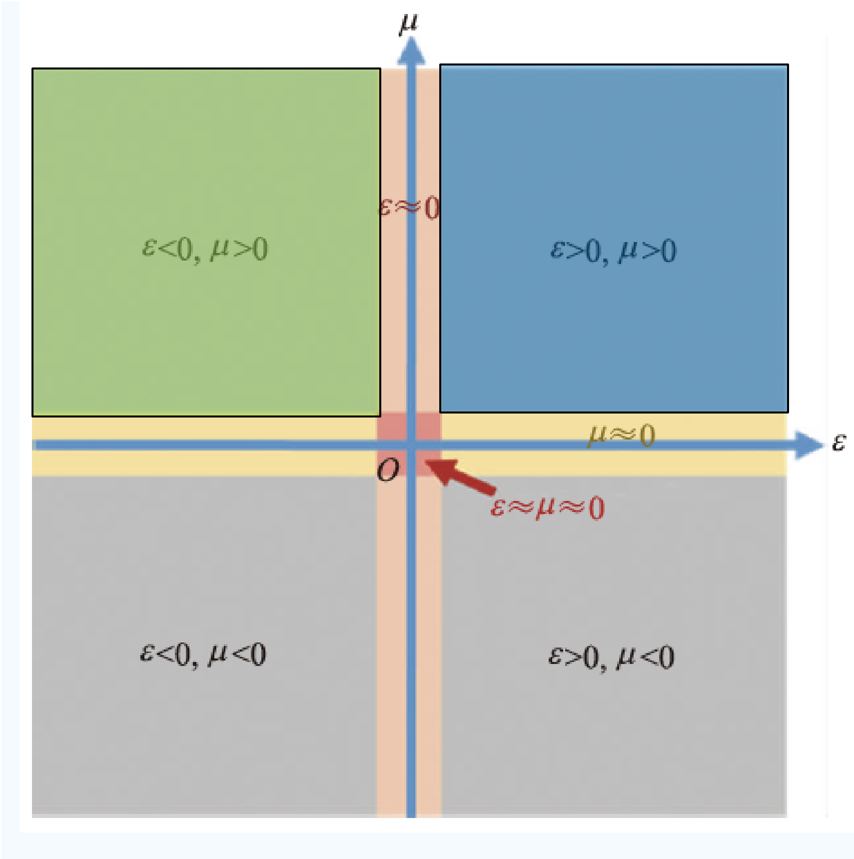
介电常数和磁导率示意图



超构表面：

绿色和蓝色区域的材料

但有微小、超薄的平面周期

性或准周期结构

零折射率材料：黄粉色区域

图取自“罗杰，赖耘，零折射率材料的物理与应用，物理， 48卷7 期，426 （2019）”

第四章 零折射率材料及超构表面

4.1 零折射率材料及其应用

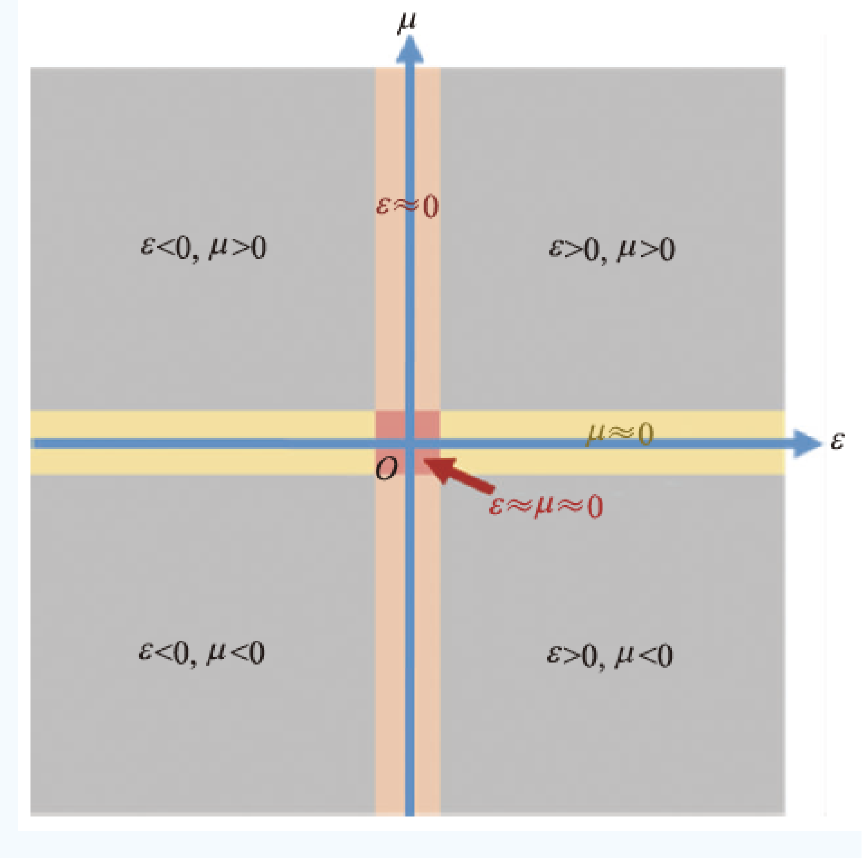
4.1.1 零折射率材料分类及实现

4.1.2 零折射率材料物理及应用

4.2 超构表面及其应用

20250412 古英　ygu@pku.edu.cn

4.1.1 零折射率材料分类及实现



折射率 $n = \pm\sqrt{\varepsilon\mu}$

粉色区域和橙黄色区域：零折射率材料

zero-index media，ZIM

单零材料：只有ε或μ接近零

介电常数近零

epsilon-near-zero，ENZ

磁导率近零材料

mu-near-zero，MNZ

双零材料：ε和μ同时接近零

EMNZ

ε near zero 和 n near zero, 不是一回事

罗杰，赖耘，零折射率材料的物理与应用，物理，48卷7 期，426（2019）

零折射率材料中的电磁场特性

无源电磁场

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = -i\omega\varepsilon\mathbf{E}$$

$$\varepsilon = 0$$
$$\mu = 0$$

1、电场和磁场的解耦合
2、无限长的波长
3、空间（波长）和时间（频率）解耦
4、 静止相（相位不随空间变化）

相速度和群速度

$$v_c = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

$$\varepsilon = 0$$
$$\mu = 0$$

相速度无限大

$$v_g = \frac{\partial \omega}{\partial k}$$

群速度如何呢？

零折射率材料的例子

1、单零材料

$$\varepsilon(\omega) = 1 - \omega_p^2/\omega(\omega + i\omega_c), \text{ and } \mu(\omega) = 1$$

无损情况下且$\omega = \omega_p$时，**ENZ材料，** $v_g \to 0$

Ciattoni, A., Marini, A., Rizza, C., Scalora, M. & Biancalana, F. Polariton excitation in epsilon-near-zero slabs: transient trapping of slow light. *Phys. Rev. A* 87, 053853 (2013).

2、双零材料

$$\varepsilon(\omega) = 1 - \omega_p^2/\omega(\omega + i\omega_c);$$

$$\mu(\omega) = (\omega^2 - \omega_p^2 + i\gamma\omega)/(\omega^2 - \omega_0^2 + i\gamma\omega)$$

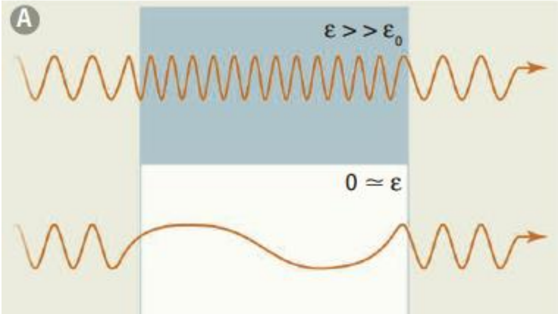
无损情况下且$\omega = \omega_p$时，**EMNZ材料，** $v_g = \frac{c}{2}\sqrt{1 - {\omega_0}^2/{\omega_p}^2}$

Mahmoud, A. M. & Engheta, N. Wave–matter interactions in epsilon-and-mu-near-zero structures. *Nat. Commun.* 5, 5638 (2014).

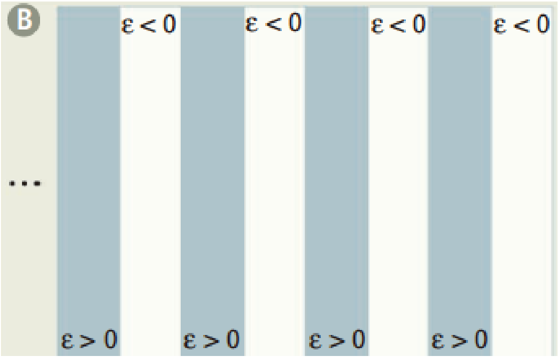
零折射率材料的直观理解

A class of metamaterials designed with low permittivity provides a platform for developing optical devices with unconventional properties.

1. 波长无穷大

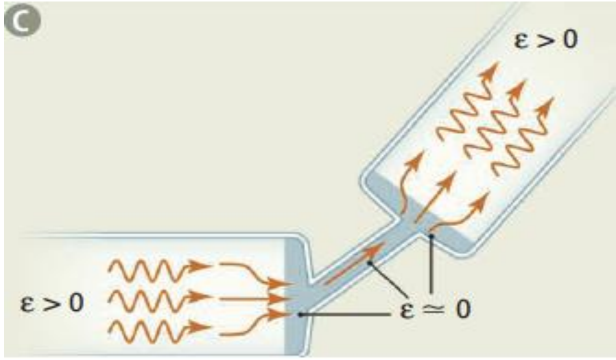


2. 等效ENZ材料



3. ENZ中隧穿效应

等相位、场均匀、局域增强



Nader Engheta，Pursuing Near-Zero Response，19 APRIL 2013 VOL 340 SCIENCE，340，286（2013）

零折射率材料的物理实现

大体分为两类

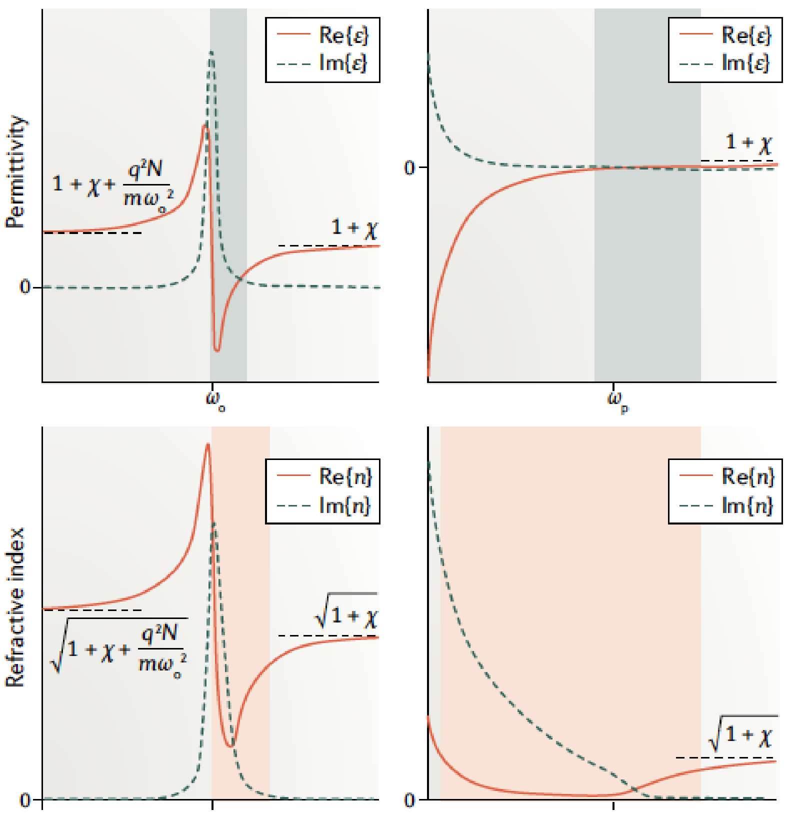
1. **天然材料**

2. **等效零折射率材料 (*都叫做等效折射率*)**
   2.a 材料本身的等效
   2.b 传播常数的等效
   2.c 群速度转变的等效

重点介绍：epsilon-near-zero，ENZ

1. 天然零折射率材料

介电常数的洛伦兹模型和特鲁德模型



$$\varepsilon_{r,\mathrm{Lorentz}} = 1 + \chi + \sum_i \frac{A_i}{\omega_{o,i}^2 - \omega^2 - i\Gamma_i\omega}$$

$$\varepsilon_{r,\mathrm{Drude}} = 1 + \chi - \frac{q^2 N}{\varepsilon_o m}\frac{1}{\omega^2 + i\Gamma\omega}$$

Near-zero-index materials for photonics

Nathaniel Kinsey[1], Clayton DeVault[2], Alexandra Boltasseva[3] and Vladimir M. Shalaev[2,3]**

NATURE REVIEWS | MATERIALS

742 | DECEMBER 2019 | VOLUME 4

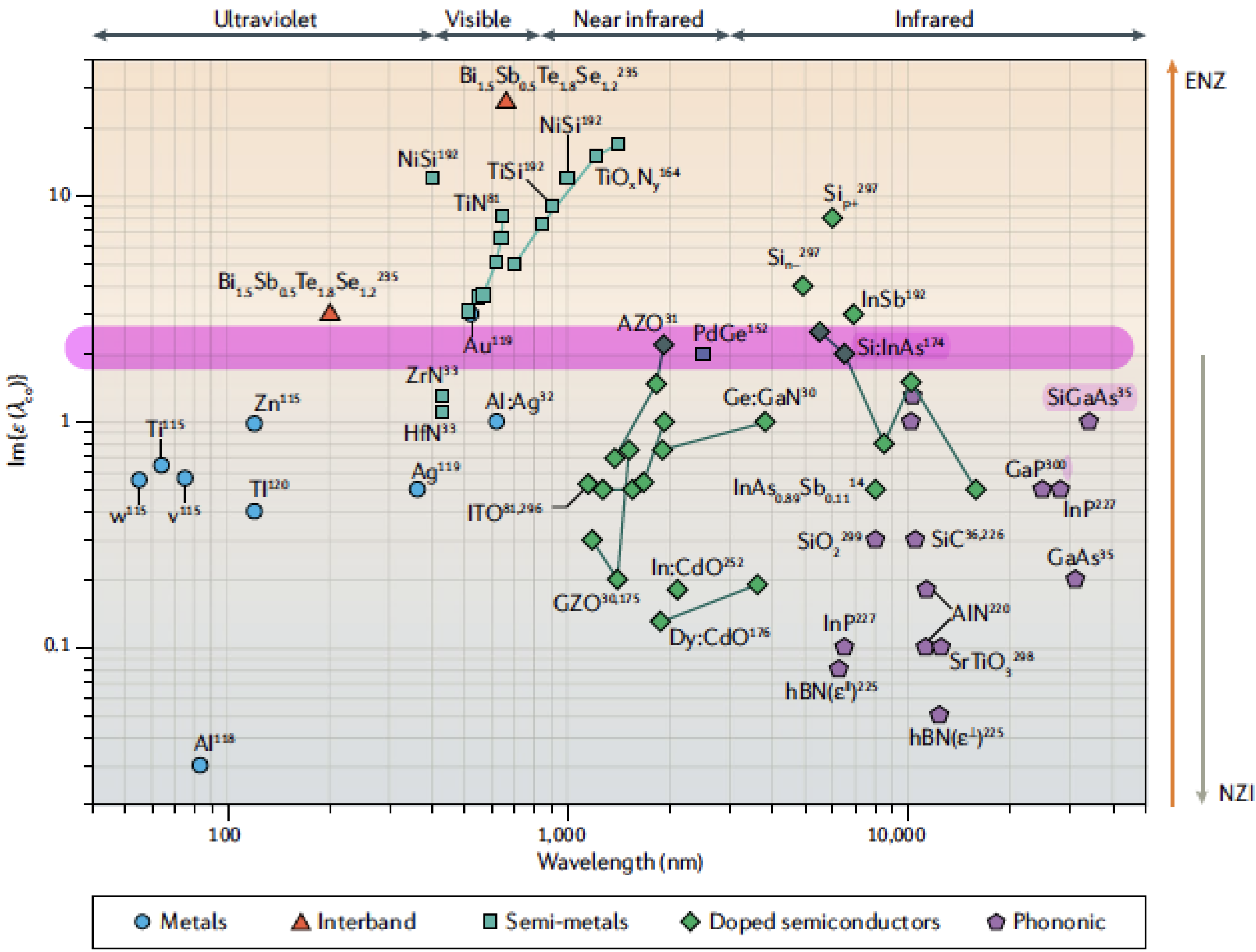


Fig. 2 | **Comparison of experimentally realized bulk homogeneous ε-near-zero and near-zero-index materials.** The value of the imaginary permittivity ε is plotted at the crossover wavelength (λ_{co}) at which the real permittivity is zero. Materials with an imaginary permittivity smaller than 2 at the crossover wavelength exhibit near-zero-index (NZI) properties. The lines connect the different measured values of Im{ε} for a given statically tunable material and illustrate their variability. ENZ, ε-near-zero; hBN, hexagonal boron nitride.

