{i\phi}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix}$$

根据输入输出之间的变换关系，有

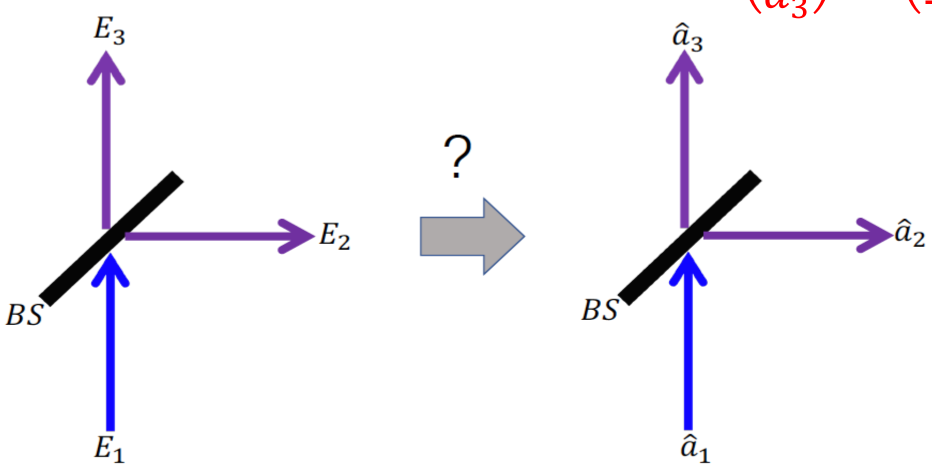
$$\begin{aligned}
&\left[\hat{a}_2, \hat{a}_2^\dagger\right] \\
&= \left[\cos\theta\,\hat{a}_0 + e^{-i\phi}\sin\theta\,\hat{a}_1, \cos\theta\,\hat{a}_0^\dagger + e^{i\phi}\sin\theta\,\hat{a}_1^\dagger\right] \\
&\quad = \cos^2\theta\left[\hat{a}_0, \hat{a}_0^\dagger\right] + \sin^2\theta\left[\hat{a}_1, \hat{a}_1^\dagger\right] + e^{-i\phi}\sin\theta\cos\theta\left[\hat{a}_1, \hat{a}_0^\dagger\right] \\
&\qquad + e^{i\phi}\sin\theta\cos\theta\left[\hat{a}_0, \hat{a}_1^\dagger\right] \\
&\quad = \cos^2\theta + \sin^2\theta \\
&\quad = 1
\end{aligned}$$

容易验证在分束器的变换下，$\left[\hat{a}_3, \hat{a}_3^\dagger\right] = 1$也满足

思考：有损耗的时候，对易关系是否满足？

如果经典分束时只有一束输入光，那么按下面的方式对应到量子分束的情况**是否正确**呢？

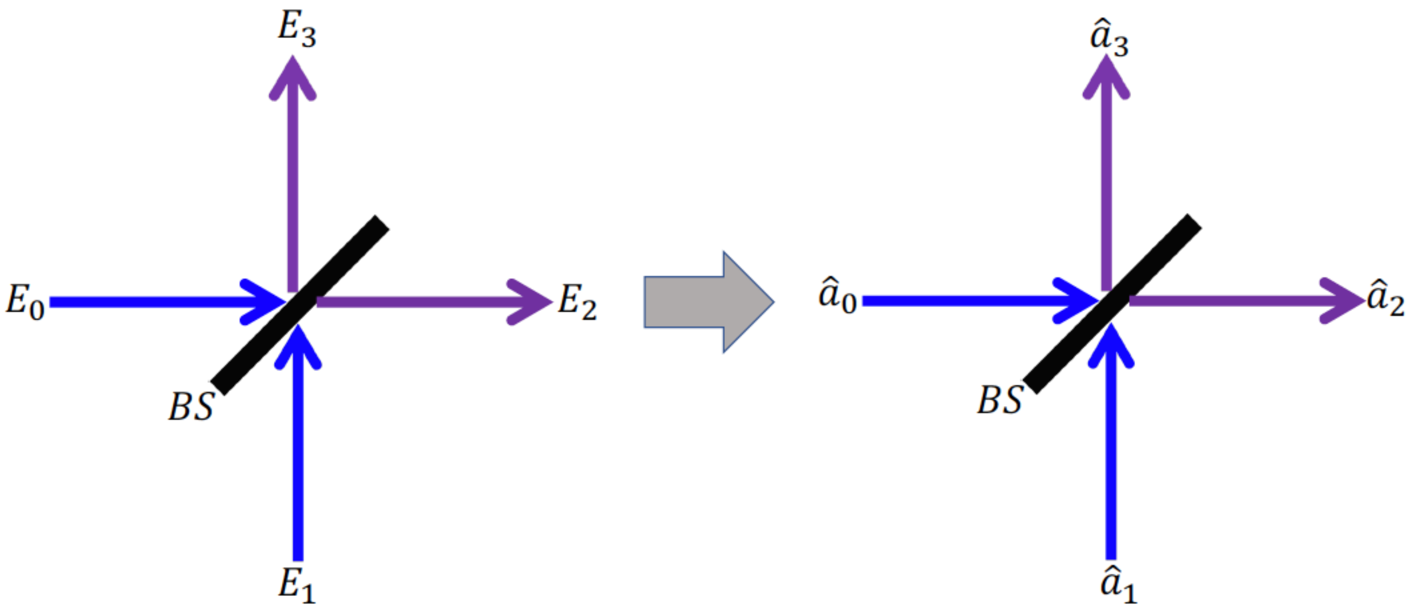
$$\begin{pmatrix}\hat{a}_2\\ \hat{a}_3\end{pmatrix}=\begin{pmatrix}\cos\theta & e^{-i\phi}\sin\theta\\ -e^{i\phi}\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}0\\ \hat{a}_1\end{pmatrix}\ ?$$



我们可以通过验证输出光场的算符之间的对易关系进行检验。

按照量子分束的变换关系，上图中$\hat{a}_2=e^{-i\phi}\sin\theta\,\hat{a}_1$，则$[\hat{a}_2,\hat{a}_2^\dagger]=[e^{-i\phi}\sin\theta\,\hat{a}_1,e^{i\phi}\sin\theta\,\hat{a}_1^\dagger]=\sin^2\theta$，不满足玻色对易关系

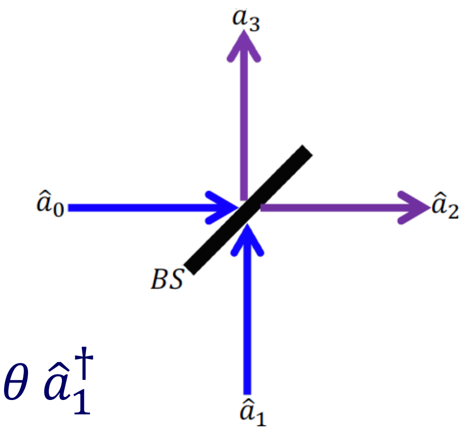
为了满足对易关系，经典中无输入的一端，在量子中也仍然对应一算符，对应真空态的输入，正确的形式如下：



$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ -e^{i\phi}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix}$$

☞输入和输出的对应关系（自己推一下）

当$\phi = -\frac{\pi}{2}$时，$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix}$



$$\begin{cases} \hat{a}_2 = \cos\theta\,\hat{a}_0 + \boldsymbol{i}\sin\theta\,\hat{a}_1 \\ \hat{a}_3 = \boldsymbol{i}\sin\theta\,\hat{a}_0 + \cos\theta\,\hat{a}_1 \end{cases} \Rightarrow \begin{cases} \hat{a}_2^\dagger = \cos\theta\,\hat{a}_0^\dagger - \boldsymbol{i}\sin\theta\,\hat{a}_1^\dagger \\ \hat{a}_3^\dagger = -\boldsymbol{i}\sin\theta\,\hat{a}_0^\dagger + \cos\theta\,\hat{a}_1^\dagger \end{cases}$$

由此得到

$$\begin{cases} \hat{a}_0 = \cos\theta\,\hat{a}_2 - \boldsymbol{i}\sin\theta\,\hat{a}_3 \\ \hat{a}_1 = -\boldsymbol{i}\sin\theta\,\hat{a}_2 + \cos\theta\,\hat{a}_3 \end{cases} \Rightarrow \begin{cases} \hat{a}_0^\dagger = \cos\theta\,\hat{a}_2^\dagger + \boldsymbol{i}\sin\theta\,\hat{a}_3^\dagger \\ \hat{a}_1^\dagger = \boldsymbol{i}\sin\theta\,\hat{a}_2^\dagger + \cos\theta\,\hat{a}_3^\dagger \end{cases}$$

$\boldsymbol{\theta} \to \boldsymbol{t}$,得到

$$\begin{cases} \hat{a}_2 = \hat{a}_0(\boldsymbol{t}) = \cos\boldsymbol{t}\,\hat{a}_0 + \boldsymbol{i}\sin\boldsymbol{t}\,\hat{a}_1 \\ \hat{a}_3 = \hat{a}_1(\boldsymbol{t}) = \boldsymbol{i}\sin\boldsymbol{t}\,\hat{a}_0 + \cos\boldsymbol{t}\,\hat{a}_1 \end{cases}$$

☞ 利用算符的演化关系构造有效哈密顿量

在海森堡表象下

$$\begin{cases} \hat{a}_2 = \hat{a}_0(t) = \cos t\, \hat{a}_0 + i \sin t\, \hat{a}_1 \\ \hat{a}_3 = \hat{a}_1(t) = i \sin t\, \hat{a}_0 + \cos t\, \hat{a}_1 \end{cases}$$