文献与上页同

2. 等效零折射率材料

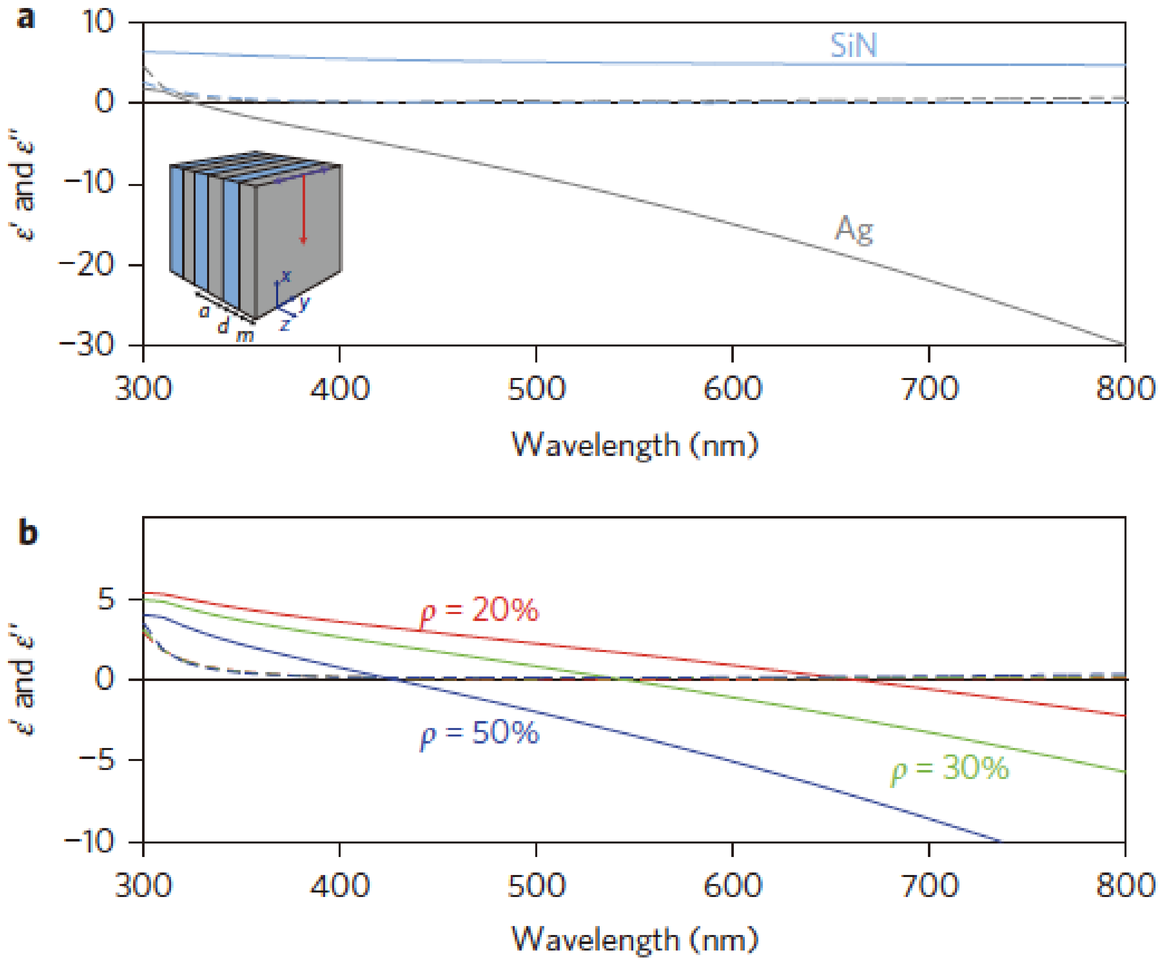
2.a 材料本身的等效：

nature photonics ARTICLES
PUBLISHED ONLINE: 13 OCTOBER 2013 | DOI: 10.1038/NPHOTON.2013.256

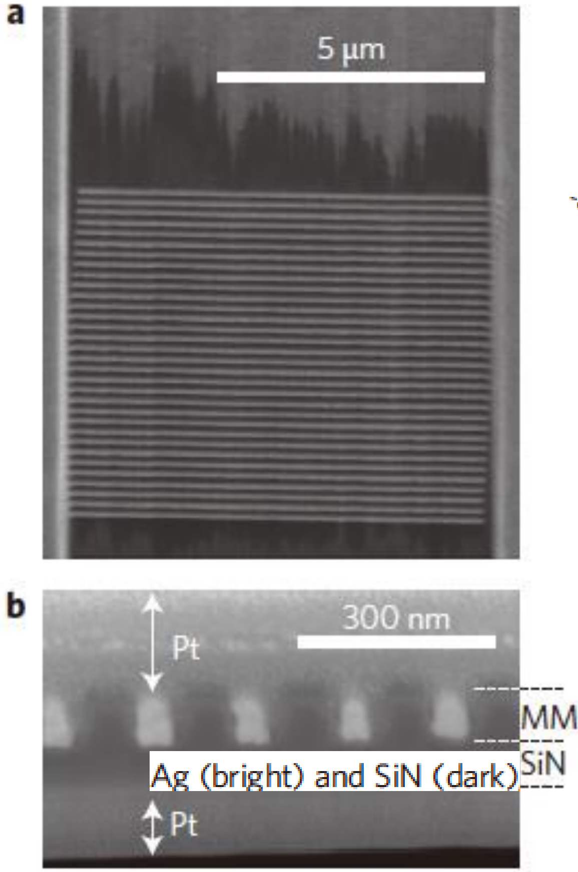
Experimental realization of an epsilon-near-zero metamaterial at visible wavelengths

Ruben Maas[1]*, James Parsons[1], Nader Engheta[2] and Albert Polman[1]

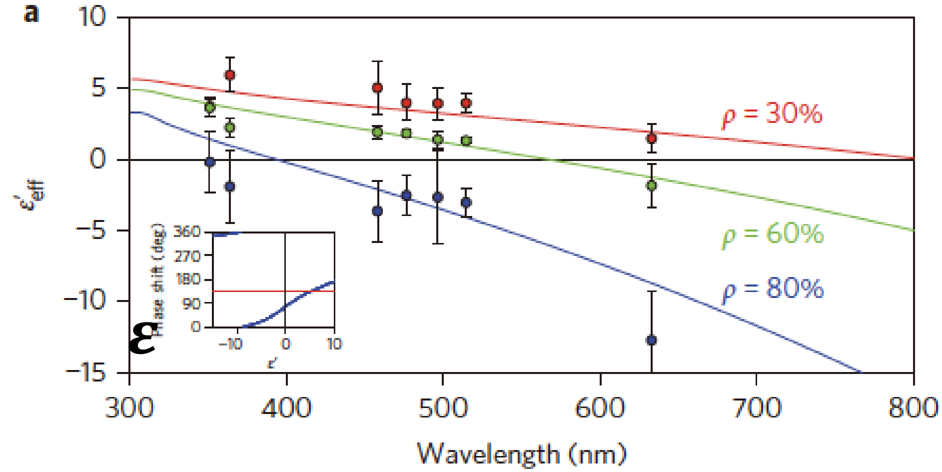
用材料的平均折射率来等效 $\varepsilon_{av} = \rho\varepsilon_m + (1-\rho)\varepsilon_d$

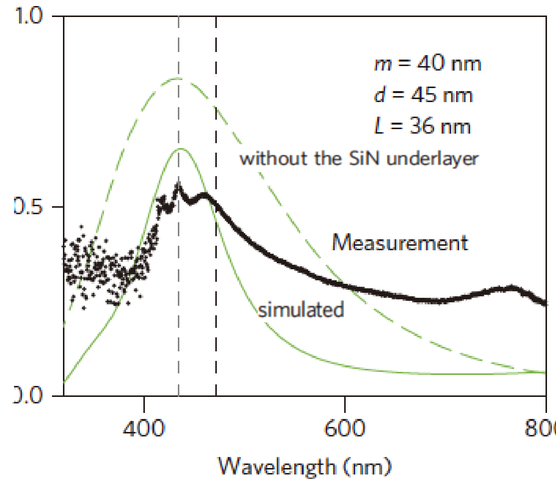


电镜图



平均折射率





透射谱

直灰线：$\varepsilon = 1$

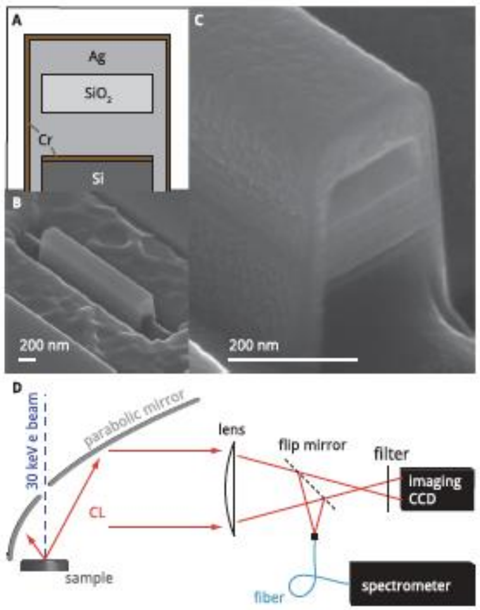
直黑线：$\varepsilon = 0$

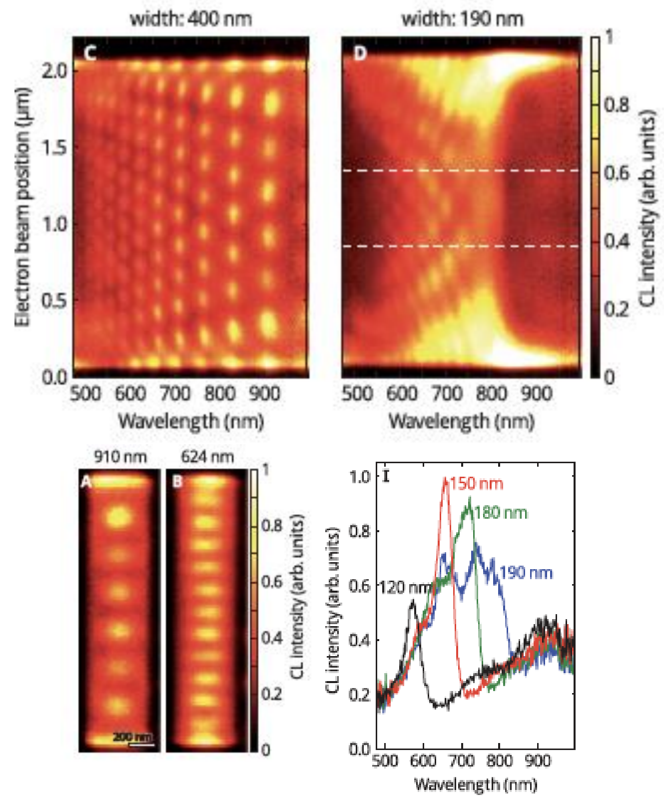
文献与上页同

2.b 传播常数的等效：

对应模式截止频率处的等效折射率 （width指SiO_2)

色散关系图？





910和624nm是激发波长

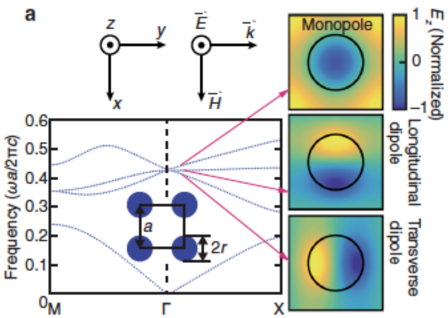
Experimental Verification of $n = 0$ Structures for Visible Light PRL **110,** 013902 (2013)

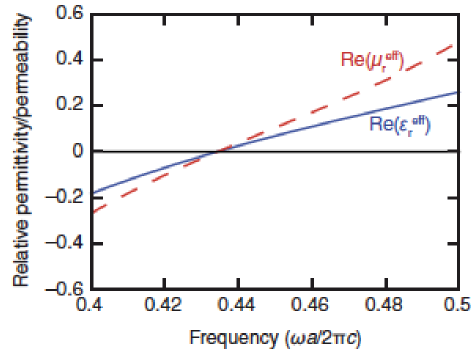
Ernst Jan R. Vesseur,[1] Toon Coenen,[1] Humeyra Caglayan,[2] Nader Engheta,[2] and Albert Polman[1,*]

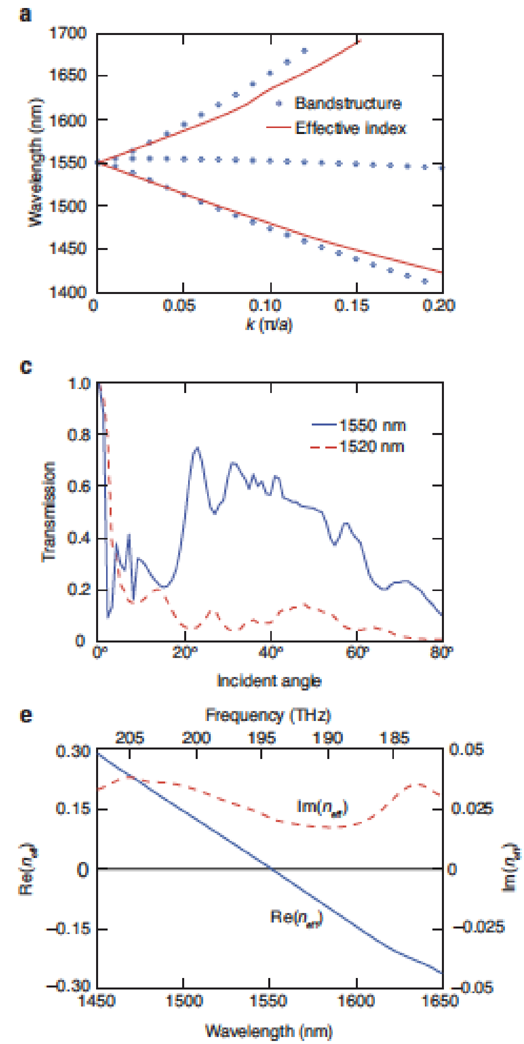
2.c 群速度转变的等效

在群速度转变处寻找等效的零折射率

(狄拉克锥处) $v_g = \dfrac{\partial \omega}{\partial k}$







Li et al. *Light: Science & Applications* (2021)10:203
https://doi.org/10.1038/s41377-021-00642-2

Official journal of the CIOMP 2047-7538
www.nature.com/lsa

REVIEW ARTICLE Open Access

Dirac-like cone-based electromagnetic zero-index metamaterials

Yang Li[1], C. T. Chan[2] and Eric Mazur[3]