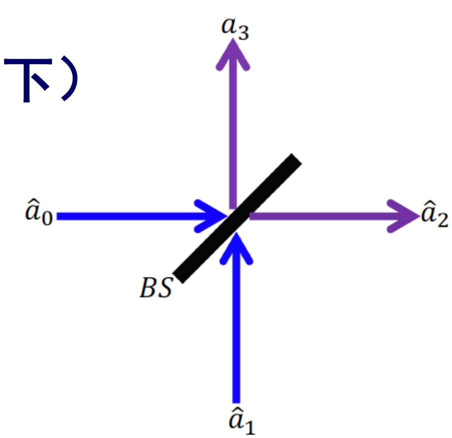
$$\begin{cases} \dfrac{d\hat{a}_0(t)}{dt} = -sin\, t\, \hat{a}_0 + icos\, t\, \hat{a}_1 = i\hat{a}_1(t) = i\hat{a}_3 \\ \dfrac{d\hat{a}_1(t)}{dt} = icos\, t\, \hat{a}_0 - sin\, t\, \hat{a}_1 = i\hat{a}_0(t) = i\hat{a}_2 \end{cases}$$

由此构造有效哈密顿量

$$H_{eff} = -\hbar g_0(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_0 \hat{a}_1^\dagger) \qquad g_0 = 1$$

可以验证满足海森堡运动方程（自己验证一下）

$$\begin{cases} \dfrac{d\hat{a}_0(t)}{dt} = \dfrac{i}{\hbar}[H_{eff}, \hat{a}_0(t)] \\ \dfrac{d\hat{a}_1(t)}{dt} = \dfrac{i}{\hbar}[H_{eff}, \hat{a}_1(t)] \end{cases}$$

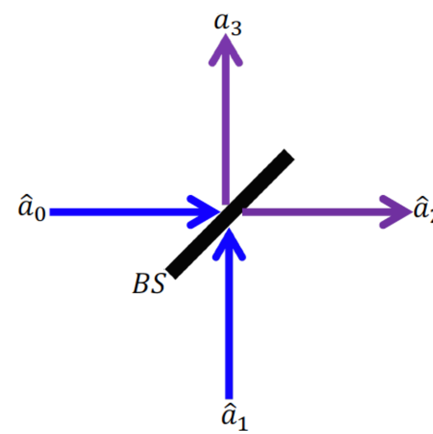


☞ 薛定谔表象下**波函数的演化**

$$i\hbar\frac{\partial}{\partial t}\psi(t)=H_{eff}\,\psi(t) \qquad\qquad H_{eff}=-\hbar g_0(\hat{a}_0^{\dagger}\hat{a}_1+\hat{a}_0\hat{a}_1^{\dagger})$$

在无损体系中， 且g_0=1

$$\psi(t)=U\psi(0) \qquad \mathrm{U}=\exp\left(-\frac{iH_{eff}\,t}{\hbar}\right)$$



于是时间演化算符

$$U=S(t)=exp[it(\hat{a}_0^{\dagger}\hat{a}_1+\hat{a}_0\hat{a}_1^{\dagger})]$$

其共轭形式为

$$U^{\dagger}=S^{\dagger}(t)=exp[-it(\hat{a}_1^{\dagger}\hat{a}_0+\hat{a}_1\hat{a}_0^{\dagger})]$$

满足$\boldsymbol{S^{\dagger}(t)=S(-t)}$

经过分束器之后，初末态之间的关系可以通过演化算符$\boldsymbol{S(t)}$得到

☞ **薛定谔表象和海森堡表象的联系**

两种表象下算符得到的期望值相等：

$$\langle\psi(0)|\mathrm{A}(\mathrm{t})|\psi(0)\rangle = \langle\psi(t)|\mathrm{A}(0)|\psi(t)\rangle = \left\langle\psi(0)\middle|U^{\dagger}A(0)U\middle|\psi(0)\right\rangle$$

海森堡表象　　　　薛定谔表象　　　　海森堡表象

因此，在海森堡表象下，算符的演化可以表示为

$$\mathrm{A}(t) = U^{\dagger}A(0)U = S^{\dagger}(t)A(0)S(t)$$

在分束器中，以θ取代t，可以得到

$$\begin{cases} \hat{a}_2 = \hat{a}_0(\theta) = S^{\dagger}(\theta)\hat{a}_0\mathrm{S}(\theta) \\ \hat{a}_3 = \hat{a}_1(\theta) = S^{\dagger}(\theta)\hat{a}_1\mathrm{S}(\theta) \end{cases}$$

$$\begin{pmatrix}\hat{a}_2\\ \hat{a}_3\end{pmatrix} = S^{\dagger}(\theta)\begin{pmatrix}\hat{a}_0\\ \hat{a}_1\end{pmatrix}\mathrm{S}(\theta),\quad \begin{pmatrix}\hat{a}_2^{\dagger}\\ \hat{a}_3^{\dagger}\end{pmatrix} = S^{\dagger}(\theta)\begin{pmatrix}\hat{a}_0^{\dagger}\\ \hat{a}_1^{\dagger}\end{pmatrix}\mathrm{S}(\theta)$$

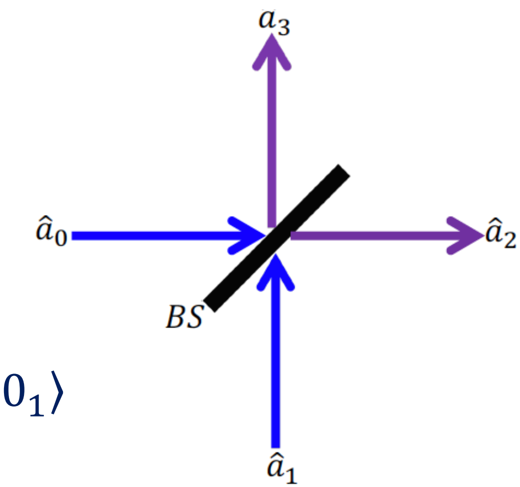
可以验证 $\mathrm{S}(\theta)|0{,}0\rangle = exp\left[i\theta\left(\hat{a}_0^{\dagger}\hat{a}_1 + \hat{a}_0\hat{a}_1^{\dagger}\right)\right]|0{,}0\rangle = |0{,}0\rangle$（提问）

即：如果初态是真空态，末态也是真空态

6.3 基于量子分束的量子态输出

6.3.1 输入Fock态

$$\begin{pmatrix}\hat{a}_2\\ \hat{a}_3\end{pmatrix}=\begin{pmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\hat{a}_0\\ \hat{a}_1\end{pmatrix}$$



1. 输入态为 $|0_0,1_1\rangle$

输出态为$|0_0,1_1\rangle_{out}=S(\theta)\big|0_0,1_1\rangle=S(\theta)\hat{a}_1^\dagger S^\dagger(\theta)S(\theta)\big|0_0,0_1\rangle$

$$=S^\dagger(-\theta)\hat{a}_1^\dagger S(-\theta)|0_0,0_1\rangle$$

$$=\hat{a}_1^\dagger(-\theta)\big|0_0,0_1\rangle=\big(i\sin\theta\,\hat{a}_0^\dagger+\cos\theta\,\hat{a}_1^\dagger\big)\big|0_0,0_1\rangle$$

$$=i\sin\theta\,|1_0,0_1\rangle+\cos\theta\,|0_0,1_1\rangle$$

2. 输入态为$|1_0,0_1\rangle$（自己算）

输出态为$|1_0,0_1\rangle_{out}=S(\theta)\big|1_0,0_1\rangle=S(\theta)\hat{a}_0^\dagger S^\dagger(\theta)S(\theta)\big|0_0,0_1\rangle$

$$=S^\dagger(-\theta)\hat{a}_0^\dagger S(-\theta)|0_0,0_1\rangle$$

$$=\hat{a}_0^\dagger(-\theta)\big|0_0,0_1\rangle=\big(\cos\theta\,\hat{a}_0^\dagger+i\sin\theta\,\hat{a}_1^\dagger\big)\big|0_0,0_1\rangle$$

$$=i\sin\theta\,|0_0,1_1\rangle+\cos\theta\,|1_0,0_1\rangle$$

$$\psi(\theta)=S(\theta)\psi(0)=exp\big[i\theta\big(\hat{a}_0^\dagger\hat{a}_1+\hat{a}_0\hat{a}_1^\dagger\big)\big]\psi(0)$$

3、输入态为$|1,1\rangle$（自己算）

输出态为$|1,1\rangle_{out} = S(\theta)|1,1\rangle$

$$= S(\theta)\hat{a}_1^\dagger S^\dagger(\theta)S(\theta)\hat{a}_0^\dagger S^\dagger(\theta)S(\theta)|0,0\rangle$$

$$= S^\dagger(-\theta)\hat{a}_1^\dagger S(-\theta)S^\dagger(-\theta)\hat{a}_0^\dagger S(-\theta)|0,0\rangle$$

$$= \hat{a}_1^\dagger(-\theta)\hat{a}_0^\dagger(-\theta)|0,0\rangle$$

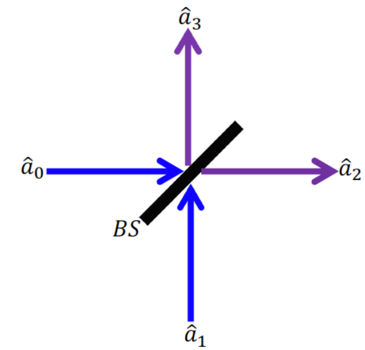
$$= \left(i\,sin\,\theta\,\hat{a}_0^\dagger + cos\,\theta\,\hat{a}_1^\dagger\right)\left(cos\,\theta\,\hat{a}_0^\dagger + i\,sin\,\theta\,\hat{a}_1^\dagger\right)|0,0\rangle$$

$$= \left[i\,cos\theta\,sin\theta\,\hat{a}_0^\dagger\hat{a}_0^\dagger + i\,cos\theta\,sin\theta\,\hat{a}_1^\dagger\hat{a}_1^\dagger + (cos^2\theta - sin^2\theta)\hat{a}_0^\dagger\hat{a}_1^\dagger\right]|0,0\rangle$$

$$= \frac{i}{\sqrt{2}}sin\,2\theta(|2,0\rangle + |0,2\rangle) + (cos^2\theta - sin^2\theta)|1,1\rangle$$

$$|1,1\rangle_{out} = S(\theta)|1,1\rangle$$

$$= \frac{i}{\sqrt{2}} sin\ 2\theta(|2,0\rangle + |0,2\rangle) + (cos^2\theta - sin^2\theta)|1,1\rangle$$



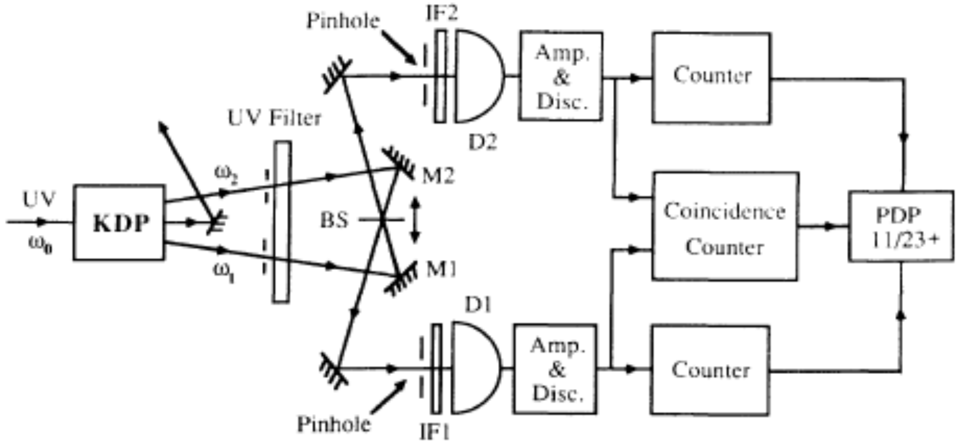
当$\boldsymbol{\theta = \frac{\pi}{4}}$时，归一化输出态为

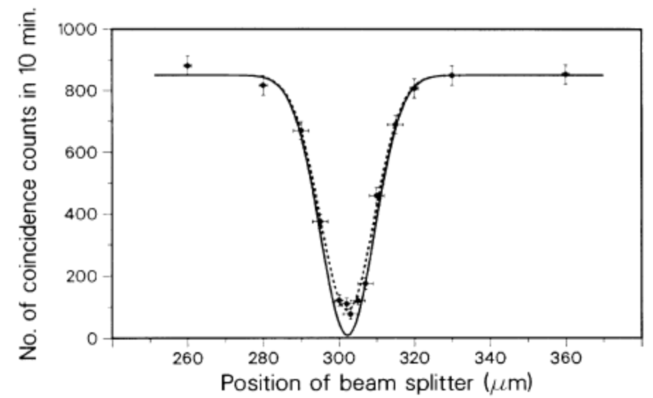
$$|1,1\rangle_{out} = S(\theta)|1,1\rangle = \frac{i}{\sqrt{2}}(|2,0\rangle + |0,2\rangle)$$

即2端口输出双光子及3端口输出双光子的叠加态。此时，对该态进行测量，总是在同一个端口探测到两个光子（*著名的Hong_Ou_mandel实验*），不违背光子数守恒条件的$|1\rangle_2|1\rangle_3$态却不能被探测到，这是一种量子干涉现象 (*Quantum interference effect*)

Hong-Ou-Mandel实验

相当于 $g^{(2)}(\tau)$





$$|\psi_{\text{out}}\rangle = (R-T)|1_1,1_2\rangle + i(2RT)^{1/2}|2_1,0_2\rangle + i(2RT)^{1/2}|0_1,2_2\rangle, \qquad R/T = 0.95$$

where R and T are the reflectivity and transmissivity of the beam splitter, with $R+T=1$.

HONG, CK (HONG, CK) OU, ZY (OU, ZY) MANDEL, L (MANDEL, L），**MEASUREMENT OF SUBPICOSECOND TIME INTERVALS BETWEEN 2 PHOTONS BY INTERFERENCE**，Phys. Rev. Lett. **59**, 2044 – Published 2 November 1987，**引用2700余次（截止到2021），原理性实验**

4、输入态为$|2,1\rangle$

输出态为$|2,1\rangle_{out} = S(\theta)|2,1\rangle = S(\theta)\frac{1}{\sqrt{2}}\hat{a}_1^{\dagger}\hat{a}_0^{\dagger}\hat{a}_0^{\dagger}|0,0\rangle$

$$= \frac{1}{\sqrt{2}} S(\theta)\hat{a}_1^{\dagger}S^{\dagger}(\theta)S(\theta)\hat{a}_0^{\dagger}S^{\dagger}(\theta)S(\theta)\hat{a}_0^{\dagger}S^{\dagger}(\theta)S(\theta)|0,0\rangle$$

$$= \frac{1}{\sqrt{2}} S^{\dagger}(-\theta)\hat{a}_1^{\dagger}S(-\theta)S^{\dagger}(-\theta)\hat{a}_0^{\dagger}S(-\theta)S^{\dagger}(-\theta)\hat{a}_0^{\dagger}S(-\theta)|0,0\rangle$$

$$= \frac{1}{\sqrt{2}} \hat{a}_1^{\dagger}(-\theta)\hat{a}_0^{\dagger}(-\theta)\hat{a}_0^{\dagger}(-\theta)|0,0\rangle$$

$$= \frac{1}{\sqrt{2}} \left(i\,sin\,\theta\,\hat{a}_0^{\dagger} + cos\,\theta\,\hat{a}_1^{\dagger}\right)(cos\,\theta\,\hat{a}_0^{\dagger} + i\,sin\,\theta\,\hat{a}_1^{\dagger})^2|0,0\rangle$$

$$= \frac{1}{\sqrt{2}}\ (icos^2\theta\ sin\theta\hat{a}_0^{\dagger 3} + cos^3\theta\hat{a}_0^{\dagger 2}\hat{a}_1^{\dagger} - isin^3\theta\hat{a}_0^{\dagger}\hat{a}_1^{\dagger 2} -$$

$$cos\theta sin^2\theta\hat{a}_1^{\dagger 3} - 2cos\theta sin^2\theta\hat{a}_0^{\dagger 2}\hat{a}_1^{\dagger} + 2icos^2\theta sin\theta\hat{a}_0^{\dagger}\hat{a}_1^{\dagger 2})|0,0\rangle$$

$$=\frac{1}{\sqrt{2}}\left(icos^2\theta\, sin\theta \hat{a}_0^{\dagger 3} + cos^3\theta \hat{a}_0^{\dagger 2}\hat{a}_1^{\dagger} - isin^3\theta \hat{a}_0^{\dagger}\hat{a}_1^{\dagger 2} - cos\theta sin^2\theta \hat{a}_1^{\dagger 3} - 2cos\theta sin^2\theta \hat{a}_0^{\dagger 2}\hat{a}_1^{\dagger} + 2icos^2\theta sin\theta \hat{a}_0^{\dagger}\hat{a}_1^{\dagger 2}\right)|0,0\rangle$$

$$= \frac{1}{\sqrt{2}}[icos^2\theta\, sin\theta|3,0\rangle + (cos^3\theta - 2cos\theta sin^2\theta)|2,1\rangle$$

$$+ (-isin^3\theta + 2icos^2\theta sin\theta)|1,2\rangle - cos\theta sin^2\theta|0,3\rangle]$$

输出的态是四种叠加态，典型的量子干涉

可以态重构、制备纠缠态

按照此原则，可以计算任意两个Fock态输入，经过分束器的输出态

5、输入态为$|N,M\rangle$

输出为$|N,M\rangle_{out} = S(\theta)|N,M\rangle = S(\theta)\frac{1}{\sqrt{N!}}\frac{1}{\sqrt{M!}}\hat{a}_{\mathbf{1}}^{\dagger^M}\hat{a}_0^{\dagger^N}|0,0\rangle$

$$= \frac{1}{\sqrt{N!}}\frac{1}{\sqrt{M!}}(i\, sin\,\theta\, \hat{a}_0^\dagger + cos\,\theta\, \hat{a}_1^\dagger)^M (cos\,\theta\, \hat{a}_0^\dagger + i\, sin\,\theta\, \hat{a}_1^\dagger)^N|0,0\rangle$$

特别的，当$\theta = \frac{\pi}{4}$且$\mathbf{N = M}$时，输出态为

$$\frac{(i\hat{a}_0^\dagger + \hat{a}_1^\dagger)^N(\hat{a}_0^\dagger + i\hat{a}_1^\dagger)^N}{N!\,2^N}|0,0\rangle = \frac{i^N}{N!\,2^N}(\hat{a}_0^{\dagger 2} + \hat{a}_1^{\dagger 2})^N|0,0\rangle$$

$$= \frac{i^N}{N!\,2^N}\sum_{m=0}^{N}\binom{N}{m}\sqrt{(2m)!\,(2N-2m)!}\,|2m, 2N-2m\rangle$$