第四章 超表面及零折射率材料

4.1 零折射率材料及其应用

4.1.1 零折射率材料分类及实现

4.1.2 零折射率材料物理及应用

4.2 超表面及其应用

4.1.2 零折射率材料物理及应用

a. 波前调控与定向辐射

b. 隧穿效应

c. ENZ腔效应

d. 非线性效应

e. 掺杂调控

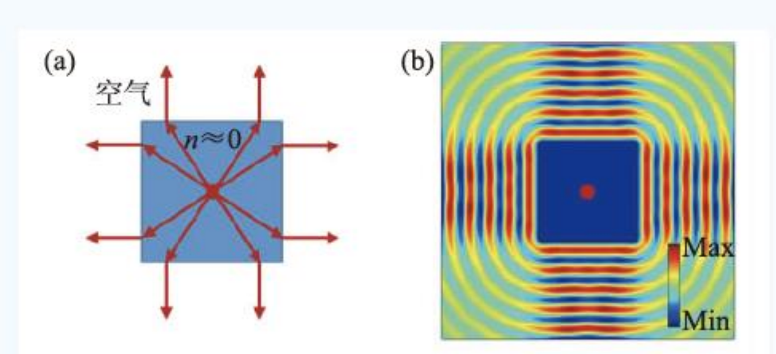
d. 其它

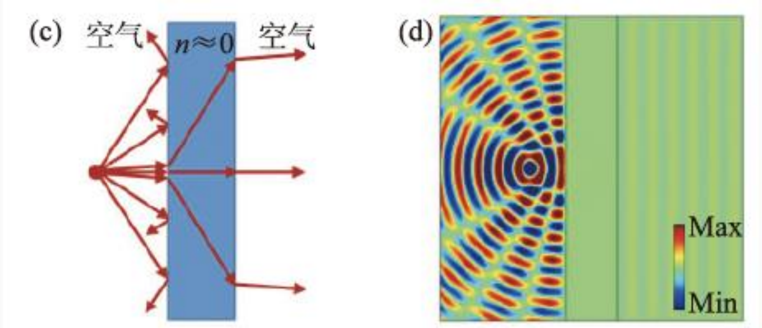
a. 定向辐射与波前调控

折射定律： $n_i \sin\theta_i = n_0 \sin\theta_0$

$n_i \approx 0$ 且 $n_0 = 1$

右图：定向辐射





$n_i = 1$ 且 $n_0 \approx 0$

只有入射角是0的光才能

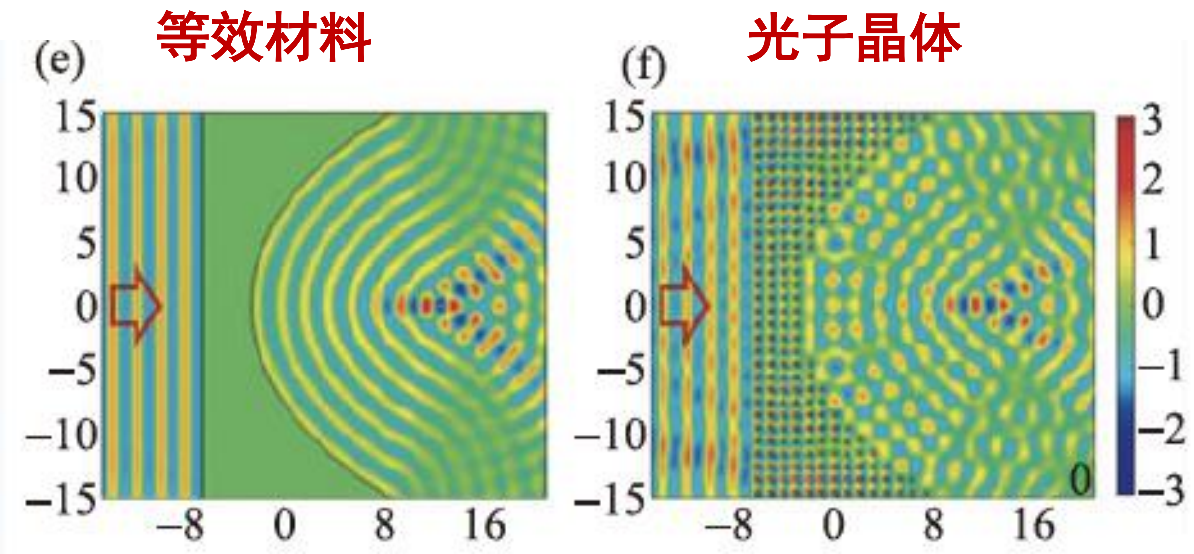
进入介质，*左图：全反射*

罗杰，赖耘，零折射率材料的物理与应用，物理， 48卷7 期，426 （2019）
and references therein.

只有光正入射时，才能进入零折射率材料；

当电磁波再次出射时，零折射率材料出射面的形状决定了出射波的波阵面，也就是说

波前调控（裁剪）



罗杰，赖耘，零折射率材料的物理与应用，物理，48卷7 期，426 （2019）
and references therein.

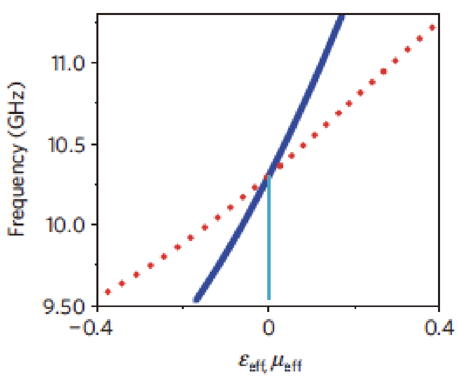
LETTERS
PUBLISHED ONLINE: 29 MAY 2011 | DOI: 10.1038/NMAT3030
nature materials

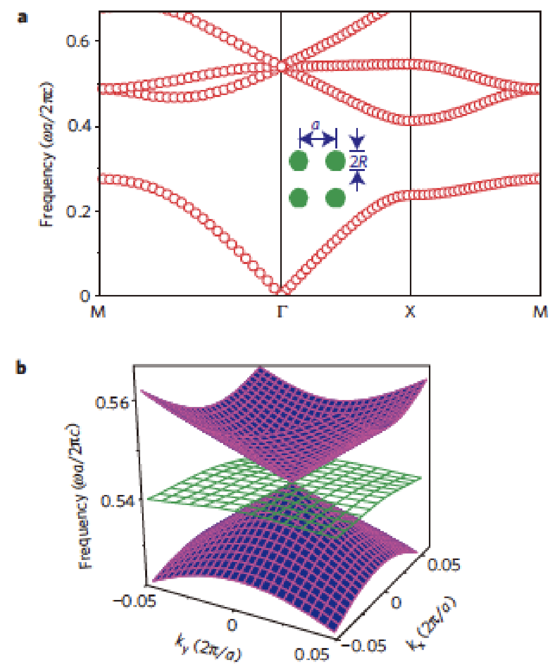
Dirac cones induced by accidental degeneracy in photonic crystals and zero-refractive-index materials

Xueqin Huang[1†], Yun Lai[1,2†], Zhi Hong Hang[1†], Huihuo Zheng[1] and C. T. Chan[1*]

先用狄拉克锥的方法

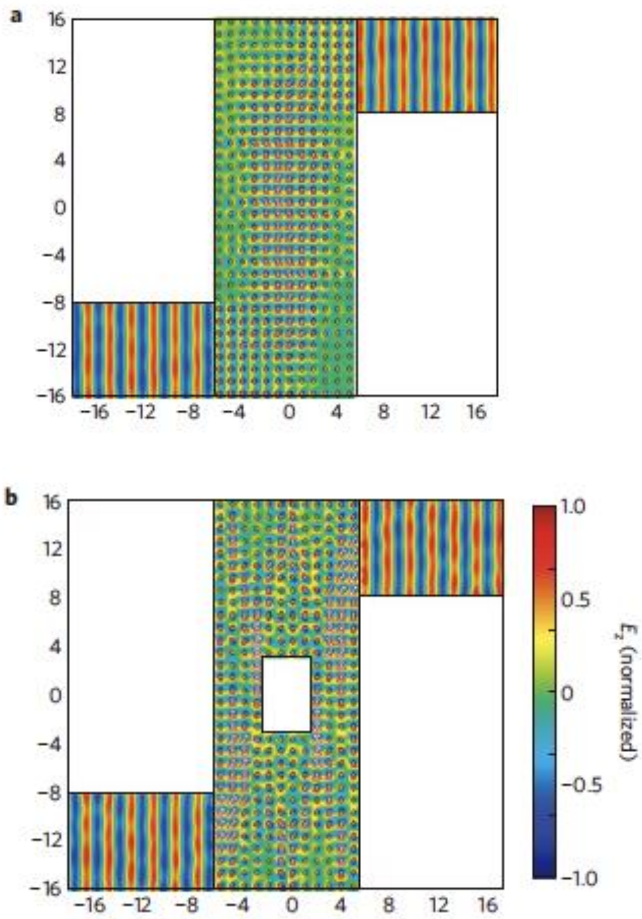
设计出 零折射率材料

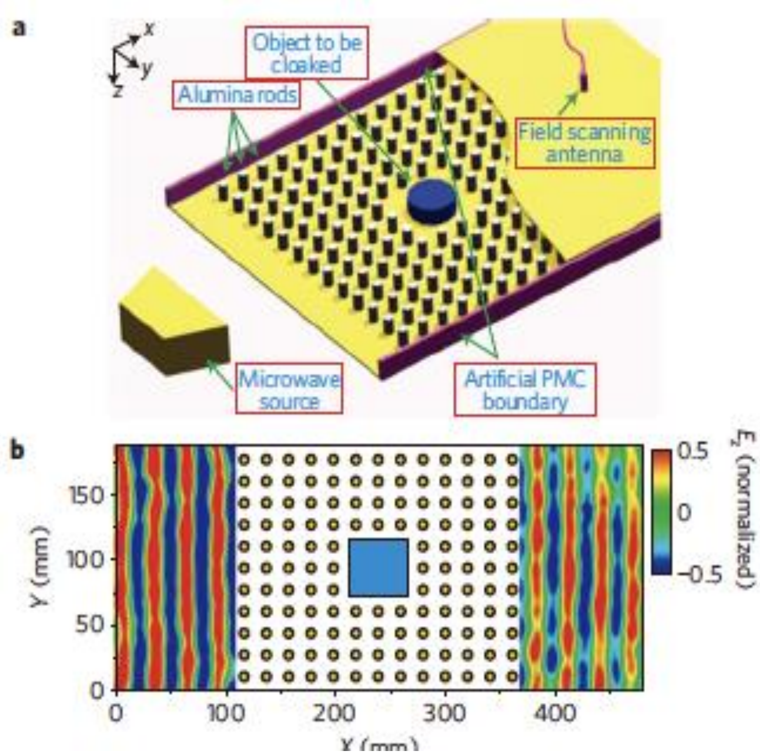




理论演示出零折射率材料中的*波前调控及隐身应用*

微波波段的实验实现





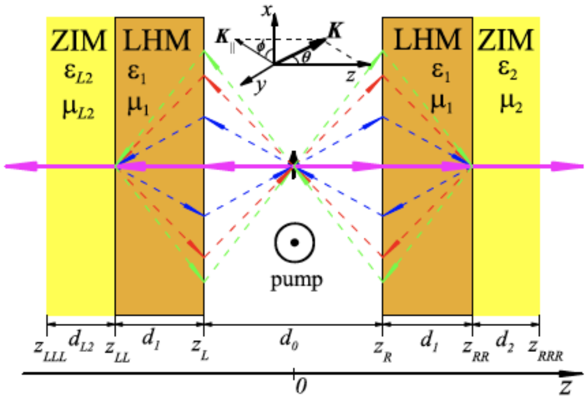
文献与上页同

RAPID COMMUNICATIONS

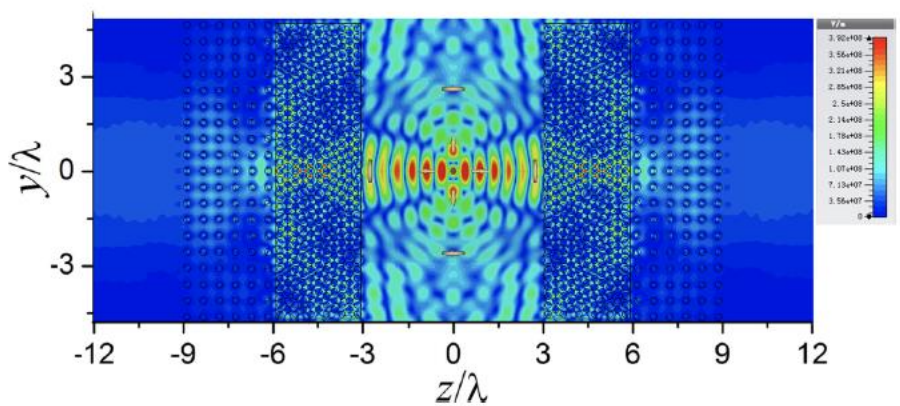
PHYSICAL REVIEW B 94, 220103(R) (2016)

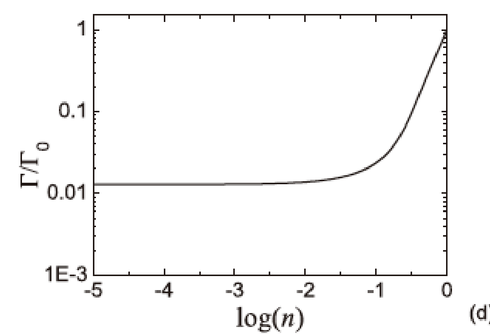
Unidirectional single-photon generation via matched zero-index metamaterials

Jingping Xu,[1,2] Ge Song,[1] Zhenqing Zhang,[1] Yaping Yang,[1,*] Hong Chen,[1] M. Suhail Zubairy,[2,3] and Shiyao Zhu[2,4,†]
[1]MOE Key Laboratory of Advanced Micro-Structured Materials, School of Physics Science and Engineering,
Tongji University, Shanghai 200092, People's Republic of China



原理：单个emitter产生单光子后，通过左手材料聚焦，然后在零折射率材料中*方向性*的出射







4.1.2 零折射率材料物理及应用

a. 波前调控与定向辐射

b. 隧穿效应

c. ENZ腔效应

d. 非线性效应

e. 掺杂调控

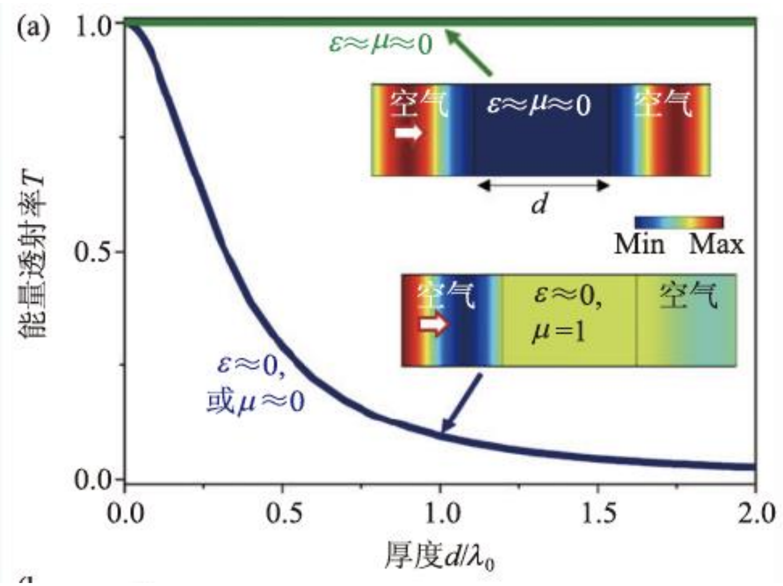
d. 其它

b. 隧穿效应

电磁波在厚度为d的折射率平板波导中透射系数

$$t=\frac{2}{2\cos\left(\sqrt{\varepsilon}\sqrt{\mu}k_0d\right)-\mathrm{i}\left(\sqrt{\varepsilon}/\sqrt{\mu}+\sqrt{\mu}/\sqrt{\varepsilon}\right)\sin\left(\sqrt{\varepsilon}\sqrt{\mu}k_0d\right)}$$

[46] Berreman D W. J. Opt. Soc. Am., 1972, 62: 502



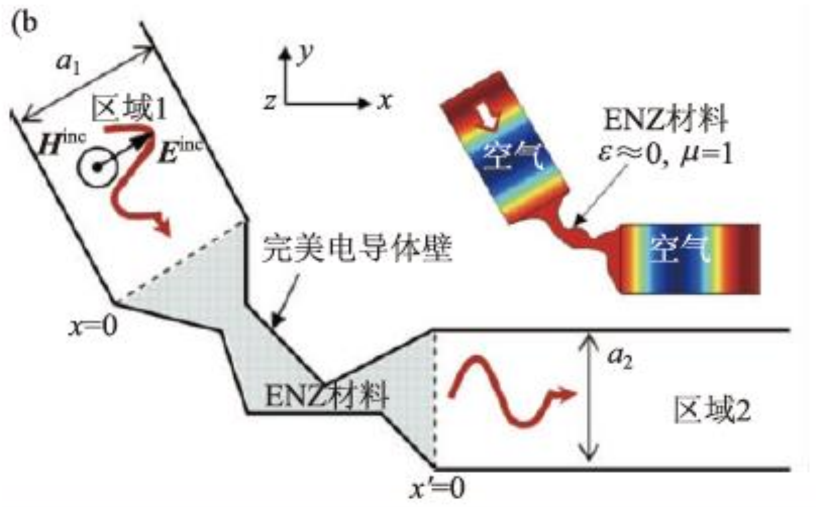
双零材料中：**透射率为1（绿线）**

单零材料中：**蓝线**

$$t=\frac{2}{2-\mathrm{i}\mu k_0 d} \text{ 或 } t=\frac{2}{2-\mathrm{i}\varepsilon k_0 d}$$

罗杰，赖耘，零折射率材料的物理与应用，物理， 48卷7 期，426 （2019）
and references therein.