可能输出的光子数均为偶数

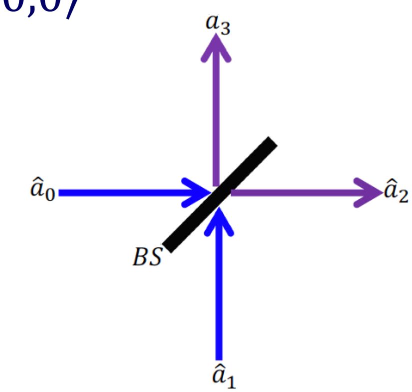
$$\begin{cases} \hat{a}_2^\dagger = \hat{a}_0^\dagger(\theta) = cos\,\theta\,\hat{a}_0^\dagger - i\,sin\,\theta\,\hat{a}_1^\dagger \\ \hat{a}_3^\dagger = \hat{a}_1^\dagger(\theta) = -i\,sin\,\theta\,\hat{a}_0^\dagger + cos\,\theta\,\hat{a}_1^\dagger \end{cases}$$

6.3.2 输入连续变量

$$\psi(\theta) = S(\theta)\psi(0) = exp\left[i\theta\left(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_0 \hat{a}_1^\dagger\right)\right]\psi(0)$$

1、输入态为$|\alpha, 0\rangle$

输出态为$|\alpha, 0\rangle_{out} = S(\theta)|\alpha, 0\rangle_{in} = S(\theta)D_0(\alpha)S^\dagger(\theta)S(\theta)|0,0\rangle$



$$= S^\dagger(-\theta)D_0(\alpha)S(-\theta)|0,0\rangle$$

$$= S^\dagger(-\theta)exp\left(\alpha\hat{a}_0^\dagger - \alpha^*\hat{a}_0\right)S(-\theta)|0,0\rangle$$

$$= exp\left[\alpha\hat{a}_0^\dagger(-\theta) - \alpha^*\hat{a}_0(-\theta)\right]|0,0\rangle$$

$$= exp\left[\alpha\left(\cos\theta\,\hat{a}_0^\dagger + i\sin\theta\,\hat{a}_1^\dagger\right) - \alpha^*(\cos\theta\,\hat{a}_0 - i\sin\theta\,\hat{a}_1)\right]|0,0\rangle$$

$$= exp\left(cos\,\theta\,\alpha\hat{a}_0^\dagger - \cos\theta\,\alpha^*\hat{a}_0\right)exp\left(isin\theta\,\alpha\hat{a}_1^\dagger - i\sin\theta\,\alpha^*\hat{a}_1\right)|0,0\rangle$$

$$= |\alpha cos\theta, i\alpha sin\theta\rangle$$

类似于经典分束（提问）

$$\begin{cases} \hat{a}_2 = \hat{a}_0(\theta) = cos\,\theta\,\hat{a}_0 + i\,sin\,\theta\,\hat{a}_1 \\ \hat{a}_3 = \hat{a}_1(\theta) = i\,sin\,\theta\,\hat{a}_0 + cos\,\theta\,\hat{a}_1 \end{cases} \qquad \begin{cases} \hat{a}_2^\dagger = \hat{a}_0^\dagger(\theta) = cos\,\theta\,\hat{a}_0^\dagger - i\,sin\,\theta\,\hat{a}_1^\dagger \\ \hat{a}_3^\dagger = \hat{a}_1^\dagger(\theta) = -i\,sin\,\theta\,\hat{a}_0^\dagger + cos\,\theta\,\hat{a}_1^\dagger \end{cases}$$

2、输入态为$|0,\alpha\rangle$

输出态为$|0,\alpha\rangle_{out}=S(\theta)|0,\alpha\rangle_{in}=S(\theta)D_1(\alpha)S^{\dagger}(\theta)S(\theta)|0,0\rangle$

$$=S^{\dagger}(-\theta)D_1(\alpha)S(-\theta)|0,0\rangle$$

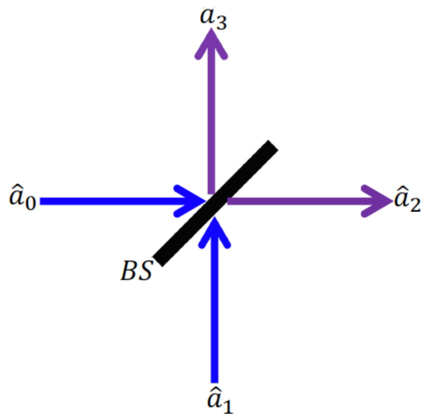
$$=S^{\dagger}(-\theta)exp\left(\alpha\hat{a}_1^{\dagger}-\alpha^*\hat{a}_1\right)S(-\theta)|0,0\rangle$$

$$=exp\left[\alpha\hat{a}_1^{\dagger}(-\theta)-H.c.\right]|0,0\rangle$$

$$=exp\left[\alpha(i\sin\theta\,\hat{a}_0^{\dagger}+\cos\theta\,\hat{a}_1^{\dagger})-H.c.\right]|0,0\rangle$$

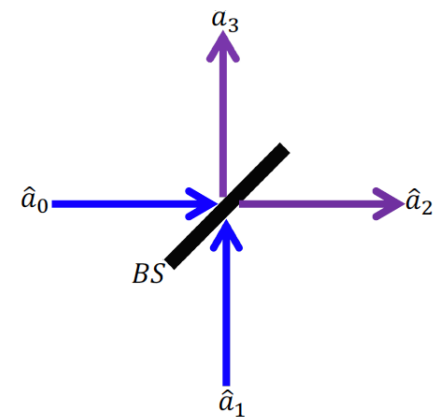
$$=exp\left(isin\theta\,\alpha\hat{a}_0^{\dagger}-H.c.\right)exp\left(cos\theta\,\alpha\hat{a}_1^{\dagger}-H.c.\right)|0,0\rangle$$

$$=|i\alpha sin\theta,\alpha cos\theta\rangle$$



类似于经典分束

3、输入态为 $|\alpha,\beta\rangle$



输出态为 $|\alpha,\beta\rangle_{out} = S(\theta)|\alpha,\beta\rangle_{in}$

$$= S(\theta)D_0(\alpha)D_1(\beta)S^{\dagger}(\theta)S(\theta)|0,0\rangle$$

$$= S^{\dagger}(-\theta)exp\left[\alpha\hat{a}_0^{\dagger} - \alpha^*\hat{a}_0 + \beta\hat{a}_1^{\dagger} - \beta^*\hat{a}_1\right]S(-\theta)|0,0\rangle$$

$$= exp\left[\alpha\hat{a}_0^{\dagger}(-\theta) - \alpha^*\hat{a}_0(-\theta) + \beta\hat{a}_1^{\dagger}(-\theta) - \beta^*\hat{a}_1(-\theta)\right]|0,0\rangle$$

$$= exp\left[\alpha\left(cos\,\theta\,\hat{a}_0^{\dagger} + i\,sin\,\theta\,\hat{a}_1^{\dagger}\right) - \alpha^*(\cos\theta\,\hat{a}_0 - i\sin\theta\,\hat{a}_1)\right.$$

$$\left. + \beta\left(i\,sin\,\theta\,\hat{a}_0^{\dagger} + cos\,\theta\,\hat{a}_1^{\dagger}\right) - \beta^*(-i\sin\theta\,\hat{a}_0 + \cos\theta\,\hat{a}_1)\right]|0,0\rangle$$

$$= exp\left(i\beta sin\theta\,\hat{a}_0^{\dagger} + \alpha\cos\theta\,\hat{a}_0^{\dagger} - H.c.\right)exp\left(\beta cos\theta\,\hat{a}_1^{\dagger} + i\alpha\sin\theta\,\hat{a}_1^{\dagger}\right.$$

$$\left. - H.c.\right)|0,0\rangle = |\alpha cos\theta + i\beta sin\theta, \beta cos\theta + i\alpha\sin\theta\rangle = |\alpha(\theta),\beta(\theta)\rangle$$

$$\begin{pmatrix} E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} cos\,\theta & isin\,\theta \\ isin\,\theta & cos\,\theta \end{pmatrix}\begin{pmatrix} E_0 \\ E_1 \end{pmatrix}$$

惊现经典结果

4、输入双模压缩态

$$|in\rangle = exp\left(\xi \hat{a}_0^\dagger \hat{a}_1^\dagger - \xi^* \hat{a}_0 \hat{a}_1\right)|0,0\rangle$$

输出态为 $|out\rangle = S(\theta)|in\rangle = S(\theta)exp\left(\xi \hat{a}_0^\dagger \hat{a}_1^\dagger - \xi^* \hat{a}_0 \hat{a}_1\right)S^\dagger(\theta)S(\theta)|0,0\rangle$

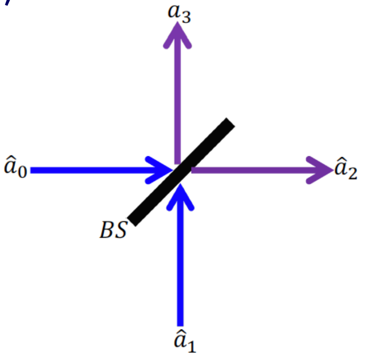
$$= exp[\xi \hat{a}_0^\dagger(-\theta)\hat{a}_1^\dagger(-\theta) - H.c.]|0,0\rangle$$

$$= exp\left[\xi\left(cos\,\theta\,\hat{a}_0^\dagger + i\,sin\,\theta\,\hat{a}_1^\dagger\right)\left(i\,sin\,\theta\,\hat{a}_0^\dagger + cos\,\theta\,\hat{a}_1^\dagger\right) - H.c.\right]|0,0\rangle$$

$$= exp\{\xi\left[\hat{a}_0^\dagger \hat{a}_1^\dagger\,cos2\theta + \frac{i}{2}\left(\hat{a}_0^{\dagger 2} + \hat{a}_1^{\dagger 2}\right)sin2\theta\right] - H.c.\}|0,0\rangle$$

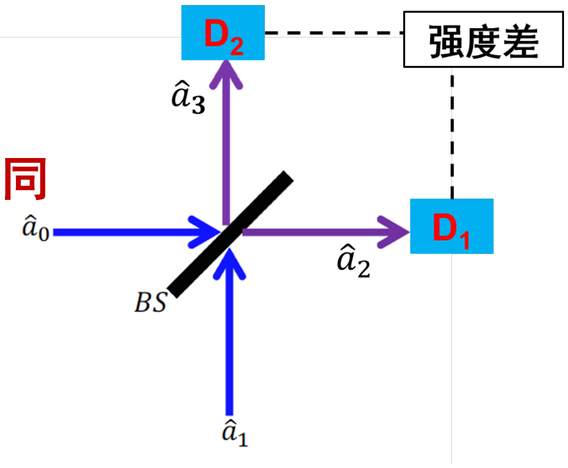
特别的，当 $\theta = \frac{\pi}{4}$ **时，**

$$|out\rangle = exp\left[\frac{i}{2}(\xi \hat{a}_0^{\dagger 2} + \xi^* \hat{a}_0^2)\right] exp\left[\frac{i}{2}(\xi \hat{a}_1^{\dagger 2} + \xi^* \hat{a}_1^2)\right]|0,0\rangle$$



双模压缩态变为两个单模压缩态的直积并从两个端口分别输出

6.3.3 量子分束的应用之一：平衡零拍探测



Balanced homodyne interferometer

平衡：分束器半透半反；零拍：信号光和本地光频率同

- **经过50:50分束器** $\begin{pmatrix}\hat{a}_2\\ \hat{a}_3\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\hat{a}_0\\ \hat{a}_1\end{pmatrix}$

于是$\hat{a}_2 = \frac{1}{\sqrt{2}}(\hat{a}_0 + \mathrm{i}\hat{a}_1)$ ， $\hat{a}_3 = \frac{1}{\sqrt{2}}(i\hat{a}_0 + \hat{a}_1)$

$$\hat{n}_{23} = \hat{n}_2 - \hat{n}_3 = \hat{a}_2^\dagger\hat{a}_2 - \hat{a}_3^\dagger\hat{a}_3$$

$$= \frac{1}{2}\left[(\hat{a}_0^\dagger - \mathrm{i}\hat{a}_1^\dagger)(\hat{a}_0 + \mathrm{i}\hat{a}_1) - (-i\hat{a}_0^\dagger + \hat{a}_1^\dagger)(i\hat{a}_0 + \hat{a}_1)\right]$$

$$= i(\hat{a}_0^\dagger\hat{a}_1 - \hat{a}_1^\dagger\hat{a}_0)$$

- **当$\hat{a}_1$端输入的态为相干态$|\beta\rangle$，而$\hat{a}_0$输入的态为未知量子态$|\psi\rangle$时**

$$\langle\hat{n}_{23}\rangle = \langle\beta,\psi|i(\hat{a}_0^\dagger\hat{a}_1 - \hat{a}_1^\dagger\hat{a}_0)|\psi,\beta\rangle$$

$$\langle\hat{n}_{23}\rangle = i\langle\beta,\psi|\hat{a}_0^\dagger\beta - \beta^*\hat{a}_0|\psi,\beta\rangle$$

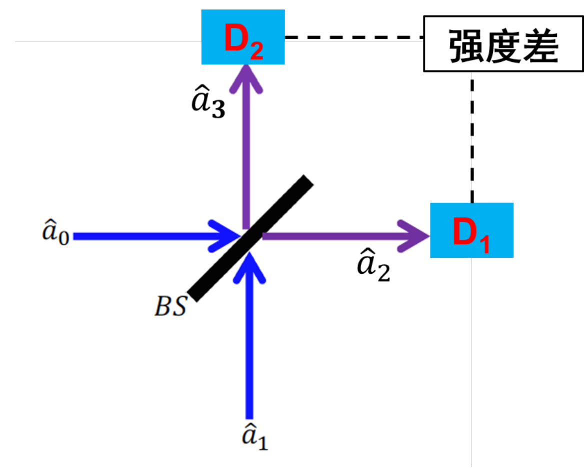
➢ **不妨设**$\beta = |\beta| e^{ix}$,**则**$\beta^* = |\beta| e^{-ix}$

因此

$$\langle \hat{n}_{23} \rangle = i \langle \beta, \psi | \hat{a}_0^\dagger \beta - \beta^* \hat{a}_0 | \psi, \beta \rangle$$

$$= |\beta| \left\langle \psi \left| \left(\hat{a}_0^\dagger e^{i\left(x+\frac{\pi}{2}\right)} + \hat{a}_0 e^{-i\left(x+\frac{\pi}{2}\right)} \right) \right| \psi \right\rangle = 2|\beta| \langle X(\theta) \rangle$$

其中$X(\theta) = \frac{1}{2}\left(\hat{a}_0^\dagger e^{i\theta} + \hat{a}_0 e^{-i\theta}\right)$ $\theta = x + \frac{\pi}{2}$ and $\langle \hat{n}_0 \rangle$=0



$$\langle \Delta \hat{n}_{23} \rangle^2 = \langle \hat{n}_{23}^2 \rangle - \langle \hat{n}_{23} \rangle^2$$

$$= 4|\beta|^2 (\langle X^2(\theta) \rangle - \langle X(\theta) \rangle^2)$$

$$= 4|\beta|^2 \langle \Delta X(\theta) \rangle^2$$

➢ **通过改变本地相干光场的相位**x**，进而改变**θ**，可以测量光场在任意方向的正交分量**

➢ **当**$\hat{a}_0$**端口输入的态为**$|\psi\rangle = |0\rangle$**时** $\langle X(\theta) \rangle^2 = \frac{1}{4}$

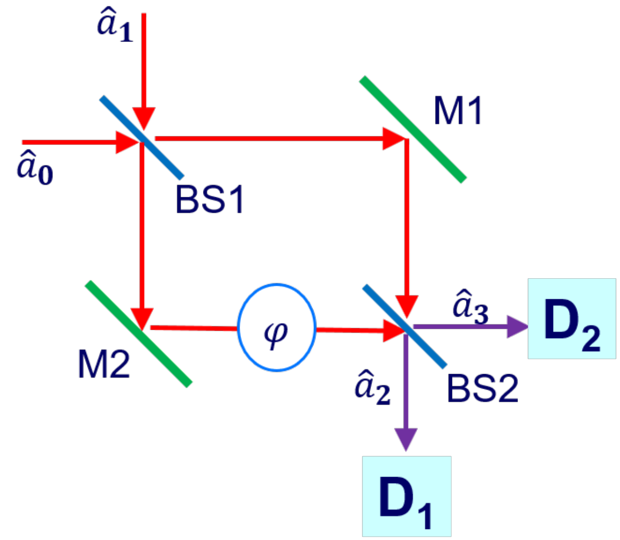
➢ 若测出来$\langle X(\theta) \rangle^2 < \frac{1}{4}$ **时，则对应的**$\hat{a}_0$**输入的为压缩光场**

6.3.4 标准量子极限

➢ **M-Z干涉仪输出光场与输入光场之间的关系：**

$$\begin{pmatrix}\hat{a}_2\\ \hat{a}_3\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{i\varphi}\end{pmatrix}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}\hat{a}_0\\ \hat{a}_1\end{pmatrix}$$

（自己算一下）



$$\hat{a}_2=\frac{1}{2}\left(1-e^{i\varphi}\right)\hat{a}_0+\frac{i}{2}\left(1+e^{i\varphi}\right)\hat{a}_1$$

$$\hat{a}_3=\frac{i}{2}\left(1+e^{i\varphi}\right)\hat{a}_0-\frac{1}{2}\left(1-e^{i\varphi}\right)\hat{a}_1$$

➢ **特别的**

当$\varphi=0$时，$\hat{a}_2=i\hat{a}_1$，$\hat{a}_3=i\hat{a}_0$ **（置换）**

当$\varphi=\pi$时，$\hat{a}_2=\hat{a}_0$，$\hat{a}_3=\hat{a}_1$ **（通过）**

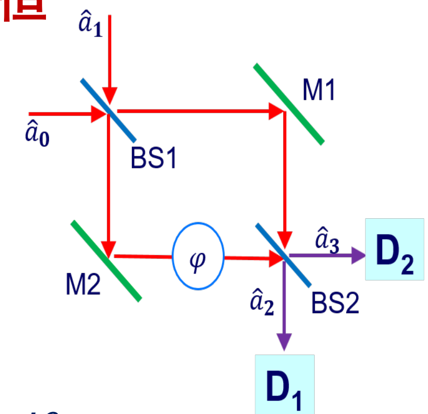
- **对于一般的φ（自己算一下）**

$$\hat{n}_2=\hat{a}_2^\dagger\hat{a}_2=sin^2\frac{\varphi}{2}\hat{a}_0^\dagger\hat{a}_0+cos^2\frac{\varphi}{2}\hat{a}_1^\dagger\hat{a}_1-cos\frac{\varphi}{2}sin\frac{\varphi}{2}(\hat{a}_0^\dagger\hat{a}_1+\hat{a}_1^\dagger\hat{a}_0)$$

$$\hat{n}_3=\hat{a}_3^\dagger\hat{a}_3=cos^2\frac{\varphi}{2}\hat{a}_0^\dagger\hat{a}_0+sin^2\frac{\varphi}{2}\hat{a}_1^\dagger\hat{a}_1+cos\frac{\varphi}{2}sin\frac{\varphi}{2}(\hat{a}_0^\dagger\hat{a}_1+\hat{a}_1^\dagger\hat{a}_0)$$