零折射率材料中的隧穿效应



特点：

1. 可以通过不规则、弯曲波导

2. 电场的压缩性、局域性

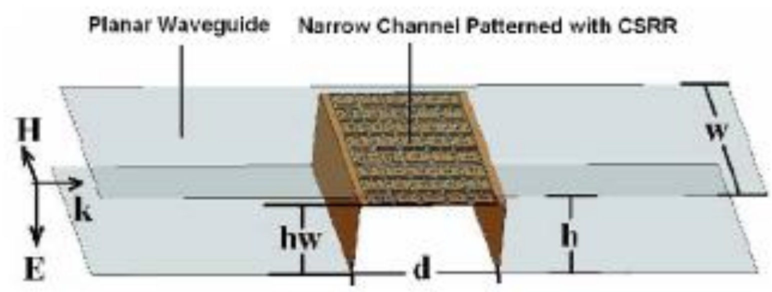
3. 或者叫做supercoupling

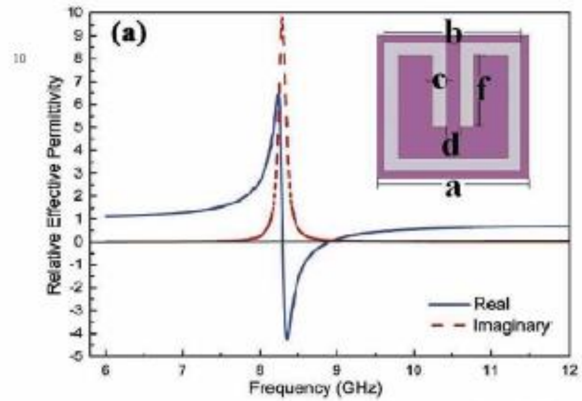
PRL **97,** 157403 (2006) PHYSICAL REVIEW LETTERS week ending 13 OCTOBER 2006

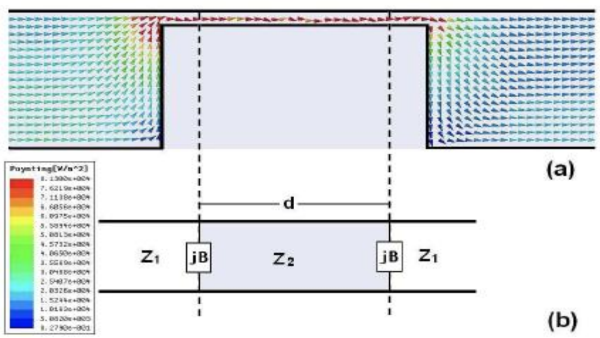
Tunneling of Electromagnetic Energy through Subwavelength Channels and Bends using ε-Near-Zero Materials

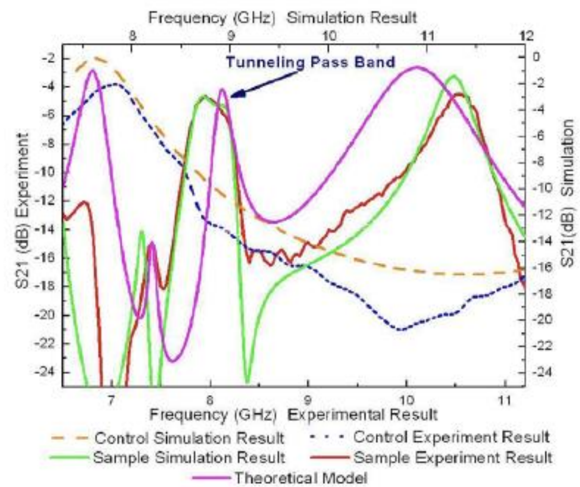
Mário Silveirinha* and Nader Engheta†

隧穿效应在微波波段的实验实现









PRL 100, 023903 (2008) PHYSICAL REVIEW LETTERS week ending 18 JANUARY 2008

Experimental Demonstration of Electromagnetic Tunneling Through an Epsilon-Near-Zero Metamaterial at Microwave Frequencies

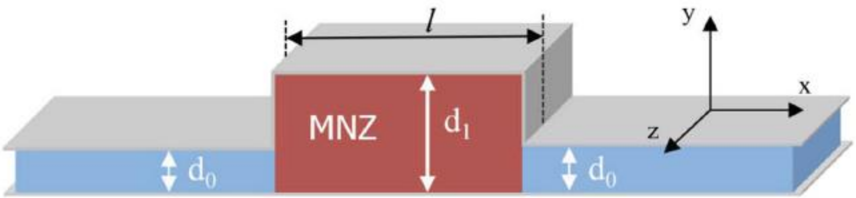
Ruopeng Liu 等

隧穿效应在MNZ材料中的理论和微波实验

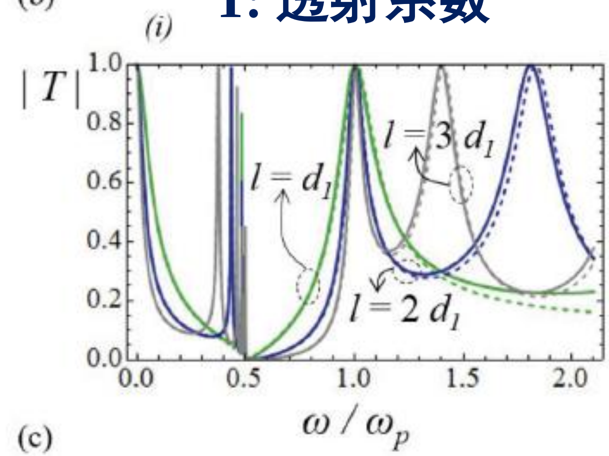
PHYSICAL REVIEW B **91**, 195112 (2015)

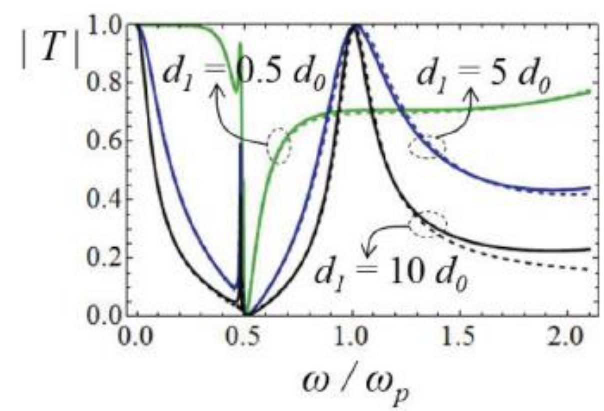
μ-near-zero supercoupling

João S. Marcos,[1] Mário G. Silveirinha,[1,*] and Nader Engheta[2]



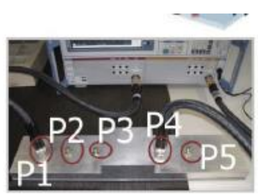
T: 透射系数





the permeability of the MNZ channel has a Drude-Lorentz dispersion model such that $\mu_r = 1 + \omega_m^2/(\omega_r^2 - \omega^2 - 2i\omega\omega_c)$, where ω_r and ω_c are the resonant and damping frequencies, and $\omega_m^2 = \omega_p^2 - \omega_r^2$, where ω_p is the frequency wherein the permeability of the material is near zero $\mu(\omega_p) \approx 0$.

(d)



(e)

4.1.2 零折射率材料物理及应用

a. 波前调控与定向辐射

b. 隧穿效应

c. ENZ腔效应

d. 非线性效应

e. 掺杂调控

d. 其它

c. ENZ腔效应：ENZ形变后，腔的性质不变（2D）

示意图

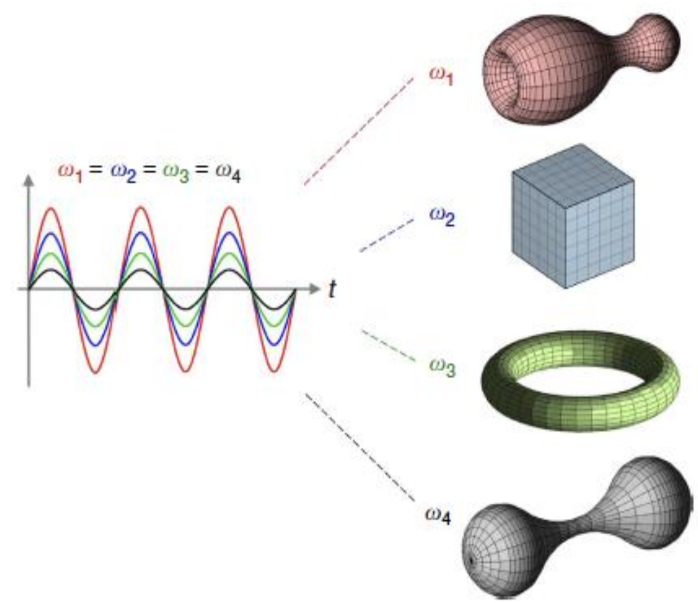
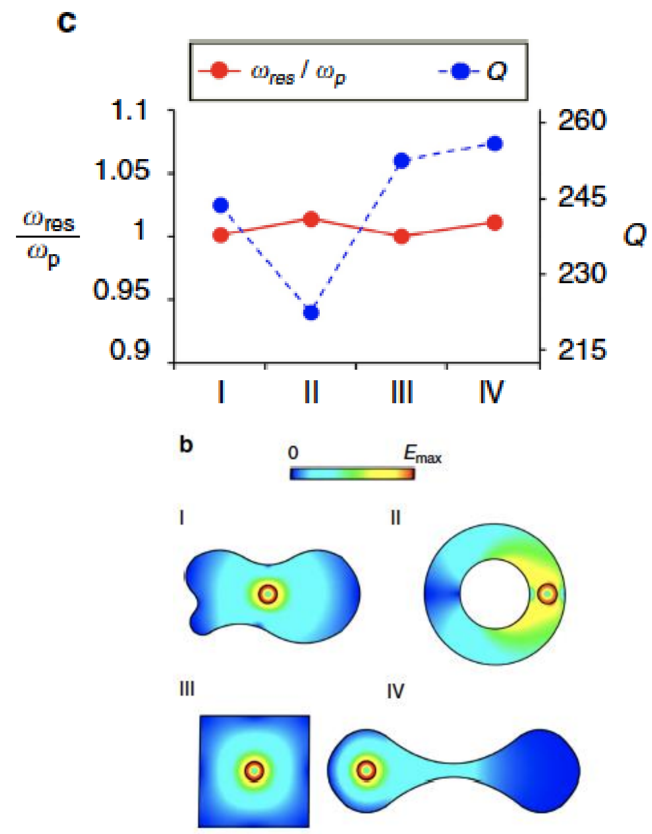


Figure 1 | Geometry-invariant resonant cavities. Conceptual sketch of

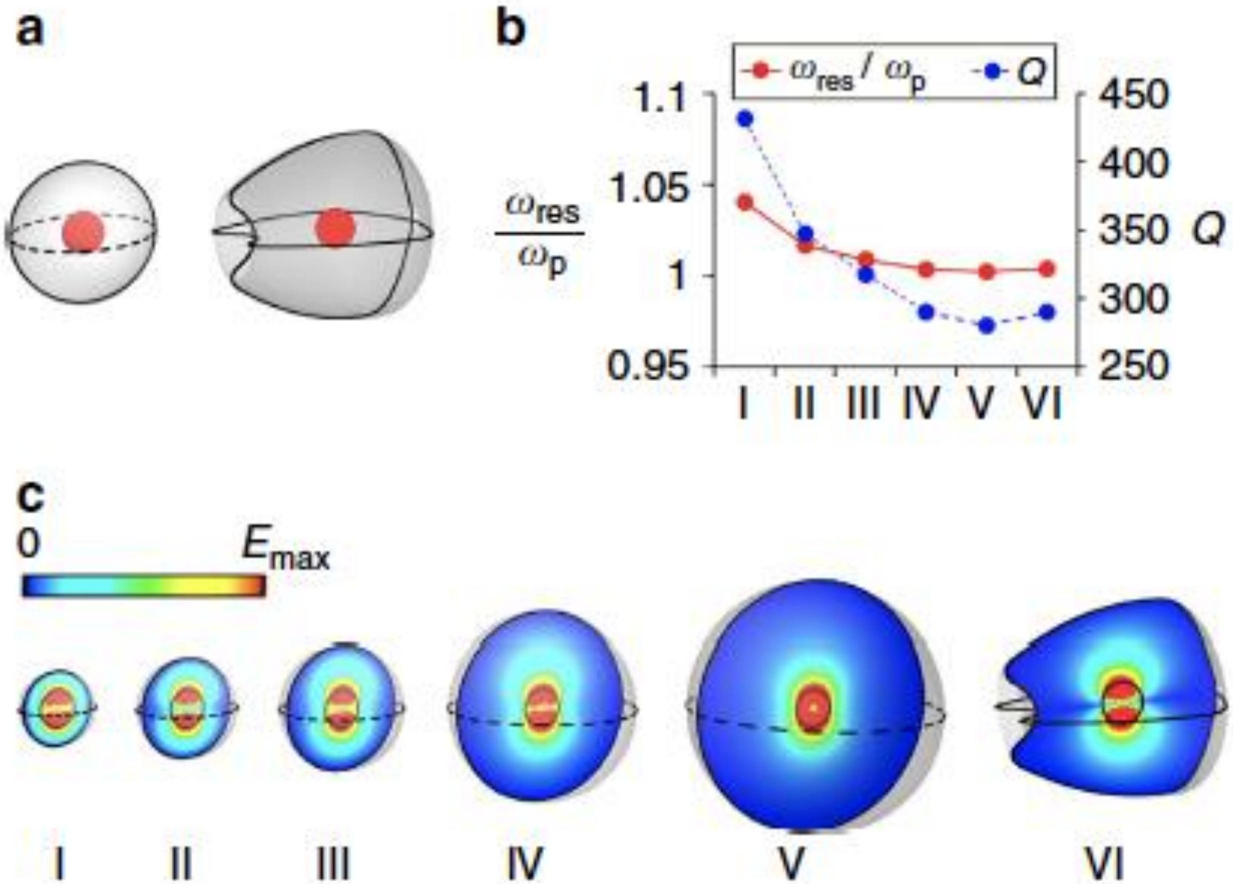


Geometry-invariant resonant cavities

I. Liberal[1,2], A.M. Mahmoud[2] & N. Engheta[2]

NATURE COMMUNICATIONS | 7:10989 | DOI: 10.1038/ 2016

3D情况下的结果



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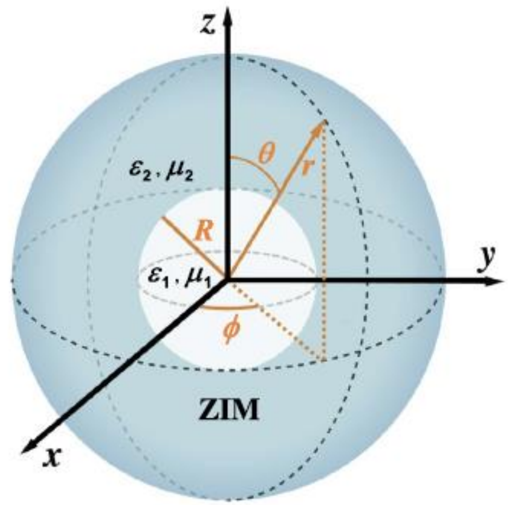
IOP Publishing Journal of Optics
J. Opt. 24 (2022) 025401 (9pp) https://doi.org/10.1088/2040-8986/ac440c