可以验证：$\boldsymbol{\hat{n}_2+\hat{n}_3=\hat{a}_0^\dagger\hat{a}_0+\hat{a}_1^\dagger\hat{a}_1=\hat{n}_0+\hat{n}_1}$ **能量守恒**

$$\hat{n}_{23}=\hat{n}_2-\hat{n}_3=cos\varphi\left(\hat{a}_1^\dagger\hat{a}_1-\hat{a}_0^\dagger\hat{a}_0\right)-sin\varphi\left(\hat{a}_0^\dagger\hat{a}_1+\hat{a}_1^\dagger\hat{a}_0\right)$$



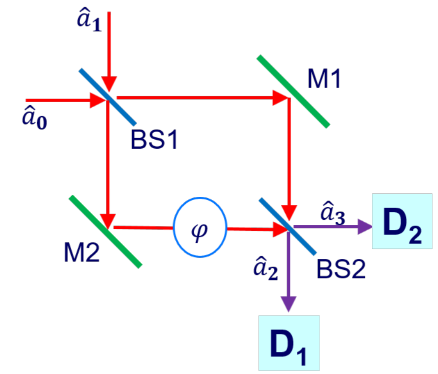
- **当$\boldsymbol{\varphi=\frac{\pi}{2}}$时，上式简化为**$\hat{n}_{23}=-(\hat{a}_0^\dagger\hat{a}_1+\hat{a}_1^\dagger\hat{a}_0)$

- **如果此时$\hat{a}_1$端口输入相干光场使得**$\langle\hat{a}_1\rangle=\beta=|\beta|e^{i\theta}$

则$\langle\hat{n}_{23}\rangle=-|\beta|\langle(\hat{a}_0^\dagger e^{i\theta}+\hat{a}_0e^{-i\theta})\rangle=-2|\beta|\langle X(\theta)\rangle$

对应于之前平衡零拍探测的结果

$$\hat{n}_{23} = cos\varphi\,(\hat{a}_1^\dagger\hat{a}_1 - \hat{a}_0^\dagger\hat{a}_0) - sin\varphi\,(\hat{a}_0^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_0)$$



➢ **另一种考虑，如果此时$\hat{a}_0$端口输入真空态**

则$\langle\hat{n}_{23}\rangle = \langle\beta,0|cos\varphi\,(\hat{a}_1^\dagger\hat{a}_1 - \hat{a}_0^\dagger\hat{a}_0) - sin\varphi\,(\hat{a}_0^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_0)|0,\beta\rangle$

$= |\beta|^2 cos\varphi = n_l cos\varphi$

在这种情况下，$\varphi = \pi/2$时， $\hat{n}_{23} = -$（$\hat{a}_0^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_0$）， 而$\langle\hat{n}_{23}\rangle = n_l cos\varphi = 0$

$$\langle\Delta\hat{n}_{23}\rangle^2_{\varphi=\pi/2} = \langle\beta,0|(\hat{a}_0^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_0)(\hat{a}_0^\dagger\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_0)|0,\beta\rangle - (n_l cos\varphi\,)^2 = n_l$$

上述结果是在$\hat{a}_0$端口输入真空态， $\hat{a}_1$端口输入相干光场且$\varphi = \frac{\pi}{2}$下得到的

➢ **另一方面, 测量相位时灵敏度可以表示为**$\left|\frac{\partial\langle\hat{n}_{23}\rangle}{\partial\varphi}\right| = n_l sin\varphi \le n_l$

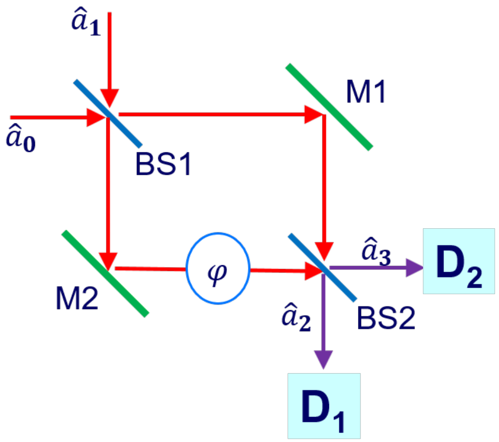
当$\varphi = \pi/2$时，灵敏度最大， n_l越大，灵敏度越高

- 相位不确定度$\Delta\varphi = \frac{\Delta\hat{n}_{23}}{\left|\frac{\partial\langle\hat{n}_{23}\rangle}{\partial\varphi}\right|} = \frac{\sqrt{n_l}}{n_l} = \frac{1}{\sqrt{n_l}}$ 标准量子极限

- 如果$\hat{a}_0$端口输入真空压缩态，且压缩度为e^{-r}

则相位不确定度$\Delta\varphi = \frac{e^{-r}}{\sqrt{n_l}}$

当r很大时，可以提高标准量子极限



6.4. 其它相关内容

量子分束中损耗的考虑

$$\hat{a}_{\rm out}(\omega)=t(\omega)\hat{a}_{\rm in}(\omega)+r(\omega)\hat{b}_{\rm in}(\omega)+\boxed{\hat{F}_a(\omega),}$$

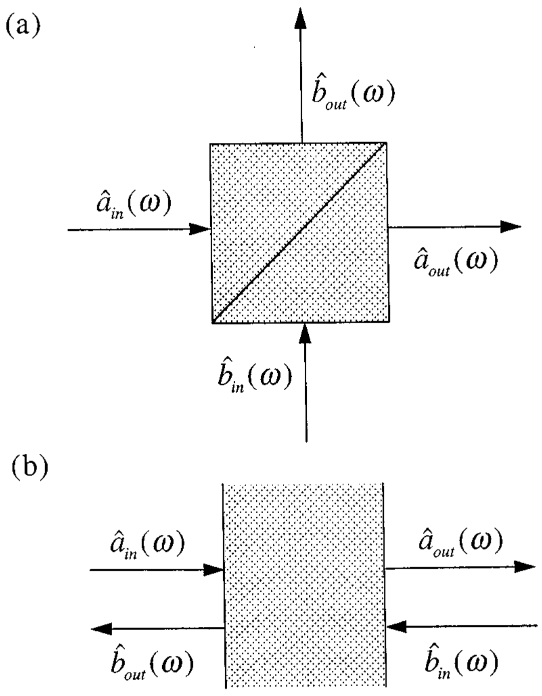
$$\hat{b}_{\rm out}(\omega)=t(\omega)\hat{b}_{\rm in}(\omega)+r(\omega)\hat{a}_{\rm in}(\omega)+\boxed{\hat{F}_b(\omega).} \quad (2.5)$$

$$|t(\omega)|^2+|r(\omega)|^2\leqslant 1$$

$$[\hat{a}_{\rm in}(\omega),\hat{F}_a^\dagger(\omega')]=[\hat{a}_{\rm in}(\omega),\hat{F}_b^\dagger(\omega')]=[\hat{a}_{\rm in}(\omega),\hat{F}_a(\omega')]$$
$$=[\hat{a}_{\rm in}(\omega),\hat{F}_b(\omega')]=0 \quad (2.6)$$

$$[\hat{F}_a(\omega),\hat{F}_a^\dagger(\omega')]=\delta(\omega-\omega')\{1-|t(\omega)|^2-|r(\omega)|^2\}$$
$$=[\hat{F}_b(\omega),\hat{F}_b^\dagger(\omega')],$$

$$[\hat{F}_a(\omega),\hat{F}_b^\dagger(\omega')]=-\delta(\omega-\omega')\{t(\omega)r^*(\omega)+r(\omega)t^*(\omega)\}$$
$$=[\hat{F}_b(\omega),\hat{F}_a^\dagger(\omega')]. \quad (2.7)$$



如果损耗是对称的，以后基本上都能去掉

Stephen M. Barnett, John Jeffers, and Alessandra Gatti，Quantum optics of lossy beam splitters，PHYSICAL REVIEW A VOLUME 57, NUMBER 3，2134 （1998）

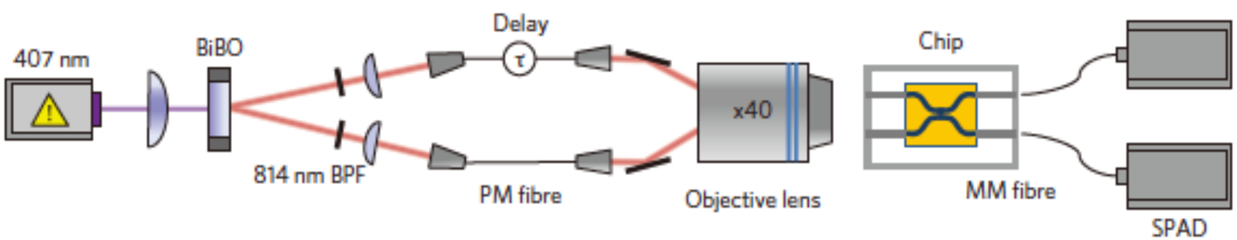
nature photonics

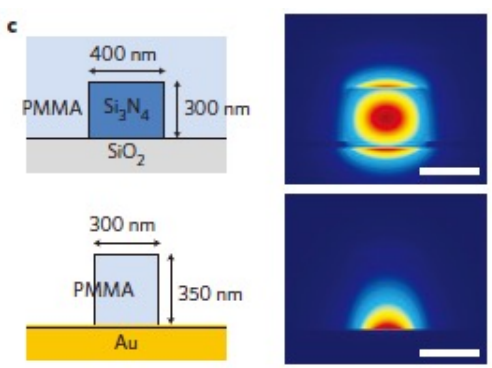
LETTERS

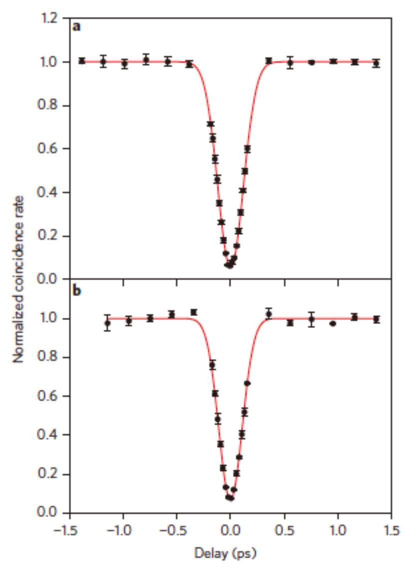
PUBLISHED ONLINE: 2 MARCH 2014 | DOI: 10.1038/NPHOTON.2014.40

Two-plasmon quantum interference

James S. Fakonas[1,2], Hyunseok Lee[2], Yousif A. Kelaita[2] and Harry A. Atwater[1,2]*







如果损耗不对称呢？
PT对称时候如何？

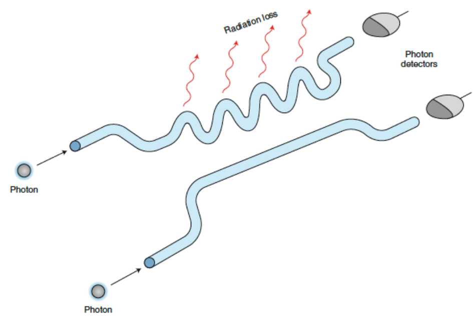
nature photonics

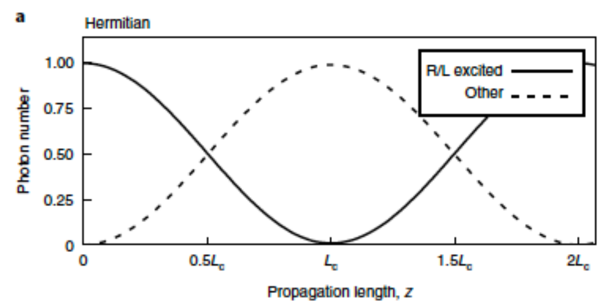
ARTICLES

https://doi.org/10.1038/s41566-019-0517-0

Observation of PT-symmetric quantum interference

F. Klauck[1,3], L. Teuber[1,3], M. Ornigotti[2], M. Heinrich[1], S. Scheel[1] and A. Szameit[1*]



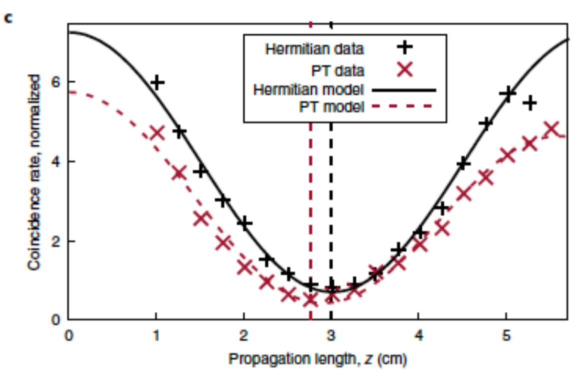


$$H = \kappa\left(a_{\mathrm{L}}^{\dagger} a_{\mathrm{R}} + a_{\mathrm{L}} a_{\mathrm{R}}^{\dagger}\right)$$

$$\partial_z \rho(z) = -\frac{i}{\hbar}[H,\rho] + \gamma\left(2a_{\mathrm{L}}\rho a_{\mathrm{L}}^{\dagger} + \left\{a_{\mathrm{L}}^{\dagger} a_{\mathrm{L}}, \rho\right\}_{+}\right) = \mathcal{L}\rho$$

$$\rho(z) = \mathrm{e}^{\mathcal{L}z}\rho_0$$

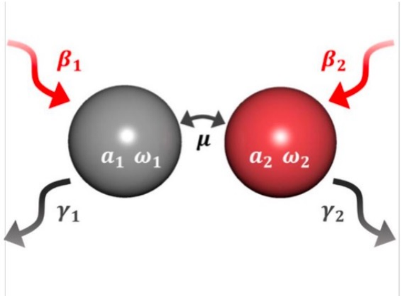
$$\Gamma(z) = \mathrm{e}^{-2\gamma z}\left(\frac{\gamma^2 - 4\kappa^2\cos(\omega z)}{\omega^2}\right)^2$$



微纳光子结构中量子PT对称

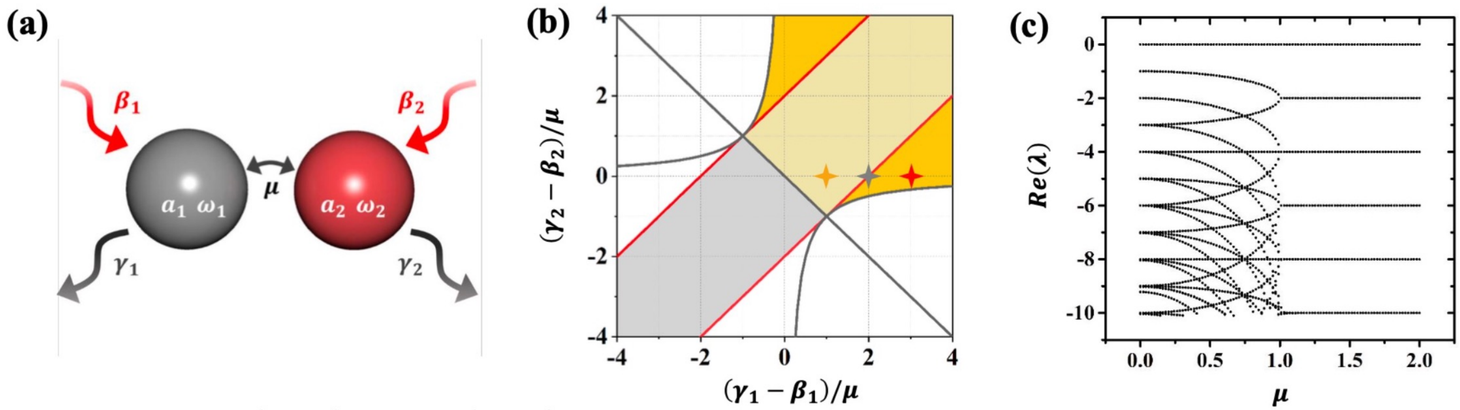
量子PT相图及量子态制备

- The existence of quantum jumping term
- In classical, there is an average effect in gain and loss
  But in quantum, the role of loss and gain is different:
  The gain will bring the noise, but the loss can not decrease the noise



$$\hat{H} = \hbar\omega_1 \hat{a}_1^\dagger \hat{a}_1 + \hbar\omega_2 \hat{a}_2^\dagger \hat{a}_2 + \hbar\mu(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)$$

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \sum_{i=1,2} \gamma_i(2\hat{a}_i\hat{\rho}\hat{a}_i^\dagger - \hat{\rho}\hat{a}_i^\dagger\hat{a}_i - \hat{a}_i^\dagger\hat{a}_i\hat{\rho}) + \sum_{i=1,2} \beta_i(2\hat{a}_i^\dagger\hat{\rho}\hat{a}_i - \hat{\rho}\hat{a}_i\hat{a}_i^\dagger - \hat{a}_i\hat{a}_i^\dagger\hat{\rho})$$



$$i\frac{d}{dt}\begin{pmatrix}\langle\hat{a}_1\rangle\\ \langle\hat{a}_2\rangle\end{pmatrix} = H_{\text{eff}}\begin{pmatrix}\langle\hat{a}_1\rangle\\ \langle\hat{a}_2\rangle\end{pmatrix}$$

where

$$H_{\text{eff}} = \begin{pmatrix}\omega - i\gamma_1 + i\beta_1 & \mu\\ \mu & \omega - i\gamma_2 + i\beta_2\end{pmatrix}.$$

The eigenvalues of H_{eff} are

$$\omega_{\pm} = \omega - \frac{i}{2}(\gamma_1 - \beta_1 + \gamma_2 - \beta_2) \pm \frac{1}{2}\sqrt{4\mu^2 - [(\gamma_1-\beta_1)-(\gamma_2-\beta_2)]^2}.$$

Steady-state formulas

$$\langle\hat{n}_1\rangle_{ss} = \frac{\Delta_1 - \beta_1\left(\Delta_2 + 2\beta_2\gamma_2 - \gamma_2^2\right)}{\Delta_3}$$

$$\langle\hat{n}_2\rangle_{ss} = \frac{\Delta_1 - \beta_2\left(\Delta_2 + 2\beta_1\gamma_1 - \gamma_1^2\right)}{\Delta_3}$$

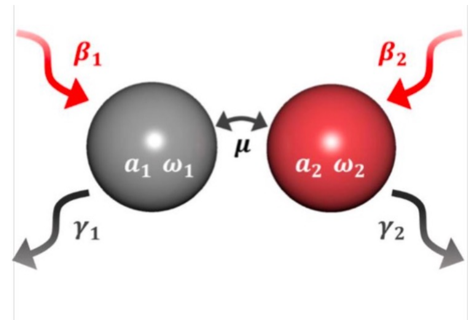
$$\langle\hat{\eta}\rangle_{ss} = \frac{2\mu\left(\beta_2\gamma_1 - \beta_1\gamma_2\right)}{\Delta_3}$$

$$\boxed{\langle\hat{\eta}\rangle = \langle i(\hat{a}_2^{\dagger}\hat{a}_1 - \hat{a}_1^{\dagger}\hat{a}_2)\rangle}$$

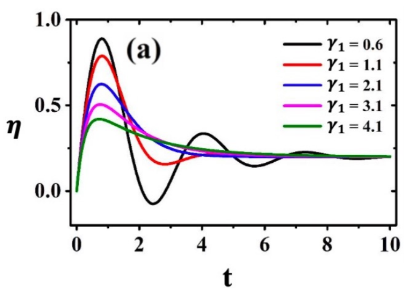
Xinchen Zhang, Yun Ma, and Ying Gu et al., *Quantum Phase Diagram of PT-Symmetry or Broken in a Non-Hermitian Photonic Structure,* ArXiv.2303.00189v2 (2023).