Nesting and degeneracy of Mie resonances of dielectric cavities within zero-index materials

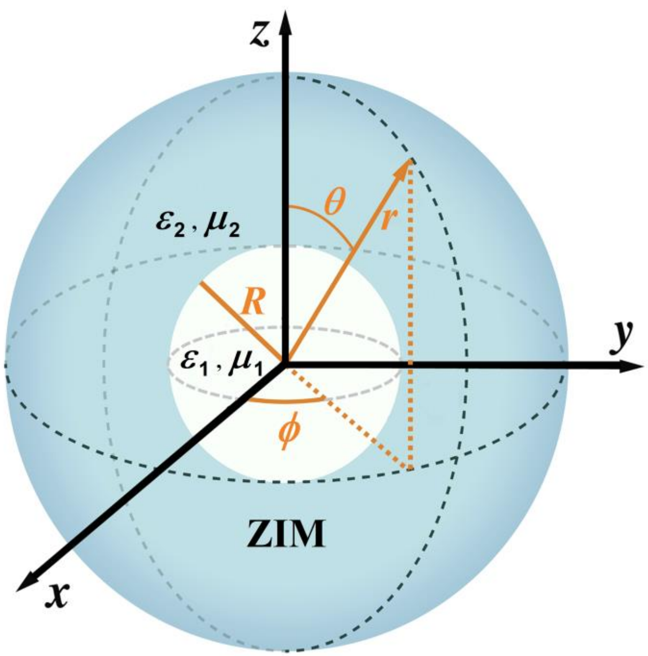
Xueke Duan[1,6], Haoxiang Chen[1,6], Yun Ma[1,6], Zhiyuan Qian[1], Qi Zhang[1], Yun Lai[5], Ruwen Peng[5], Qihuang Gong[1,2,3,4] and Ying Gu[1,2,3,4,*]

ZIM中球形腔的特点：

1. 超高的内外折射率对比
2. MIE理论解析解



模式分类：具有不同极次 *l* 的模式被分成两类



球形腔的模式

- 2^l-TM 模式 横磁
- 2^l-TE 模式 横电

零折射率环境：$s = \sqrt{\tilde{\varepsilon}\tilde{\mu}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}\frac{\mu_2}{\mu_1}} \to 0$

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

2^l-TM 模式的米共振条件

电磁场表达式

$$\mathbf{H}_{\mathrm{TM}}^{l}=\begin{cases}\mathbf{M}_l^{(2)}+a\mathbf{M}_l^{(1)}, & r<R,\\ c\mathbf{M}_l^{(3)}, & r\geq R,\end{cases}$$

$$\mathbf{E}_{\mathrm{TM}}^{l}=\begin{cases}-\dfrac{k_1}{i\varepsilon_1\varepsilon_0\omega}\left(\mathbf{N}_l^{(2)}+a\mathbf{N}_l^{(1)}\right), & r<R,\\ -\dfrac{k_2}{i\varepsilon_2\varepsilon_0\omega}\left(c\mathbf{N}_l^{(3)}\right), & r\geq R,\end{cases}$$

边界连续

待求解系数关系式

$$a=\frac{\begin{vmatrix}-\tilde{\varepsilon}\zeta_l'(\rho) & -\xi_l'(s\rho)\\ -s\zeta_l(\rho) & -\xi_l(s\rho)\end{vmatrix}}{\begin{vmatrix}\tilde{\varepsilon}\eta_l'(\rho) & -\xi_l'(s\rho)\\ s\eta_l(\rho) & -\xi_l(s\rho)\end{vmatrix}},\quad c=\frac{\begin{vmatrix}\tilde{\varepsilon}\eta_l'(\rho) & -\tilde{\varepsilon}\zeta_l'(\rho)\\ s\eta_l(\rho) & -s\zeta_l(\rho)\end{vmatrix}}{\begin{vmatrix}\tilde{\varepsilon}\eta_l'(\rho) & -\xi_l'(s\rho)\\ s\eta_l(\rho) & -\xi_l(s\rho)\end{vmatrix}}.$$

共振

分母为0

$$\tilde{\varepsilon}\eta_l'(\rho)\xi_l(s\rho)=s\eta_l(\rho)\xi_l'(s\rho)$$

形式简洁

零折射率

$s\rightarrow 0$

2^l-TM 模式

共振条件

$$\tilde{\varepsilon}\rho\eta_l'(\rho)+l\eta_l(\rho)=0$$

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

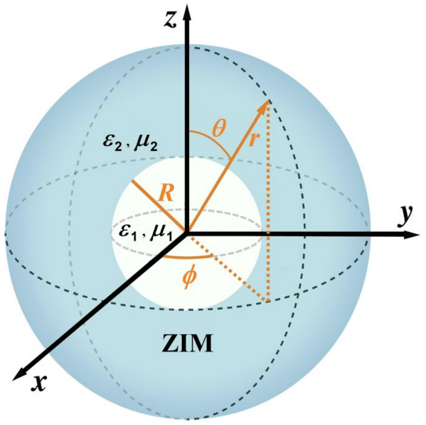
米基矢

球谐函数和连带勒让德函数的组合

$$\mathbf{M}_l^{(j=1,2,3)} = -\frac{\partial \mathrm{P}_l}{\partial \theta} z_l^{(j)}(x)\hat{\mathbf{e}}_\phi$$

$$\mathbf{N}_l^{(j=1,2,3)} = \frac{z_l^{(j)}(x)}{x} l(l+1)\mathrm{P}_l \hat{\mathbf{e}}_r + \frac{1}{x}\frac{\partial\left[x z_l^{(j)}(x)\right]}{\partial x}\frac{\partial \mathrm{P}_l}{\partial \theta}\hat{\mathbf{e}}_\theta$$

$$\rho = 2\pi\frac{R}{\lambda}$$



$\eta_l(\rho) = \rho j_l(\rho)$， $j_l(\rho)$ *球谐 Besell 函数*

$\varsigma_l(\rho) = \rho n_l(\rho)$， $n_l(\rho)$ *球谐 Neumann 函数*

$\xi_l(\rho) = \rho h_l(\rho)$， $h_l(\rho)$ *球谐 Hankel 函数*

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

2^l-TE 模式的米共振条件

电磁场表达式

$$\mathbf{E}_{\mathrm{TE}}^{l}=\begin{cases}\mathbf{M}_l^{(2)}+b\mathbf{M}_l^{(1)}, & r<R,\\ d\mathbf{M}_l^{(3)}, & r\geq R,\end{cases}$$

$$\mathbf{H}_{\mathrm{TE}}^{l}=\begin{cases}\dfrac{k_1}{i\mu_1\mu_0\omega}\left(\mathbf{N}_l^{(2)}+b\mathbf{N}_l^{(1)}\right), & r<R,\\ \dfrac{k_2}{i\mu_2\mu_0\omega}\left(d\mathbf{N}_l^{(3)}\right), & r\geq R,\end{cases}$$

边界连续

待求解系数关系式

$$b=\frac{\begin{vmatrix}-\tilde{\mu}\zeta_l'(\rho) & -\xi_l'(s\rho)\\ -s\zeta_l(\rho) & -\xi_l(s\rho)\end{vmatrix}}{\begin{vmatrix}\tilde{\mu}\eta_l'(\rho) & -\xi_l'(s\rho)\\ s\eta_l(\rho) & -\xi_l(s\rho)\end{vmatrix}},\quad d=\frac{\begin{vmatrix}\tilde{\mu}\eta_l'(\rho) & -\tilde{\mu}\zeta_l'(\rho)\\ s\eta_l(\rho) & -s\zeta_l(\rho)\end{vmatrix}}{\begin{vmatrix}\tilde{\mu}\eta_l'(\rho) & -\xi_l'(s\rho)\\ s\eta_l(\rho) & -\xi_l(s\rho)\end{vmatrix}}.$$

共振

分母为0

$$\tilde{\mu}\eta_l'(\rho)\xi_l(s\rho)=s\eta_l(\rho)\xi_l'(s\rho)$$

零折射率

$s\rightarrow 0$

形式简洁

2^l-TE 模式

共振条件

$$\tilde{\mu}\rho\eta_l'(\rho)+l\eta_l(\rho)=0$$

零折射率条件下米共振条件的具体形式

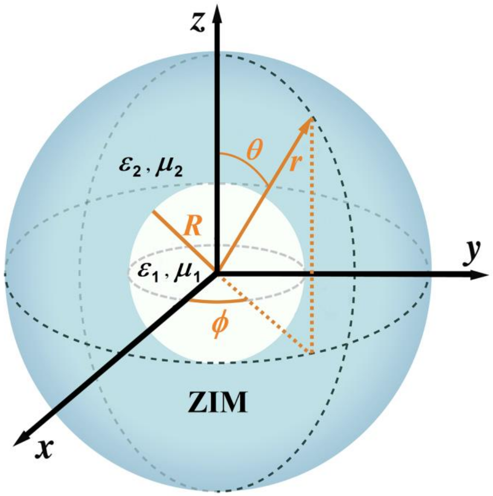
	ENZ	MNZ	EMNZ
2^l-TM mode	$\eta_l(\rho)=0$	$\tilde{\varepsilon}\rho\eta_l'(\rho)+l\eta_l(\rho)=0$	$\eta_l(\rho)=0$
2^l-TE mode	$\tilde{\mu}\rho\eta_l'(\rho)+l\eta_l(\rho)=0$	$\eta_l(\rho)=0$	$\eta_l(\rho)=0$

$$\tilde{\varepsilon}=\frac{\varepsilon_2}{\varepsilon_1}$$

$$\tilde{\mu}=\frac{\mu_2}{\mu_1}$$

$$\rho=2\pi\frac{R}{\lambda}$$

$$\eta_l(\rho)=\rho j_l(\rho)$$
$$\eta_l'(\rho)=j_l(\rho)+\rho j_l'(\rho)$$

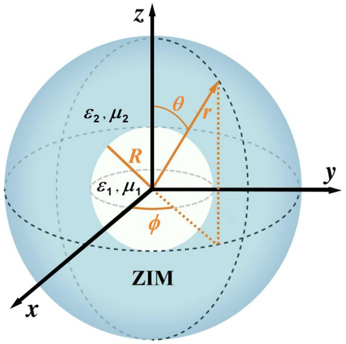


$$\eta_1(\rho)=\rho^{-1}(\sin\rho-\rho\cos\rho),$$
$$\eta_2(\rho)=\rho^{-2}\left[(3-\rho^2)\sin\rho-3\rho\cos\rho\right],$$
$$\eta_3(\rho)=\rho^{-3}\left[(15-6\rho^2)\sin\rho-(15\rho-\rho^3)\cos\rho\right].$$

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

球形腔中的共振简并和嵌套

R/λ	2-TM	2-TE	4-TM	4-TE
ENZ	0.7151 1.2295 …	0.5 1.0 …	0.9173 1.4475 …	0.7151 1.2295 …
MNZ	0.5 1.0 …	0.7151 1.2295 …	0.7151 1.2295 …	0.9173 1.4475 …
EMNZ	0.7151 1.2295 …	0.7151 1.2295 …	0.9173 1.4475 …	0.9173 1.4475 …



$$\rho = 2\pi\frac{R}{\lambda}$$

$$\sin\rho - \rho\cos\rho = 0$$

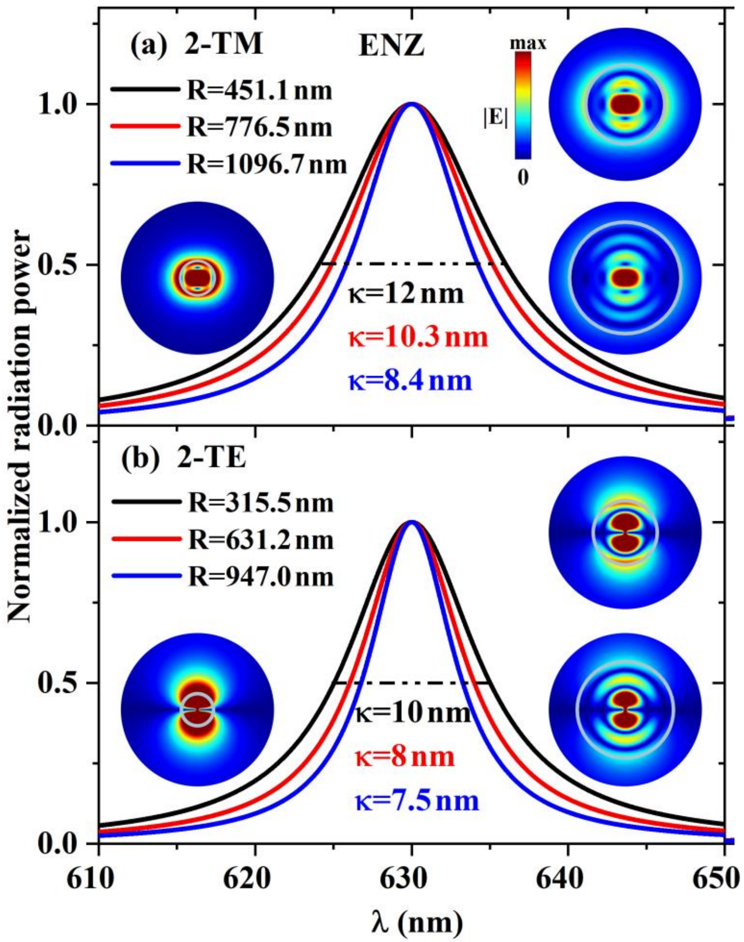
$$\sin\rho = 0$$

$$(3-\rho^2)\sin\rho - 3\rho\cos\rho = 0$$

共振嵌套： 在固定的波长，不同的腔半径下，可以得到一系列的同极（相同的l）米共振，就像是俄罗斯套娃。

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

ENZ 环境中的共振嵌套



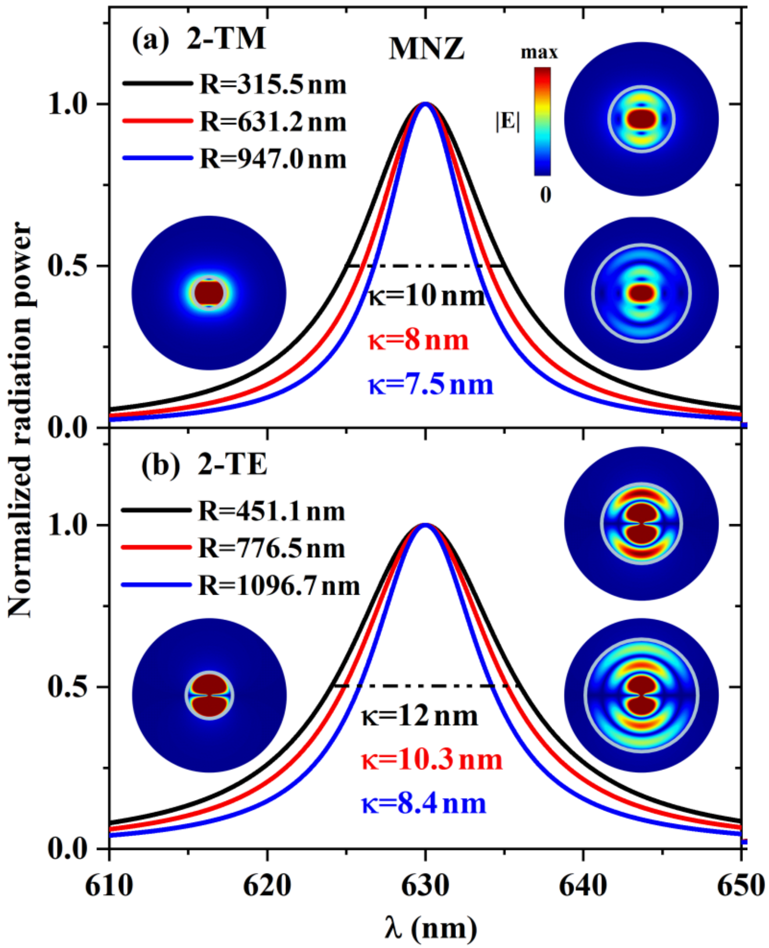
不同腔半径的电场形式一致

腔越大，模式损耗越小

电场大多分布在球腔中，
TM 模式在边界上有跃变

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

MNZ 环境中的共振嵌套



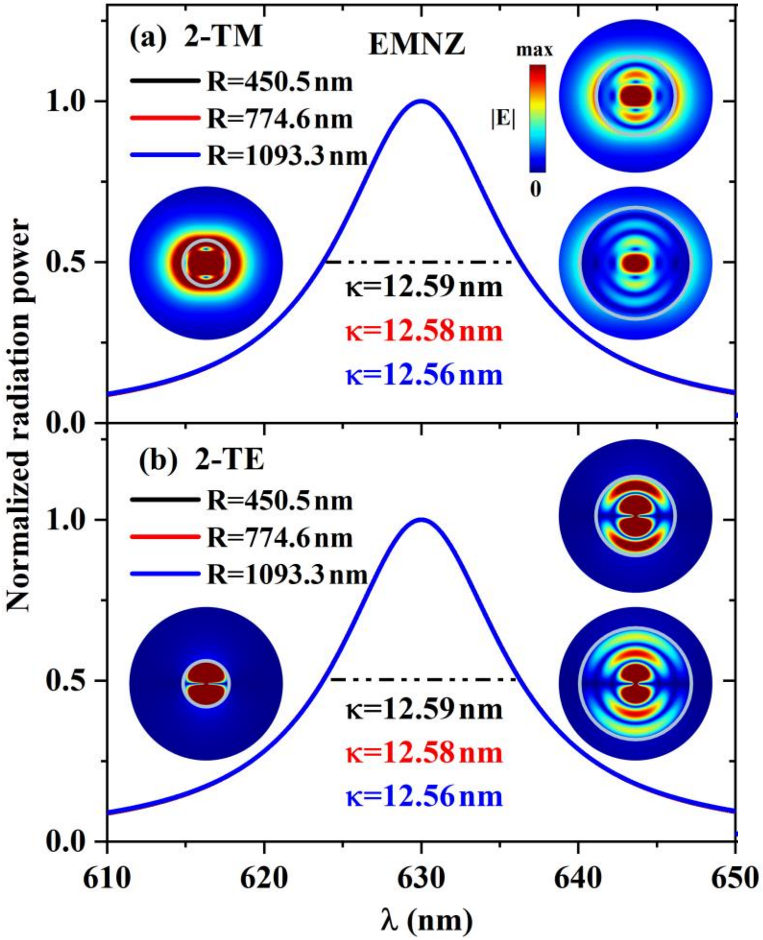
不同腔半径的电场形式一致

腔越大，模式损耗越小

电场：TE 模式在腔外为0；
对应ENZ中的磁场分布

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

EMNZ 环境中的共振嵌套



不同腔半径的电场形式一致

腔变大，模式损耗基本不变

电场：TM 模式与ENZ中相同；
TE 模式与MNZ中相同。

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

球形腔中的米共振简并

不同极模式间发生米共振简并

a	2-TM	2-TE	4-TM	4-TE	8-TM	8-TE	…
ENZ ($\mu_2 = \mu_1$)	A	B	C	A	D	C	…
MNZ ($\varepsilon_2 = \varepsilon_1$)	B	A	A	C	C	D	…
EMNZ	A	A	C	C	D	D	…

A: $\sin\rho - \rho\cos\rho = 0$;
B: $\sin\rho = 0$;
C: $(3-\rho^2)\sin\rho - 3\rho\cos\rho = 0$;
D: $(15-6\rho^2)\sin\rho - (15\rho-\rho^3)\cos\rho = 0$

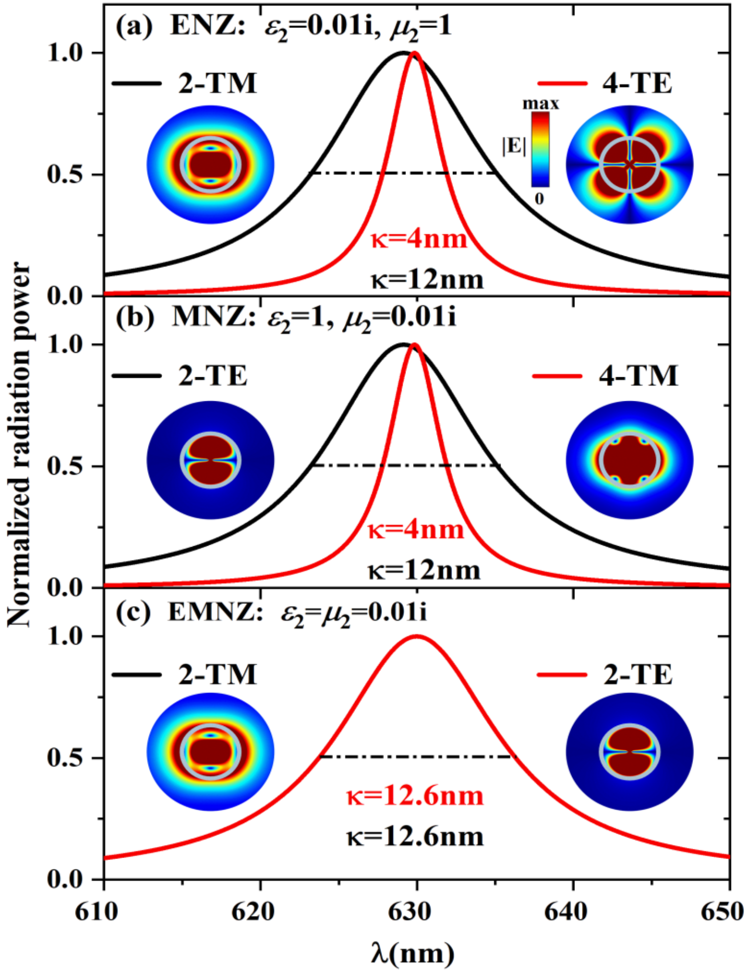
ENZ
- 2^l-TM
- 2^{l+1}-TE

MNZ
- 2^l-TE
- 2^{l+1}-TM

EMNZ
- 2^l-TE
- 2^l-TM

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

球形腔中的米共振简并



同一个腔中，在相同波长下支持两种不同极的模式

简并模式损耗不同（EMNZ相同）

为调控腔量子电动力学提供平台

Duan, Chen, Ma et al., J. Opt. 24, 02540 (2022)

4.1.2 零折射率材料物理及应用

a. 波前调控与定向辐射

b. 隧穿效应

c. ENZ腔效应

d. 非线性效应

e. 掺杂调控

d. 其它

为什么要研究零折射率材料中的非线性？

光学非线性及典型的非线性光学现象？*提问*

非线性光学效应的重要性：广泛的应用

由于*一般材料的非线性系数低*，强的非线性效应需要：

强的光场、相位匹配、长的相互作用长度、体材料

由于器件小型化（芯片）的要求，
微纳尺度或结构中的非线性效应成为发展趋势

而零折射率材料另辟蹊径：有超高的非线性系数

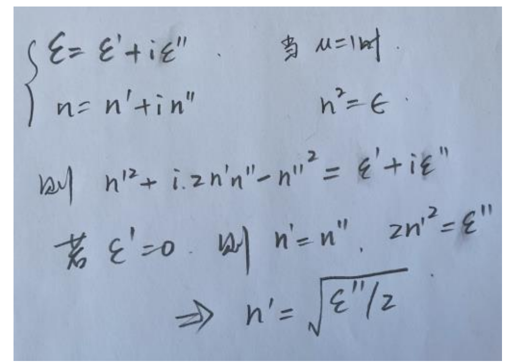
零折射率材料中的非线性效应的特点

1. 宽松的波矢匹配条件，只需要满足能量守恒

因为在零折射率材料中，所有的 $k \approx 0$, *所以 $\Delta k=0$ 自动满足*

2. 超高的非线性系数

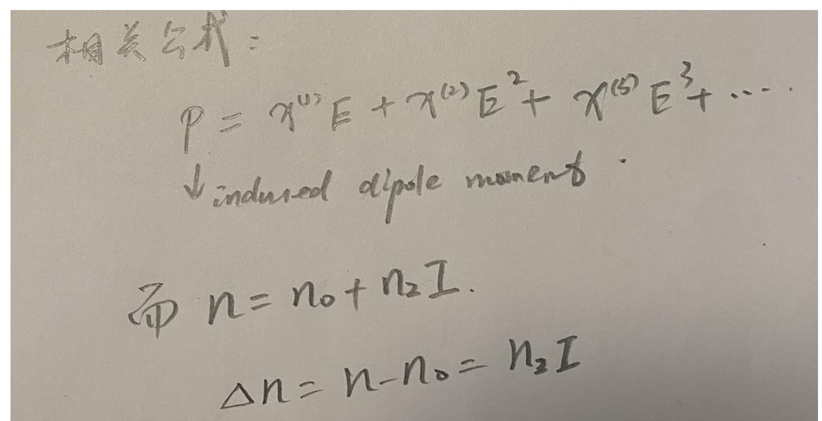
首先 $\boldsymbol{n = \sqrt{\varepsilon\mu}}$, $\boldsymbol{n}$ 和 $\boldsymbol{\varepsilon}$不能等同看待



然后

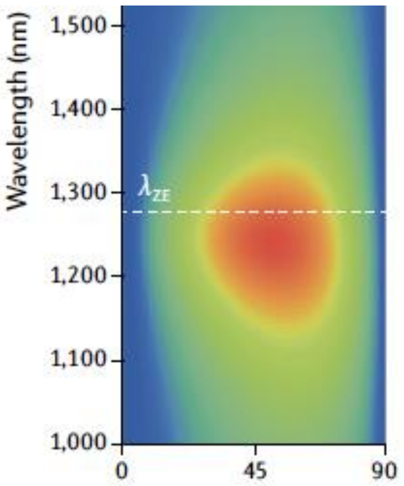
$$n' = \sqrt{\frac{|\epsilon''|}{2}}$$

$$n_2 = \frac{3\chi^{(3)}(\omega;\omega,\omega,-\omega)}{4n_0\mathrm{Re}(n_0)\epsilon_0 c}$$



3. 由边界上电场连续而带来的巨大电场增益

$$|E| = |E_0|\sqrt{\cos^2\theta + \frac{\sin^2\theta}{\epsilon}}$$



Review article

Nonlinear optical effects in epsilon-near-zero media

Orad Reshef[1], Israel De Leon[2], M. Zahirul Alam[1] and Robert W. Boyd[1,3]**

NATURE REVIEWS | MATERIALS VOLUME 4 | AUGUST 2019 | 535

Phase Mismatch–Free Nonlinear Propagation in Optical Zero-Index Materials

SCIENCE VOL 342 6 DECEMBER 2013 1223

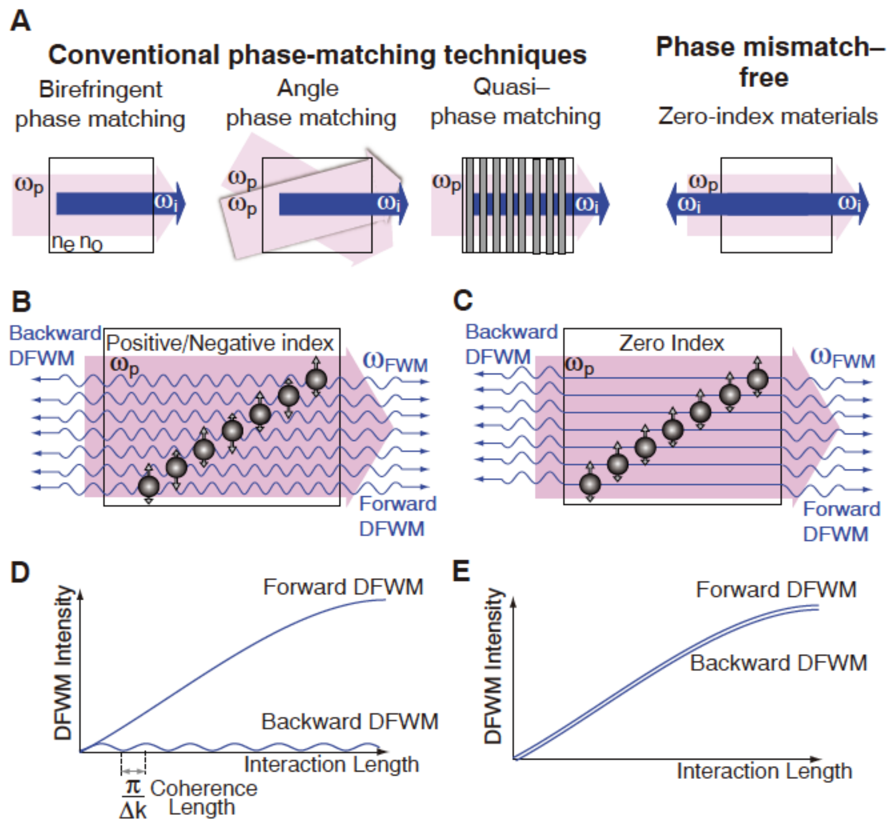
Haim Suchowski,[1*] Kevin O'Brien,[1*] Zi Jing Wong,[1*] Alessandro Salandrino,[1]
Xiaobo Yin,[1,2] Xiang Zhang[1,2]†