Exchange operator $\boxed{\hat{\eta} = 2(\hat{X}_2\hat{Y}_1 - \hat{X}_1\hat{Y}_2)}$

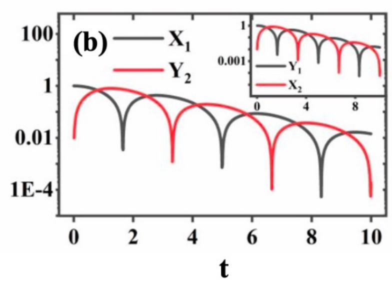
with $\hat{X}_{1,2} = (\hat{a}_{1,2} + \hat{a}^{\dagger}_{1,2})/2, \hat{Y}_{1,2} = i(\hat{a}_{1,2} - \hat{a}^{\dagger}_{1,2})/2$



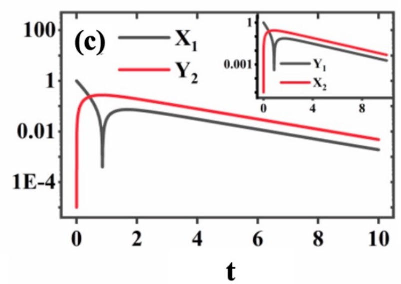
EP: blue curve

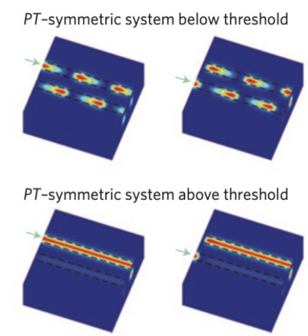


PT-symmetry

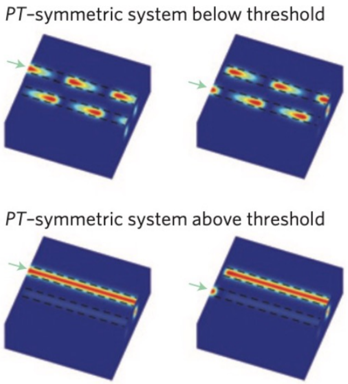


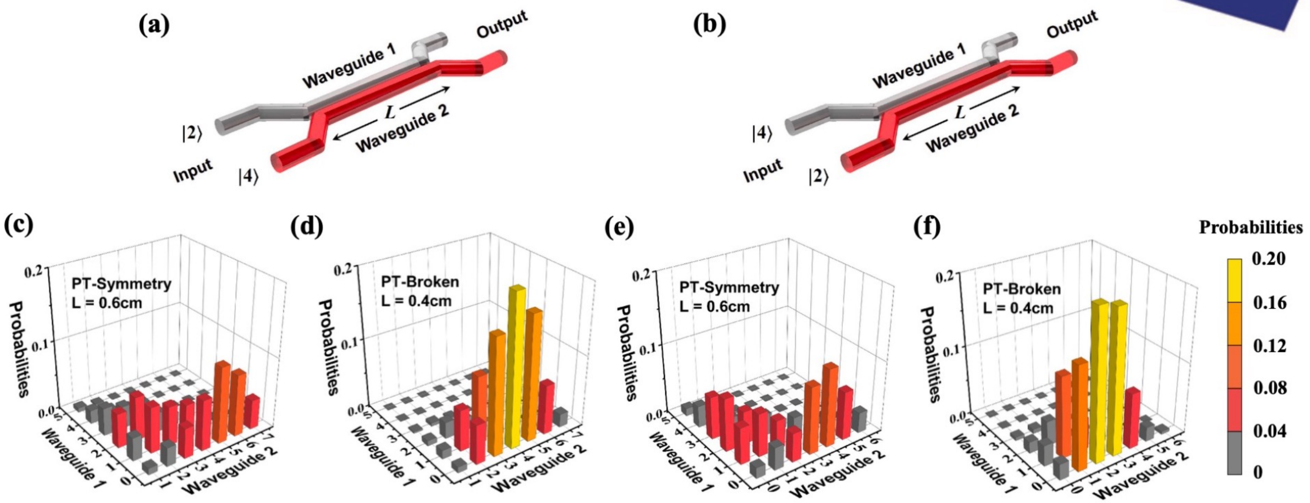
PT-broken





Most photons concentrate on superior waveguide with some Fock state distributions





延伸阅读：

Research Article Vol. 6, No. 2 / February 2019 / *Optica* 181

optica

Quantum coherent absorption of squeezed light

A. Ü. C. Hardal[1,*] and Martijn Wubs[1,2]

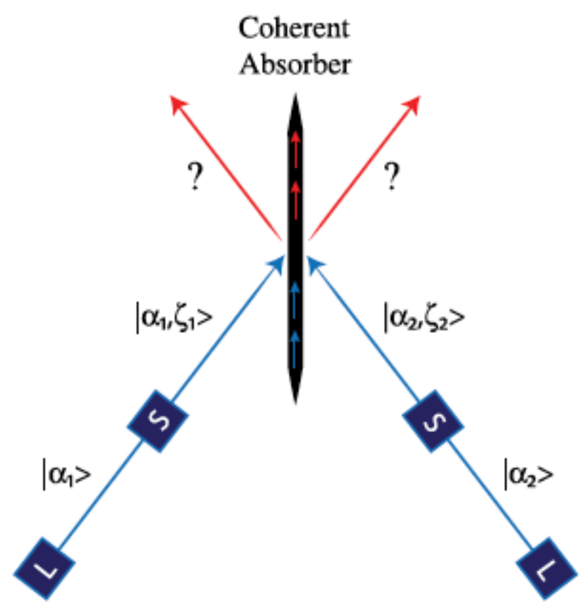
在有损耗时：

如果输入两个相干态，

和经典情况类似；

如果输入两个压缩态，

会因为吸收而产生纠缠



IOP Publishing

Reports on Progress in Physics

Rep. Prog. Phys. **84** (2021) 012402 (25pp)

https://doi.org/10.1088/1361-6633/abcd7a

Key Issues Review

Two-photon interference: the Hong–Ou–Mandel effect

Frédéric Bouchard[1,5], Alicia Sit[1], Yingwen Zhang[2], Robert Fickler[1,6], Filippo M Miatto[3], Yuan Yao[3], Fabio Sciarrino[4] and Ebrahim Karimi[1,2,*]

Nearly 30 years ago, two-photon interference was observed, marking the beginning of a new quantum era. Indeed, two-photon interference has no classical analogue, giving it a distinct advantage for a range of applications. The peculiarities of quantum physics may now be used to our advantage to outperform classical computations, securely communicate information, simulate highly complex physical systems and increase the sensitivity of precise measurements. This separation from classical to quantum physics has motivated physicists to study two-particle interference for both fermionic and bosonic quantum objects. So far, two-particle interference has been observed with massive particles, among others, such as electrons and atoms, in addition to plasmons, demonstrating the extent of this effect to larger and more complex quantum systems. A wide array of novel applications to this quantum effect is to be expected in the future. This review will thus cover the progress and applications of two-photon (two-particle) interference over the last three decades.

七、思考题

1. Michelson Stellar干涉仪和HBT exp的工作目的、工作原理？
2. 光电效应用a还是a^+来表示？为什么？
3. 利用光电效应原理的光子计数器中，counting rate ω_1，joint counting rate ω_2的含义？
4. 量子关联函数$G^{(1)}, G^{(2)}$的定义，与ω_1，ω_2的联系？
5. 相干度$g^{(1)}$，$g^{(2)}$的定义，与$G^{(1)}, G^{(2)}$的联系？
6. 热光场、相干态下，$g^{(1)}(0)$，$g^{(2)}(0)$的表达式？
7. HBT实验中，两个原子发出的光子之间的干涉？制备和未制备时，$G^{(2)}$的异同及原因？
8. photon bunching，antibunching的概念及$g^{(2)}(\tau)$在热光场、相干态和量子光场中不同？
9. 光子计数率P_m与$P(\alpha, \alpha^*)$间对应关系？

10. 没有损耗情况下，经典分束和量子分束的变换矩阵？

11. 量子分束特点及各种量子态制备

12. Hong_Ou_Mandel实验及其原理

13. 什么是平衡零拍探测？

14. 标准量子极限是多少？

作业：

围绕“量子分束（前沿）”

写一个完整的report，基础和前沿各占50%

要求：自己的语言、逻辑合理、图文并茂、公式和例子皆有