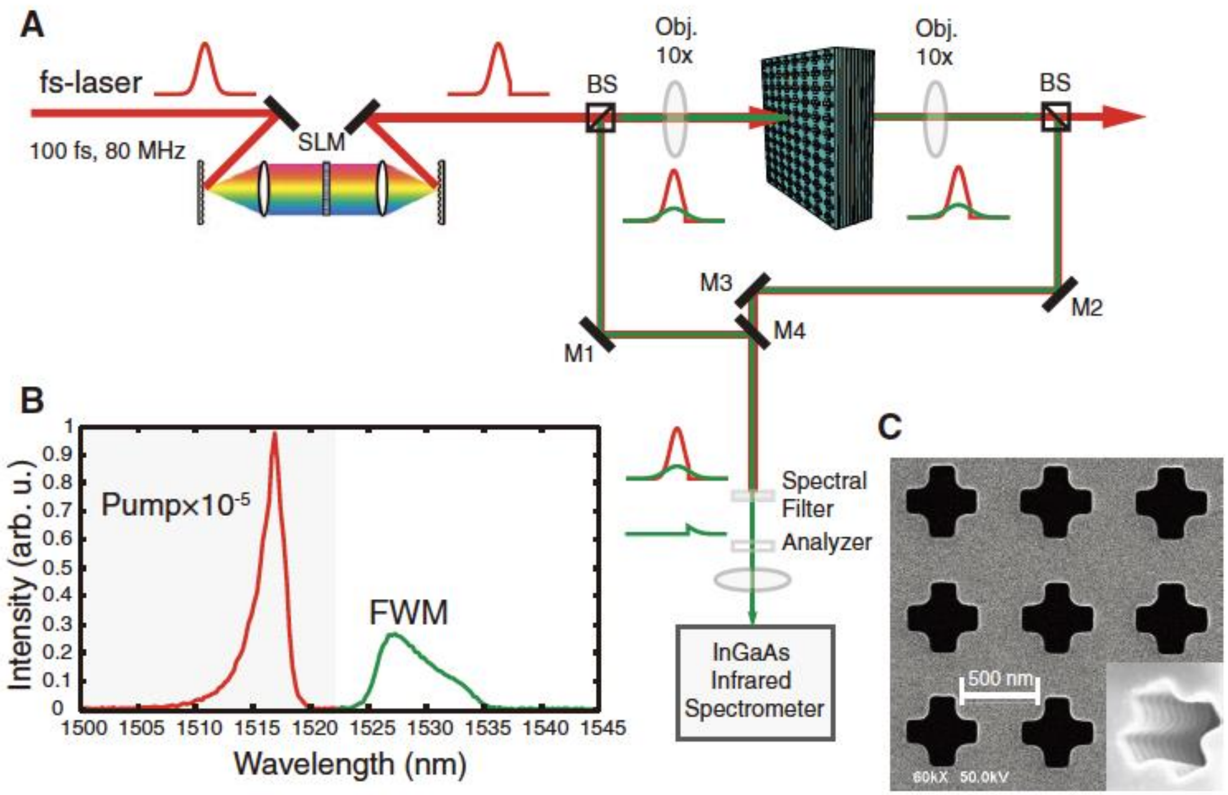
传统和零折射率材料中*波矢匹配、四波混频信号传输示意图*

A: 三种传统相位匹配和ZIM中的相位匹配自由

B和D：传统中，前向和背向散射

C和E：ZIM中，前向和背向散射自由





在*鱼网型、多层金属和介质膜*零折射率材料中得到了*很强的四波混频信号*

文献与上页同

PRL 116, 233901 (2016) PHYSICAL REVIEW LETTERS week ending 10 JUNE 2016

Enhanced Nonlinear Refractive Index in ϵ-Near-Zero Materials

L. Caspani,[1] R. P. M. Kaipurath,[1] M. Clerici,[1,2] M. Ferrera,[1] T. Roger,[1] J. Kim,[3] N. Kinsey,[3] M. Pietrzyk,[4]
A. Di Falco,[4] V. M. Shalaev,[3] A. Boltasseva,[3] and D. Faccio[1,*]

文章的目的：在ENZ中增强 n_2

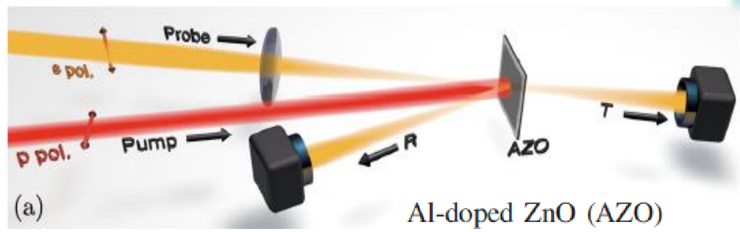
$$n_2 = \frac{3\chi^{(3)}(\omega;\omega,\omega,-\omega)}{4n_0\mathrm{Re}(n_0)\epsilon_0 c}$$

$$n_2 = \frac{3}{2\varepsilon_0 c}\frac{\chi^{(3)}}{n_r^{\mathrm{pump}}(n_r + i n_i)},$$

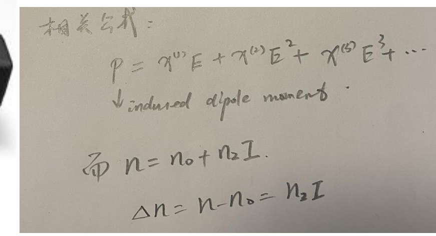
$$n_{2r} = \frac{3}{2\varepsilon_0 c}\frac{n_r\chi_r^{(3)} + n_i\chi_i^{(3)}}{D}$$

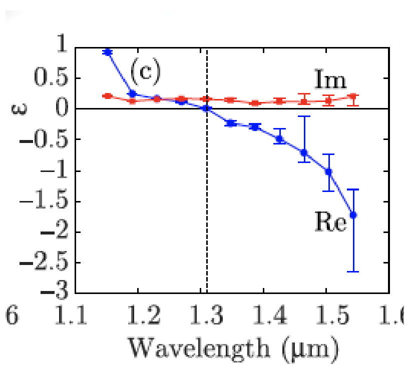
$$n_{2i} = \frac{3}{2\varepsilon_0 c}\frac{n_r\chi_i^{(3)} - n_i\chi_r^{(3)}}{D},$$

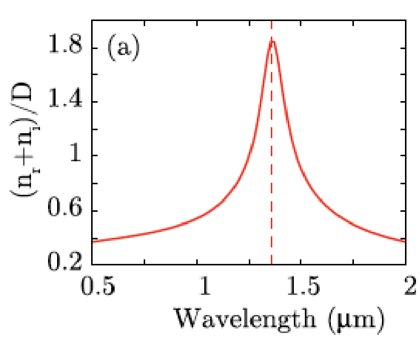
$$D = n_r^{\mathrm{pump}}(n_r^2 + n_i^2) \qquad \chi_r^{(3)} = \chi_i^{(3)} = \mathrm{const.}$$

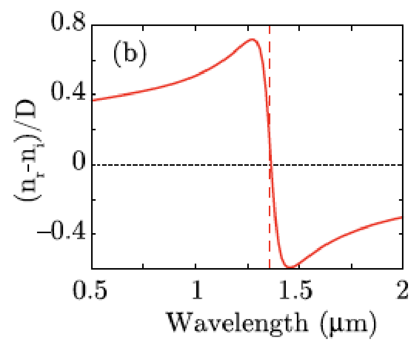


(a)
Al-doped ZnO (AZO)









SCIENCE 13 MAY 2016 • VOL 352 ISSUE 6287 795

Large optical nonlinearity of indium tin oxide in its epsilon-near-zero region

M. Zahirul Alam,[1] Israel De Leon,[1,3*] Robert W. Boyd[1,2]

和上一篇文章的相同之处：在ENZ中增强 n_2

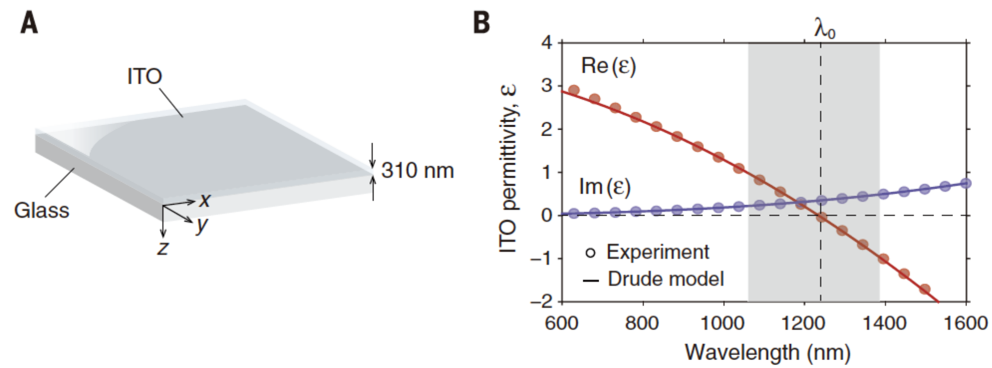
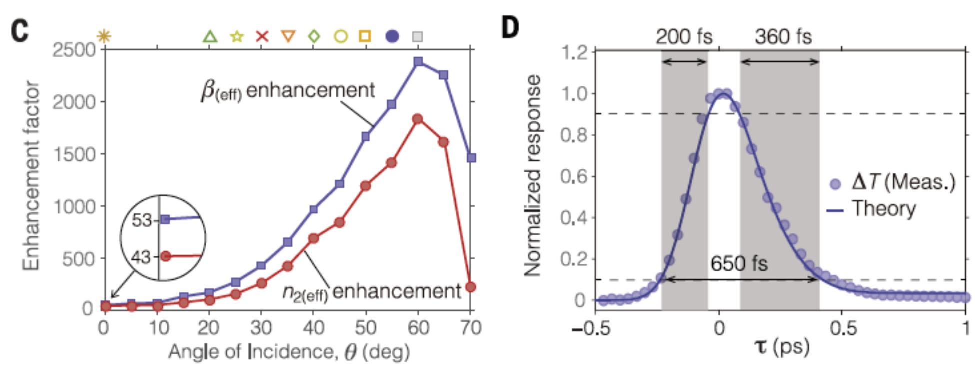


Fig. 1. The ITO sample under investigation and its linear optical response. (**A**) Structure under investigation. (**B**) Linear relative permittivity of the ITO film measured via spectroscopic ellipsometry (symbols) and estimated by the Drude model (lines). The condition Re(ε) = 0 occurs at λ_0 = 1240 nm. The shaded region shows the spectral range investigated in our nonlinear optical characterization.

不同之处：*n_2的增强 和角度有关，并且可以达到360fs的切换时间*

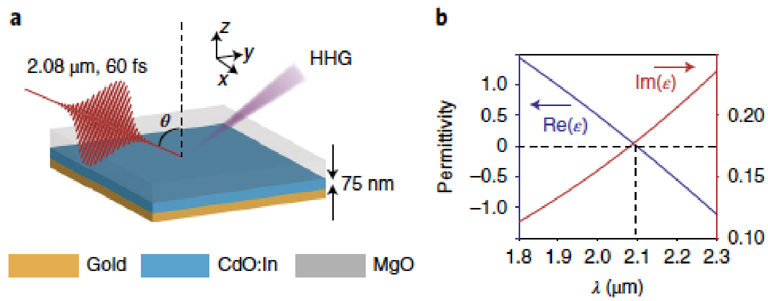
衰减系数：$\beta_{(eff)} = \Delta\alpha / I$



High-harmonic generation from an epsilon-near-zero material

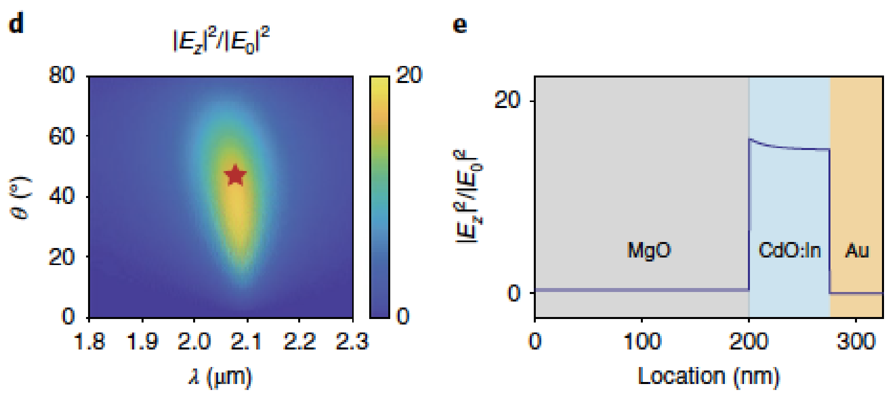
Yuanmu Yang[1*], Jian Lu[2], et al.

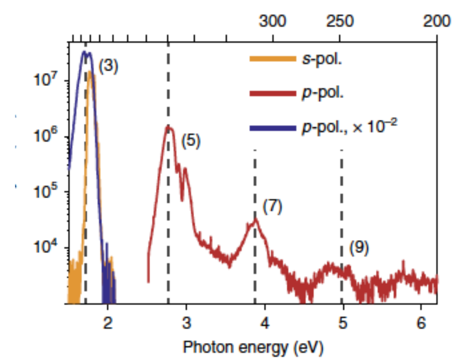
NATURE PHYSICS | VOL 15 | OCTOBER 2019 | 1022–1026 |



CdO那一层是零折射率材料，

观察到*线宽加宽*的高次谐波





4.1.2 零折射率材料物理及应用

a. 波前调控与定向辐射

b. 隧穿效应

c. ENZ腔效应

d. 非线性效应

e. 掺杂调控

d. 其它

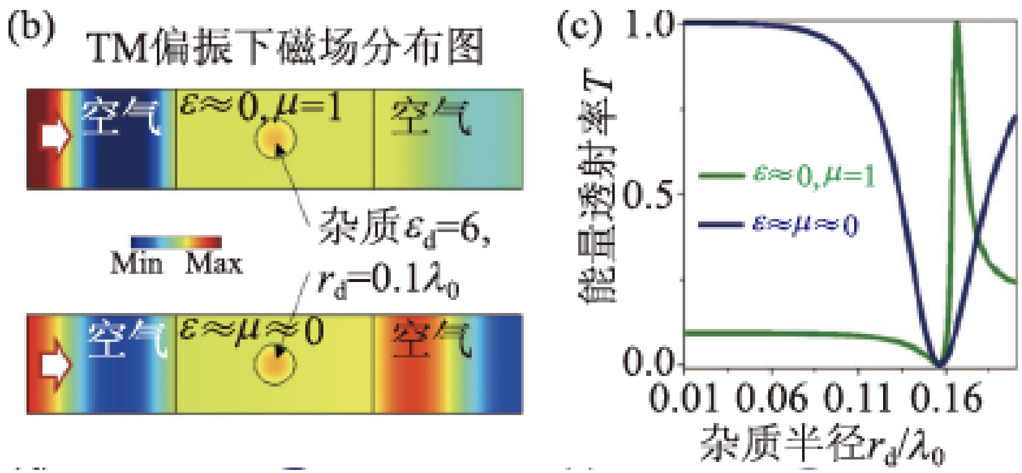
核心想法：通过在零折射率材料（ZIM）中掺入杂质，改变整块材料的电磁场发布，从而调节透射、反射、散射和吸收等

总体感觉：情况复杂，比如2D和3D差别显著、调控敏感性，*也就是说，还未有一般性规律*

我们相信，随着更多零折射率材料的实现，掺杂的ZIM体系的性质将会不断发展和完善。

下面几页给出一些有代表性的特例

例子一：2D零折射率材料中，通过介质半径的改变调控光透射

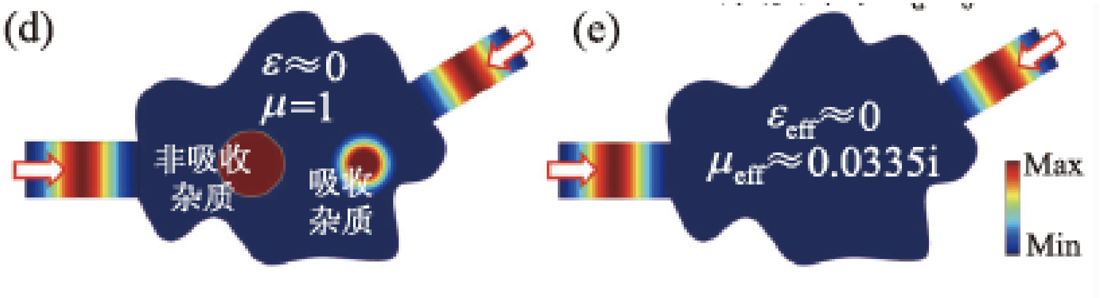


通过有效介质理论（公式略去，因为个案各例），发现ENZ和EMNZ中介质调控的根本不同

罗杰，赖耘，零折射率材料的物理与应用，物理， 48卷7 期，426 （2019）and references therein.

例子二：2D零折射率材料中，通过掺杂实现的相干完美吸收

相干完美吸收： (coherent perfect absorption，CPA）
是指多束光照到同一吸收材料时，通过相位相干，所有光被同时吸收



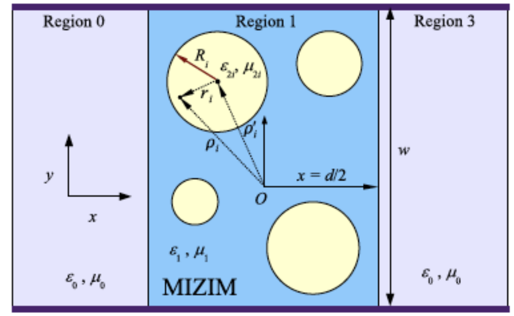
而零折射率材料中的相干完美吸收是一种新的原理，不受材料参数、形状和尺寸的影响，是一种等效机制

上图中：通过模拟发现，有效介质的磁导率为一纯虚数

罗杰，赖耘，零折射率材料的物理与应用，物理， 48卷7 期，426 （2019）
and references therein.

例子三：2D零折射率材料中，通过缺陷调节的ZIM中全反射和全透射

机制：通过调节几个ZIM中的介质块的介电常数，实现光场的全反和全透



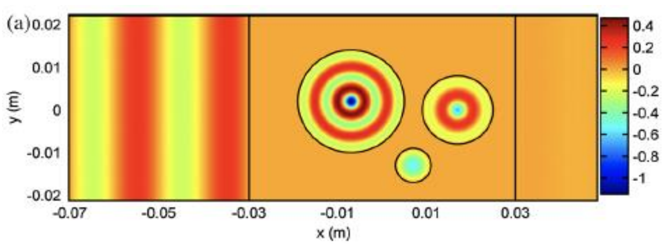


FIG. 2 (color online). Perfect reflection. (a) The magnetic field distribution for an arrangement of three defects embedded in a MIZIM. The radii are $R_1 = 4$ mm, $R_2 = 8$ mm, and $R_3 = 12$ mm with respective materials $\epsilon_{2,1} = 3.7$, $\epsilon_{2,2} = 11.9$, and $\epsilon_{2,3} = 15.7$. The **E** field magnitude is shown in (b) with its

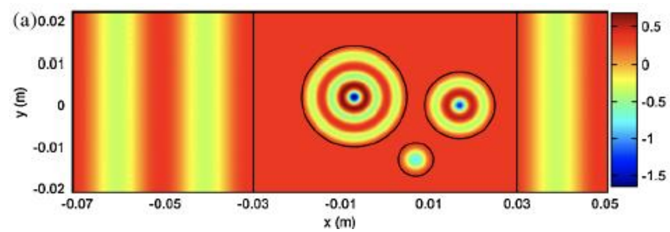


FIG. 3 (color online). Various field profiles illustrating perfect transmission. In (a) the magnetic field pattern is shown for the same setup as Fig. 2 but with the modifications $\epsilon_{2,1} = 9.3$, $\epsilon_{2,2} = 16.4$, and $\epsilon_{2,3} = 19.1$. In (b) the corresponding **E** field is shown

PRL **105**, 233908 (2010) PHYSICAL REVIEW LETTERS week ending 3 DECEMBER 2010

Total Transmission and Total Reflection by Zero Index Metamaterials with Defects

Viet Cuong Nguyen,[1] Lang Chen,[1,*] and Klaus Halterman[2]

延伸阅读：

ZIM中的*doping*和*transformation Optics*的结合

PHYSICAL REVIEW LETTERS **123,** 067701 (2019)

Transformation-Invariant Metamaterials

Youming Zhang,[1,*] Yu Luo,[2,*,†] J. B. Pendry,[3] and Baile Zhang[1,4,‡]

PHYSICAL REVIEW LETTERS **127,** 123902 (2021)

Three-Dimensional Electromagnetic Void Space

Changqing Xu,[1,*] Hongchen Chu,[2,*] Jie Luo,[3] Zhi Hong Hang,[3] Ying Wu,[1,†] and Yun Lai[2,‡]

4.1.2 零折射率材料物理及应用

a. 波前调控与定向辐射

b. 隧穿效应

c. ENZ腔效应

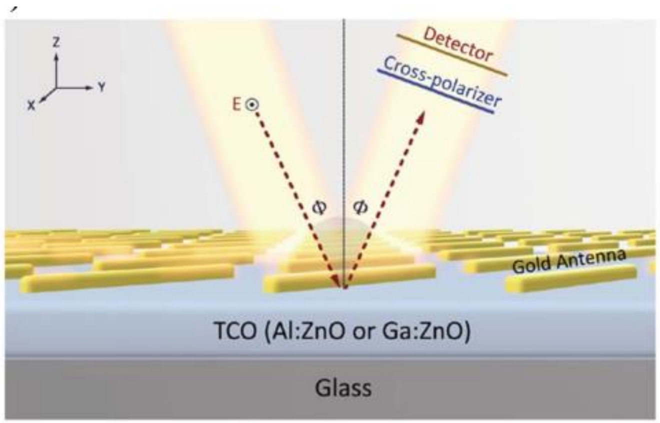
d. 非线性效应

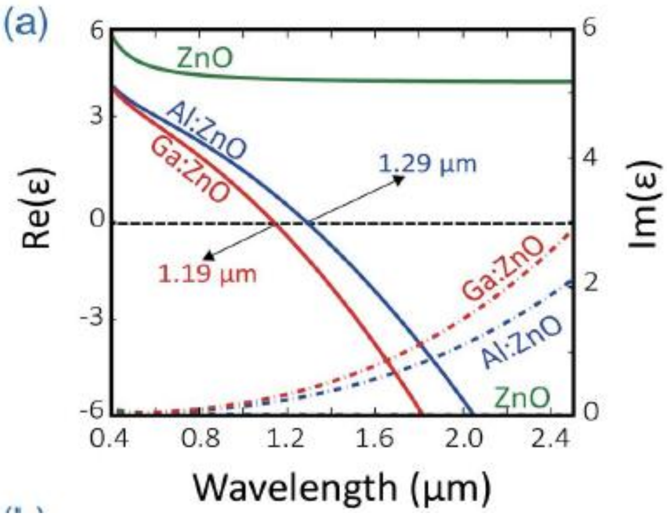
e. 掺杂调控

d. 其它

零折射率衬底的共振堆积

结构如下，首先制备ENZ衬底

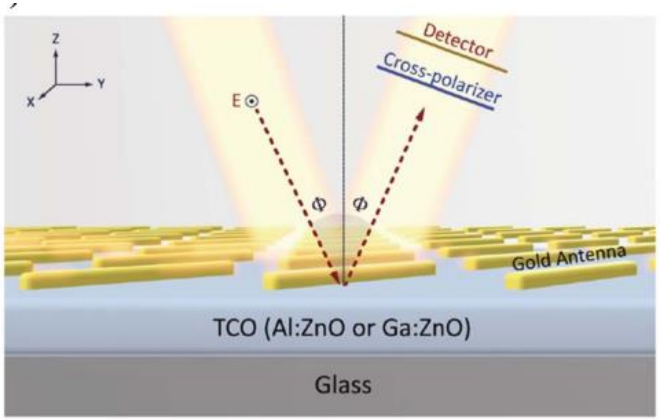


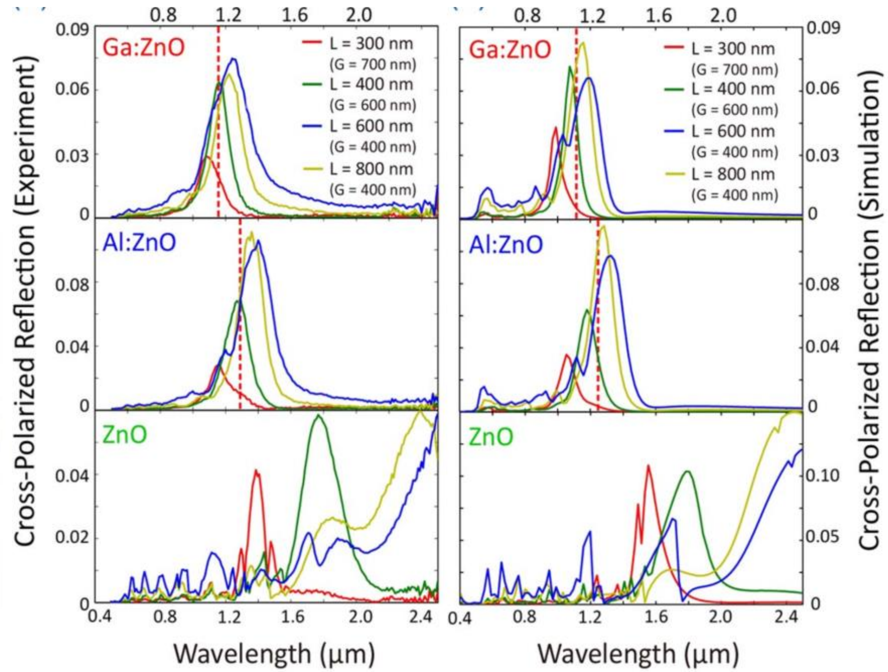


ENZ points are at 1.19 and 1.29 µm for Ga:ZnO and Al:ZnO

JONGBUM KIM and AVEEK DUTTA et al. , Role of epsilon-near-zero substrates in the optical response of plasmonic antennas, Optica 3, 339 (2016)

零折射率衬底的共振堆积

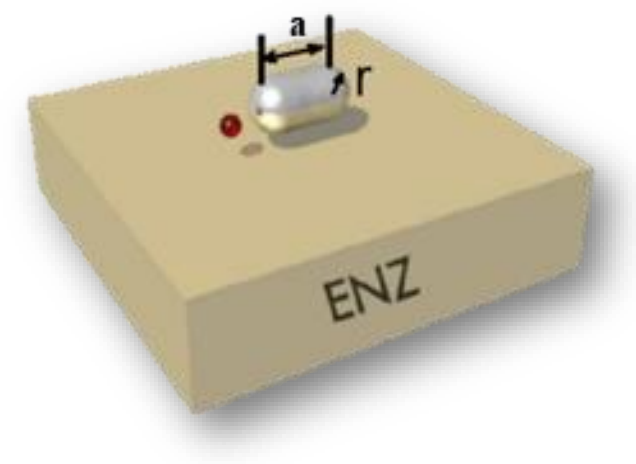


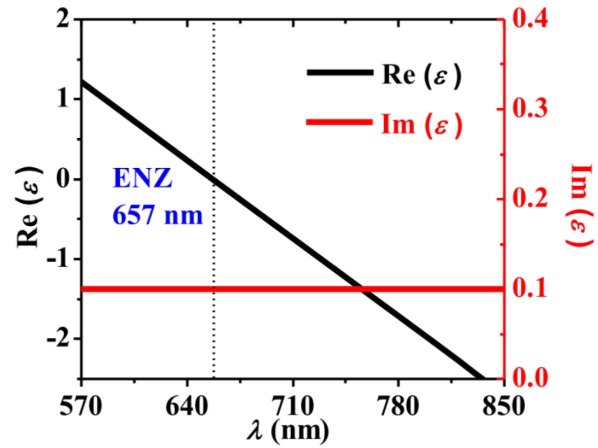


随着衬底介电常数减小，长纳米天线的响应谱线红移变慢，多种长度的纳米天线均在零介电常数频率附近产生共振，*即共振波长堆积在零折射率波长附近*

JONGBUM KIM and AVEEK DUTTA et al. , Role of epsilon-near-zero substrates in the optical response of plasmonic antennas, Optica 3, 339 (2016)

零折射率衬底-表面等离激元纳腔耦合结构

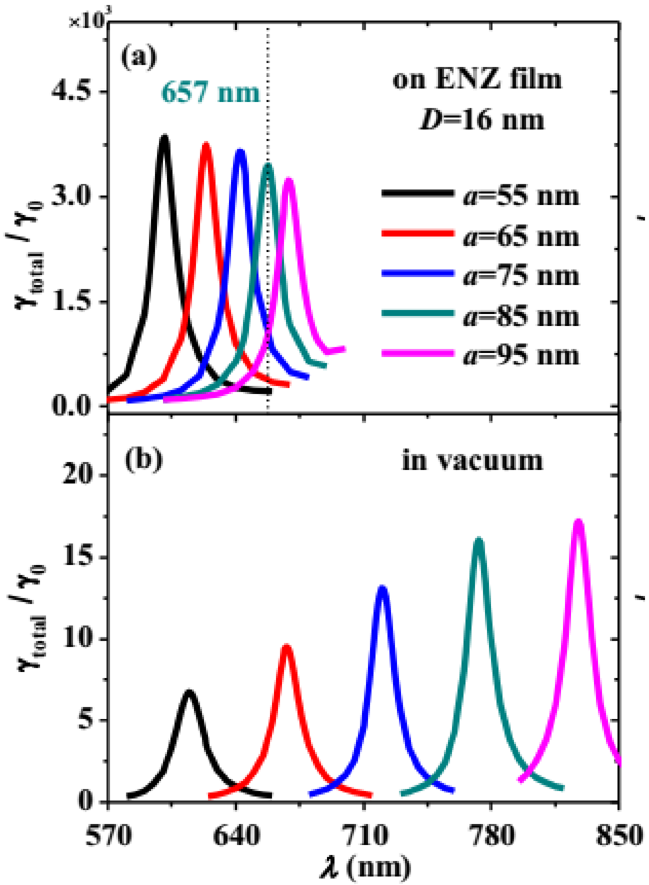




银纳米棒局域电磁场	→	大的自发辐射增强
衬底共振堆积效应	→	自发辐射谱堆积
衬底波矢失配宽松	→	方向性光子辐射

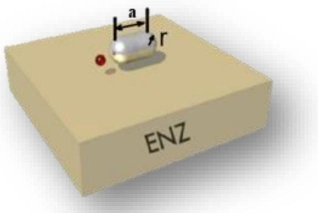
Xueke Duan and Ying Gu et al, Accumulation and directionality of large spontaneous emission enabled by epsilon-near-zero film, Opt. Express 27, 7426 (2019)

零折射率衬底的共振堆积效应，获得自发辐射谱堆积



有ENZ衬底：

长腔结构的自发辐射谱红移变慢，并在零折射率波长附近形成自发辐射谱堆积



没有ENZ衬底：

随着纳腔结构变长，其自发辐射谱线移动明显

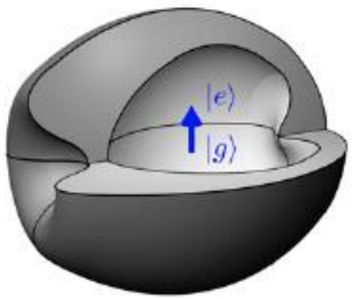
Xueke Duan and Ying Gu et al, Accumulation and directionality of large spontaneous emission enabled by epsilon-near-zero film, Opt. Express 27, 7426 (2019)

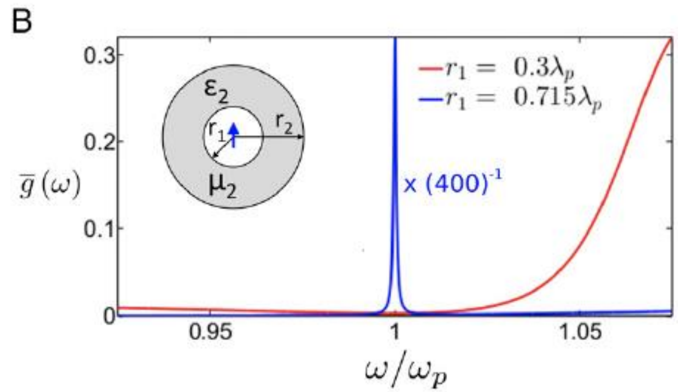
总结：

本节中，首先介绍了零折射率材料的*概念、分类及物理实现*，然后介绍了零折射率材料诸多性质，包括：*波前调控与定向辐射、隧穿效应、ENZ腔效应、以及零折射率材料的非线性效应和掺杂调控*。可以看出，做为一种特殊的超材料，零折射率材料大大丰富了微纳光学的研究内容，并将在光场调控方面有诸多可能的应用

Zero-index structures as an alternative platform for quantum optics

Iñigo Liberal[a] and Nader Engheta[a,1]
822–827 | PNAS | January 31, 2017 | vol. 114 | no. 5





由于腔的共振效应，

态密度增加，并且emitter

和腔周期性的交换能量